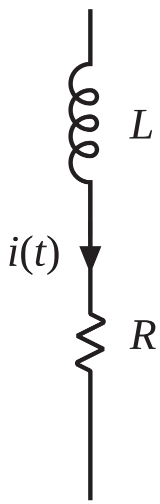
Chapter 14 Inductor Design

14.1 Filter inductor design constraints



Objective:

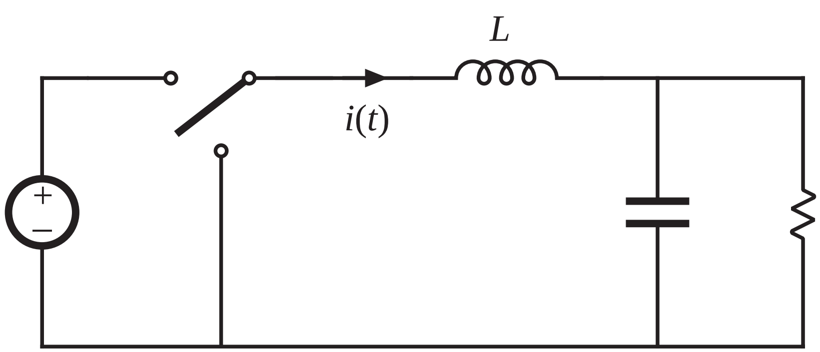
Design inductor having a given inductance L,

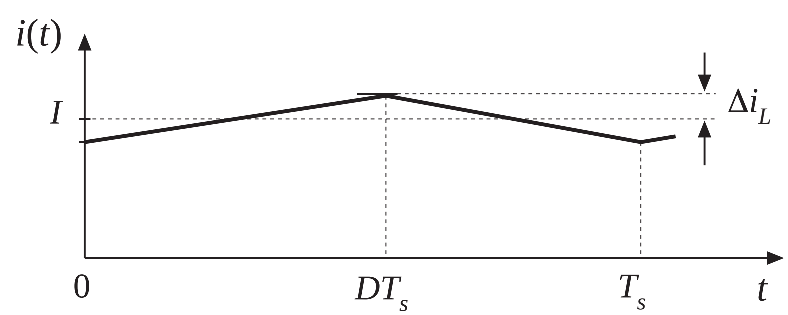
which carries worst-case current I_{max} without saturating,

and which has a given winding resistance R, or, equivalently, exhibits a worst-case copper loss of

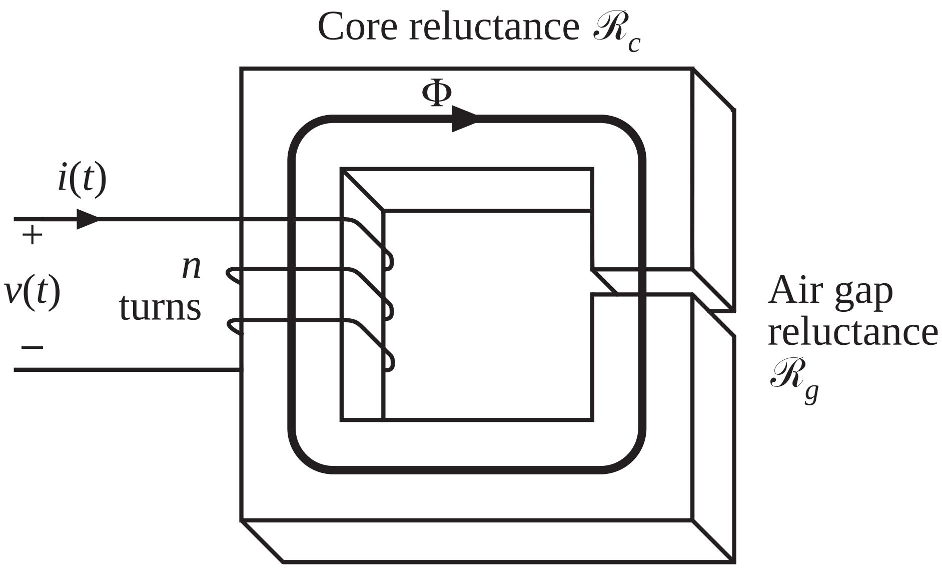
$$P_{cu} = I_{rms}^{2} R$$

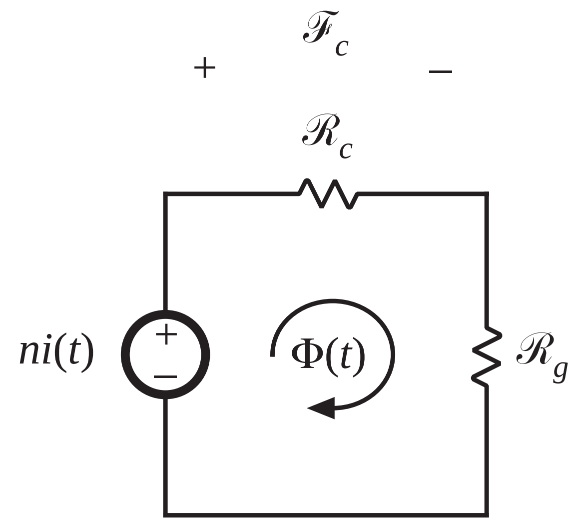
Example: filter inductor in CCM buck converter





Assumed filter inductor geometry





$$\mathcal{R}_c = \frac{\ell_c}{\mu_c A_c}$$

$$\mathcal{R}_g = \frac{\ell_g}{\mu_0 A_c}$$

Solve magnetic circuit:

$$ni = \Phi\left(\mathcal{R}_c + \mathcal{R}_g\right)$$

Usually $\mathcal{R}_c \ll \mathcal{R}_g$ and hence

$$ni \approx \Phi \mathcal{R}_g$$

14.1.1 Constraint: maximum flux density

Given a peak winding current I_{max}, it is desired to operate the core flux density at a peak value B_{max}. The value of B_{max} is chosen to be less than the worst-case saturation flux density B_{sat} of the core material.

From solution of magnetic circuit:

$$ni = BA_c\mathcal{R}_g$$

Let $I = I_{max}$ and $B = B_{max}$:

$$nI_{max} = B_{max} A_c \mathcal{R}_g = B_{max} \frac{\ell_g}{\mu_0}$$

This is constraint #1. The turns ratio n and air gap length ℓ_g are unknown.

14.1.2 Constraint: inductance

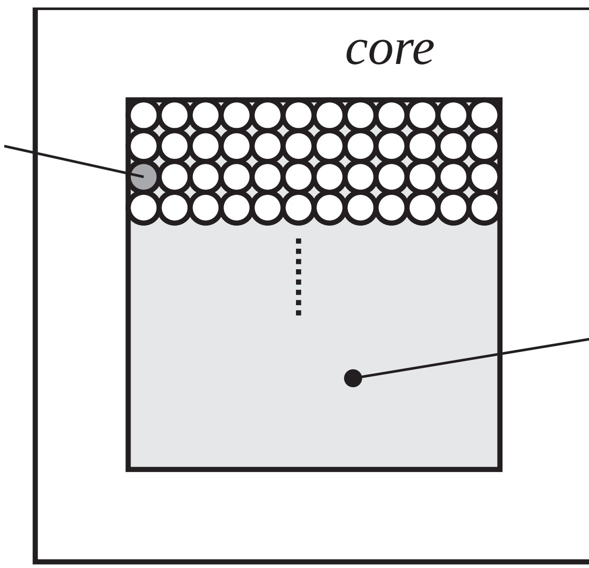
Must obtain specified inductance L. We know that the inductance is

$$L = \frac{n^2}{\mathcal{R}_g} = \frac{\mu_0 A_c n^2}{\ell_g}$$

This is constraint #2. The turns ratio n, core area A_c, and air gap length ℓ_g are unknown.

14.1.3 Constraint: winding area

Wire must fit through core window (i.e., hole in center of core)



Total area of copper in window:

$$nA_W$$

Area available for winding conductors:

$$K_u W_A$$

Third design constraint:

$$K_u W_A \geq nA_W$$

The window utilization factor K_u

also called the "fill factor"

K_u is the fraction of the core window area that is filled by copper

Mechanisms that cause K_u to be less than 1:

- Round wire does not pack perfectly, which reduces K_u by a factor of 0.7 to 0.55 depending on winding technique
- Insulation reduces K_u by a factor of 0.95 to 0.65, depending on wire size and type of insulation
- Bobbin uses some window area
- Additional insulation may be required between windings

Typical values of K_u :

0.5 for simple low-voltage inductor

0.25 to 0.3 for off-line transformer

0.05 to 0.2 for high-voltage transformer (multiple kV)

0.65 for low-voltage foil-winding inductor

14.1.4 Winding resistance

The resistance of the winding is

$$R = \rho \frac{\ell_b}{A_W}$$

where ρ is the resistivity of the conductor material, ℓ_b is the length of the wire, and A_W is the wire bare area. The resistivity of copper at room temperature is $1.724 \cdot 10^{-6}\ \Omega\text{-cm}$. The length of the wire comprising an n-turn winding can be expressed as

$$\ell_b = n\,(MLT)$$

where (MLT) is the mean-length-per-turn of the winding. The mean-length-per-turn is a function of the core geometry. The above equations can be combined to obtain the fourth constraint:

$$R = \rho \frac{n\,(MLT)}{A_W}$$

14.1.5 The core geometrical constant K_g

The four constraints:

$$nI_{max} = B_{max} A_c \mathcal{R}_g = B_{max} \frac{\ell_g}{\mu_0}$$

$$L = \frac{n^2}{\mathcal{R}_g} = \frac{\mu_0 A_c n^2}{\ell_g}$$

$$K_u W_A \geq n A_W$$

$$R = \rho \frac{n\,(MLT)}{A_W}$$

These equations involve the quantities

A_c, W_A, and MLT, which are functions of the core geometry,

I_{max}, B_{max} , μ_0, L, K_u, R, and ρ, which are given specifications or other known quantities, and

n, ℓ_g, and A_W, which are unknowns.

Eliminate the three unknowns, leading to a single equation involving the remaining quantities.

Core geometrical constant K_g

Elimination of n, ℓ_g, and A_W leads to

$$\frac{A_c^2 W_A}{(MLT)} \geq \frac{\rho L^2 I_{max}^2}{B_{max}^2 R K_u}$$

- Right-hand side: specifications or other known quantities
- Left-hand side: function of only core geometry

So we must choose a core whose geometry satisfies the above equation.

The core geometrical constant K_g is defined as

$$K_g = \frac{A_c^2 W_A}{(MLT)}$$

Discussion

$$K_g = \frac{A_c^2 W_A}{(MLT)} \geq \frac{\rho L^2 I_{max}^2}{B_{max}^2 R K_u}$$

K_g is a figure-of-merit that describes the effective electrical size of magnetic cores, in applications where the following quantities are specified:

- Copper loss
- Maximum flux density

How specifications affect the core size:

A smaller core can be used by increasing

$B_{max} \Rightarrow$ use core material having higher B_{sat}

$R \Rightarrow$ allow more copper loss

How the core geometry affects electrical capabilities:

A larger K_g can be obtained by increase of

$A_c \Rightarrow$ more iron core material, or

$W_A \Rightarrow$ larger window and more copper

14.2 A step-by-step procedure

The following quantities are specified, using the units noted:

Wire resistivity	ρ	(Ω-cm)
Peak winding current	I_{max}	(A)
Inductance	L	(H)
Winding resistance	R	(Ω)
Winding fill factor	K_u	
Core maximum flux density	B_{max}	(T)

The core dimensions are expressed in cm:

Core cross-sectional area	A_c	(cm^2)
Core window area	W_A	(cm^2)
Mean length per turn	MLT	(cm)

The use of centimeters rather than meters requires that appropriate factors be added to the design equations.

Determine core size

$$K_g \ge \frac{\rho L^2 I_{max}^2}{B_{max}^2 R K_u} 10^8 \qquad (\mathrm{cm}^5)$$

Choose a core which is large enough to satisfy this inequality (*see Appendix D for magnetics design tables*).

Note the values of A_c, W_A, and MLT for this core.

Determine air gap length

$$\ell_g = \frac{\mu_0 L I_{max}^2}{B_{max}^2 A_c} 10^4 \qquad \text{(m)}$$

with A_c expressed in cm^2. $\mu_0 = 4\pi 10^{-7}$ H/m.

The air gap length is given in meters.

The value expressed above is approximate, and neglects fringing flux and other nonidealities.

A_L

Core manufacturers sell gapped cores. Rather than specifying the air gap length, the equivalent quantity A_L is used.

A_L is equal to the inductance, in mH, obtained with a winding of 1000 turns.

When A_L is specified, it is the core manufacturer's responsibility to obtain the correct gap length.

The required A_L is given by:

$$A_L = \frac{10 B_{max}^2 A_c^2}{L I_{max}^2} \qquad \text{(mH/1000 turns)}$$

Units:

A_c cm²,
L Henries,
B_{max} Tesla.

$$L = A_L\, n^2\, 10^{-9} \qquad \text{(Henries)}$$

Determine number of turns n

$$n = \frac{LI_{max}}{B_{max}A_c} \; 10^4$$

Evaluate wire size

$$A_W \le \frac{K_u W_A}{n} \qquad (\mathrm{cm}^2)$$

Select wire with bare copper area A_W less than or equal to this value. An American Wire Gauge table is included in Appendix D.

As a check, the winding resistance can be computed:

$$R = \frac{\rho n\,(MLT)}{A_w} \qquad (\Omega)$$

14.3 Multiple-winding magnetics design using the K_g method

The K_g design method can be extended to multiple-winding magnetic elements such as transformers and coupled inductors.

This method is applicable when

- Copper loss dominates the total loss (i.e. core loss is ignored), or
- The maximum flux density B_{max} is a specification rather than a quantity to be optimized

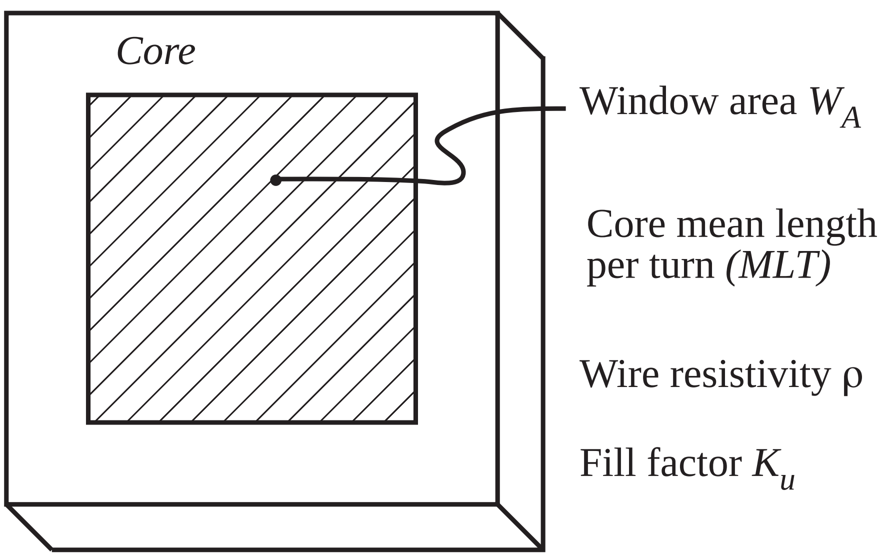
To do this, we must

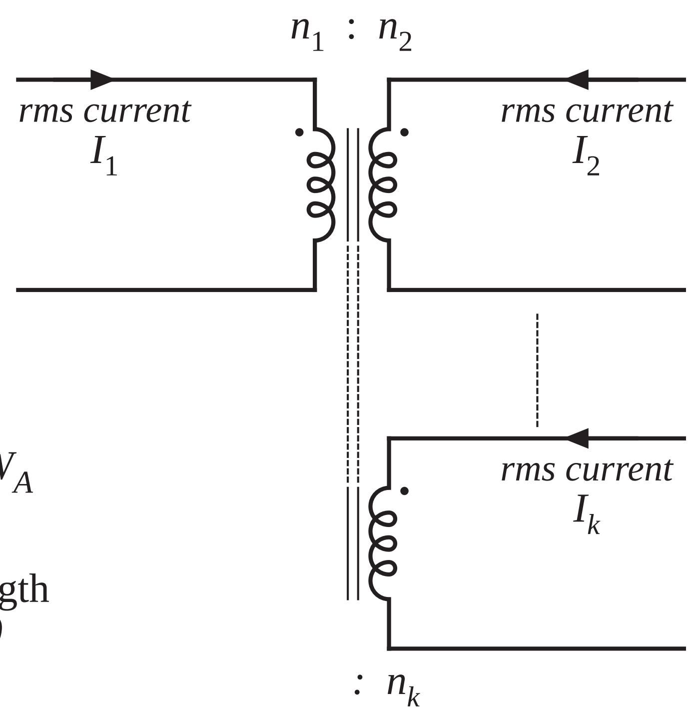
- Find how to allocate the window area between the windings
- Generalize the step-by-step design procedure

14.3.1 Window area allocation

Given: application with k windings having known rms currents and desired turns ratios

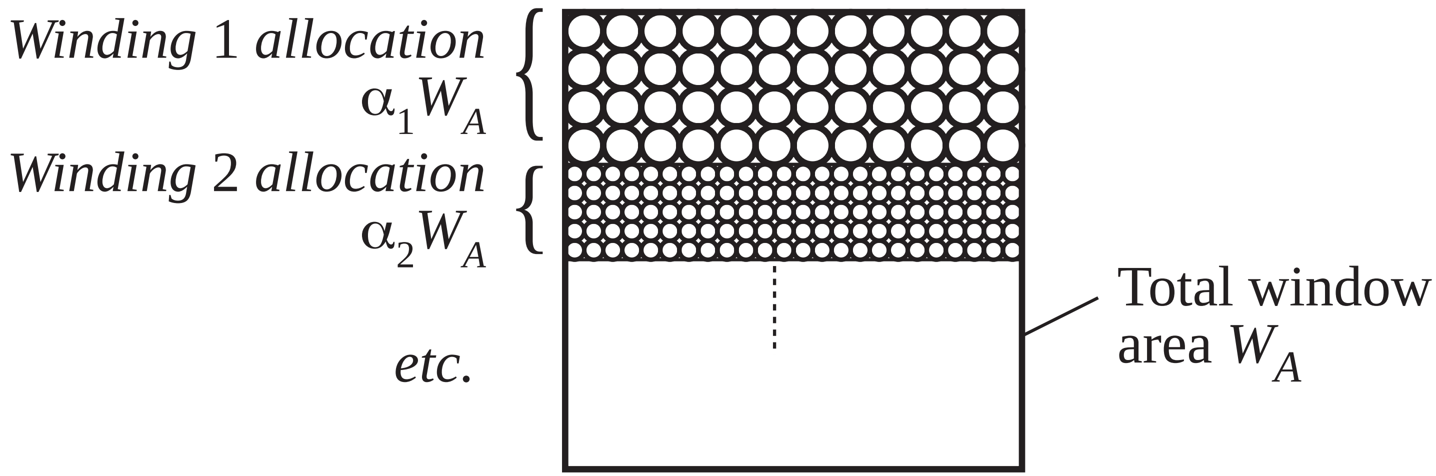
$$\frac{v_1(t)}{n_1} = \frac{v_2(t)}{n_2} = \cdots = \frac{v_k(t)}{n_k}$$





Q: how should the window area W_A be allocated among the windings?

Allocation of winding area



$$0 < \alpha_j < 1$$

$$\alpha_1 + \alpha_2 + \cdots + \alpha_k = 1$$

Copper loss in winding j

Copper loss (not accounting for proximity loss) is

$$P_{cu,j} = I_j^2 R_j$$

Resistance of winding j is

$$R_j = \rho \frac{\ell_j}{A_{W,j}}$$

with

$$\ell_j = n_j (MLT)$$ length of wire, winding j

$$A_{W,j} = \frac{W_A K_u \alpha_j}{n_j}$$ wire area, winding j

Hence

$$R_j = \rho \frac{n_j^2 (MLT)}{W_A K_u \alpha_j}$$

$$P_{cu,j} = \frac{n_j^2 i_j^2 \rho (MLT)}{W_A K_u \alpha_j}$$

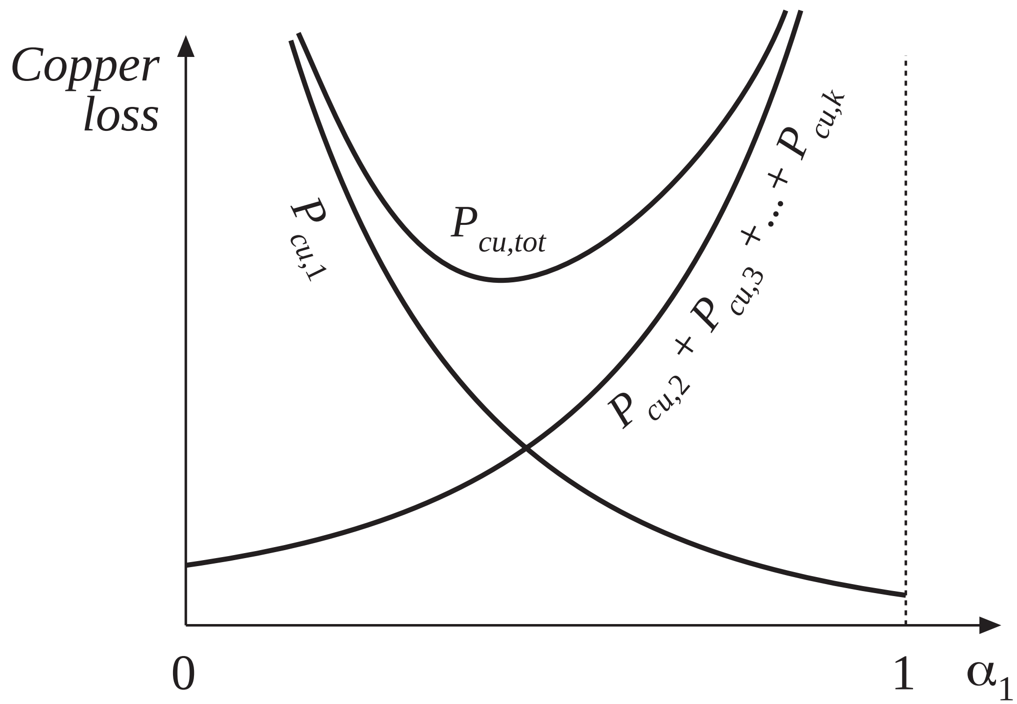
Total copper loss of transformer

Sum previous expression over all windings:

$$P_{cu,tot} = P_{cu,1} + P_{cu,2} + \cdots + P_{cu,k} = \frac{\rho\,(MLT)}{W_A K_u} \sum_{j=1}^{k} \left(\frac{n_j^2 I_j^2}{\alpha_j} \right)$$

Need to select values for $\alpha_1, \alpha_2, \ldots, \alpha_k$ such that the total copper loss is minimized

Variation of copper losses with α_1



For $\boldsymbol{\alpha_1}$ **= 0:** wire of winding 1 has zero area. $P_{cu,1}$ tends to infinity

For $\boldsymbol{\alpha_1}$ **= 1:** wires of remaining windings have zero area. Their copper losses tend to infinity

There is a choice of α_1 that minimizes the total copper loss

Method of Lagrange multipliers

to minimize total copper loss

Minimize the function

$$P_{cu,tot} = P_{cu,1} + P_{cu,2} + \cdots + P_{cu,k} = \frac{\rho\,(MLT)}{W_A K_u} \sum_{j=1}^{k} \left(\frac{n_j^2 I_j^2}{\alpha_j} \right)$$

subject to the constraint

$$\alpha_1 + \alpha_2 + \cdots + \alpha_k = 1$$

Define the function

$$f(\alpha_1, \alpha_2, \cdots, \alpha_k, \xi) = P_{cu,tot}(\alpha_1, \alpha_2, \cdots, \alpha_k) + \xi\, g(\alpha_1, \alpha_2, \cdots, \alpha_k)$$

where

$$g(\alpha_1, \alpha_2, \cdots, \alpha_k) = 1 - \sum_{j=1}^{k} \alpha_j$$

is the constraint that must equal zero

and ξ is the Lagrange multiplier

Lagrange multipliers
continued

Optimum point is solution of the system of equations

$$\frac{\partial f(\alpha_1, \alpha_2, \cdots, \alpha_k, \xi)}{\partial \alpha_1} = 0$$

$$\frac{\partial f(\alpha_1, \alpha_2, \cdots, \alpha_k, \xi)}{\partial \alpha_2} = 0$$

$$\vdots$$

$$\frac{\partial f(\alpha_1, \alpha_2, \cdots, \alpha_k, \xi)}{\partial \alpha_k} = 0$$

$$\frac{\partial f(\alpha_1, \alpha_2, \cdots, \alpha_k, \xi)}{\partial \xi} = 0$$

Result:

$$\xi = \frac{\rho\,(MLT)}{W_A K_u} \left(\sum_{j=1}^{k} n_j I_j \right)^2 = P_{cu,tot}$$

$$\alpha_m = \frac{n_m I_m}{\sum_{n=1}^{\infty} n_j I_j}$$

An alternate form:

$$\alpha_m = \frac{V_m I_m}{\sum_{n=1}^{\infty} V_j I_j}$$

Interpretation of result

$$\alpha_m = \frac{V_m I_m}{\sum_{n=1}^{\infty} V_j I_j}$$

Apparent power in winding j is

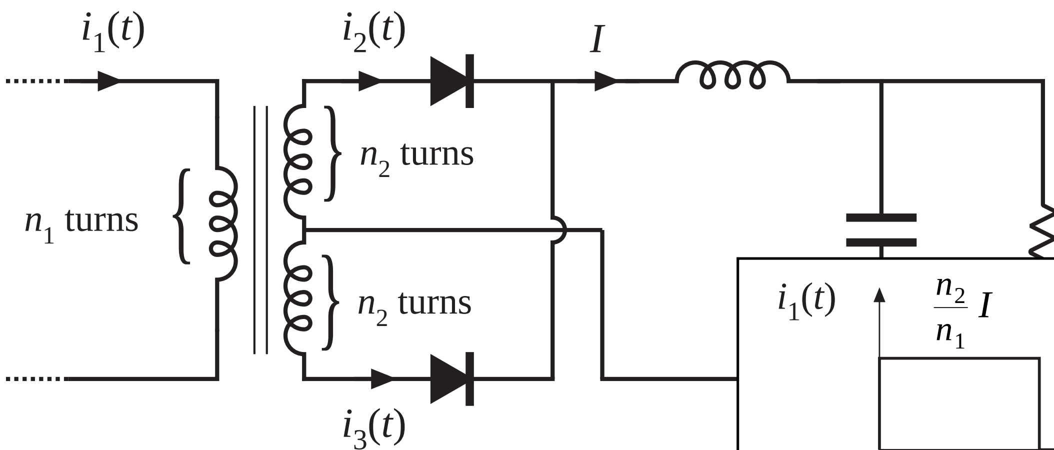
$V_j I_j$

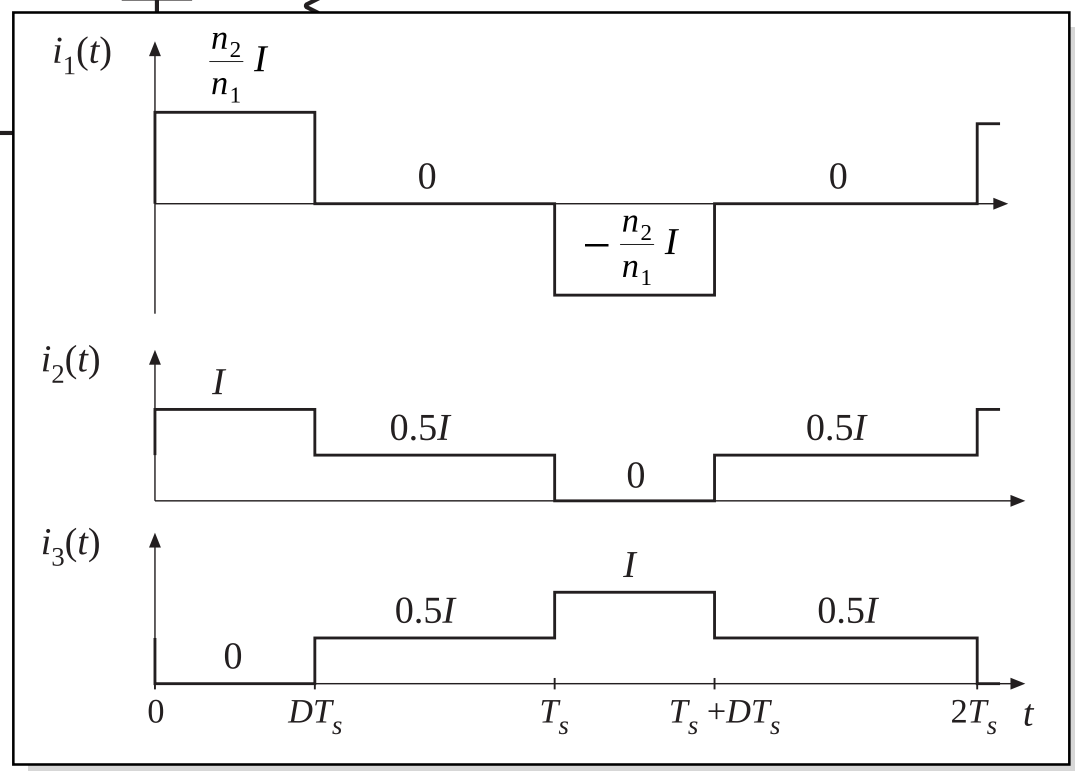
where V_j is the rms or peak applied voltage

I_j is the rms current

Window area should be allocated according to the apparent powers of the windings

Example
PWM full-bridge transformer





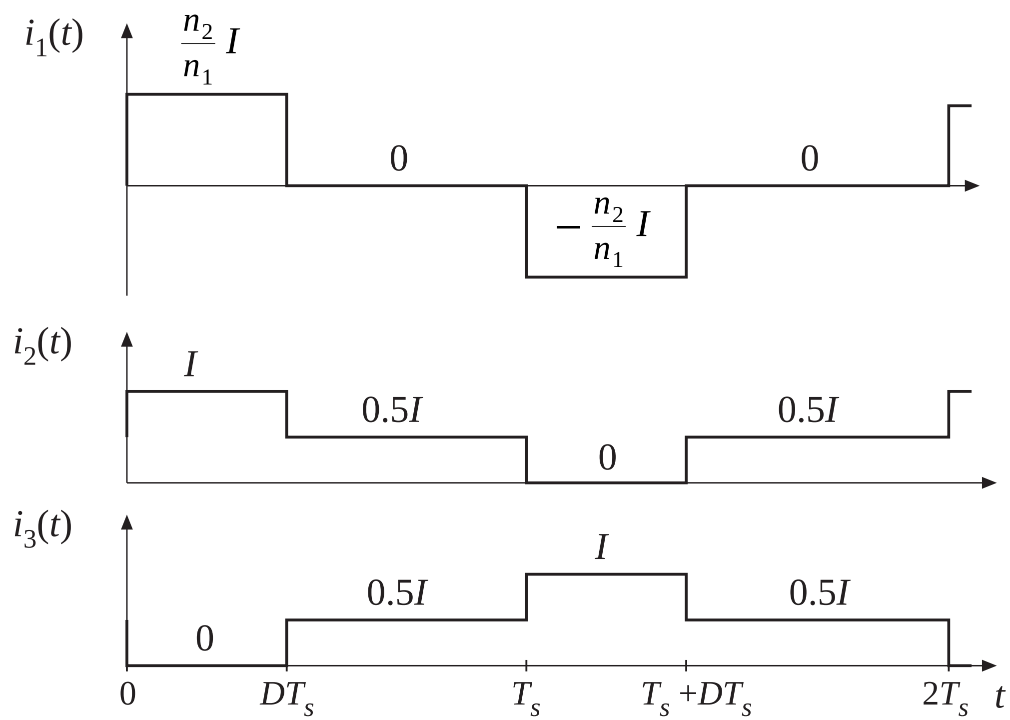
- Note that waveshapes (and hence rms values) of the primary and secondary currents are different
- Treat as a three-winding transformer

Expressions for RMS winding currents

$$I_1 = \sqrt{\frac{1}{2T_s}\int_0^{2T_s} i_1^2(t)dt} = \frac{n_2}{n_1} I \sqrt{D}$$

$$I_2 = I_3 = \sqrt{\frac{1}{2T_s}\int_0^{2T_s} i_2^2(t)dt} = \frac{1}{2} I \sqrt{1 + D}$$

see Appendix A



Allocation of window area: $\alpha_m = \dfrac{V_m I_m}{\sum_{n=1}^{\infty} V_j I_j}$

Plug in rms current expressions. Result:

$$\alpha_1 = \frac{1}{\left(1 + \sqrt{\dfrac{1+D}{D}}\right)}$$

Fraction of window area allocated to primary winding

$$\alpha_2 = \alpha_3 = \frac{1}{2}\,\frac{1}{\left(1 + \sqrt{\dfrac{D}{1+D}}\right)}$$

Fraction of window area allocated to each secondary winding

Numerical example

Suppose that we decide to optimize the transformer design at the worst-case operating point $D = 0.75$. Then we obtain

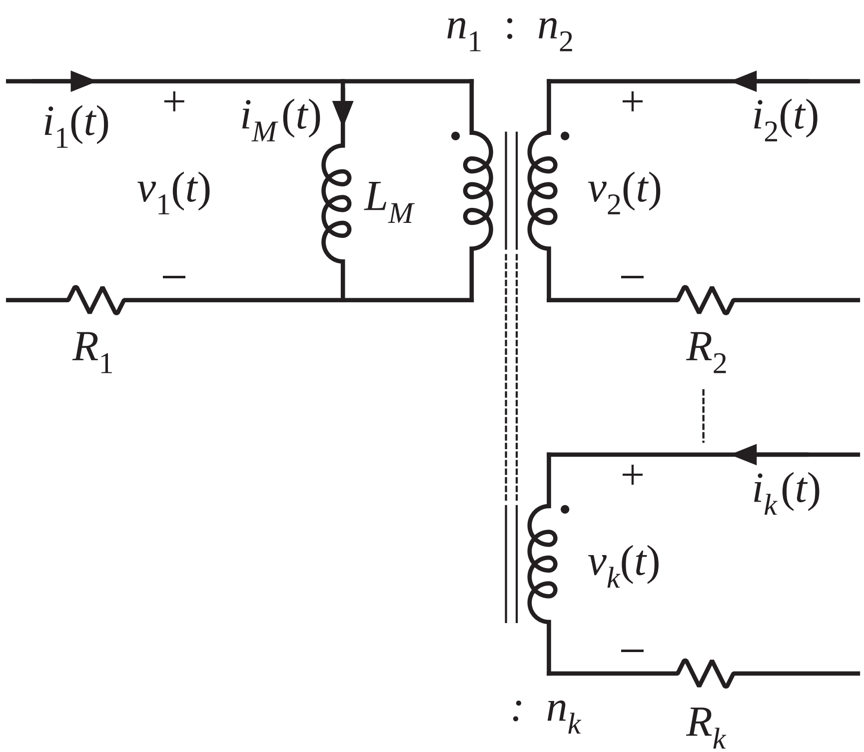
$$\alpha_1 = 0.396$$
$$\alpha_2 = 0.302$$
$$\alpha_3 = 0.302$$

The total copper loss is then given by

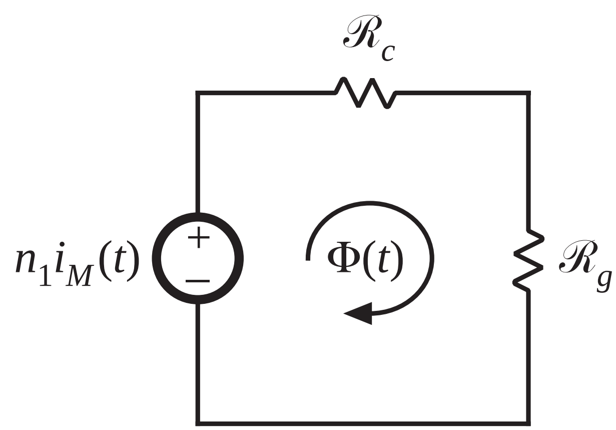
$$P_{cu,tot} = \frac{\rho(MLT)}{W_A K_u} \left(\sum_{j=1}^{3} n_j I_j \right)^2$$
$$= \frac{\rho(MLT) n_2^2 I^2}{W_A K_u} \left(1 + 2D + 2\sqrt{D(1+D)} \right)$$

14.3.2 Coupled inductor design constraints

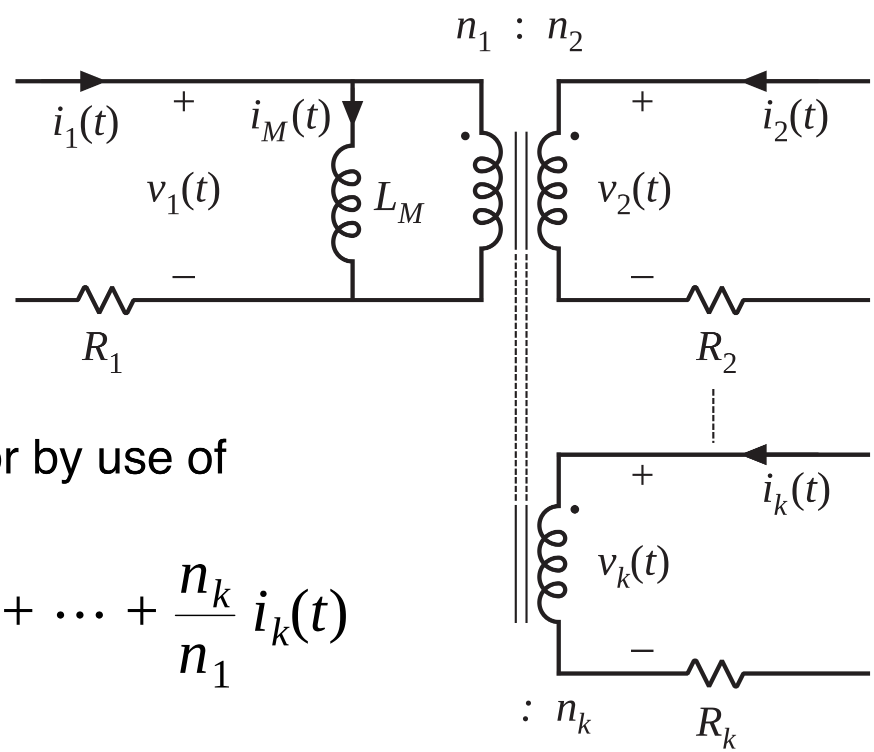
Consider now the design of a coupled inductor having k windings. We want to obtain a specified value of magnetizing inductance, with specified turns ratios and total copper loss.



Magnetic circuit model:



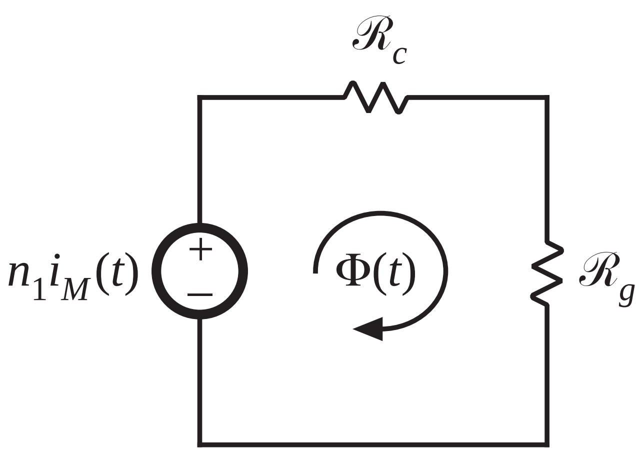
Relationship between magnetizing current and winding currents



Solution of circuit model, or by use of Ampere's Law:

$$i_M(t) = i_1(t) + \frac{n_2}{n_1}\, i_2(t) + \cdots + \frac{n_k}{n_1}\, i_k(t)$$

Solution of magnetic circuit model:
Obtain desired maximum flux density



Assume that gap reluctance is much larger than core reluctance:

$$n_1 i_M(t) = B(t) A_c \mathcal{R}_g$$

Design so that the maximum flux density B_{max} is equal to a specified value (that is less than the saturation flux density B_{sat}). B_{max} is related to the maximum magnetizing current according to

$$n_1 I_{M,max} = B_{max} A_c \mathcal{R}_g = B_{max} \frac{\ell_g}{\mu_0}$$

Obtain specified magnetizing inductance

By the usual methods, we can solve for the value of the magnetizing inductance L_M (referred to the primary winding):

$$L_M = \frac{n_1^2}{\mathcal{R}_g} = n_1^2 \frac{\mu_0 A_c}{\ell_g}$$

Copper loss

Allocate window area as described in Section 14.3.1. As shown in that section, the total copper loss is then given by

$$P_{cu} = \frac{\rho (MLT) n_1^2 I_{tot}^2}{W_A K_u}$$

with

$$I_{tot} = \sum_{j=1}^{k} \frac{n_j}{n_1} I_j$$

Eliminate unknowns and solve for K_g

Eliminate the unknowns ℓ_g and n_1:

$$P_{cu} = \frac{\rho (MLT) L_M^2 I_{tot}^2 I_{M,max}^2}{B_{max}^2 A_c^2 W_A K_u}$$

Rearrange equation so that terms that involve core geometry are on RHS while specifications are on LHS:

$$\frac{A_c^2 W_A}{(MLT)} = \frac{\rho L_M^2 I_{tot}^2 I_{M,max}^2}{B_{max}^2 K_u P_{cu}}$$

The left-hand side is the same K_g as in single-winding inductor design. Must select a core that satisfies

$$K_g \geq \frac{\rho L_M^2 I_{tot}^2 I_{M,max}^2}{B_{max}^2 K_u P_{cu}}$$

14.3.3 Step-by-step design procedure:
Coupled inductor

The following quantities are specified, using the units noted:

Wire resistivity	ρ	(Ω-cm)
Total rms winding currents	$I_{tot} = \sum_{j=1}^{k} \frac{n_j}{n_1} I_j$	(A) (referred to winding 1)
Peak magnetizing current	$I_{M,\,max}$	(A) (referred to winding 1)
Desired turns ratios	n_2/n_1. n_3/n_2. etc.	
Magnetizing inductance	L_M	(H) (referred to winding 1)
Allowed copper loss	P_{cu}	(W)
Winding fill factor	K_u	
Core maximum flux density	B_{max}	(T)

The core dimensions are expressed in cm:

Core cross-sectional area	A_c	(cm^2)
Core window area	W_A	(cm^2)
Mean length per turn	MLT	(cm)

The use of centimeters rather than meters requires that appropriate factors be added to the design equations.

1. Determine core size

$$K_g \geq \frac{\rho L_M^2 I_{tot}^2 I_{M,max}^2}{B_{max}^2 P_{cu} K_u} 10^8 \qquad (\mathrm{cm}^5)$$

Choose a core that satisfies this inequality. Note the values of A_c, W_A, and MLT for this core.

The resistivity ρ of copper wire is $1.724 \cdot 10^{-6}\ \Omega\ \mathrm{cm}$ at room temperature, and $2.3 \cdot 10^{-6}\ \Omega\ \mathrm{cm}$ at 100˚C.

2. Determine air gap length

$$\ell_g = \frac{\mu_0 L_M I_{M,max}^2}{B_{max}^2 A_c} 10^4 \qquad \text{(m)}$$

(value neglects fringing flux, and a longer gap may be required)

The permeability of free space is $\mu_0 = 4\pi \cdot 10^{-7}$ H/m

3. Determine number of turns

For winding 1:

$$n_1 = \frac{L_M I_{M,max}}{B_{max} A_c} 10^4$$

For other windings, use the desired turns ratios:

$$n_2 = \left(\frac{n_2}{n_1}\right) n_1$$

$$n_3 = \left(\frac{n_3}{n_1}\right) n_1$$

$$\vdots$$

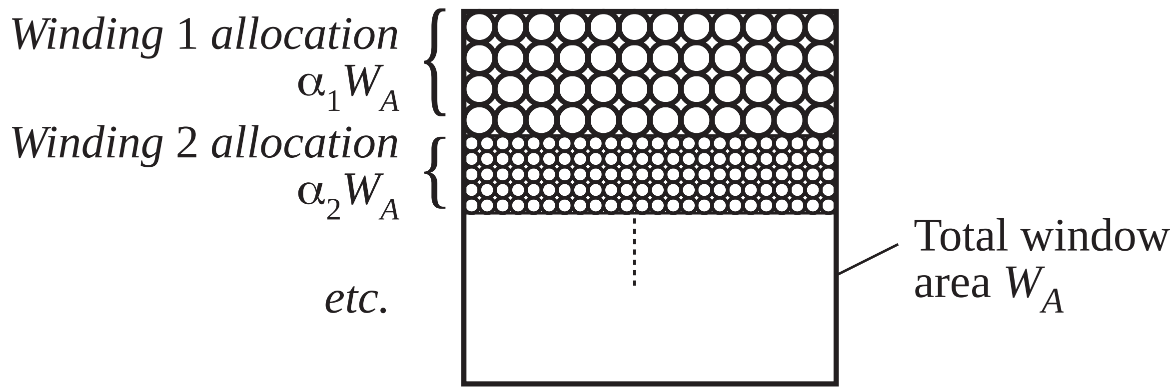
4. Evaluate fraction of window area allocated to each winding

$$\alpha_1 = \frac{n_1 I_1}{n_1 I_{tot}}$$

$$\alpha_2 = \frac{n_2 I_2}{n_1 I_{tot}}$$

$$\vdots$$

$$\alpha_k = \frac{n_k I_k}{n_1 I_{tot}}$$



$$0 < \alpha_j < 1$$

$$\alpha_1 + \alpha_2 + \cdots + \alpha_k = 1$$

5. Evaluate wire sizes

$$A_{w1} \le \frac{\alpha_1 K_u W_A}{n_1}$$

$$A_{w2} \le \frac{\alpha_2 K_u W_A}{n_2}$$

$$\vdots$$

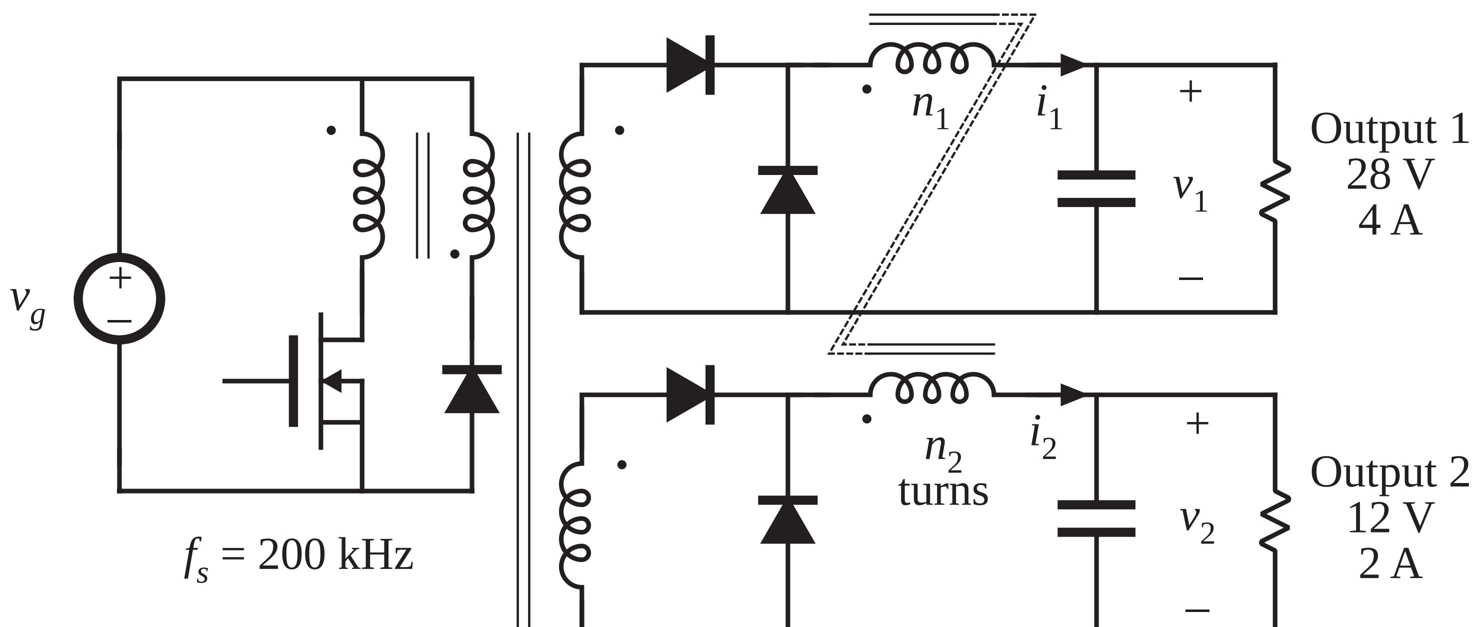
See American Wire Gauge (AWG) table at end of Appendix D.

14.4 Examples

14.4.1 Coupled Inductor for a Two-Output Forward Converter

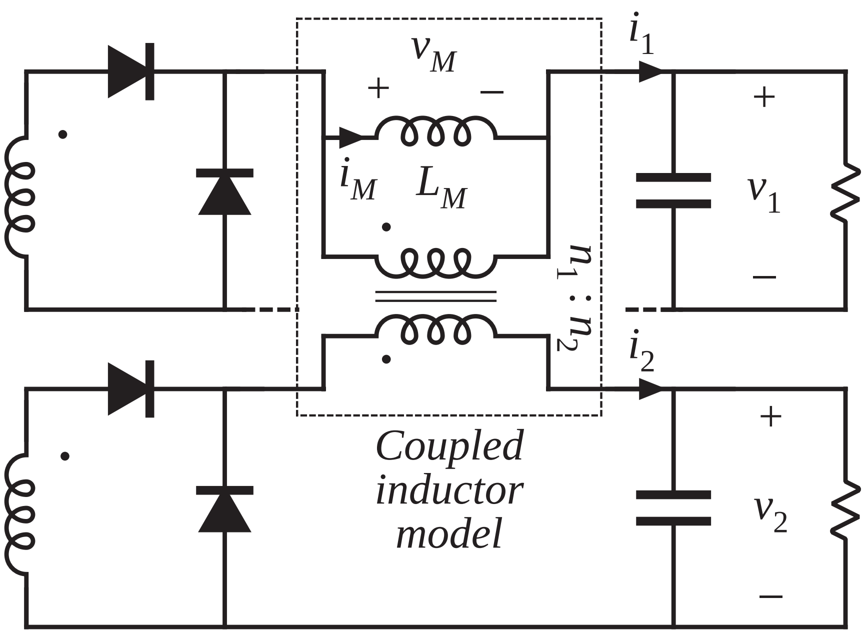
14.4.2 CCM Flyback Transformer

14.4.1 Coupled Inductor for a Two-Output Forward Converter

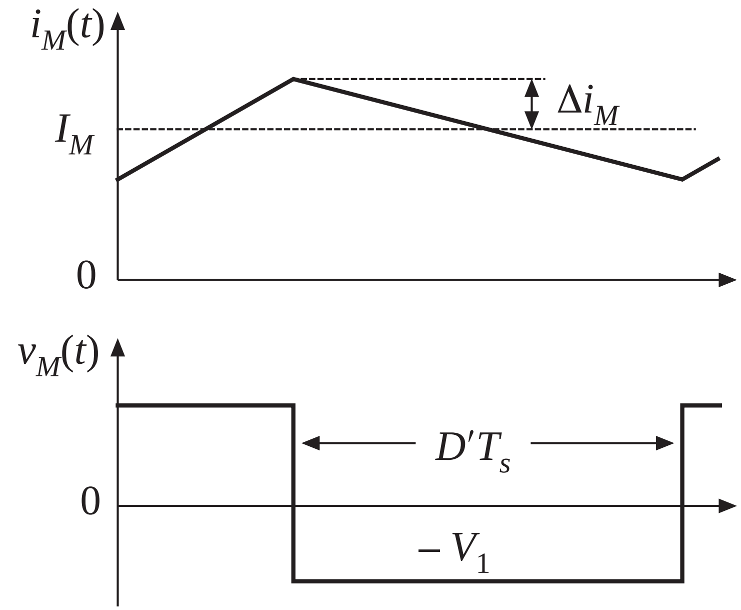


The two filter inductors can share the same core because their applied voltage waveforms are proportional. Select turns ratio n_2/n_1 approximately equal to $v_2/v_1 = 12/28$.

Coupled inductor model and waveforms



Secondary-side circuit, with coupled inductor model



Magnetizing current and voltage waveforms. $i_M(t)$ is the sum of the winding currents $i_1(t) + i_2(t)$.

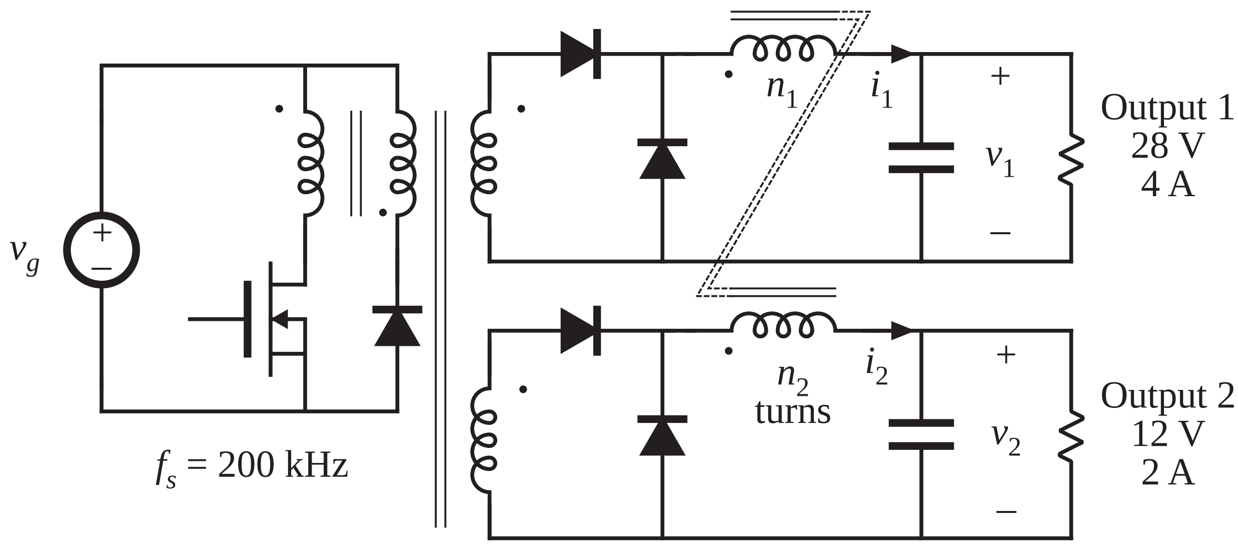
Nominal full-load operating point

Design for CCM operation with

$D = 0.35$

$\Delta i_M = 20\%$ of I_M

$f_s = 200$ kHz



DC component of magnetizing current is

$$I_M = I_1 + \frac{n_2}{n_1} I_2$$

$$= (4\ \text{A}) + \frac{12}{28}(2\ \text{A})$$

$$= 4.86\ \text{A}$$

Magnetizing current ripple

$$\Delta i_M = \frac{V_1 D' T_s}{2 L_M}$$

To obtain

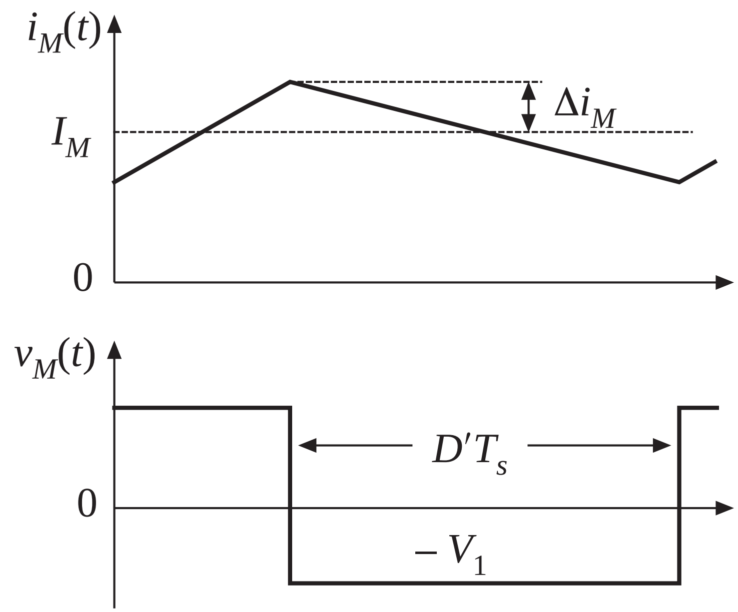
$\Delta i_M = 20\%$ of I_M

choose

$$L_M = \frac{V_1 D' T_s}{2 \Delta i_M}$$

$$= \frac{(28\ \text{V})(1 - 0.35)(5\ \mu\text{s})}{2(4.86\ \text{A})(20\%)}$$

$$= 47\ \mu\text{H}$$



This leads to a peak magnetizing current (referred to winding 1) of

$$I_{M,max} = I_M + \Delta i_M = 5.83\ \text{A}$$

RMS winding currents

Since the winding current ripples are small, the rms values of the winding currents are nearly equal to their dc comonents:

$I_1 = 4\ \text{A} \qquad\qquad I_2 = 2\ \text{A}$

Hence the sum of the rms winding currents, referred to the primary, is

$$I_{tot} = I_1 + \frac{n_2}{n_1} I_2 = 4.86\ \text{A}$$

Evaluate K_g

The following engineering choices are made:

- Allow 0.75 W of total copper loss (a small core having thermal resistance of less than 40 ˚C/W then would have a temperature rise of less than 30 ˚C)
- Operate the core at $B_{max} = 0.25\ \mathrm{T}$ (which is less than the ferrite saturation flux density of 0.3 ot 0.5 T)
- Use fill factor $K_u = 0.4$ (a reasonable estimate for a low-voltage inductor with multiple windings)

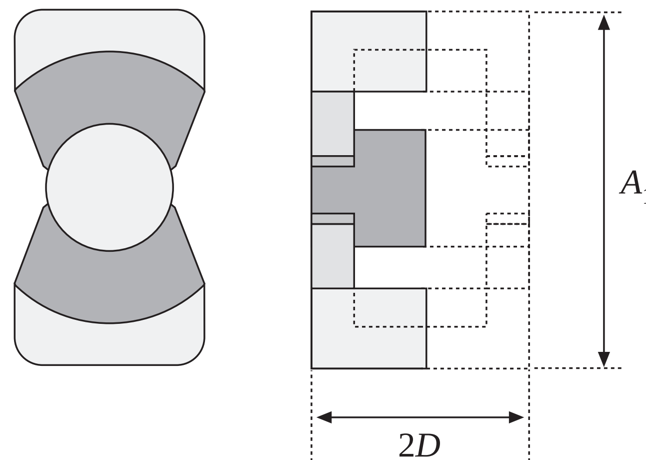
Evaluate K_g:

$$K_g \geq \frac{(1.724 \cdot 10^{-6}\ \Omega - \mathrm{cm})(47\ \mu\mathrm{H})^2 (4.86\ \mathrm{A})^2 (5.83\ \mathrm{A})^2}{(0.25\ \mathrm{T})^2 (0.75\ \mathrm{W})(0.4)} 10^8$$

$$= 16 \cdot 10^{-3}\ \mathrm{cm}^5$$

Select core

It is decided to use a ferrite PQ core. From Appendix D, the smallest PQ core having $K_g \geq 16 \cdot 10^{-3}$ cm^5 is the PQ 20/16, with K_g = $22.4 \cdot 10^{-3}$ cm^5 . The data for this core are:



$$A_c = 0.62 \text{ cm}^2$$

$$W_A = 0.256 \text{ cm}^2$$

$$MLT = 4.4 \text{ cm}$$

Air gap length

$$\ell_g = \frac{\mu_0 L_M I_{M,max}^2}{B_{max}^2 A_c} 10^4$$

$$= \frac{(4\pi \cdot 10^{-7} \mathrm{H/m})(47\ \mu\mathrm{H})(5.83\ \mathrm{A})^2}{(0.25\ \mathrm{T})^2 (0.62\ \mathrm{cm}^2)} 10^4$$

$$= 0.52\ \mathrm{mm}$$

Turns

$$n_1 = \frac{L_M I_{M,max}}{B_{max} A_c} 10^4$$
$$= \frac{(47\ \mu\text{H})(5.83\ \text{A})}{(0.25\ \text{T})(0.62\ \text{cm}^2)} 10^4$$
$$= 17.6\ \text{turns}$$

$$n_2 = \left(\frac{n_2}{n_1}\right) n_1$$
$$= \left(\frac{12}{28}\right)(17.6)$$
$$= 7.54\ \text{turns}$$

Let's round off to

$n_1 = 17$ $\qquad$ $n_2 = 7$

Wire sizes

Allocation of window area:

$$\alpha_1 = \frac{n_1 I_1}{n_1 I_{tot}} = \frac{(17)(4\text{ A})}{(17)(4.86\text{ A})} = 0.8235$$

$$\alpha_2 = \frac{n_2 I_2}{n_1 I_{tot}} = \frac{(7)(2\text{ A})}{(17)(4.86\text{ A})} = 0.1695$$

Determination of wire areas and AWG (from table at end of Appendix D):

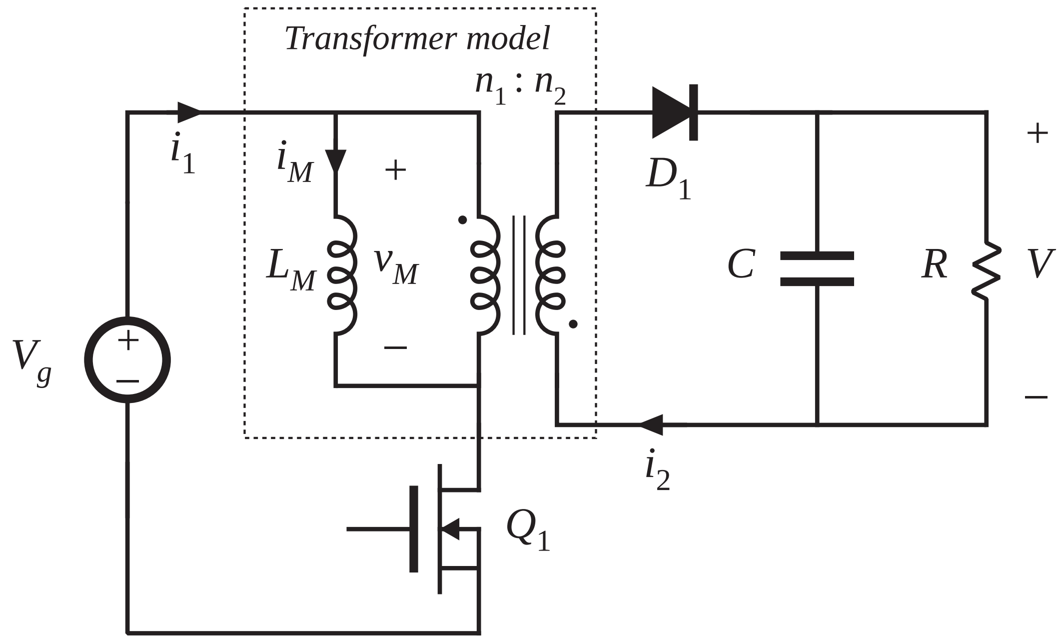
$$A_{w1} \le \frac{\alpha_1 K_u W_A}{n_1} = \frac{(0.8235)(0.4)(0.256\,\text{cm}^2)}{(17)} = 4.96 \cdot 10^{-3}\text{ cm}^2$$

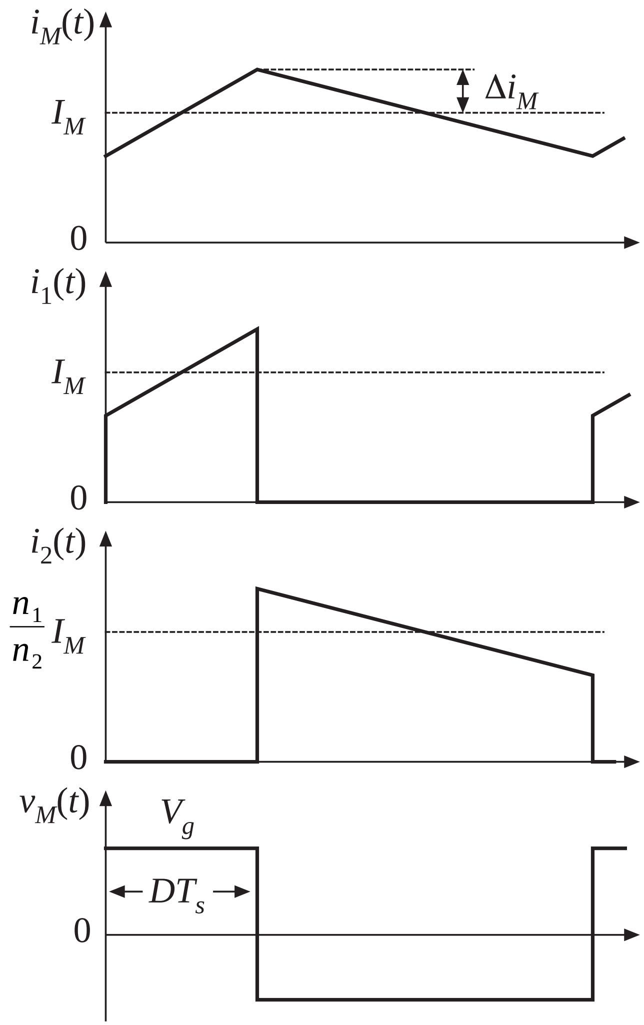
use AWG #21

$$A_{w2} \le \frac{\alpha_2 K_u W_A}{n_2} = \frac{(0.1695)(0.4)(0.256\,\text{cm}^2)}{(7)} = 2.48 \cdot 10^{-3}\text{ cm}^2$$

use AWG #24

14.4.2 Example 2: CCM flyback transformer





Specifications

Input voltage	$V_g = 200\text{V}$
Output (full load)	20 V at 5 A
Switching frequency	150 kHz
Magnetizing current ripple	20% of dc magnetizing current
Duty cycle	$D = 0.4$
Turns ratio	$n_2/n_1 = 0.15$
Copper loss	1.5 W
Fill factor	$K_u = 0.3$
Maximum flux density	$B_{max} = 0.25$ T

Basic converter calculations

Components of magnetizing current, referred to primary:

$$I_M = \left(\frac{n_2}{n_1}\right) \frac{1}{D'} \frac{V}{R} = 1.25 \text{ A}$$

$$\Delta i_M = (20\%) I_M = 0.25 \text{ A}$$

$$I_{M,max} = I_M + \Delta i_M = 1.5 \text{ A}$$

Choose magnetizing inductance:

$$L_M = \frac{V_g D T_s}{2 \Delta i_M}$$
$$= 1.07 \text{ mH}$$

RMS winding currents:

$$I_1 = I_M \sqrt{D} \sqrt{1 + \frac{1}{3}\left(\frac{\Delta i_M}{I_M}\right)^2} = 0.796 \text{ A}$$

$$I_2 = \frac{n_1}{n_2} I_M \sqrt{D'} \sqrt{1 + \frac{1}{3}\left(\frac{\Delta i_M}{I_M}\right)^2} = 6.50 \text{ A}$$

$$I_{tot} = I_1 + \frac{n_2}{n_1} I_2 = 1.77 \text{ A}$$

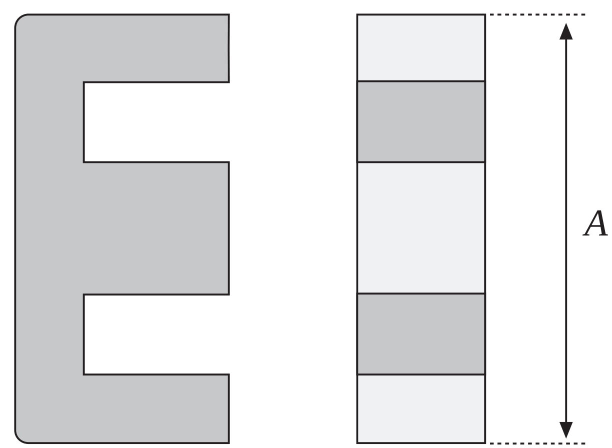
Choose core size

$$K_g \geq \frac{\rho L_M^2 I_{tot}^2 I_{M,max}^2}{B_{max}^2 P_{cu} K_u} 10^8$$

$$= \frac{\left(1.724 \cdot 10^{-6}\Omega\text{-cm}\right)\left(1.07 \cdot 10^{-3}\ \text{H}\right)^2\left(1.77\ \text{A}\right)^2\left(1.5\ \text{A}\right)^2}{\left(0.25\ \text{T}\right)^2\left(1.5\ \text{W}\right)\left(0.3\right)} 10^8$$

$$= 0.049\ \text{cm}^5$$

The smallest EE core that satisfies this inequality (Appendix D) is the EE30.



Choose air gap and turns

$$\ell_g = \frac{\mu_0 L_M I_{M,max}^2}{B_{max}^2 A_c} 10^4$$

$$= \frac{\left(4\pi \cdot 10^{-7}\text{H/m}\right)\left(1.07 \cdot 10^{-3}\ \text{H}\right)\left(1.5\ \text{A}\right)^2}{\left(0.25\ \text{T}\right)^2\left(1.09\ \text{cm}^2\right)} 10^4$$

$$= 0.44\ \text{mm}$$

$$n_1 = \frac{L_M I_{M,max}}{B_{max} A_c} 10^4$$

$$= \frac{\left(1.07 \cdot 10^{-3}\ \text{H}\right)\left(1.5\ \text{A}\right)}{\left(0.25\ \text{T}\right)\left(1.09\ \text{cm}^2\right)} 10^4$$

$$= 58.7\ \text{turns}$$

$$n_2 = \left(\frac{n_2}{n_1}\right) n_1$$

$$= \left(0.15\right) 59$$

$$= 8.81$$

Round to $n_1 = 59$

$n_2 = 9$

Wire gauges

$$\alpha_1 = \frac{I_1}{I_{tot}} = \frac{(0.796\ \text{A})}{(1.77\ \text{A})} = 0.45$$

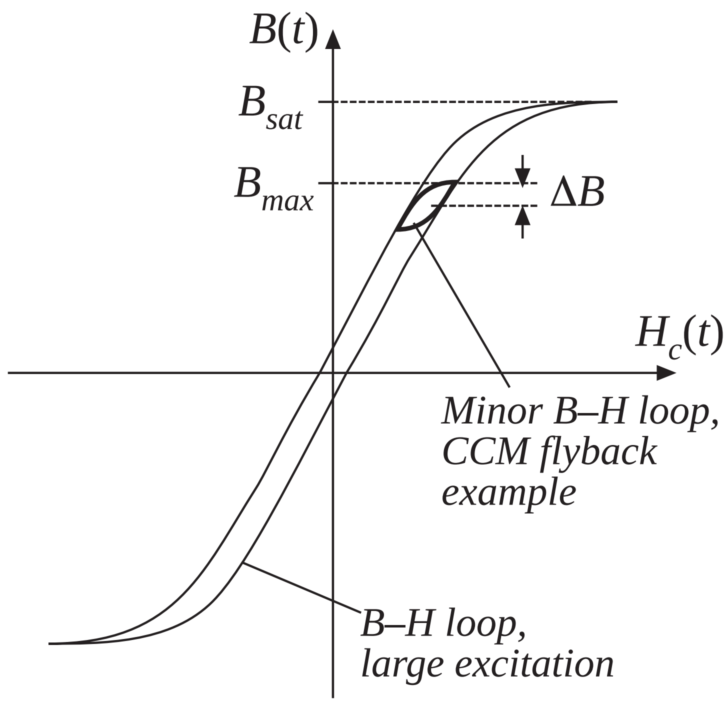
$$\alpha_2 = \frac{n_2 I_2}{n_1 I_{tot}} = \frac{(9)(6.5\ \text{A})}{(59)(1.77\ \text{A})} = 0.55$$

$$A_{W1} \le \frac{\alpha_1 K_u W_A}{n_1} = 1.09 \cdot 10^{-3}\ \text{cm}^2 \qquad \text{— use \#28 AWG}$$

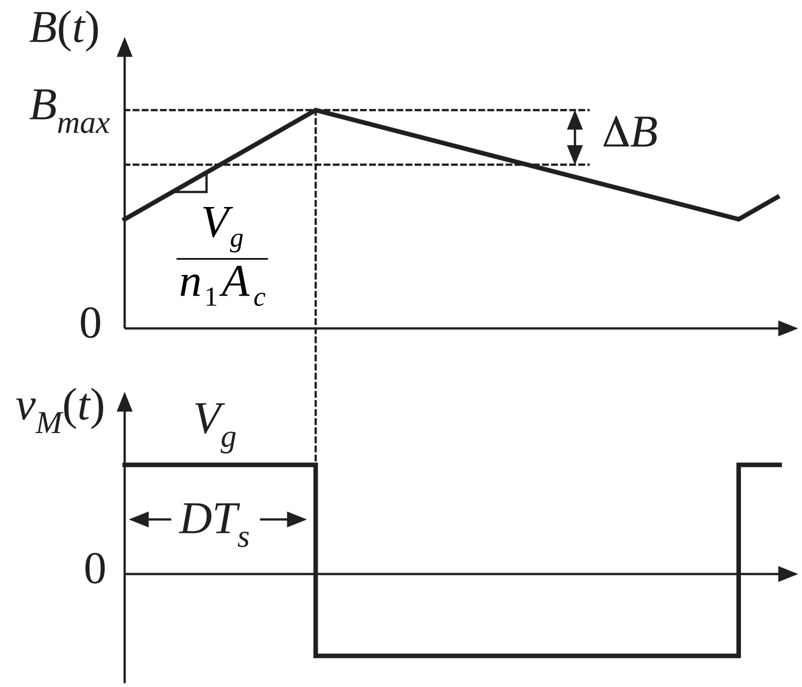
$$A_{W2} \le \frac{\alpha_2 K_u W_A}{n_2} = 8.88 \cdot 10^{-3}\ \text{cm}^2 \qquad \text{— use \#19 AWG}$$

Core loss
CCM flyback example

B-H loop for this application:



The relevant waveforms:



$B(t)$ vs. applied voltage, from Faraday's law:

$$\frac{dB(t)}{dt} = \frac{v_M(t)}{n_1 A_c}$$

For the first subinterval:

$$\frac{dB(t)}{dt} = \frac{V_g}{n_1 A_c}$$

Calculation of ac flux density and core loss

Solve for ΔB:

$$\Delta B = \left(\frac{V_g}{n_1 A_c} \right) \left(D T_s \right)$$

Plug in values for flyback example:

$$\Delta B = \frac{\left(200\ \text{V}\right)\left(0.4\right)\left(6.67\ \mu\text{s}\right)}{2\left(59\right)\left(1.09\ \text{cm}^2\right)}\ 10^4$$

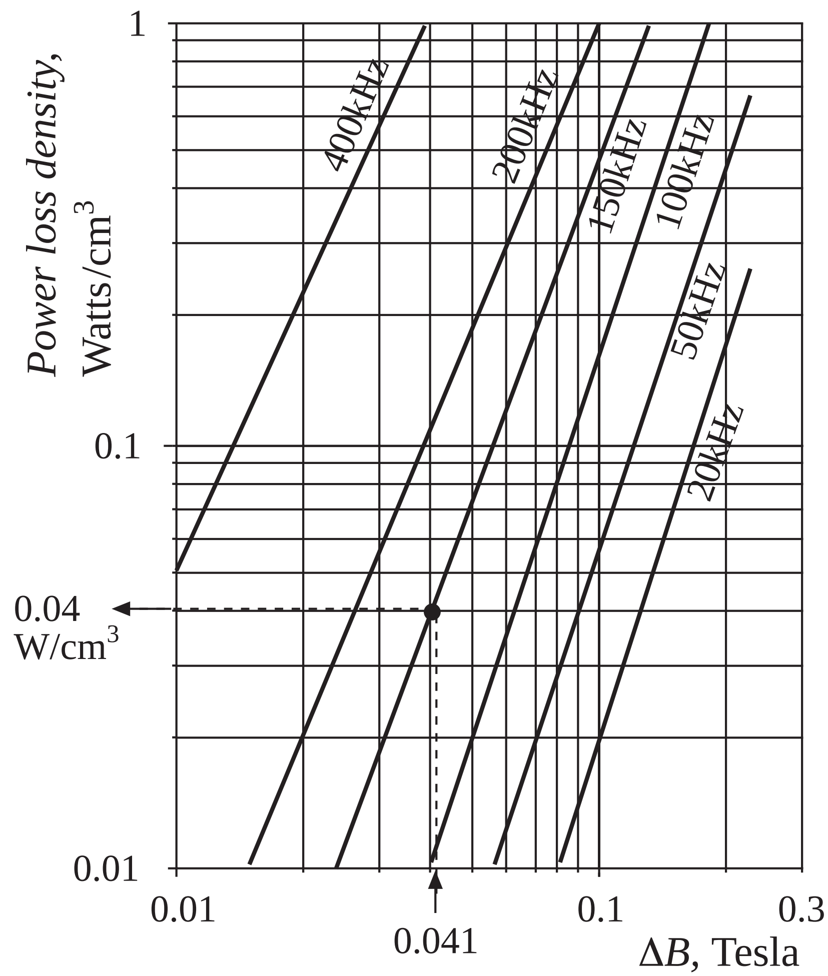
$$= 0.041\ \text{T}$$

From manufacturer's plot of core loss (at left), the power loss density is 0.04 W/cm³. Hence core loss is

$$P_{fe} = \left(0.04\ \text{W/cm}^3\right)\left(A_c \ell_m\right)$$

$$= \left(0.04\ \text{W/cm}^3\right)\left(1.09\ \text{cm}^2\right)\left(5.77\ \text{cm}\right)$$

$$= 0.25\ \text{W}$$



Comparison of core and copper loss

- Copper loss is 1.5 W
  - does not include proximity losses, which could substantially increase total copper loss
- Core loss is 0.25 W
  - Core loss is small because ripple and ΔB are small
  - It is not a bad approximation to ignore core losses for ferrite in CCM filter inductors
  - Could consider use of a less expensive core material having higher core loss
  - Neglecting core loss is a reasonable approximation for this application
- Design is dominated by copper loss
  - The dominant constraint on flux density is saturation of the core, rather than core loss

14.5 Summary of key points

1. A variety of magnetic devices are commonly used in switching converters. These devices differ in their core flux density variations, as well as in the magnitudes of the ac winding currents. When the flux density variations are small, core loss can be neglected. Alternatively, a low-frequency material can be used, having higher saturation flux density.

2. The core geometrical constant K_g is a measure of the magnetic size of a core, for applications in which copper loss is dominant. In the K_g design method, flux density and total copper loss are specified.