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Some Design Aspects Of Components Utilizing Symmetric 3db Hybrids

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INTRODUCTION

There are a variety of components which use a pair of symmetric three db hybrids in order to achieve certain overall operating characteristics. Typically these units employ filters, variable line lengths or non-linear elements with the hybrids in order to achieve certain desired characteristics. Typical assemblies incorporating hybrids in this fashion form phase shifters,[1-3] switches,[4-6] diplexers,[7] duplexers,[8-9] variable power dividers,[10] attenuators[11] and amplifiers.[12-14]

Certain basic requirements of the hybrids and associated components are common in all these applications. These requirements will be dealt with here. In particular, this paper will treat the effects of phase, and of variation of coupling from the 3 db value. The hybrids shall however be considered to have perfect match and directivity.

Some of the assemblies treated here require the use of only one hybrid. However, in these applications the hybrid is effectively used twice, first for splitting the incoming signal and then for recombining it. The mathematics of these devices is so similar to that for units employing two hybrids that both arrangements will be considered together.

TYPICAL ARRANGEMENTS

Some typical arrangements using hybrids are shown in Figure 1. A brief description of these devices under ideal conditions follows:

Phase Shifter — Figure 1a

Signal incident on port 1 divides equally between ports 2 and 3. If the hybrid is symmetric, it can be shown that the signals at ports 2 and 3 are in phase quadrature. These signals continue along their transmission lines to the position of the sliding shorts, 5 and 6. Here the signals are reflected back toward the hybrid. If the phase shift from 2 to 5 (θ_{2-5}) is equal to that from 3 to 6 (θ_{3-6}) the signals coupled to port 1 from ports 2 and 3 will be 180° out of phase and will cancel completely. However, the signals coupled to port 4 from ports 2 and 3 will be in phase, and hence the total signal will appear at port 4. If the shorts are moved together an electrical distance θ, the phase shift between ports 1 and 4 will change by 2θ. Hence the unit is a variable phase shifter.[1]

If θ_{2-5} and θ_{3-6} are not equal, part of the signal reflected from the shorts will recombine in port 1 and hence there will be a mismatch at this port. The magnitude of the mismatch can be made to change by changing the relative phase lengths θ_{2-5} and θ_{3-6}. The phase of the mismatch can be made to change by varying θ_{2-5} and θ_{3-6}, but maintaining a constant difference between the two, hence the unit will be a mismatch, variable both in magnitude and in phase.

Microwave Switch — Figure 1b

In a microwave switch application a pair of microwave diodes are matched to transmission lines in a given condition of bias, for example, in the region of reverse breakdown. In another condition of bias, say the reverse cutoff condition, the diodes will then present large equal mismatches to the transmission line. Operation of the switch is then as follows. Signal incident on port 1 divides equally between ports 2 and 3. If the diodes are biased so they are matched to the transmission lines the signals from ports 2 and 3 are completely absorbed in the diodes. The switch is then in the "off" position. If the bias is now changed so that the diodes are in the cutoff region, signals from 2 and 3 are reflected back to the hybrid. The hybrid then recombines the signals in arm 4 of the hybrid. The switch is now in the "on" condition. The unit is hence an electrically variable switch matched in both "on" and "off" conditions.[5]

The diodes could be replaced by varactor diodes which would act as variable susceptances dependent upon the bias applied. The phase of the signal transmitted to port 4 can be made to vary relative to that in port 1 by varying the bias. In this operation, the unit is then an electrically variable phase shifter.[2]

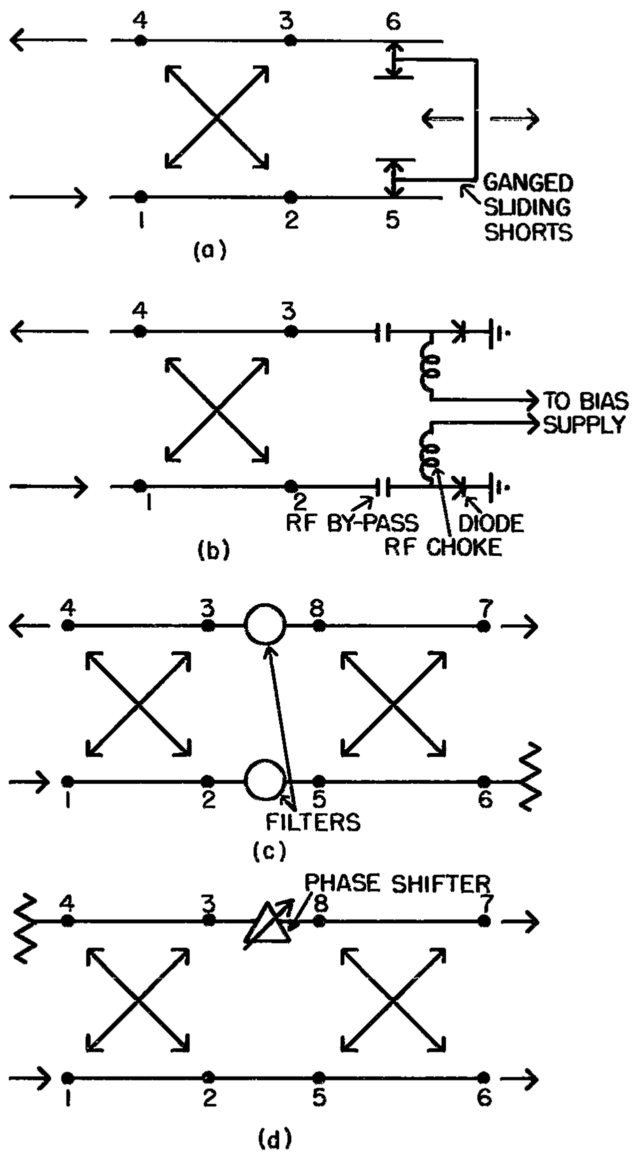

Figure 1 — Networks employing hybrids: a—phase shifter; b—microwave switch; c—diplexer; d—power divider.

Diplexer — Figure 1c

The diplexer employs a pair of identical hybrids and identical filters. Signal incident on port 1 divides equally between ports 2 and 3. If the signal frequency is in the passband of the filters, the signals pass through the filters to ports 5 and 8 of the second hybrid. Here they are caused to recombine in port 7 of this hybrid. If the signal frequency is in the rejection band of the filters, the signals at 2 and 3 are reflected back to the hybrid and are recombined in arm 4. The unit thus divides a band of frequencies incident on port 1 between ports 4 and 7 in accordance with the filter characteristics. In both pass and rejection bands, the diplexer is matched.

If the filters are replaced with gas switching tubes (TR tubes), the unit then becomes a balanced duplexer. In this condition of operation high power signal from the transmitter at port 1 divides equally between ports 2 and 3. The high level signal at ports 2 and 3 causes the TR tubes to break down. The signals are then reflected back to the hybrid where they are recombined in port 4, the antenna port. Any arc leakage signal past the gas tubes tends to recombine in port 7; a termination is placed here to absorb this power. After the tubes have deionized, echo signals from the antenna, port 4, are split by the hybrid and appear at 2 and 3. These signals then pass through the gas tubes to 5 and 8 of the second hybrid. Here they are recombined in this hybrid and appear at the receiver, port 6.

Power Divider — Figure 1d

In this application, a variable phase shifter (for example the type in Figure 1a) is connected between ports of two hybrids as shown in Figure 1d. If θ_{2-5} is equal to θ_{3-8}, signal incident on port 1 will appear at port 7. If θ_{2-5} is different from θ_{3-8} by 180° then signal incident on port 1 appears at port 6 and none goes to port 7. By varying the relative phase shift $(\theta_{2-5} - \theta_{3-8})$ between 0° and 180° any division of power between 6 and 7 can be achieved. In all conditions of operation, the input to port 1 is matched.[10]

The devices described above were completely ideal, depending on exact 3db power splits and ideal phase characteristics. In actual practice, where these conditions do not prevail, performance is in general degraded. For example, in the phase shifter a power split other than 3db will cause a net return signal to the input port with a resultant input VSWR; a difference between θ_{2-5} and θ_{3-6} will also cause a net return signal to the input port with a further increase in input VSWR. Similar degradation will occur in the other circuits in the form of input VSWR; increased insertion loss and decreased isolation.

In the next section the operation of these circuits will be analyzed. It will be seen that in the general case there is interdependence of the effects of hybrid unbalance, phase differences and component differences on overall performance. By reducing the general case, with certain assumptions, to conditions closely met in practice it will be possible to simplify the equations so that they may be conveniently presented in graphical form. These graphs can be used to evaluate a variety of circuits which utilize hybrids.

DETAILED CONSIDERATIONS

Figure 2 shows a generalized component which utilizes hybrids. The hybrids and transmission line components are represented by their voltage scattering coefficients. For convenience the voltage scattering coefficients will be represented by a magnitude S_{i-j}, and associated phase $e^{-j\theta i\text{-}j}$. From inspection of Figure 2, the pertinent scattering coefficients may be written as follows:

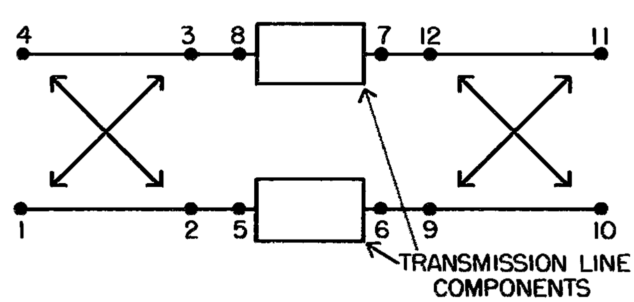

Figure 2 — Generalized network using hybrids.

$$S_{1\text{-}11}e^{-j\theta_{1\text{-}11}} = S_{1\text{-}2}S_{2\text{-}5}S_{5\text{-}6}S_{6\text{-}9}S_{9\text{-}11}e^{-j(\theta_{1\text{-}2}+\theta_{2\text{-}5}+\theta_{5\text{-}6}+\theta_{6\text{-}9}+\theta_{9\text{-}11})} + S_{1\text{-}3}S_{3\text{-}8}S_{8\text{-}7}S_{7\text{-}12}S_{12\text{-}11}e^{-j(\theta_{1\text{-}3}+\theta_{3\text{-}8}+\theta_{8\text{-}7}+\theta_{7\text{-}12}+\theta_{12\text{-}11})} \quad (1)$$

$$S_{1\text{-}10}e^{-j\theta_{1\text{-}10}} = S_{1\text{-}2}S_{2\text{-}5}S_{5\text{-}6}S_{6\text{-}9}S_{9\text{-}10}e^{-j(\theta_{1\text{-}2}+\theta_{2\text{-}5}+\theta_{5\text{-}6}+\theta_{6\text{-}9}+\theta_{9\text{-}10})} + S_{1\text{-}3}S_{3\text{-}8}S_{8\text{-}7}S_{7\text{-}12}S_{12\text{-}10}e^{-j(\theta_{1\text{-}3}+\theta_{3\text{-}8}+\theta_{8\text{-}7}+\theta_{7\text{-}12}+\theta_{12\text{-}10})} \quad (2)$$

$$S_{1\text{-}4}e^{-j\theta_{1\text{-}4}} = S_{1\text{-}2}S_{2\text{-}5}S_{5\text{-}5}S_{5\text{-}2}S_{2\text{-}4}e^{-j(\theta_{1\text{-}2}+\theta_{2\text{-}5}+\theta_{5\text{-}5}+\theta_{5\text{-}2}+\theta_{2\text{-}4})} + S_{1\text{-}3}S_{3\text{-}8}S_{8\text{-}8}S_{8\text{-}3}S_{3\text{-}4}e^{-j(\theta_{1\text{-}3}+\theta_{3\text{-}8}+\theta_{8\text{-}8}+\theta_{8\text{-}3}+\theta_{3\text{-}4})} \quad (3)$$

$$S_{1\text{-}1}e^{-j\theta_{1\text{-}1}} = S_{1\text{-}2}S_{2\text{-}5}S_{5\text{-}5}S_{5\text{-}2}S_{2\text{-}1}e^{-j(\theta_{1\text{-}2}+\theta_{2\text{-}5}+\theta_{5\text{-}5}+\theta_{5\text{-}2}+\theta_{2\text{-}1})} + S_{1\text{-}3}S_{3\text{-}8}S_{8\text{-}8}S_{8\text{-}3}S_{3\text{-}1}e^{-j(\theta_{1\text{-}3}+\theta_{3\text{-}8}+\theta_{8\text{-}8}+\theta_{8\text{-}3}+\theta_{3\text{-}1})} \quad (4)$$

If the hybrids are assumed to be identical, a condition usually closely approached in practice, then $S_{1\text{-}2} = S_{9\text{-}10}$; $S_{1\text{-}3} = S_{9\text{-}11}$; $\theta_{1\text{-}2} = \theta_{9\text{-}10}$, etc.

From conservation of energy and assuming loss-less hybrids, it can be shown that:

$$S_{1\text{-}3} = \sqrt{|1 - S^2_{1\text{-}2}|} \quad (5)$$

Further if the hybrids are symmetric,

$$\theta_{1\text{-}3} = \theta_{1\text{-}2} \pm \frac{\pi}{2} \quad (6)$$

The sign chosen in 6 above, is dependent on the hybrid used. For the analysis here the sign is not critical and we will arbitrarily use the positive sign. Assuming lossless transmission lines, $S_{2\text{-}5} = S_{6\text{-}9} = S_{3\text{-}8} = S_{7\text{-}12} = 1$. Equations 1 through 4 may now be rewritten as follows:

$$S_{1\text{-}11}e^{-j\theta_{1\text{-}11}} = S_{1\text{-}2}\sqrt{|1-S^2_{1\text{-}2}|}\,e^{-j(2\theta_{1\text{-}2}+\frac{\pi}{2})}\left[S_{5\text{-}6}e^{-j(\theta_{2\text{-}5}+\theta_{5\text{-}6}+\theta_{6\text{-}9})} + S_{8\text{-}7}e^{-j(\theta_{3\text{-}8}+\theta_{8\text{-}7}+\theta_{7\text{-}12})}\right] \quad (7)$$

$$S_{1\text{-}10}e^{-j\theta_{1\text{-}10}} = S^2_{1\text{-}2}e^{-j2\theta_{1\text{-}2}}S_{5\text{-}6}e^{-j(\theta_{2\text{-}5}+\theta_{5\text{-}6}+\theta_{6\text{-}9})} - |1-S^2_{1\text{-}2}|\,e^{-j2\theta_{1\text{-}2}}S_{8\text{-}7}e^{-j(\theta_{3\text{-}8}+\theta_{8\text{-}7}+\theta_{7\text{-}12})} \quad (8)$$

$$S_{1\text{-}4}e^{-j\theta_{1\text{-}4}} = S_{1\text{-}2}\sqrt{|1-S^2_{1\text{-}2}|}\,e^{-j(2\theta_{1\text{-}2}+\frac{\pi}{2})}\left[S_{5\text{-}5}e^{-j(2\theta_{2\text{-}5}+\theta_{5\text{-}5})} + S_{8\text{-}8}e^{-j(2\theta_{3\text{-}8}+\theta_{8\text{-}8})}\right] \quad (9)$$

$$S_{1\text{-}1}e^{-j\theta_{1\text{-}1}} = S^2_{1\text{-}2}e^{-j2\theta_{1\text{-}2}}S_{5\text{-}5}e^{-j(2\theta_{2\text{-}5}+\theta_{5\text{-}5})} - |1-S^2_{1\text{-}2}|\,e^{-j2\theta_{1\text{-}2}}S_{8\text{-}8}e^{-j(2\theta_{3\text{-}8}+\theta_{8\text{-}8})} \quad (10)$$

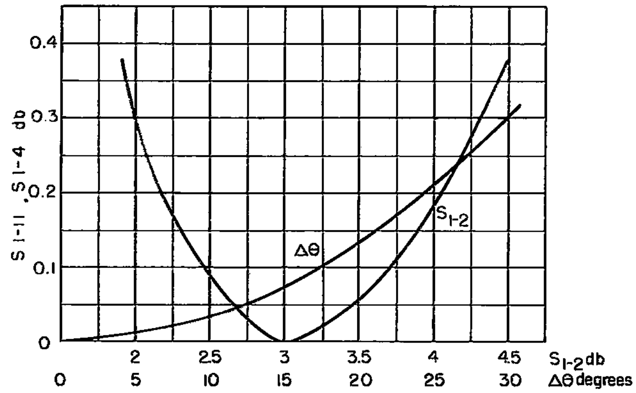

Figure 3 — Effect of coupling and phase difference on S_{1-11} and S_{1-4}.

The above equations can be used directly to analyze practical hybrid circuits. A study of these equations reveals that further simplification is possible with negligible loss of general usefulness. Equations 7 and 9 show that the effects of hybrids and components are independent and conveniently separable. Therefore, the total insertion loss is the sum of the two components of loss, in db. Further if the components are assumed to be identical and either lossless ($S_{5-6} = S_{7-8} = 1$), or completely reflective ($S_{5-5} = S_{8-8} = 1$), it is possible to isolate the effect of phase difference on loss and conveniently present it graphically. The insertion loss of the components can then be added to the losses due to coupling and phase differences.

Factors in Equations 8 and 10 which cause degradation in isolation or input VSWR are not so conveniently separable. However, by making assumptions as above and simply considering the effects of hybrids and phase separately, it is possible to obtain useful graphs which show the individual effects on isolation and VSWR. The combined effects of hybrids and phase can be conveniently presented graphically by means of an intermediate step. If the transmission line components are assumed identical and lossless or completely reflective as above, it is clear that isolation of port 10 is tied-in directly with insertion loss to port 11, and that insertion loss to port 4 is tied-in directly with reflected signal at port 1. Hence, once the insertion loss is found from the curves for Equations 7 and 9, the isolation to 10 and VSWR at 1 is determined. The effect of actual insertion loss in the components can then be added directly to the isolation loss. For incomplete reflection from the components the input VSWR will be less than that obtained from the curves.

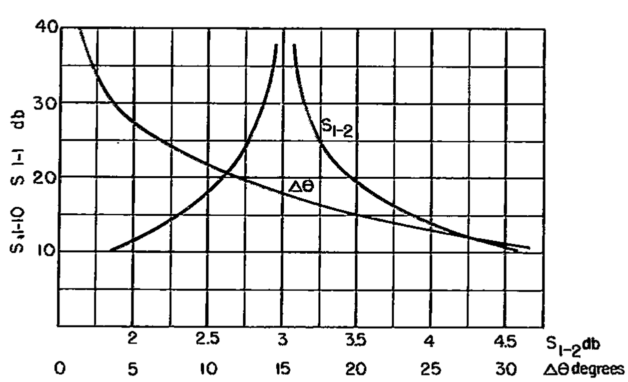

Figure 4 — Effect of coupling and phase difference on S_{1-10} and S_{1-1}.

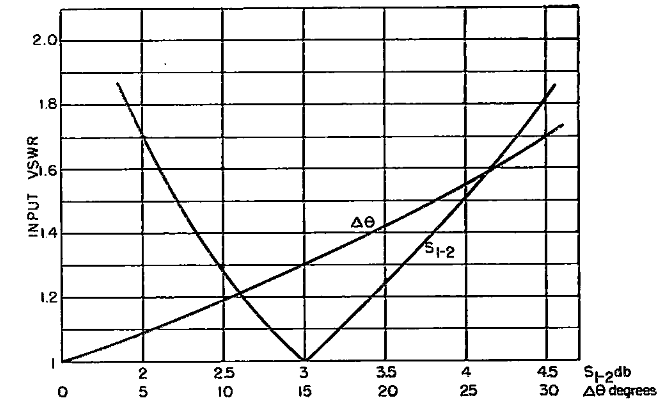

Figure 5 — Effect of coupling and phase difference on input VSWR.

With the above considerations, Equations 7 - 10 are plotted in Figures 3, and 4. The parameter $\triangle\theta$ is defined in Table 1. Figure 3 shows minimum insertion loss achievable; Figure 4 shows maximum isolation achievable as a function of hybrid coupling and phase difference. Figure 5 shows S_{11}, expressed as a VSWR, as a function of hybrid coupling and phase difference. Figure 6 shows combined effects of hybrid coupling and phase difference on input VSWR and isolation as a function of insertion loss obtained from Figure 3. Table I summarizes the equations for the various coefficients and also lists typical situations in which the curves apply.

The curves can be used to rapidly estimate requirements of hybrids and associated circuitry to achieve a desired performance or conversely, to estimate the overall performance characteristics with given hybrids and components. An example of the latter procedure follows:

Estimate the performance of a frequency diplexer utilizing components having the following characteristics.

Hybrid coupling S_{1-2}	3 ± 0.5db
Filter loss $S_{5-6} = S_{8-7}$	0.5db
Differences in phase shift through filters	$\pm 20°$
Differences in phase shift of filter reflection coefficients in rejection band.	$\pm 10°$
Pass Band Loss	
From Figure 3	
Loss due to coupling	0.088db
Loss due to $\triangle\theta = 20°$	0.134db
Filter loss	0.5 db
Pass band insertion loss	0.722db
Rejection Band Insertion Loss	
From Figure 3 Loss due to coupling	0.088db
Loss due to $\triangle\theta = 10°$	0.034db
Rejection band insertion loss	0.122db

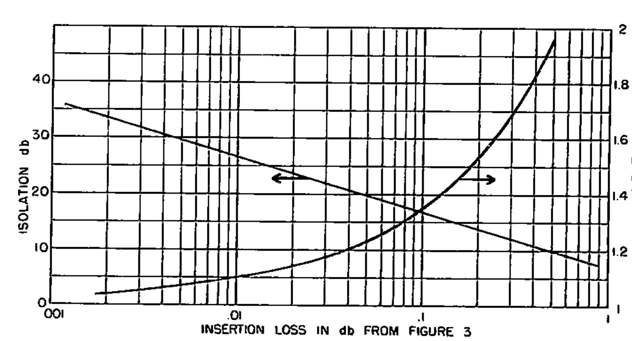

Figure 6 — Isolation and VSWR vs. Insertion Loss.

VSWR in Rejection Band	
From Figure 5 VSWR due to S_{1-2}	1.28
VSWR due to $\Delta\theta = 10°$	1.19
From Figure 6 using the rejection band loss of 0.122db:	
Total input VSWR =	1.4
Isolation in Pass Band	
From Figure 4	
Isolation due to S_{12}	18db
Isolation due to $\Delta\theta = 20°$	15.2db
From Figure 6	
Maximum isolation found from pass band loss of 0.222db =	13db
Filter insertion loss	0.5db
Total isolation	13.5db

CONCLUSIONS

An analysis has been made which is applicable to a wide variety of circuits utilizing hybrids. Assuming identical lossless components, it has been possible to present equations and graphs relating hybrid coupling and phase effects to insertion loss, isolation and input VSWR. An example of the use of the curves shows how the effect of component loss can be taken into consideration.

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References

1. J. Y. Wong, "A Mechanical Waveguide Hybrid Phase Changer," *IRE Transactions on Microwave Theory and Techniques,* Vol. MTT-9, pp. 364-365, July, 1961.
2. R. H. Hardin, E. J. Downey, J. Munushian, "Electronically-Variable Phase Shifters Utilizing Variable Capacitance Diodes," *Proceedings IRE,* Vol. 48, No. 5, pp. 994-945; May, 1960.
3. C. F. Augustine and J. Cheal, "The Design and Measurement of Two Broad Band Coaxial Phase Changers," *IRE Transactions on Microwave Theory and Techniques,* Vol. MTT-8, pp. 398-402; July, 1960.
4. Harry C. Berland and Fred W. Vortmeier, "Broad Band, Multi-Port Waveguide Switches," *the microwave journal,* Vol. 2, No. 9, pp. 30-37; September, 1959.
5. R. V. Garver, E. G. Spencer, and M. A. Harper, "Microwave Semiconductor Switching Techniques," *IRE Transactions on Microwave Theory and Techniques,* Vol. MTT-6, pp. 378-383; October, 1958.
6. W. L. Teeter, "A High Speed Broadband Microwave Waveguide Switch," *IRE Transactions on Microwave Theory and Techniques,* Vol. MTT-3, pp. 11-14; October, 1955.
7. Robert D. Wanselow, "A Nonreflective Hybrid Stop-Band Filter," *IRE Transactions on Microwave Theory and Techniques,* Vol. MTT-10, p. 91; January, 1962.
8. Kenneth Eakin and Robert Rapuano, "A Hybrid Duplexer," *the microwave journal,* Vol. 4, no. 1, pp. 47-49; January, 1961.
9. A. F. Harvey, "Duplexing Systems at Microwave Frequencies," *IRE Transactions on Microwave Theory and Techniques,* Vol. MTT-8, pp. 415-431; July, 1960.
10. R. M. Vaillancourt, "Analysis of the Variable-Ratio Microwave Power Divider," *IRE Transaction on Microwave Theory and Techniques,* Vol. MTT-6, No. 2, pp. 238-239; April, 1958.
11. P. L. Fleming, "X-Band Electronically-Variable Attenuator," *Proceedings IRE,* Vol. 49, No. 11, pp. 1690-1691; November, 1961.
12. S. H. Autler, "Proposal for a Master-Amplifier System without Nonreciprocal Elements," *Proceedings IRE,* Vol. 46, pp. 1880-1881; November, 1958.
13. L. U. Kibler, "Directional Bridge Parametric Amplifier," *Proceedings IRE,* Vol. 47, No. 4, pp. 583-584; April, 1959.
14. John J. Sie, "Absolutely Stable Hybrid Coupled Tunnel-Diode Amplifier," *Proceedings IRE,* Vol. 48, No. 7, p. 1321; July, 1960.

TABLE I

Summary For Figures 3-5

Coefficient	Figure	Conditions	Typical Applications
$S_{1-11} = 2S_{12}\sqrt{\lvert 1 - S^2_{1-2}\rvert}$	3	$S_{5-6} = S_{8-7} = 1$; $-\theta_{2-5} - \theta_{5-6} - \theta_{6-9} = -\theta_{3-8} - \theta_{8-7} - \theta_{7-2}$	Minimum Loss in: 1. Diplexer pass band. 2. Power Divider 3. Duplexer, Antenna to receiver.
$S_{1-11} = \left\lvert \cos\frac{\Delta\theta}{2} \right\rvert$	3	$S_{1-2} = \frac{1}{\sqrt{2}}$; $S_{5-6} = S_{8-7} = 1$; $-\theta_{2-5} - \theta_{5-6} - \theta_{6-9} - [-\theta_{3-8} - \theta_{8-7} - \theta_{7-12}] = \Delta\theta$	
$S_{1-4} = 2S_{12}\sqrt{\lvert 1 - S^2_{1-2}\rvert}$	3	$S_{5-5} = S_{8-8} = 1$ $-2\theta_{2-5} - \theta_{5-5} = -2\theta_{3-8} - \theta_{8-8}$	Minimum Loss in: 1. Phase Shifter. 2. Switch in "ON" position. 3. Directional Filter in Rejection Band. 4. Duplexer, Transmitter to antenna.
$S_{1-4} = \left\lvert \cos\frac{\Delta\theta}{2} \right\rvert$	3	$S_{1-2} = 1/\sqrt{2}$ $S_{5-5} = S_{8-8} = 1$ $-2\theta_{2-5} - \theta_{5-5} - [-2\theta_{3-8} - \theta_{8-8}] = \Delta\theta$	
$S_{1-10} = \lvert 2S^2_{12} - 1 \rvert$	4	$S_{5-6} = S_{8-7} = 1$; $-\theta_{2-5} - \theta_{5-6} - \theta_{6-9} = -\theta_{3-8} - \theta_{8-7} - \theta_{7-12}$	Maximum isolation in: 1. Directional Filter. 2. Balanced Duplexer between Transmitter and Receiver. 3. Power Divider
$S_{1-10} = \left\lvert \sin\frac{\Delta\theta}{2} \right\rvert$	4	$S_{1-2} = 1/\sqrt{2}$; $S_{5-6} = S_{8-7} = 1$; $-\theta_{2-5} - \theta_{5-6} - \theta_{6-9} - [-\theta_{3-8} - \theta_{8-7} - \theta_{7-12}] = \Delta\theta$	
$S_{1-1} = \lvert 2S^2_{12} - 1 \rvert$; $\text{VSWR} = \frac{1 + S_{1-1}}{1 - S_{1-1}}$	4 5	$S_{5-5} = S_{8-8} = 1$ $-2\theta_{2-5} - \theta_{5-5} = -2\theta_{3-8} - \theta_{8-8}$	Minimum Reflection (VSWR) in: 1. Phase Shifter 2. Directional Filter in Rejection Band. 3. Switch in "ON" Position. 4. Duplexer in Transmitting condition.
$S_{1-1} = \left\lvert \sin\frac{\Delta\theta}{2} \right\rvert$; $\text{VSWR} = \frac{1 + S_{1-1}}{1 - S_{1-1}}$	4 5	$S_{1-2} = 1/\sqrt{2}$; $S_{5-5} = S_{8-8} = 1$; $-2\theta_{2-5} - \theta_{5-5} - [-2\theta_{3-8} - \theta_{8-8}] = \Delta\theta$	