Magnetics energy storage and air gaps

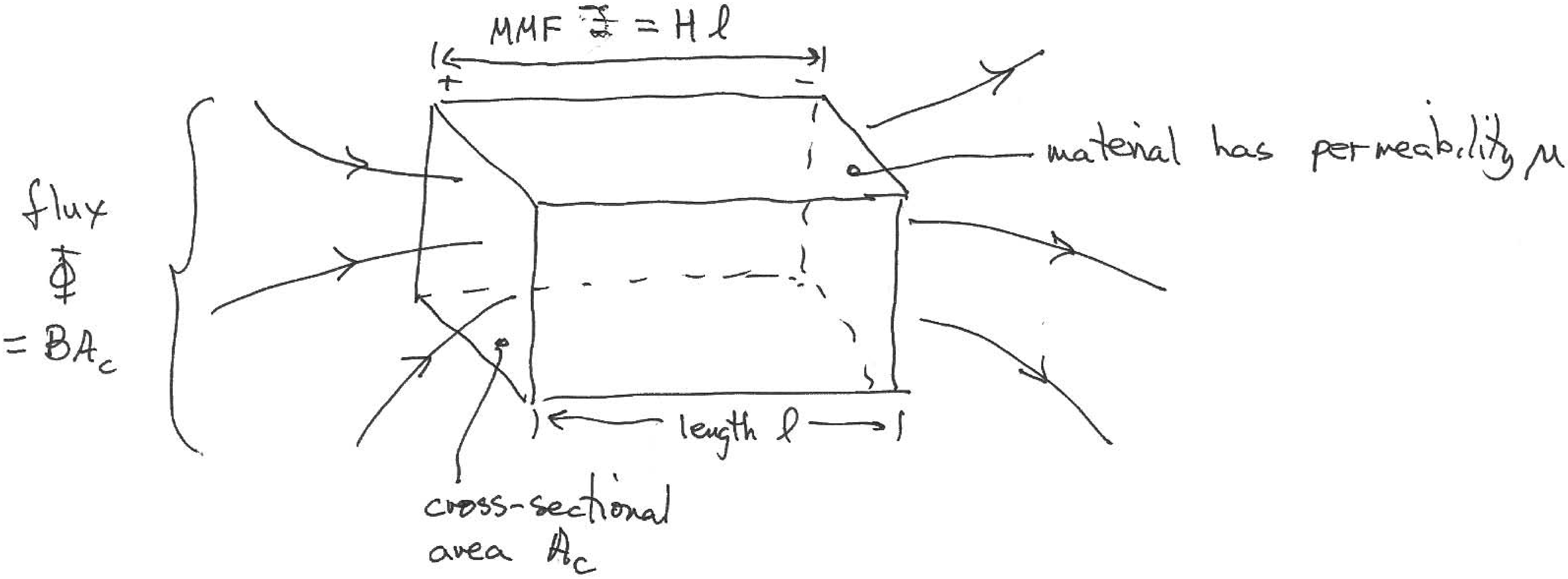
We know from magnetics theory that the energy stored in a volume of space having magnetic flux density B and field H is

$$W = \int_{\text{volume}} B \cdot H \, dV$$

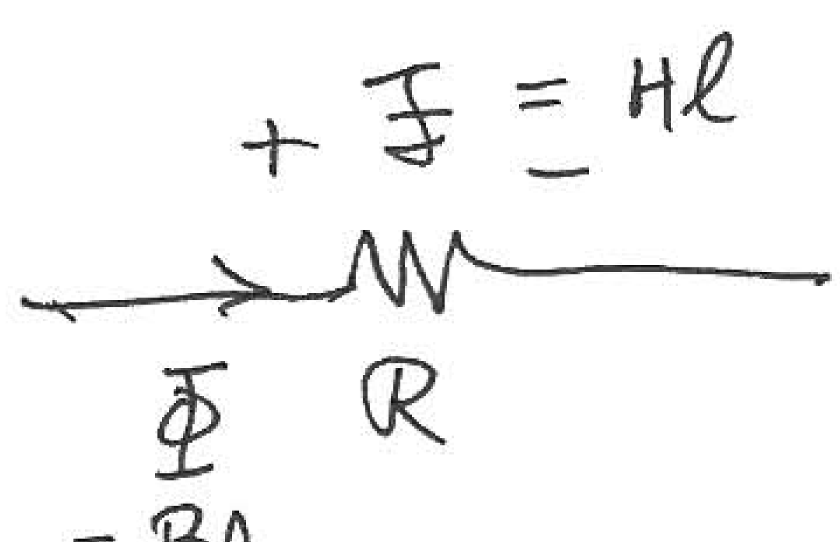
dV = volume element

can evaluate directly (see appendix, p.3) or use reluctance model (see below)

Consider a core element (or an air gap element):



Let's assume uniform B and H



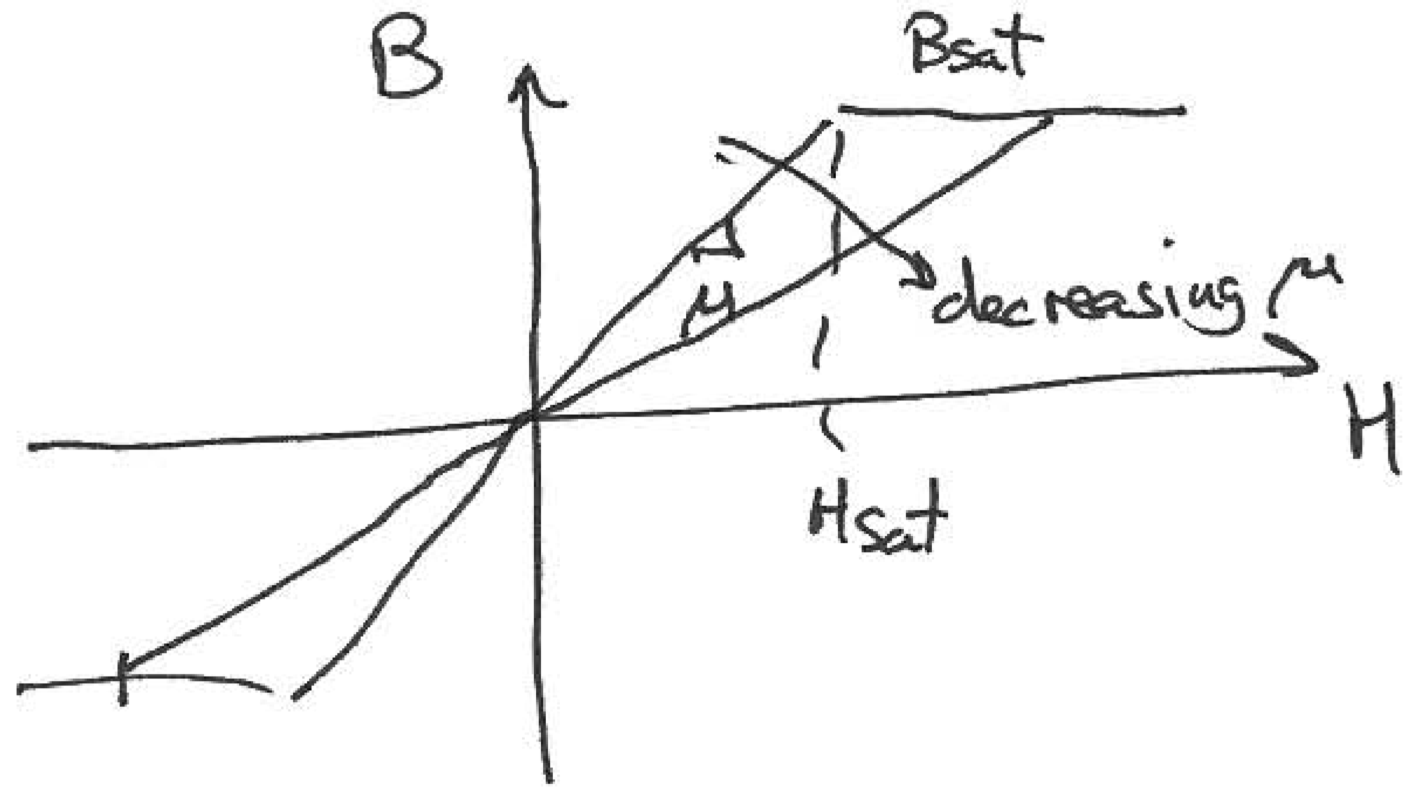
Reluctance model

The stored energy in this element is

$$W = \int_{\text{volume}} B \cdot H \, dV = BH \underbrace{A_c l}_{\text{volume}} = (BA_c)(Hl) = \underline{\underline{\Phi \mathcal{F}}}$$

How much energy can the element store before it saturates? At $B = B_{sat}$, $\Phi_{sat} = B_{sat} A_c$

and $H_{sat} = \dfrac{B_{sat}}{\mu}$



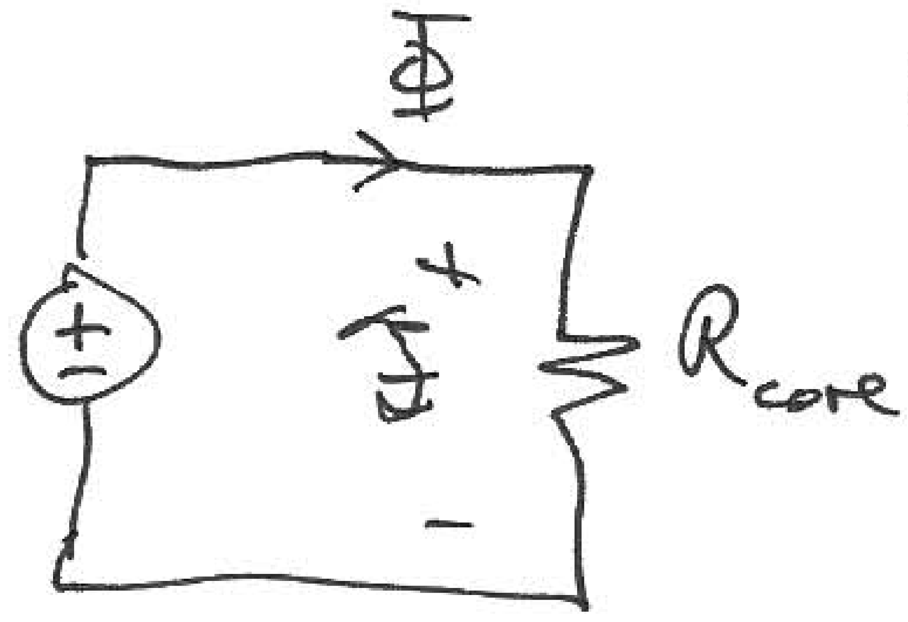
Note: decreasing μ does not change B_{sat} but it increases $H_{sat} = \dfrac{B_{sat}}{\mu}$.
So with less μ you get more H_{sat} and hence more w.

We get W_{Max} = stored energy at the onset of saturation

$= \Phi \mathcal{F}$ with $\mathcal{F} = \Phi \mathcal{R}$ and $\Phi = \Phi_{sat}$

$$W_{max} = \Phi_{sat}^2 \mathcal{R}$$

1. Basic core, no gap (Fig. 13.8)

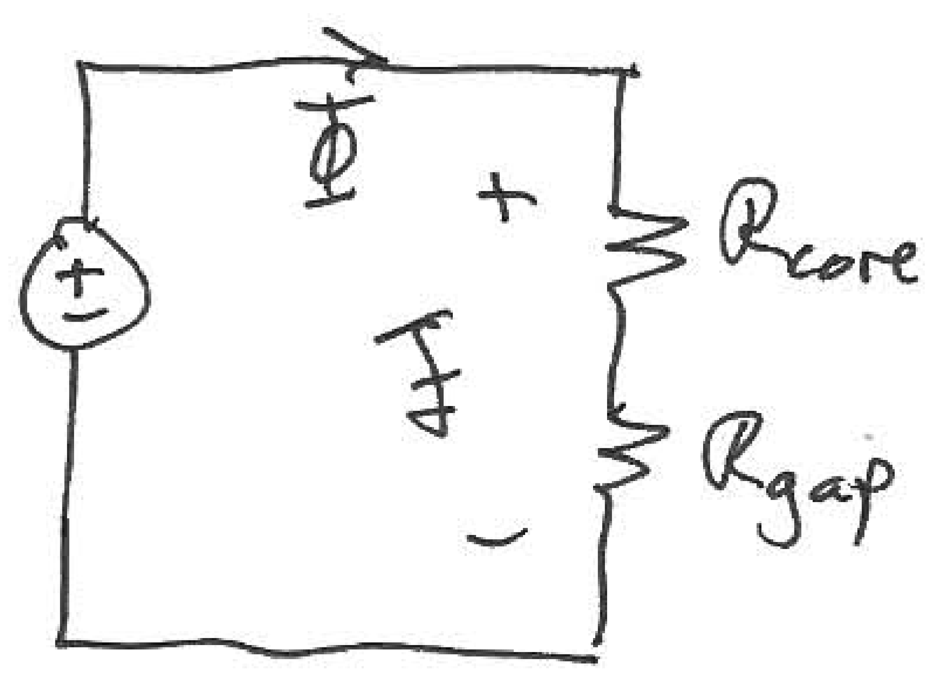


$\mathcal{R}_{core} \Phi = \mathcal{F}$, $w = \Phi \mathcal{F} = \Phi^2 \mathcal{R}_{core}$

At onset of saturation, $\Phi = \Phi_{sat} = B_{sat} A_c$

$$w = \Phi_{sat}^2 \mathcal{R}_c$$

2. Add air gap



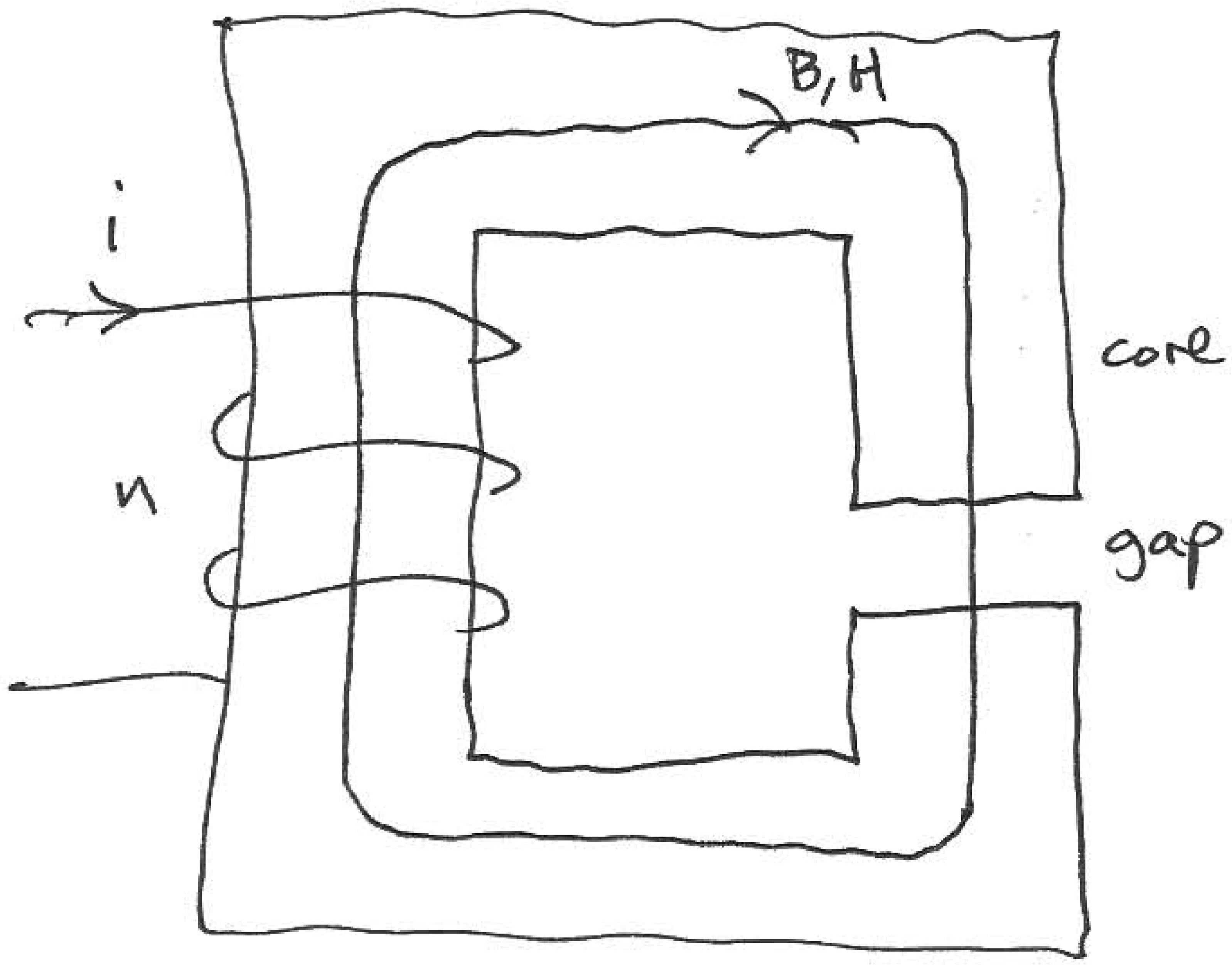
$\mathcal{F} = \Phi (\mathcal{R}_{core} + \mathcal{R}_{gap})$, $w = \Phi^2 (\mathcal{R}_{core} + \mathcal{R}_{gap})$

at onset of saturation, $\Phi = \Phi_{sat} = B_{sat} A_c$

$$w = \Phi_{sat}^2 (\mathcal{R}_{core} + \mathcal{R}_{gap})$$

$\mathcal{R}_{gap}$ increases the amount of stored energy

Appendix



Flux lines are continuous;
B and Φ in core are same as B and Φ in gap

Magnetic field: $H_{core} = \frac{B_{core}}{\mu_{core}}$

$$H_{gap} = \frac{B_{gap}}{\mu_0} = \frac{B_{core}}{\mu_0}$$

For uniform B, H:

Energy in core is

$$B_{core} \cdot H_{core} \cdot (\text{core volume})$$

$$= \frac{B^2}{\mu_{core}} (\text{core volume})$$

Energy in gap is

$$B_{gap} \cdot H_{gap} \cdot (\text{gap volume})$$

$$= \frac{B^2}{\mu_0} (\text{gap volume})$$