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INTERFERENCE COUPLING— ATTACK IT EARLY

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DON'T WAIT—He who hesitates, faced with the specter of interference coupling, may regret putting it off.

One of the biggest headaches for a designer is discovering, too late, that there are interference problems because of cross talk, either within the circuitry or between interconnecting cables. An ounce of early prevention could have saved time, money and especially trouble. This article gives you—the designer—key parameters and convenient expressions that will permit accurate determination of coupled interference in shielded and unshielded wire cables.

The Problem

Generally, the primary source of interference within electronic equipment is cross talk either within the circuits or between interfacing cables. It is vital that interference-immunizing alternates such as shielding, filtering and careful wire routing are appraised early in the design stages.

Analyses of special cases have been presented elsewhere. When the analyses are consolidated and simplified into a convenient reference form, the results can be applied to ac and transient interference problems where the coupled lines are electrically short. That is, for ac interference the lines are significantly shorter than 1/4 wavelength or for transient interference the delay time of the lines is shorter than the time constant of the interfering signal.

Wiring models with necessary equations for calculating cross talk are presented on p. 34. Data that are required for all calculations are summarized in the table and graphs starting on p. 38.

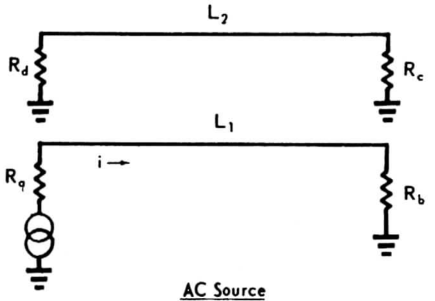
Theory and Limitations

Primarily, cross talk between lines is a combination of electric and magnetic coupling. Equations presented with each wiring model neglect cross-coupling that is a result of the ground-plane resistance. Typically, this effect is small compared with the electric and magnetic coupling except at very low frequencies.

If the terminating impedances are small compared with the open line characteristic impedance, the net coupling is dominantly magnetic with unshielded wires. Induced interference is represented as a series generator in the victim line. The induced voltage is divided proportionally by the terminating resistors and the voltage across each resistor is opposite in polarity with relation to ground. The generator has an open-circuit voltage that is directly proportional to the mutual inductance and to the amplitude and frequency of the interfering current.

If the terminating resistances are much larger than the characteristic impedance of the wires, the net coupling will be dominantly electric. Induced interference is assumed to be caused by a current generator driving the victim line. Short-circuit current of the generator is directly proportional to the source voltage amplitude, frequency and to the mutual capacitance between the open wires. The short-circuit current returns to ground through the two terminating resistors in parallel; hence, equal, in-phase voltages appear at each terminal. For cases where terminating resistors cannot be considered to be much larger nor much smaller than the characteristic impedances of the wires, net interference is a superposition of the electric and magnetic coupled interference. The voltages tend to add at one termination and to cancel at the other. **Examples 1 and 2** (p. 35) illustrate this point.

CASE 1
OPEN WIRE TO OPEN WIRE COUPLING



AC Source

$$e_c = 2\pi f L_m \ell i \frac{R_c}{R_c + R_d} \left| 1.032 \times 10^{-6} \frac{R_b R_d}{L_1 L_2} - 1 \right| \quad (1)$$

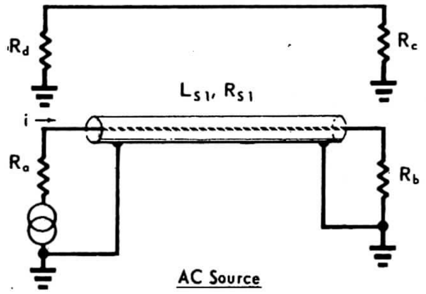
$$e_d = 2\pi f L_m \ell i \frac{R_d}{R_c + R_d} \left(1.032 \times 10^{-6} \frac{R_b R_c}{L_1 L_2} + 1 \right) \quad (2)$$

Transient Source

$$v_c = \frac{L_m \ell I}{\tau} \frac{R_c}{R_c + R_d} \left(1.032 \times 10^{-6} \frac{R_b R_d}{L_1 L_2} - 1 \right) \quad (3)$$

$$v_d = \frac{L_m \ell I}{\tau} \frac{R_d}{R_c + R_d} \left(1.032 \times 10^{-6} \frac{R_b R_c}{L_1 L_2} + 1 \right) \quad (4)$$

CASE 2
SHIELDED WIRE TO OPEN WIRE COUPLING



AC Source

$$e_c = e_i \frac{R_c}{R_c + R_d}; \quad e_d = e_i \frac{R_d}{R_c + R_d} \quad (5)$$

where

$$e_i = 2\pi f L_m \ell i \alpha_{s1} \quad (6)$$

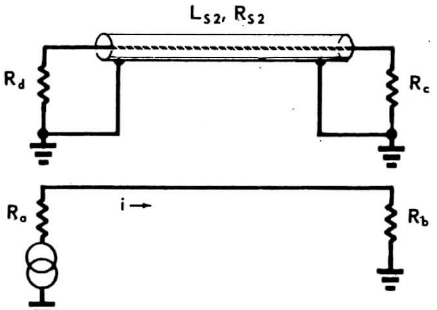
Transient Source

$$v_c = v_i \frac{R_c}{R_c + R_d}; \quad v_d = -v_i \frac{R_d}{R_c + R_d} \quad (7)$$

where

$$v_i = \frac{-L_m \ell I}{\tau} A_{s1} \quad (8)$$

CASE 3
OPEN WIRE TO SHIELDED WIRE COUPLING



AC Source

$$e_c = e_i \frac{R_c}{R_c + R_d}; \quad e_d = e_i \frac{R_d}{R_c + R_d} \quad (5)$$

where

$$e_i = 2\pi f L_m \ell i \alpha_{s2} \quad (9)$$

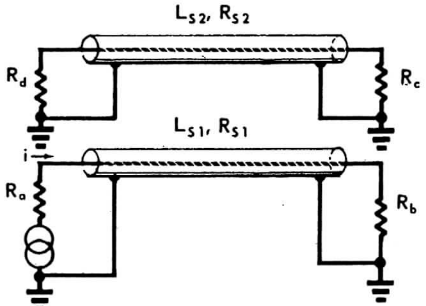
Transient Source

$$v_c = v_i \frac{R_c}{R_c + R_d}; \quad v_d = -v_i \frac{R_d}{R_c + R_d} \quad (7)$$

where

$$v_i = \frac{-L_m \ell I}{\tau} A_{s2} \quad (10)$$

CASE 4
SHIELDED WIRE TO SHIELDED WIRE COUPLING



AC Source

$$e_c = e_i \frac{R_c}{R_c + R_d}; \quad e_d = e_i \frac{R_d}{R_c + R_d} \quad (5)$$

where

$$e_i = 2\pi f L_m \ell i \alpha_{s1} \alpha_{s2} \quad (11)$$

Transient Source

$$v_c = v_i \frac{R_c}{R_c + R_d}; \quad v_d = -v_i \frac{R_d}{R_c + R_d} \quad (7)$$

where

$$v_i = \frac{-L_m \ell I}{\tau} A_{ss} \quad (12)$$

Schematics representing four cases of shielded and unshielded source/victim pairs with required expressions for cross-talk calculations.

On p. 34, Eqs. 1 through 4 include both electric and magnetic coupled components. Mutual capacitance does not appear explicitly in the equations but is contained implicitly in the interrelationships among mutual capacitance, mutual inductance and self-inductance. That is, the mutual capacitance C_m is

$$C_m = \frac{1.032\, L_m}{L_1 L_2}\ \text{pF/ft.}$$

Eqs. 1 through 4 are accurate provided that the series inductive reactance and shunt capacitive susceptance of the victim lines are negligible, i.e.

$$R_c + R_d >> 2\pi f\ell L_2$$

and

$$\frac{R_c\, R_d}{R_c + R_d} << \frac{1}{2\pi f\ell C_2} = \frac{L_2\, 10^6}{2\pi f\ell\ 1.032}$$

where C_2 is the shunt capacitance in μF/ft of the victim line.

For transient interference, the conditions are:

$$R_c + R_d >> \frac{\ell L_2}{\tau}$$

$$\frac{R_c\, R_d}{R_c + R_d} << \frac{\tau}{\ell C_2} = \frac{\tau L_2}{\ell}\ \frac{10^6}{1.032}$$

If either or both lines are shielded and their lengths are electrically short, then the impedance from any point on the shield to the ground plane will be low and the electric coupling will be small compared with the magnetic coupling. At frequencies below 3 kHz, the external magnetic field of a shielded wire is approximately equal to that of an unshielded wire because the return current essentially flows through the ground plane in both cases. For higher frequencies, the shield offers a lower return impedance and the return current flows through it, effectively canceling the external magnetic field. This shielding effect increases at 20 dB/decade up to approximately 200 kHz. Above 200 kHz to approximately 100 MHz, the shielding effectiveness increases at a more rapid rate with double-shielded line and at a reduced rate for a single-shielded line.

This frequency dependence can be explained in terms of the shield's transfer resistance, R_t, which is nearly constant and equal to the dc resistance of the shield up to about 200 kHz. At higher frequencies, skin effect becomes important with double-shielded lines and braid leakage becomes important with the single-shield. Graph V shows the frequency dependence of the transfer resistance for two commonly used coaxial cables.

Using shield resistance rather than transfer resistance in transient analysis is convenient and found to be adequate even when dealing with the fastest transients. It is valid in ac analysis up to approximately 200 kHz. Above 200 kHz, R_s is replaced with R_t.

As in the case of open wire coupling (Case 1), the equations for Cases 2 to 4 are accurate provided that the series inductive impedance and shunt capacitive susceptance of the victim lines are negligible compared with the terminating impedances.

Acknowledgment

The author thanks E. W. Karpen of AIL for his helpful suggestions in the preparation of this paper.

Databank

In-depth reference information appears in:

1. "Analysis of Cable-Coupled Interference", by L. J. Greenstein and H. J. Tobin, IEEE Trans. Radio Frequency Interference, Vol. RFI-5, March 1963, pp. 43-55.
2. "Crosstalk between Coaxial Transmission Lines", by S. A. Schelkunoff and T. M. Odarenko, Bell Systems Technical Journal, Vol. 26, April 1937, pp. 144-164. A classic—should be consulted when considering cross talk in lines comparable to or exceeding a wavelength.
3. "Coupling between Open Wires over a Ground Plane", by R. J. Mohr, presented at 1968 IEEE Symposium on EMC, July 23-25, 1968.
4. "Coupling between Open and Shielded Wire Lines over a Ground Plane", by R. J. Mohr, IEEE Trans. Electromagnetic Compatibility, Vol. EMC-9, September 1967, pp. 34-45.
5. "Coupling between Lines at High Frequencies", by R. J. Mohr, IEEE Trans. EMC, Vol. EMC-9, No. 3, December 1967, pp. 127-219. The last three references show the derivation of the expressions presented in the paper and experimental verification.
6. "Crosstalk (Noise) in Digital Systems", by Ivor Catt, IEEE Trans. Electronic Computers, Vol. EC-16, No. 6, December 1967, pp. 743-763. Should be consulted when considering crosstalk in lines in which the delay time is comparable to or exceeds the rise time of the interfering signal.

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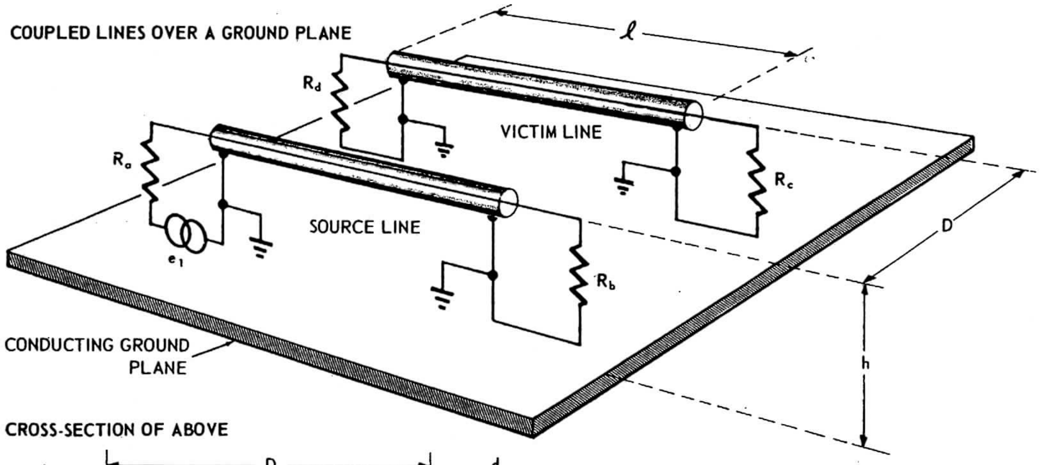
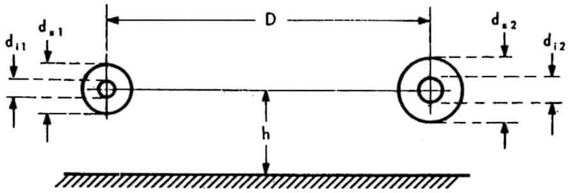
COUPLED LINES OVER A GROUND PLANE



CROSS-SECTION OF ABOVE



Model represents the bulk of system wiring where source and load are referenced to the structure (simulated by conducting ground plane). Required parameters for calculations are summarized and defined.

PARAMETERS

Subscript 1 ..... Source line parameters
Subscript 2 ..... Victim line parameters
α_s ..... AC shield attenuation factor, a function of fL_s/R_s, obtained from **GRAPH III**.
A_s ..... Transient shield attenuation factor, a function of $\frac{L_s/R_s}{\tau}$ obtained from **GRAPH IV**.
C ..... Line capacitance in μF/ft.
d ..... Diameter of unshielded wire in inches.
d_i ..... Outer diameter of inner conductor of shielded line in inches, obtained from **Table 1** for several standard coaxial cables.
d_s ..... Inner diameter of outer conductor of shielded line in inches, obtained from **Table 1** for several standard coaxial cables.
D ..... Separation of coupled wires in inches.
e_i ..... Induced ac voltage in victim line, in volts rms.
e_c, e_d ..... Coupled ac voltage at R_c, R_d, in volts rms.
f ..... Frequency in MHz.
h ..... Height of coupled lines over the ground plane in inches.
i ..... Source ac current in amperes.
I ..... Peak source transient current in amperes, with transient current of the form $i = I\,(1 - \epsilon^{-t/\tau})$
ℓ ..... Coupled line length in feet.
L ..... Inductance of wire in μH/ft, obtained from **GRAPH II** as a function of h/d.
L_s ..... Inductance of shield of a shielded wire in μH/ft, obtained from **GRAPH II** as a function of h/d_s. **Table 1** gives L_s for several standard coaxial cables at h = 2 inches.
L_m ..... Mutual inductance between coupled lines in μH/ft, obtained from **GRAPH I** as a function of h/D.
R_a, R_b, R_c, R_d ..... Terminating resistances on source and victim lines, in ohms.
R_s ..... DC and low frequency resistance of shield on shielded lines in ohms/ft. **Table 1** gives R_s for several common coaxial lines.
R_t ..... Transfer resistance of shield on shielded lines in ohms/ft. (It is a function of frequency but nearly constant and equal to R_s below about 200 kHz for typical braided cables. See **GRAPH V** for variation of typical coaxial cables.)
τ ..... Time constant of interfering transient in microseconds.
t_r ..... 10 to 90-percent rise time of interfering transient in μs, $t_r = 2.2\tau$.
v_c, v_d ..... Peak-coupled transient voltage at R_c, R_d in volts.
v_i ..... Peak transient induced voltage induced in victim line, in volts.
Z_o ..... Characteristic impedance of cable in ohms.

EXAMPLE 1

OPEN WIRE TO OPEN WIRE COUPLING (CASE 1), ac CASE.

Length of parallel wire run .... $\ell = 5$ ft
Height over ground plane .... $h = 1\text{-}3/4$ inches
Separation .... $D = 7/8$ inch
Wire size No. 20 AWG .... $d_1 = d_2 = 0.032$ inch
Terminations .... $R_a = 50\Omega$, $R_b = 150\,\Omega$
.... $R_c = 2\,\text{k}\Omega$, $R_d = 150\Omega$
Frequency .... $f = 0.15$ MHz
Source open circuit voltage .... $e = 10\text{V}$ rms

From **GRAPH I** with $h/D = 1.75/0.875 = 2$,

$$L_m = 0.086\ \mu\text{H/ft}$$

From **GRAPH II** with $h/d_1 = h/d_2 = 1.75/0.032 = 54.7$

$$L_1 = L_2 = 0.33\ \mu\text{H/ft}$$

Neglecting the small impedance due to L_1 and the shunting effect of C_1,

$$i = \frac{e}{R_a + R_b} = 10/200 = 0.05\text{A}$$

From equation 1:

$$e_c = 2\pi \times 0.15 \times 0.086 \times 5 \times 0.05 \times \frac{2000}{2150} \left| \frac{1.032 \times 10^{-6} \times 150 \times 150}{0.33 \times 0.33} - 1 \right| = 0.015\text{V}$$

From equation 2:

$$e_d = 2\pi \times 0.15 \times 0.086 \times 5 \times 0.05 \times \frac{150}{2150} \left(\frac{1.032 \times 10^{-6} \times 150 \times 2000}{0.33 \times 0.33} + 1 \right) = 0.0054\text{V}$$

EXAMPLE 2

OPEN WIRE TO OPEN WIRE COUPLING (CASE 1), TRANSIENT CASE.

Same arrangement as Example 1. Source line carries a transient current having a rise time (t_r) of 2 μs ($\tau = t_r/2.2 = 0.91\ \mu$s) and a peak level, I, of 0.5A

From equation 3:

$$v_c = \frac{0.086 \times 5 \times 0.5}{0.91} \times \frac{2000}{2150} \left(\frac{1.032 \times 10^{-6} \times 150 \times 150}{0.33 \times 0.33} - 1 \right) = -0.17\text{V}$$

From equation 4:

$$v_d = \frac{0.086 \times 5 \times 0.5}{0.91} \times \frac{150}{2150} \left(\frac{1.032 \times 10^{-6} \times 150 \times 2000}{0.33 \times 0.33} + 1 \right) = +0.064\text{V}$$

EXAMPLE 3

SHIELDED WIRE TO OPEN WIRE COUPLING (CASE 2), TRANSIENT CASE.

Coupled length .... $\ell = 10$ ft
Separation .... $D = 1$ inch
Height over ground plane .... $h = 4$ inches
Source line .... RG 58C/U
Terminating resistors .... $R_4 = 1\text{k}$, $R_d = 3\text{k}$

Interfering Current
Peak amplitude .... $I = 7\text{A}$
Rise time (10% to 90%) .... $t_r = 3\ \mu\text{s}$ $\left(\tau = \frac{3}{2.2} = 1.35\ \mu\text{s}\right)$

EXAMPLE 3 Cont'd

From equation 7:

$$v_c = v_i \frac{R_c}{R_c + R_d} \; ; \; v_d = -v_i \frac{R_d}{R_c + R_d}$$

From equation 8

$$v_i = \frac{-L_m \ell I}{\tau} A_{s1}$$

From **GRAPH I** with $h/D = 4$

$$L_m = 0.13\ \mu H/ft$$

From **TABLE 1** for RG58C/U, $d_{s1} = 0.116$ inch

$$\text{Therefore } \frac{h}{d_{s1}} = \frac{4}{0.116} = 34.5$$

From **GRAPH II** $L_{s1} = 0.3\ \mu H/ft$

From **TABLE 1** for RG58C/U

$$R_{s1} = 4.7 m\Omega/ft = 0.0047\ \Omega/ft$$

$$\text{Therefore } \frac{L_{s1}/R_{s1}}{\tau} = \frac{0.3/0.0047}{1.35} = 47.3$$

From **GRAPH IV** $A_{s1} = 0.019$.

Hence

$$v_i = \frac{-0.13 \times 10 \times 7 \times 0.019}{1.35} = -0.128V$$

$$v_c = -0.128 \times \frac{1000}{4000} = -0.032V$$

$$v_d = -(-0.128)\frac{3000}{4000} = 0.096V$$

EXAMPLE 4

OPEN WIRE TO SHIELDED WIRE COUPLING (CASE 3), TRANSIENT CASE.

Same arrangement as Example 3 except that the victim line is RG 58C/U and the source line is unshielded. From equation 10, induced voltage is

$$v_i = \frac{-L_m \ell I}{\tau} A_{s2}$$

This is the same as the equation used in Example 3 except that A_{s2}, rather than A_{s1}, is used.

Since the cable is RG58C/U both here and in Example 3, $A_{s1} = A_{s2}$. Further, L_m, ℓ, I and τ are the same in both examples. Therefore, as in Example 3,

$$v_i = -0.128V$$

The voltages at resistors R_c and R_d are $-0.032V$ and $+0.096V$, respectively, as in Example 3.

EXAMPLE 5

SHIELDED WIRE TO SHIELDED WIRE COUPLING (CASE 4), TRANSIENT CASE.

Same arrangement as Example 3, except that both source and victim lines are RG 58C/U.

From equation 7,

$$v_c = v_i \frac{R_c}{R_c + R_d} \; ; \; v_d = -v_i \frac{R_d}{R_c + R_d}$$

Where, from equation 12,

$$v_i = -\frac{L_m \ell I}{\tau} A_{ss}$$

This is the same as the equation used in Example 3, except that A_{ss}, rather than A_{s2}, is used.

As in Example 2, $\frac{L_s/R_s}{\tau} = 47.3$

From **GRAPH IV**, $A_{ss} = 0.0075$

Hence,

$$v_i = \frac{-0.13 \times 10 \times 7 \times 0.0075}{1.35} = -0.0505V$$

$$v_c = -0.0505 \times \frac{1000}{4000} = -0.0127V$$

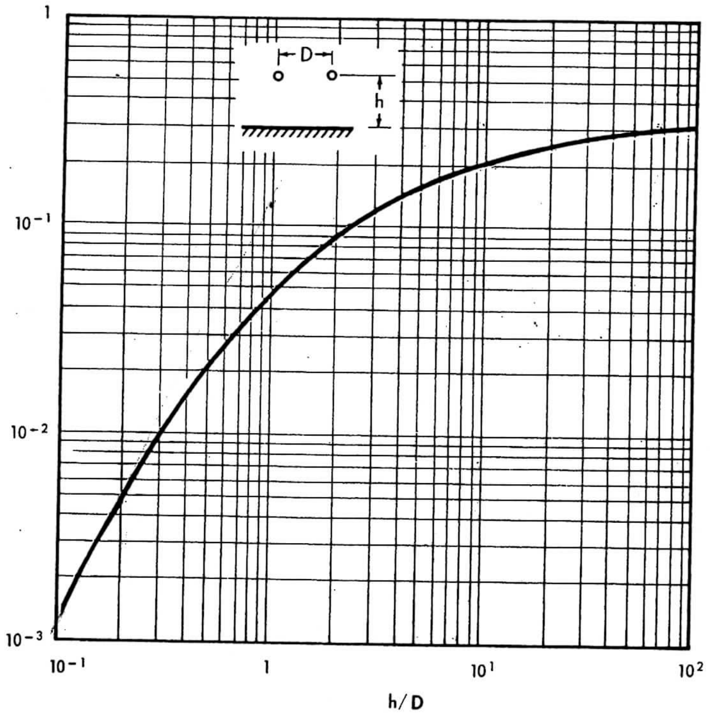
$$v_d = -(-0.0505)\frac{3000}{4000} = 0.0378V$$

$L_s =$

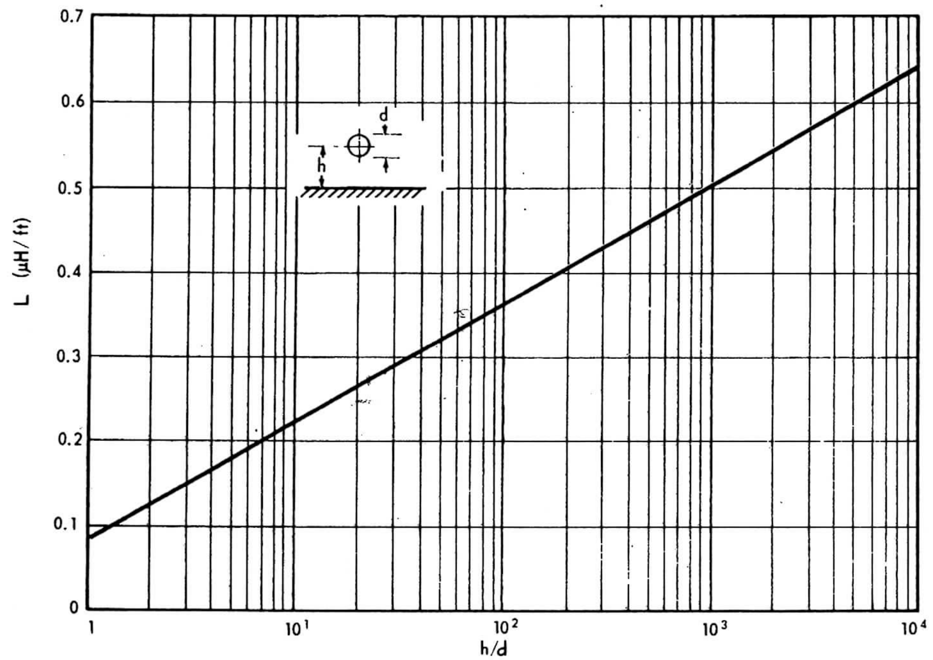
TABLE 1—COAXIAL CABLE AND SHIELDED WIRE PARAMETERS

TYPE RG /U	BRAID D-Double S-Single	Z_o (ohms)	d_s (inch)	d_i (inch)	L_s* (μH/ ft)	R_s (mΩ/ft)	L_s/R_s (μs)
9B	D	50.0	0.280	0.085	0.20	0.80	250
6A	D	75.0	0.185	0.028	0.23	1.15	200
5B	D	50.0	0.181	0.053	0.23	1.20	190
13A	D	75.0	0.280	0.043	0.20	0.95	210
8A	S	50.0	0.285	0.086	0.20	1.35	150
55A	D	50.0	0.116	0.035	0.26	2.55	100
29	S	53.5	0.116	0.032	0.26	4.75	55
58C	S	50.0	0.116	0.035	0.26	4.70	55
59A	S	75.0	0.146	0.022	0.24	3.70	65
141	S	50.0	0.116	0.036	0.26	4.70	55
142	D	50.0	0.116	0.039	0.26	2.20	118
122	S	50.0	0.096	0.029	0.27	5.70	47
223	D	50.0	0.116	0.035	0.26	2.55	100
188	S	50.0	0.060	0.019	0.30	7.60	40
180	S	95.0	0.103	0.011	0.26	6.00	43
A+	S		0.057	0.023	0.30	16.00	19
B+	S		0.056	0.030	0.30	6.20	48

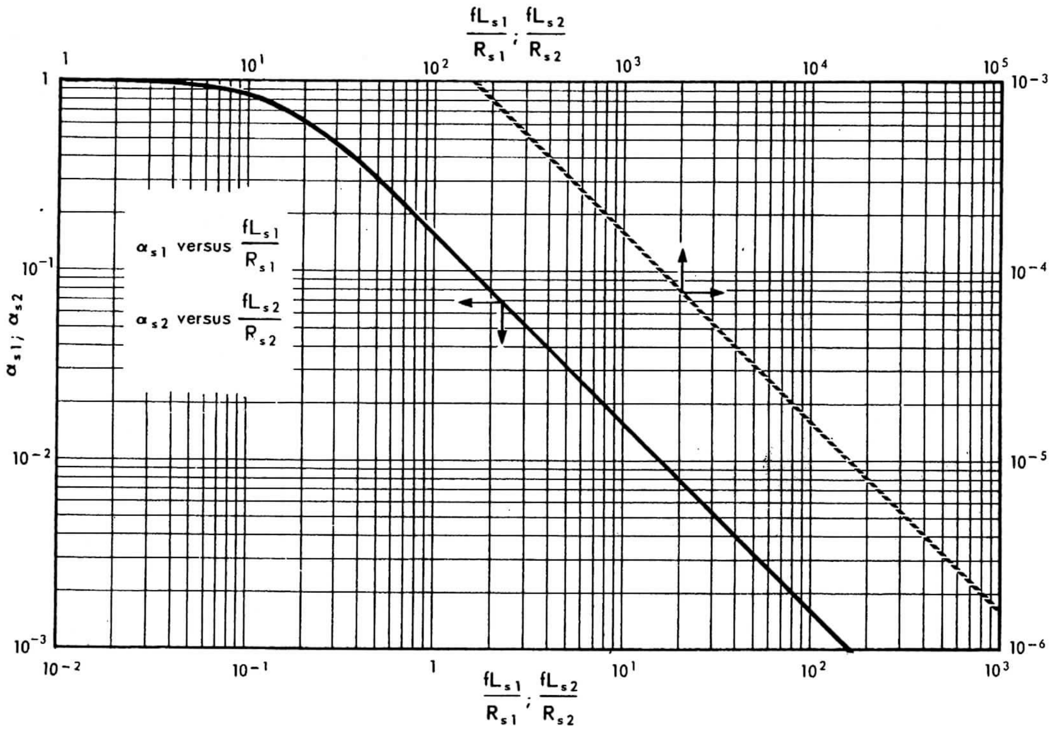
*Valid for $h = 2$ inches. For $h \neq 2$ inches, use GRAPH II to compute inductance.
+A and B are #22 and #20 shielded hook-up wires, respectively.*



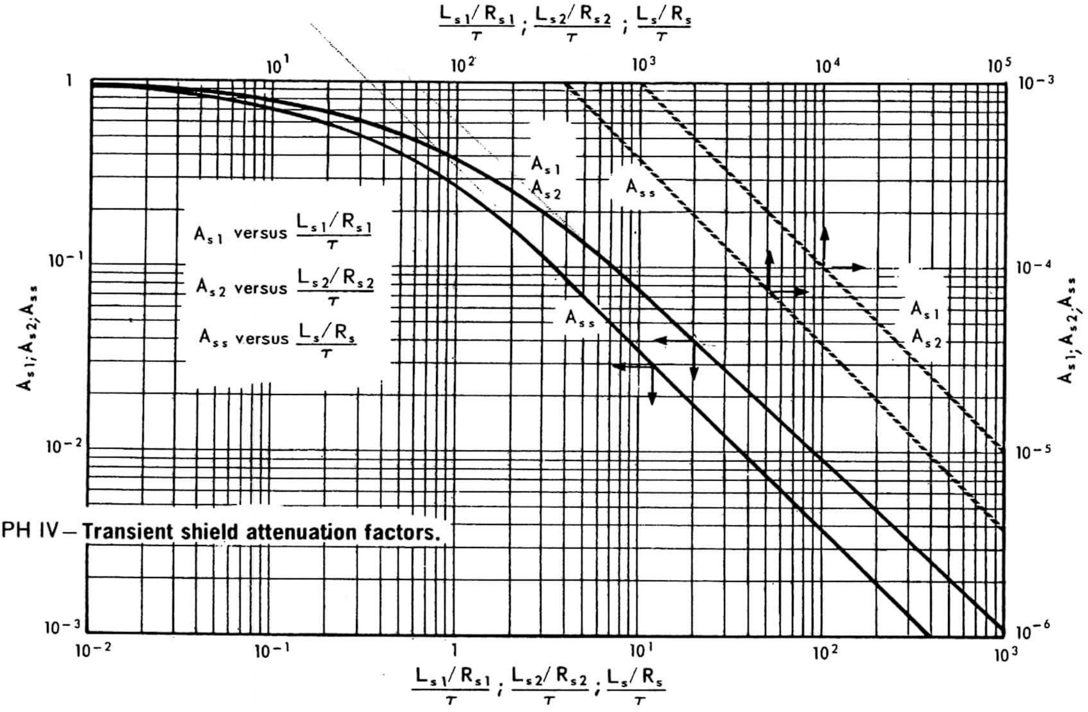
GRAPH I—**Mutual Inductance** between two wires over a ground plane.



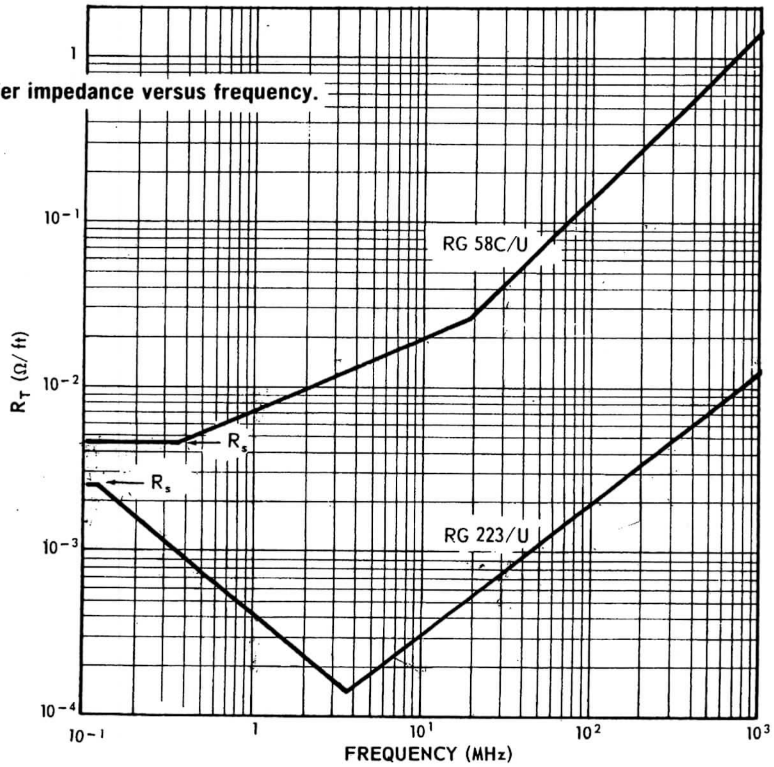
GRAPH II—**Wire inductance over a ground plane.**



GRAPH III—**AC shield attenuation factors.**



GRAPH IV—**Transient shield attenuation factors.**



GRAPH V—**Transfer impedance versus frequency.**

Transfer Impedance of Various Cable Configurations
Data extracted from RPI Report, v I