Chapter 18

Low Harmonic Rectifier Modeling and Control

18.1 Modeling losses and efficiency in CCM high-quality rectifiers

Expression for controller duty cycle $d(t)$

Expression for the dc load current

Solution for converter efficiency η

Design example

18.2 Controller schemes

Average current control

Feedforward

Current programmed control

Hysteretic control

Nonlinear carrier control

18.3 Control system modeling

Modeling the outer low-bandwidth control system

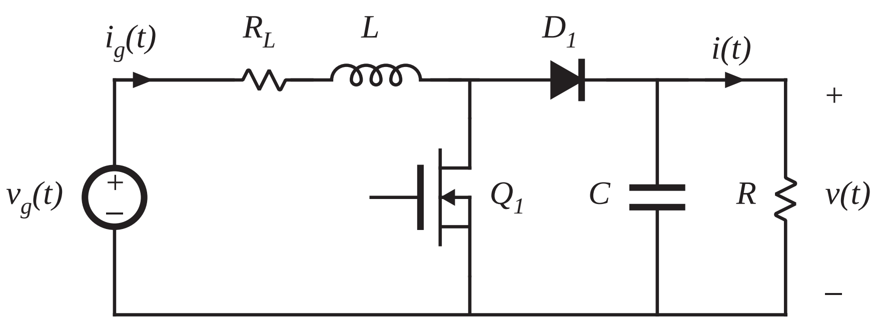
Modeling the inner wide-bandwidth average current controller

18.1 Modeling losses and efficiency in CCM high-quality rectifiers

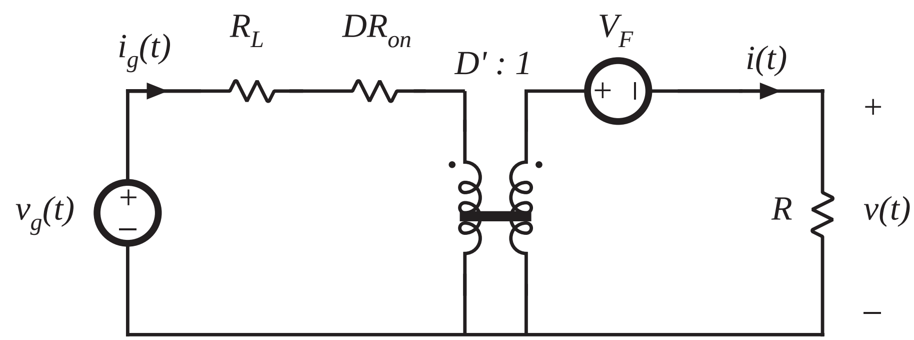
Objective: extend procedure of Chapter 3, to predict the output voltage, duty cycle variations, and efficiency, of PWM CCM low harmonic rectifiers.

Approach: Use the models developed in Chapter 3. Integrate over one ac line cycle to determine steady-state waveforms and average power.

Boost example



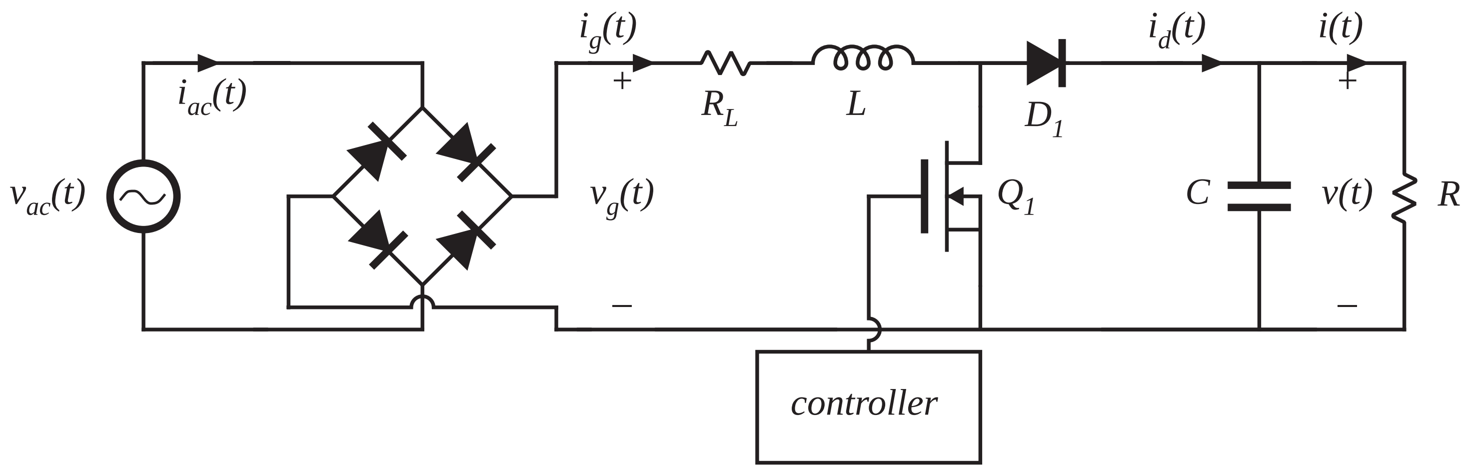
Dc-dc boost converter circuit



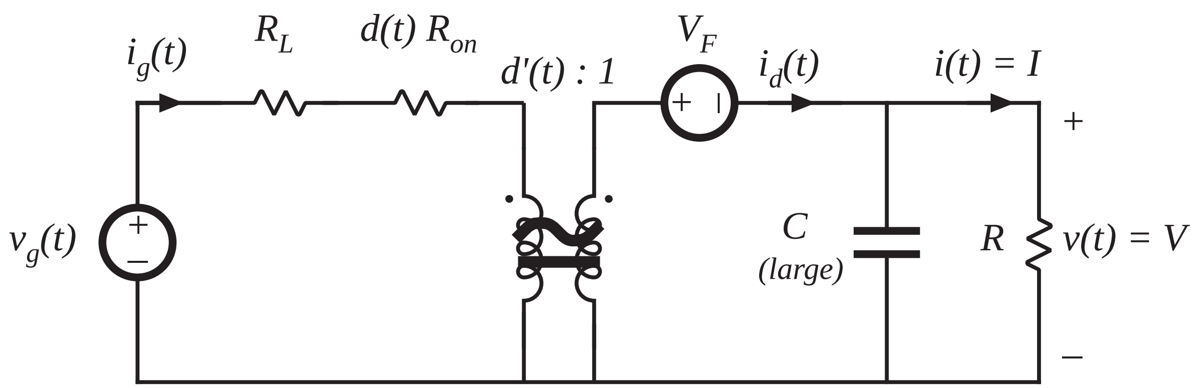
Averaged dc model

Modeling the ac-dc boost rectifier

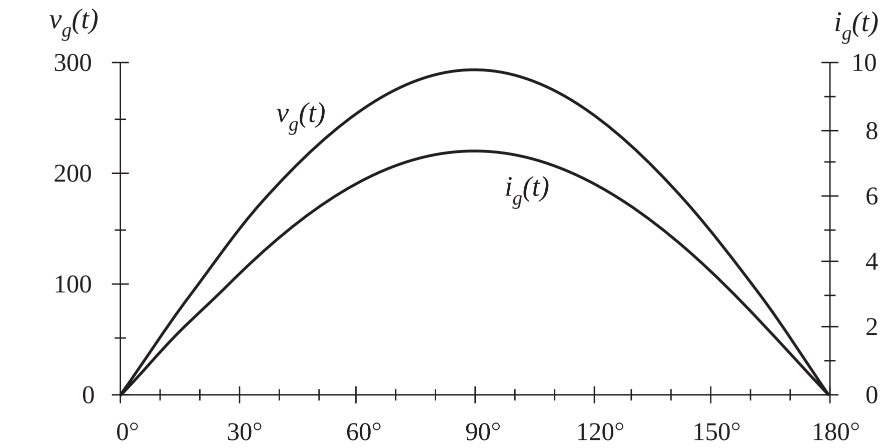
Boost rectifier circuit



Averaged model



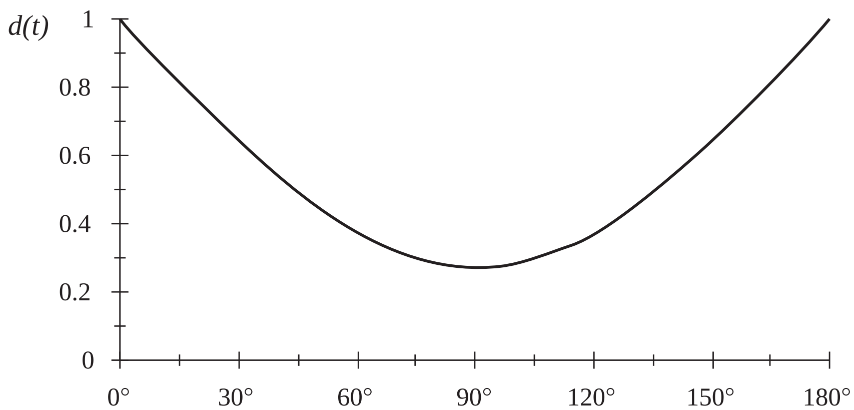
Boost rectifier waveforms

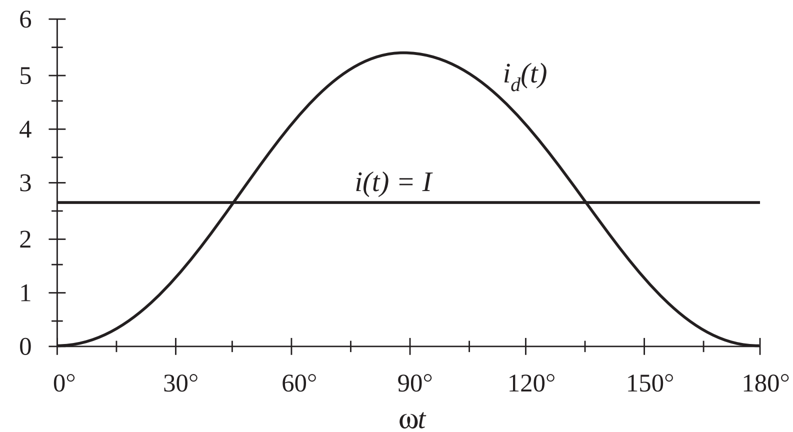


Typical waveforms

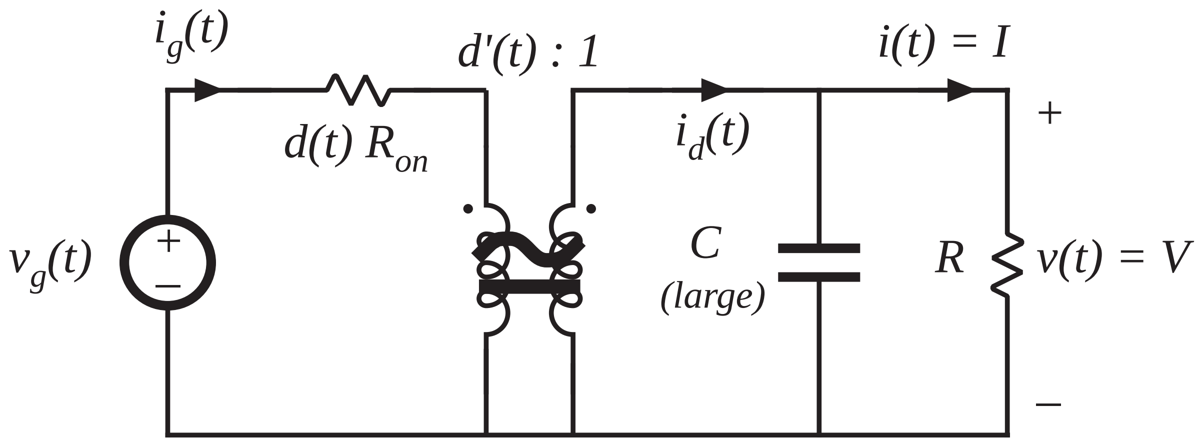
(low frequency components)

$$i_g(t) = \frac{v_g(t)}{R_e}$$





Example: boost rectifier with MOSFET on-resistance



Averaged model

Inductor dynamics are neglected, a good approximation when the ac line variations are slow compared to the converter natural frequencies

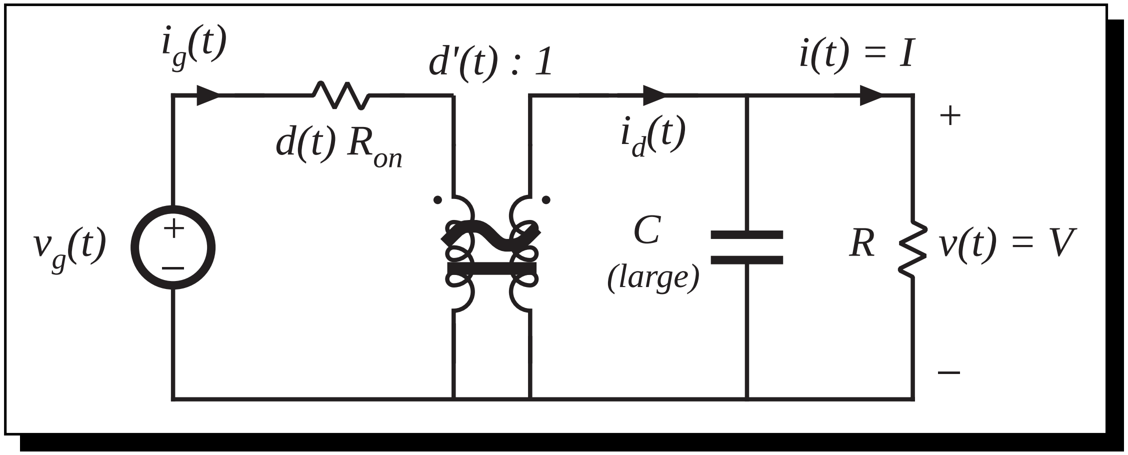
18.1.1 Expression for controller duty cycle *d(t)*

Solve input side of model:

$$i_g(t)d(t)R_{on} = v_g(t) - d'(t)v$$

with $i_g(t) = \dfrac{v_g(t)}{R_e}$

$$v_g(t) = V_M \left| \sin \omega t \right|$$



eliminate $i_g(t)$:

$$\frac{v_g(t)}{R_e} d(t)R_{on} = v_g(t) - d'(t)v$$

solve for $d(t)$:

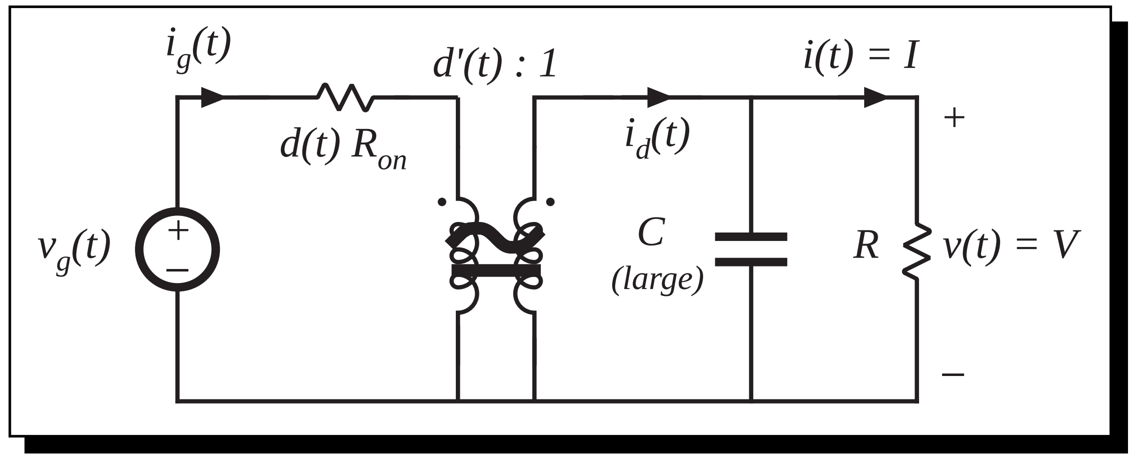
$$d(t) = \frac{v - v_g(t)}{v - v_g(t)\dfrac{R_{on}}{R_e}}$$

Again, these expressions neglect converter dynamics, and assume that the converter always operates in CCM.

18.1.2 Expression for the dc load current

Solve output side of model, using charge balance on capacitor C:

$$I = \left\langle i_d \right\rangle_{T_{ac}}$$



$$i_d(t) = d'(t)\,i_g(t) = d'(t)\,\frac{v_g(t)}{R_e}$$

But $d'(t)$ is:

$$d'(t) = \frac{v_g(t)\left(1 - \frac{R_{on}}{R_e}\right)}{v - v_g(t)\,\frac{R_{on}}{R_e}}$$

hence $i_d(t)$ can be expressed as

$$i_d(t) = \frac{v_g^2(t)}{R_e}\,\frac{\left(1 - \frac{R_{on}}{R_e}\right)}{v - v_g(t)\,\frac{R_{on}}{R_e}}$$

Next, average $i_d(t)$ over an ac line period, to find the dc load current I.

Dc load current I

Now substitute $v_g(t) = V_M \sin \omega t$, and integrate to find $\langle i_d(t) \rangle_{T_{ac}}$:

$$I = \left\langle i_d \right\rangle_{T_{ac}} = \frac{2}{T_{ac}} \int_0^{T_{ac}/2} \left(\frac{V_M^2}{R_e} \right) \frac{\left(1 - \frac{R_{on}}{R_e} \right) \sin^2 \left(\omega t \right)}{\left(v - \frac{V_M R_{on}}{R_e} \sin \left(\omega t \right) \right)} dt$$

This can be written in the normalized form

$$I = \frac{2}{T_{ac}} \frac{V_M^2}{V R_e} \left(1 - \frac{R_{on}}{R_e} \right) \int_0^{T_{ac}/2} \frac{\sin^2 \left(\omega t \right)}{1 - a \sin \left(\omega t \right)} dt$$

with $$a = \left(\frac{V_M}{V} \right) \left(\frac{R_{on}}{R_e} \right)$$

Integration

By waveform symmetry, we need only integrate from 0 to $T_{ac}/4$. Also, make the substitution $\theta = \omega t$:

$$I = \frac{V_M^2}{VR_e}\left(1 - \frac{R_{on}}{R_e}\right)\frac{2}{\pi}\int_0^{\pi/2} \frac{\sin^2(\theta)}{1 - a\sin(\theta)}\, d\theta$$

This integral is obtained not only in the boost rectifier, but also in the buck-boost and other rectifier topologies. The solution is

$$\frac{4}{\pi}\int_0^{\pi/2} \frac{\sin^2(\theta)}{1 - a\sin(\theta)}\, d\theta = F(a) = \frac{2}{a^2\pi}\left(-2a - \pi + \frac{4\sin^{-1}(a) + 2\cos^{-1}(a)}{\sqrt{1-a^2}}\right)$$

- Result is in closed form
- a is a measure of the loss resistance relative to R_e
- a is typically much smaller than unity

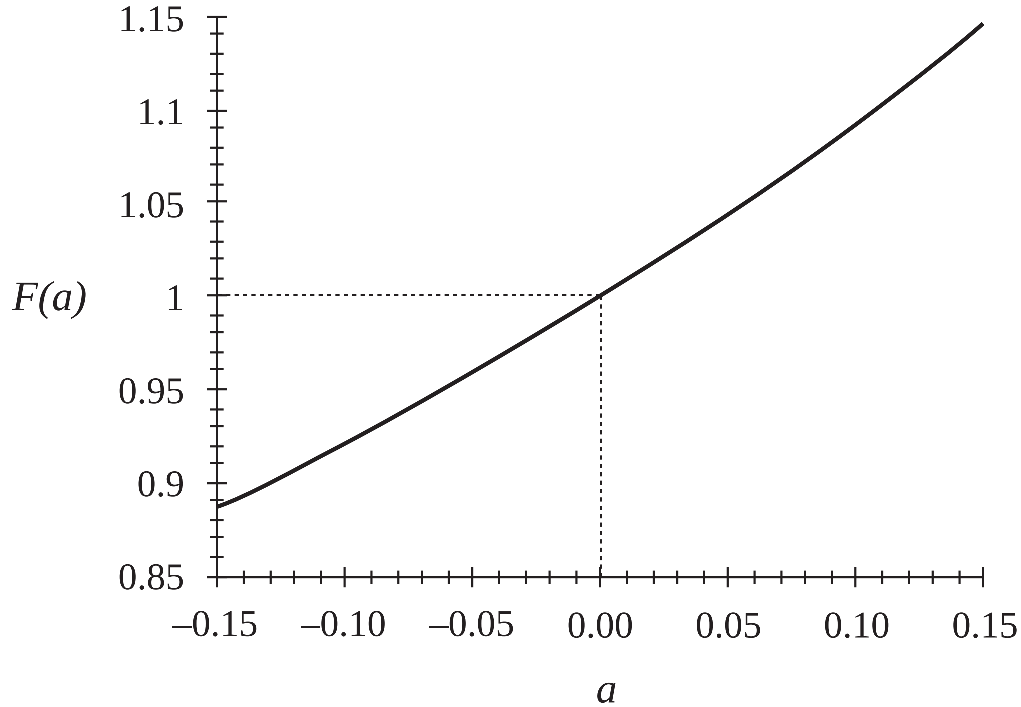
The integral $F(a)$

$$\frac{4}{\pi}\int_0^{\pi/2} \frac{\sin^2(\theta)}{1 - a\sin(\theta)}\, d\theta = F(a) = \frac{2}{a^2\pi}\left(-2a - \pi + \frac{4\sin^{-1}(a) + 2\cos^{-1}(a)}{\sqrt{1-a^2}} \right)$$

Approximation via polynomial:

$$F(a) \approx 1 + 0.862a + 0.78a^2$$

For $|\,a\,| \le 0.15$, this approximate expression is within 0.1% of the exact value. If the a^2 term is omitted, then the accuracy drops to ±2% for $|\,a\,| \le 0.15$. The accuracy of $F(a)$ coincides with the accuracy of the rectifier efficiency η.



18.1.4 Solution for converter efficiency η

Converter average input power is

$$P_{in} = \left\langle p_{in}(t) \right\rangle_{T_{ac}} = \frac{V_M^2}{2R_e}$$

Average load power is

$$P_{out} = VI = \left(V \right) \left(\frac{V_M^2}{VR_e} \left(1 - \frac{R_{on}}{R_e} \right) \frac{F(a)}{2} \right) \qquad \text{with} \qquad a = \left(\frac{V_M}{V} \right) \left(\frac{R_{on}}{R_e} \right)$$

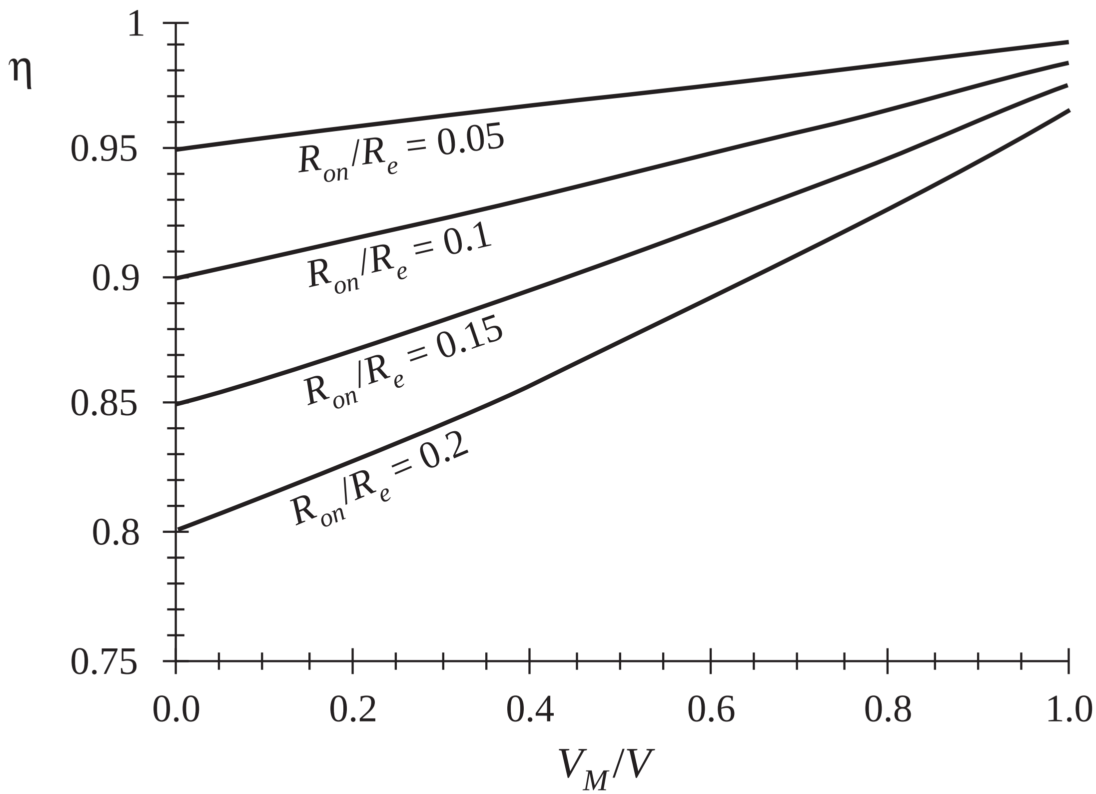
So the efficiency is

$$\eta = \frac{P_{out}}{P_{in}} = \left(1 - \frac{R_{on}}{R_e} \right) F(a)$$

Polynomial approximation:

$$\eta \approx \left(1 - \frac{R_{on}}{R_e} \right) \left(1 + 0.862 \, \frac{V_M}{V} \, \frac{R_{on}}{R_e} + 0.78 \left(\frac{V_M}{V} \, \frac{R_{on}}{R_e} \right)^2 \right)$$

Boost rectifier efficiency



$$\eta = \frac{P_{out}}{P_{in}} = \left(1 - \frac{R_{on}}{R_e}\right) F(a)$$

- To obtain high efficiency, choose V slightly larger than V_M
- Efficiencies in the range 90% to 95% can then be obtained, even with R_{on} as high as $0.2R_e$
- Losses other than MOSFET on-resistance are not included here

18.1.5 Design example

Let us design for a given efficiency. Consider the following specifications:

Output voltage	390 V
Output power	500 W
rms input voltage	120 V
Efficiency	95%

Assume that losses other than the MOSFET conduction loss are negligible.

Average input power is

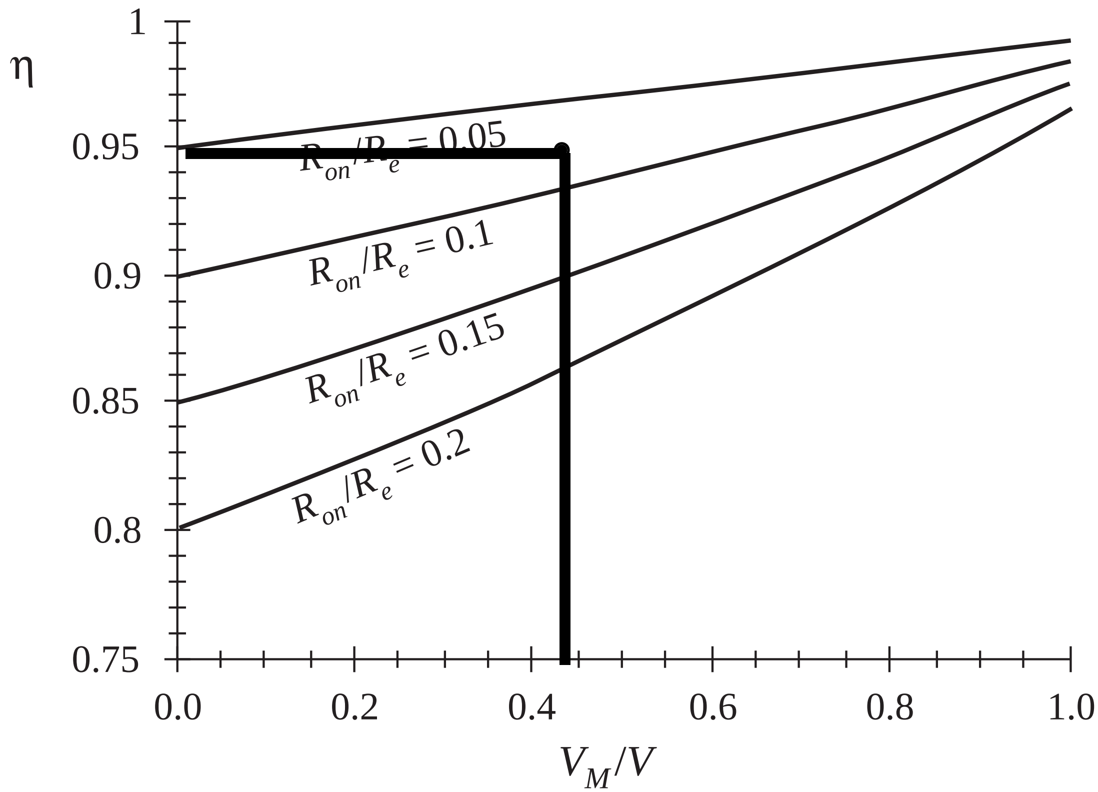
$$P_{in} = \frac{P_{out}}{\eta} = \frac{500\ \text{W}}{0.95} = 526\ \text{W}$$

Then the emulated resistance is

$$R_e = \frac{V_{g,\,rms}^2}{P_{in}} = \frac{(120\ \text{V})^2}{526\ \text{W}} = 27.4\ \Omega$$

Design example

Also, $\dfrac{V_M}{V} = \dfrac{120\sqrt{2}\ \text{V}}{390\ \text{V}} = 0.435$



95% efficiency with $V_M/V = 0.435$ occurs with $R_{on}/R_e \approx 0.075$.

So we require a MOSFET with on resistance of

$$R_{on} \le (0.075)\ R_e$$

$$= (0.075)\ (27.4\ \Omega) = 2\ \Omega$$

18.2 Controller schemes

Average current control

- Feedforward

Current programmed control

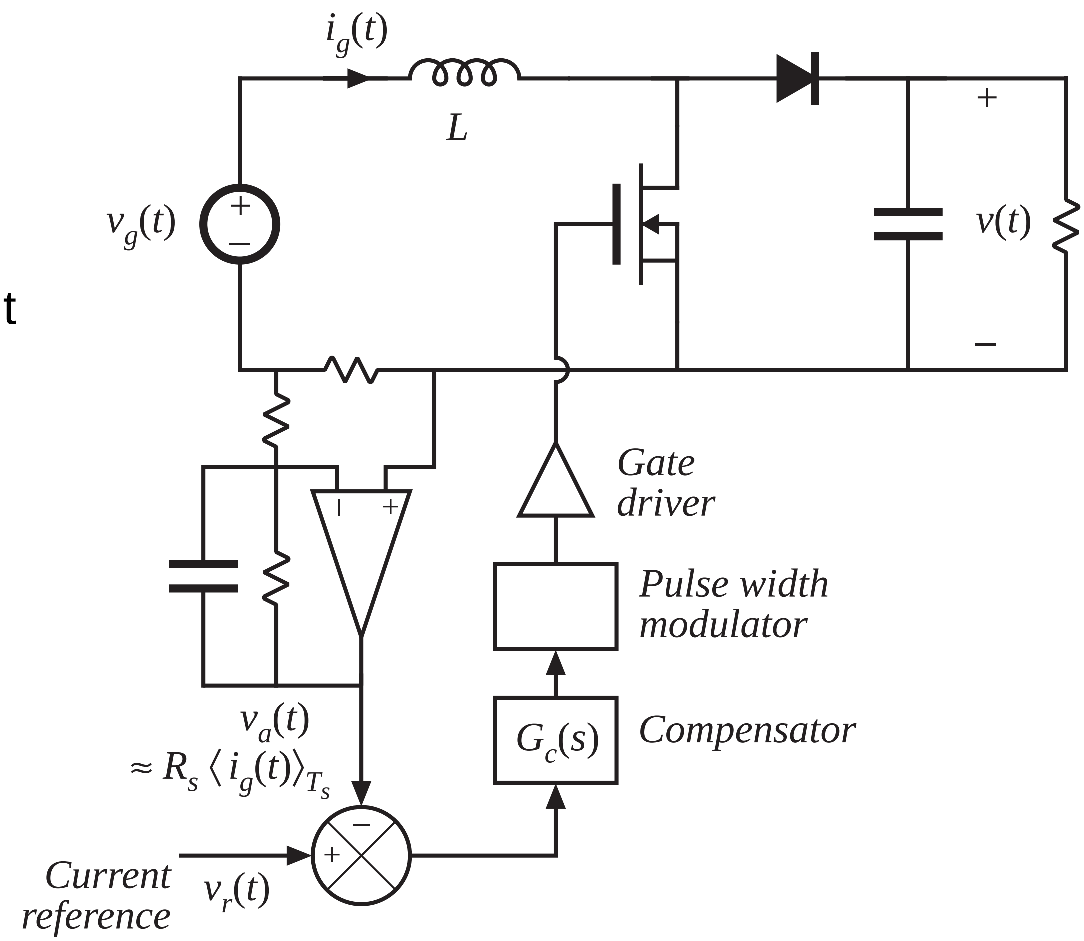
Hysteretic control

Nonlinear carrier control

18.2.1 Average current control

Boost example

Low frequency (average) component of input current is controlled to follow input voltage

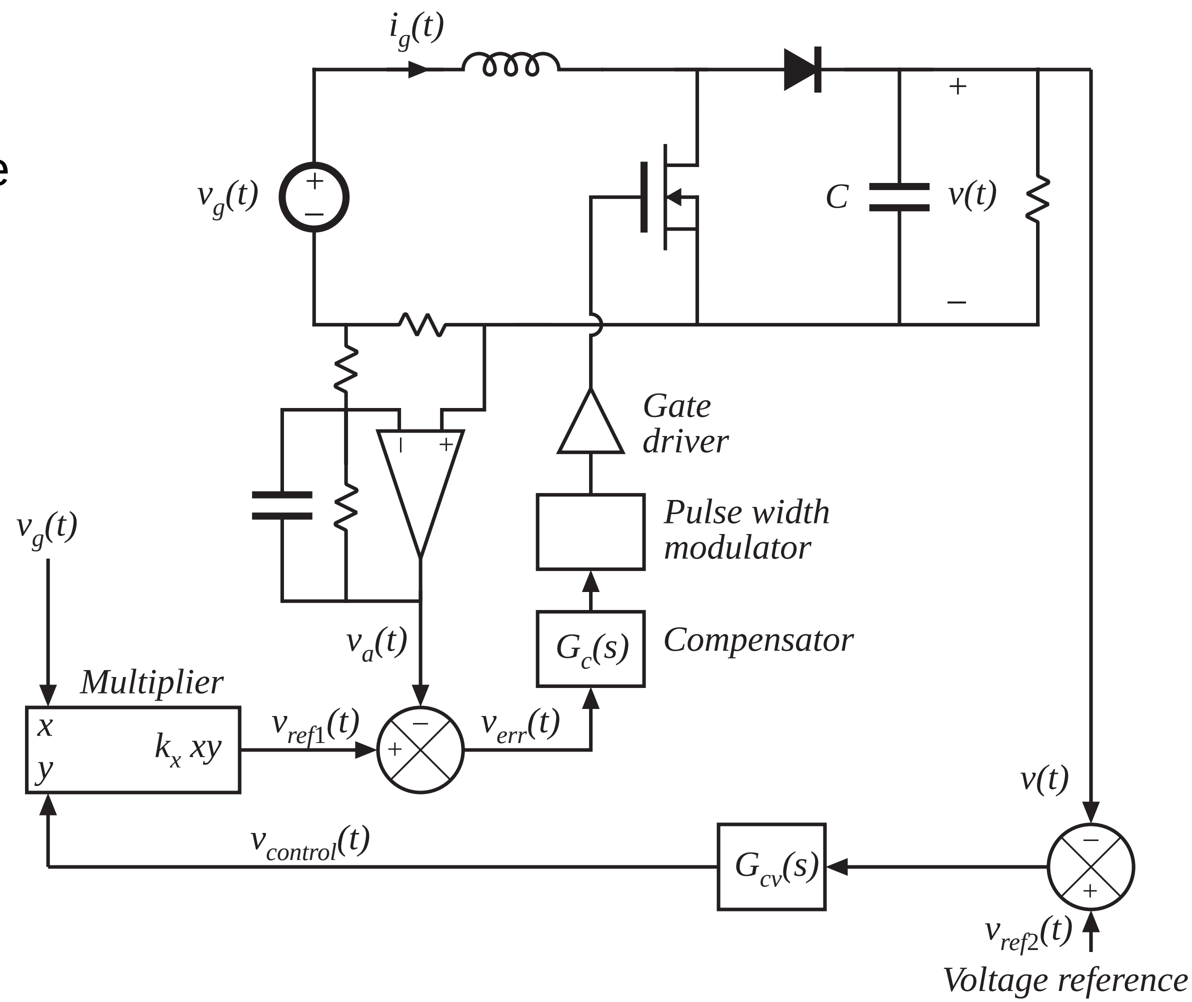


Use of multiplier to control average power

As discussed in Chapter 17, an output voltage feedback loop adjusts the emulated resistance R_e such that the rectifier power equals the dc load power:

$$P_{av} = \frac{V_{g,rms}^2}{R_e} = P_{load}$$

An analog multiplier introduces the dependence of R_e on $v(t)$.

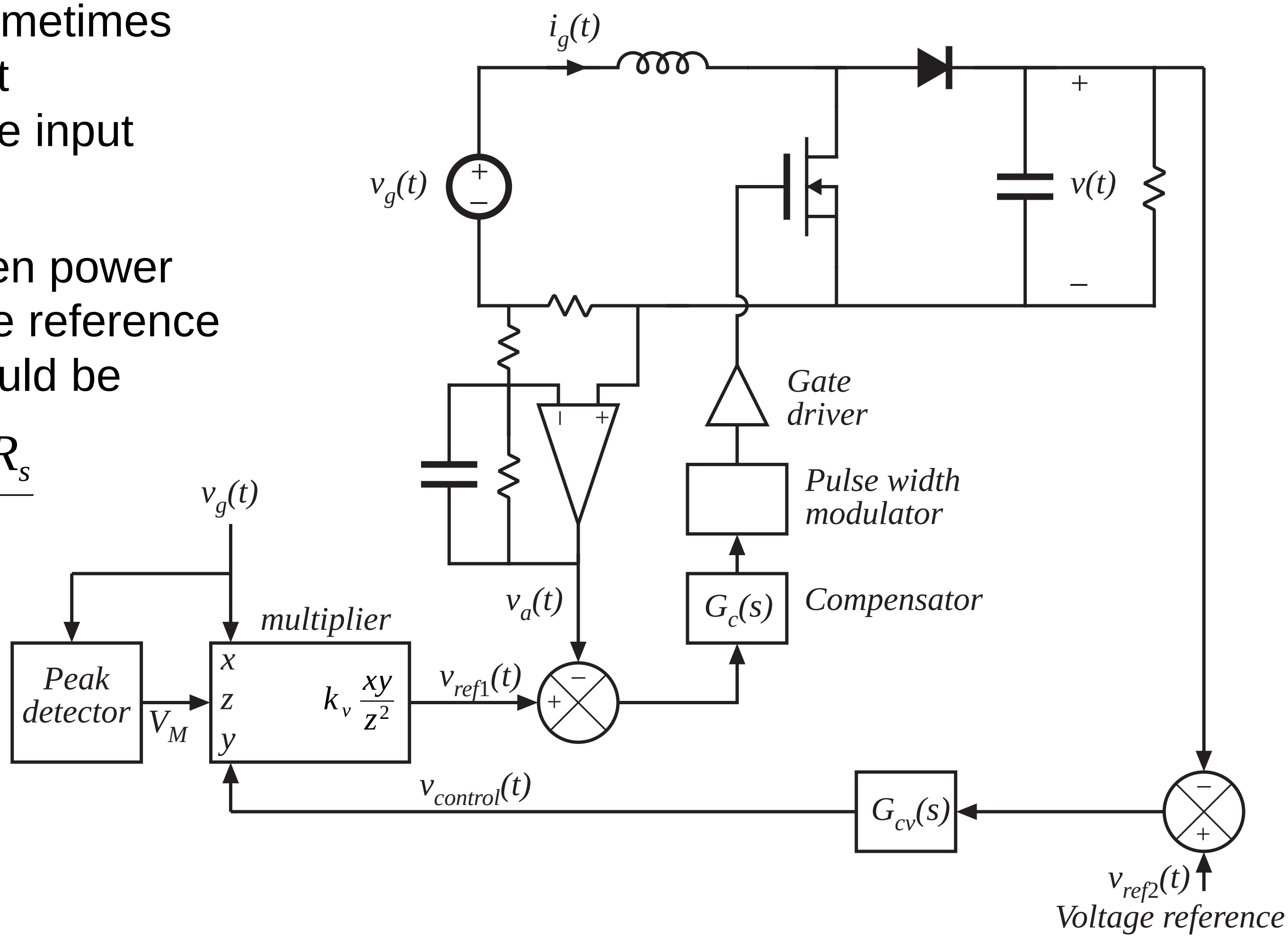


18.2.2 Feedforward

Feedforward is sometimes used to cancel out disturbances in the input voltage $v_g(t)$.

To maintain a given power throughput P_{av}, the reference voltage $v_{ref1}(t)$ should be

$$v_{ref1}(t) = \frac{P_{av} v_g(t) R_s}{V_{g,rms}^2}$$



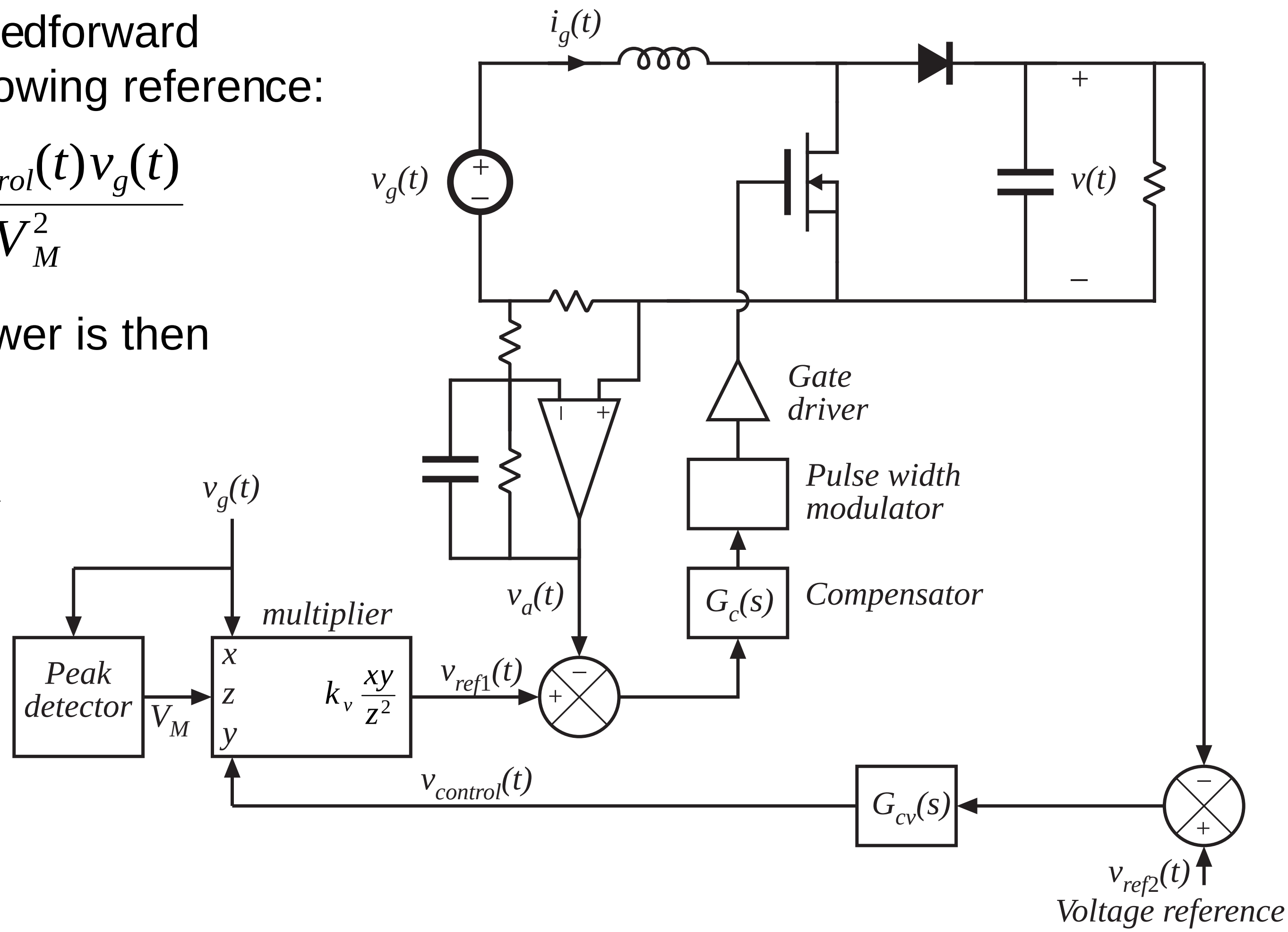
Feedforward, continued

Controller with feedforward produces the following reference:

$$v_{ref1}(t) = \frac{k_v v_{control}(t) v_g(t)}{V_M^2}$$

The average power is then given by

$$P_{av} = \frac{k_v v_{control}(t)}{2R_s}$$

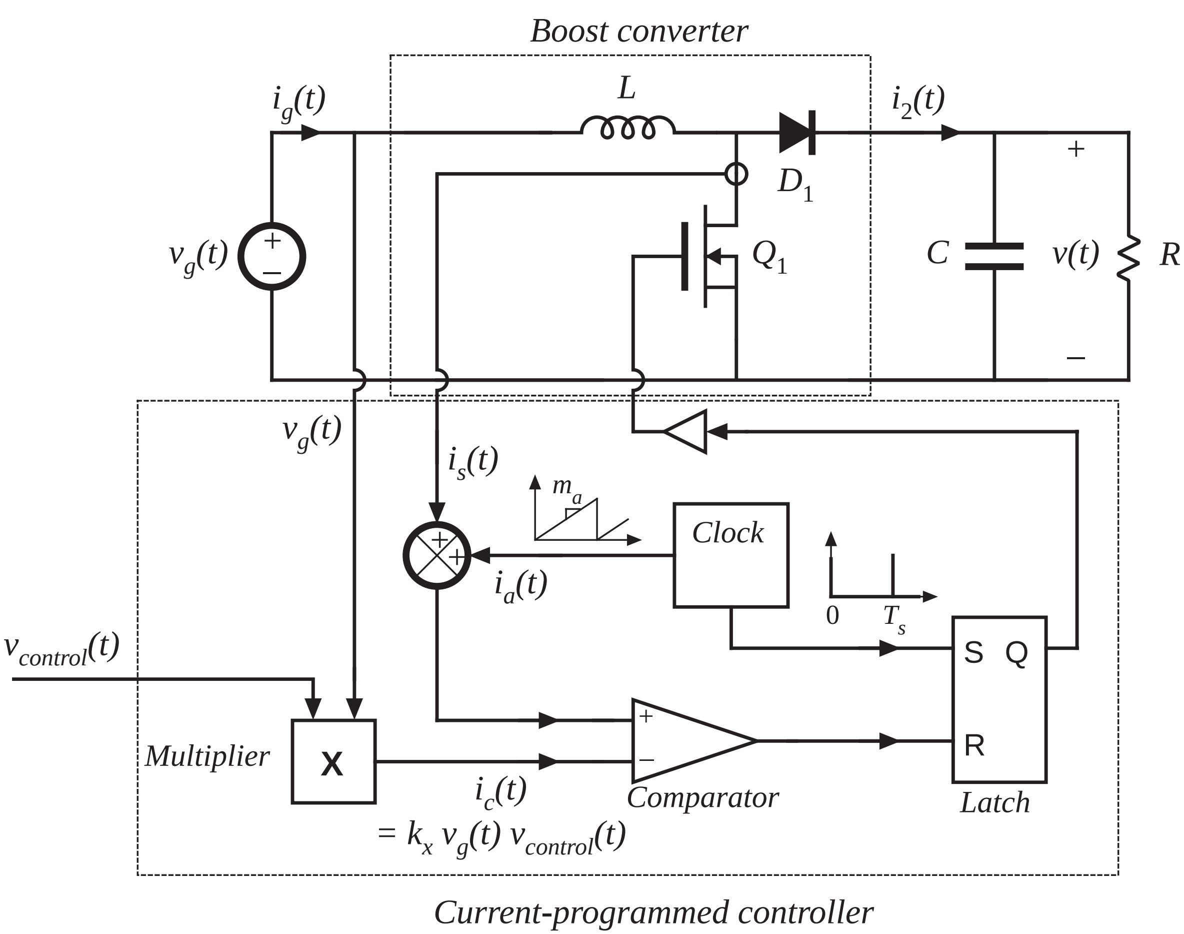


18.2.3 Current programmed control

Current programmed control is a natural approach to obtain input resistor emulation:

Peak transistor current is programmed to follow input voltage.

Peak transistor current differs from average inductor current, because of inductor current ripple and artificial ramp. This leads to significant input current waveform distortion.



CPM boost converter: Static input characteristics

$$\left\langle i_g(t) \right\rangle_{T_s} = \begin{cases} v_g(t) \dfrac{L i_c^2(t) f_s}{\left(V - v_g(t)\right)\left(\dfrac{v_g(t)}{V} + \dfrac{m_a L}{V}\right)} & \text{in DCM} \\ i_c(t) + m_a T_s \left(\dfrac{v_g(t)}{V} - 1\right) + \dfrac{v_g^2(t) T_s}{2LV} & \text{in CCM} \end{cases}$$

Mode boundary: CCM occurs when

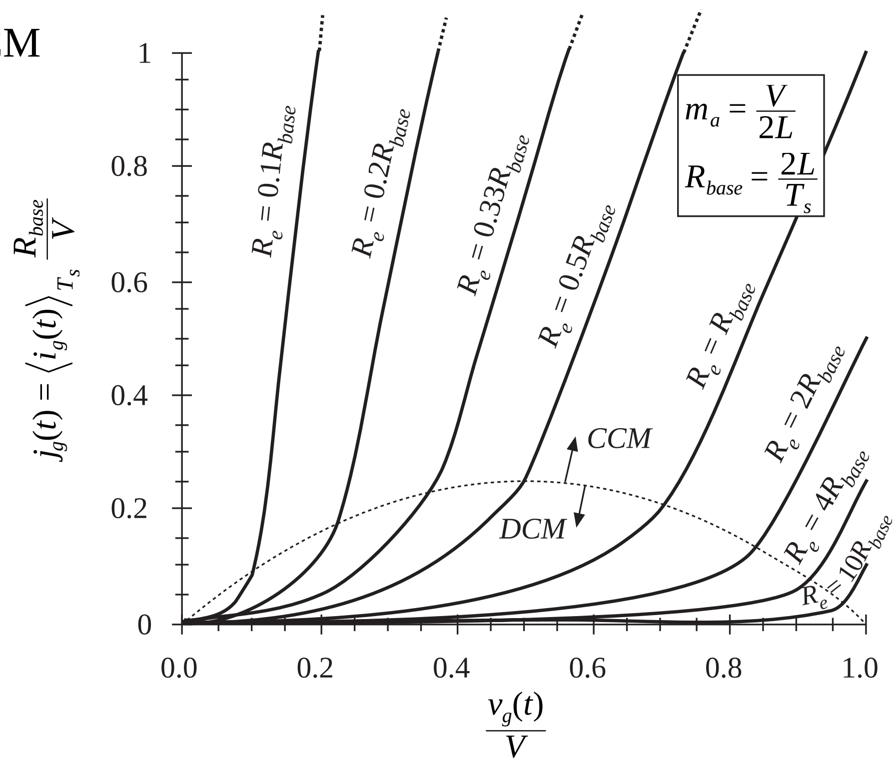
$$\left\langle i_g(t) \right\rangle_{T_s} > \frac{T_s V}{2L} \frac{v_g(t)}{V} \left(1 - \frac{v_g(t)}{V}\right)$$

or,
$$i_c(t) > \frac{T_s V}{L} \left(\frac{m_a L}{V} + \frac{v_g(t)}{V}\right)\left(1 - \frac{v_g(t)}{V}\right)$$

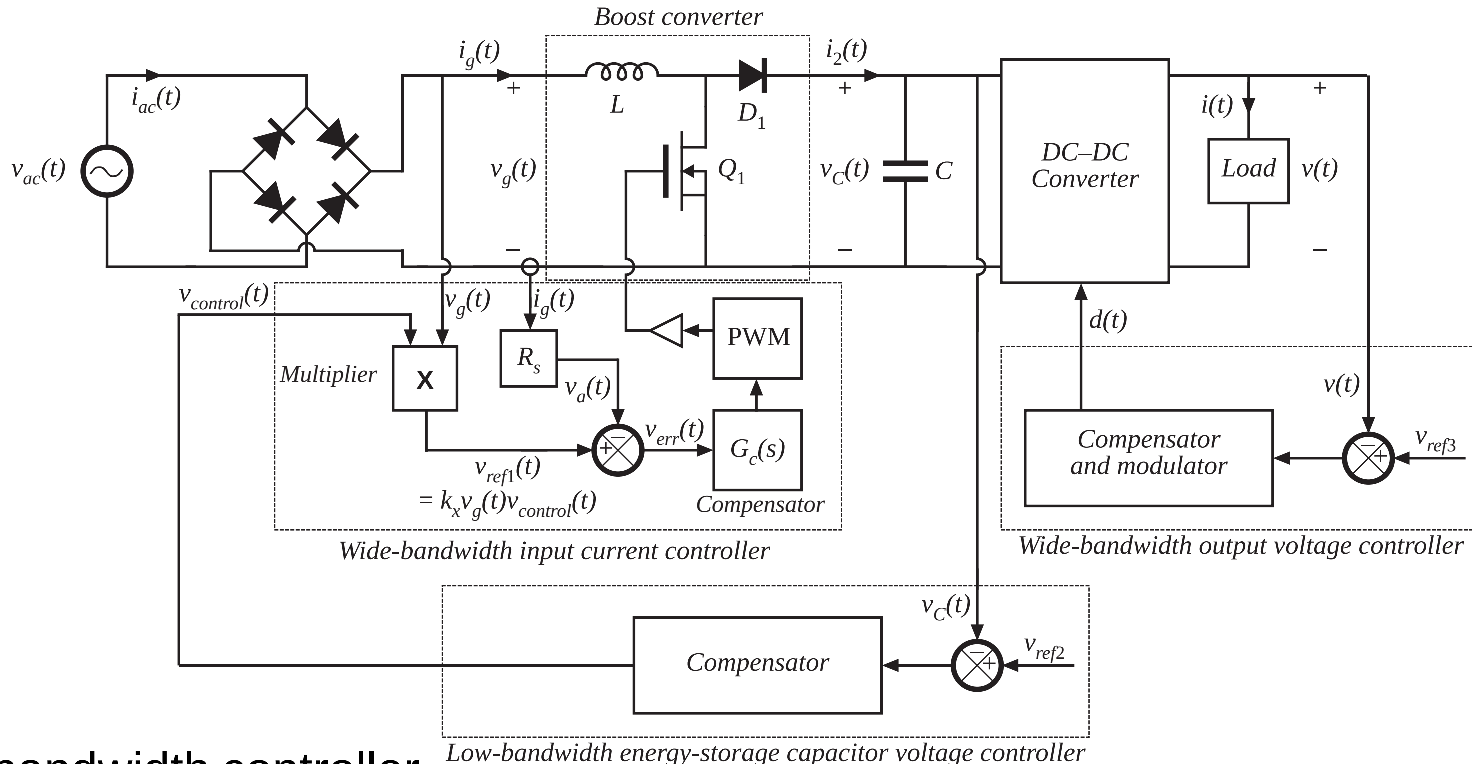
It is desired that
$$i_c(t) = \frac{v_g(t)}{R_e}$$

Minimum slope compensation:
$$m_a = \frac{V}{2L}$$

Static input characteristics of CPM boost, with minimum slope compensation:



18.3 Control system modeling
of high quality rectifiers



Two loops:

Outer low-bandwidth controller

Inner wide-bandwidth controller

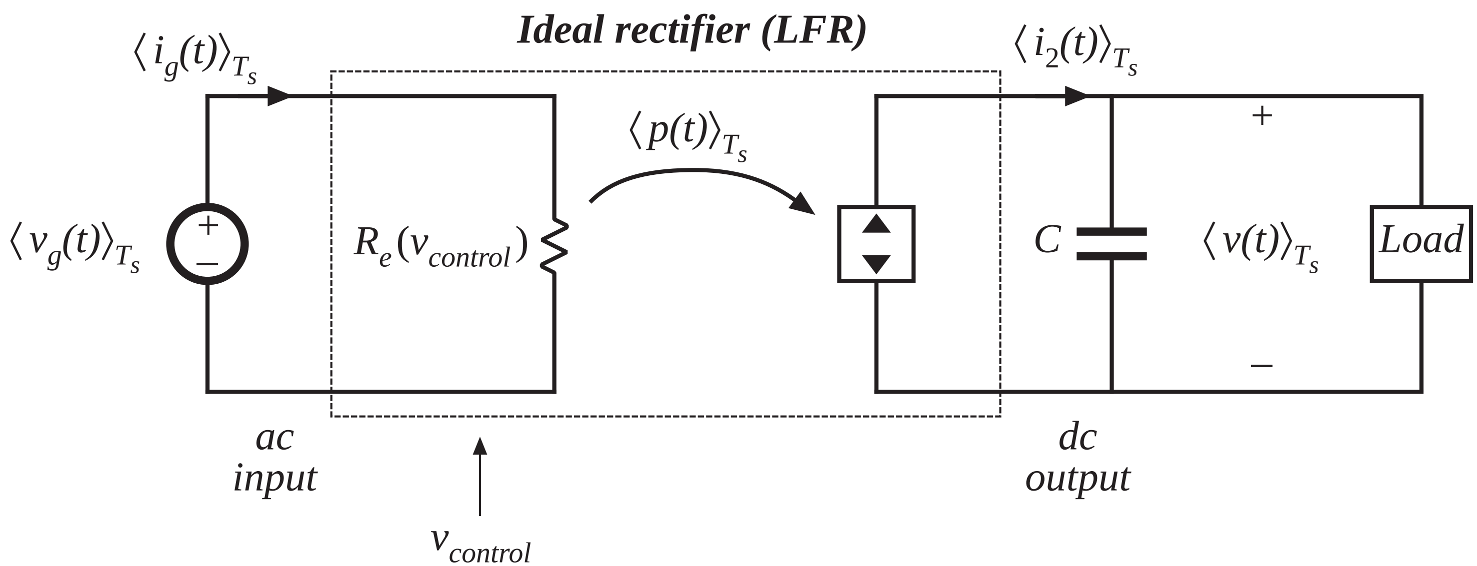
18.3.1 Modeling the outer low-bandwidth control system

This loop maintains power balance, stabilizing the rectifier output voltage against variations in load power, ac line voltage, and component values

The loop must be slow, to avoid introducing variations in R_e at the harmonics of the ac line frequency

Objective of our modeling efforts: low-frequency small-signal model that predicts transfer functions at frequencies below the ac line frequency

Large signal model
averaged over switching period T_s



Ideal rectifier model, assuming that inner wide-bandwidth loop operates ideally

High-frequency switching harmonics are removed via averaging

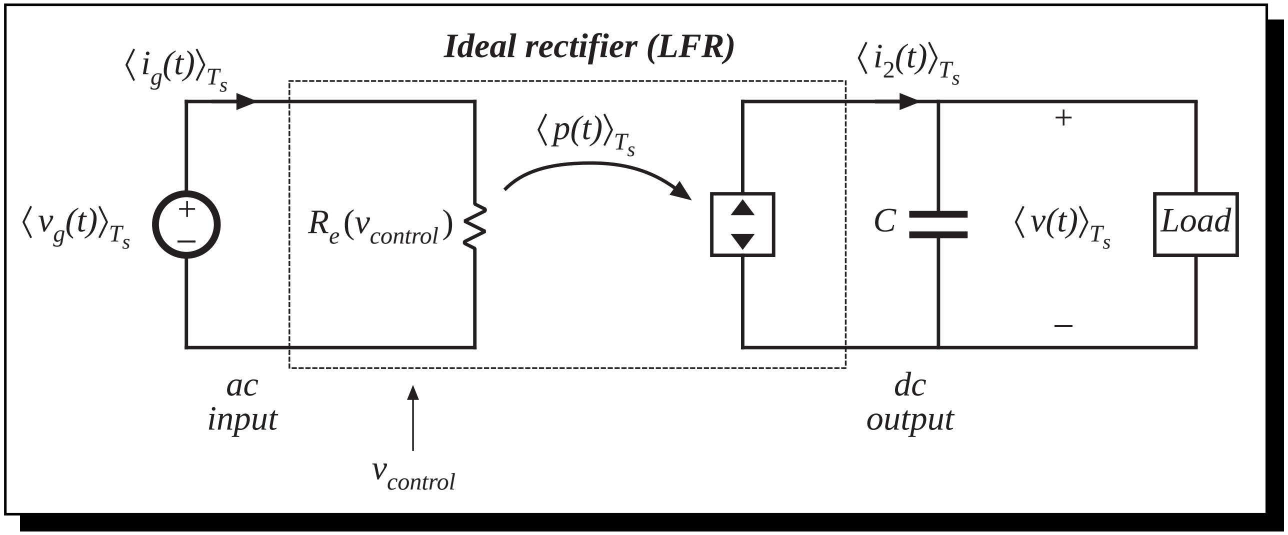
Ac line-frequency harmonics are included in model

Nonlinear and time-varying

Predictions of large-signal model

If the input voltage is

$$v_g(t) = \sqrt{2}\, v_{g,rms} \left| \sin\left(\omega t\right) \right|$$

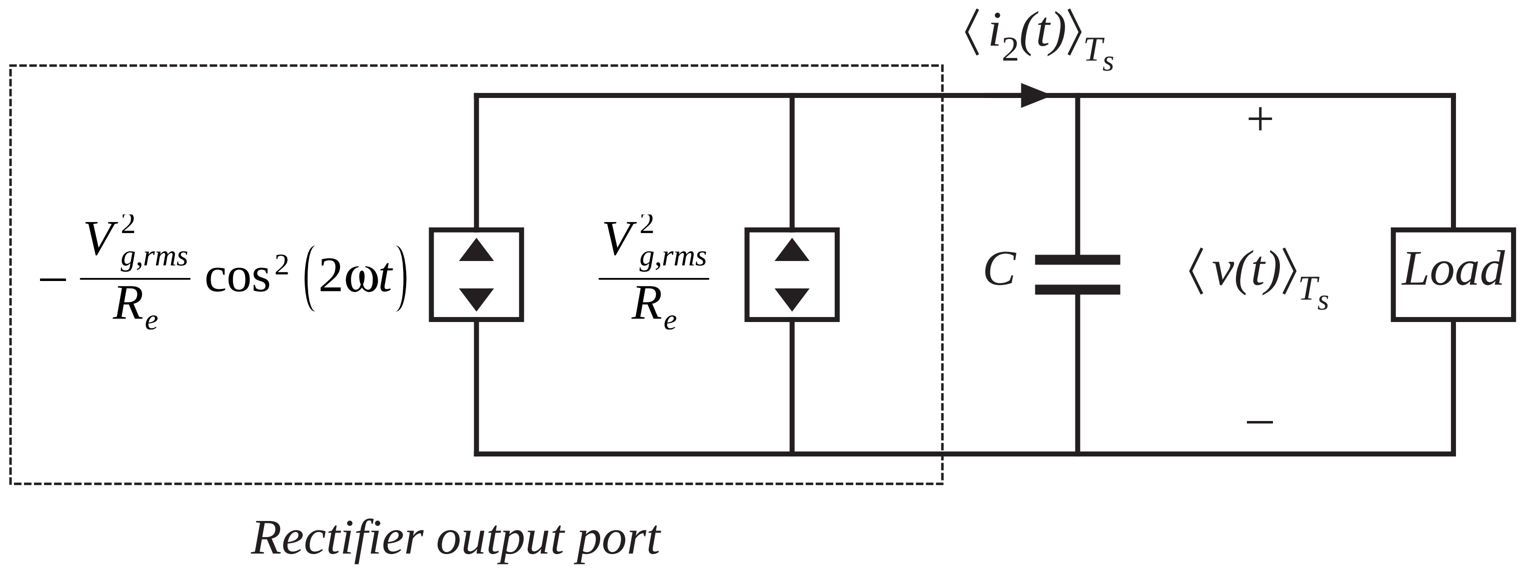


Then the instantaneous power is:

$$\left\langle p(t) \right\rangle_{T_s} = \frac{\left\langle v_g(t) \right\rangle_{T_s}^2}{R_e(v_{control}(t))} = \frac{v_{g,rms}^2}{R_e(v_{control}(t))} \left(1 - \cos\left(2\omega t\right)\right)$$

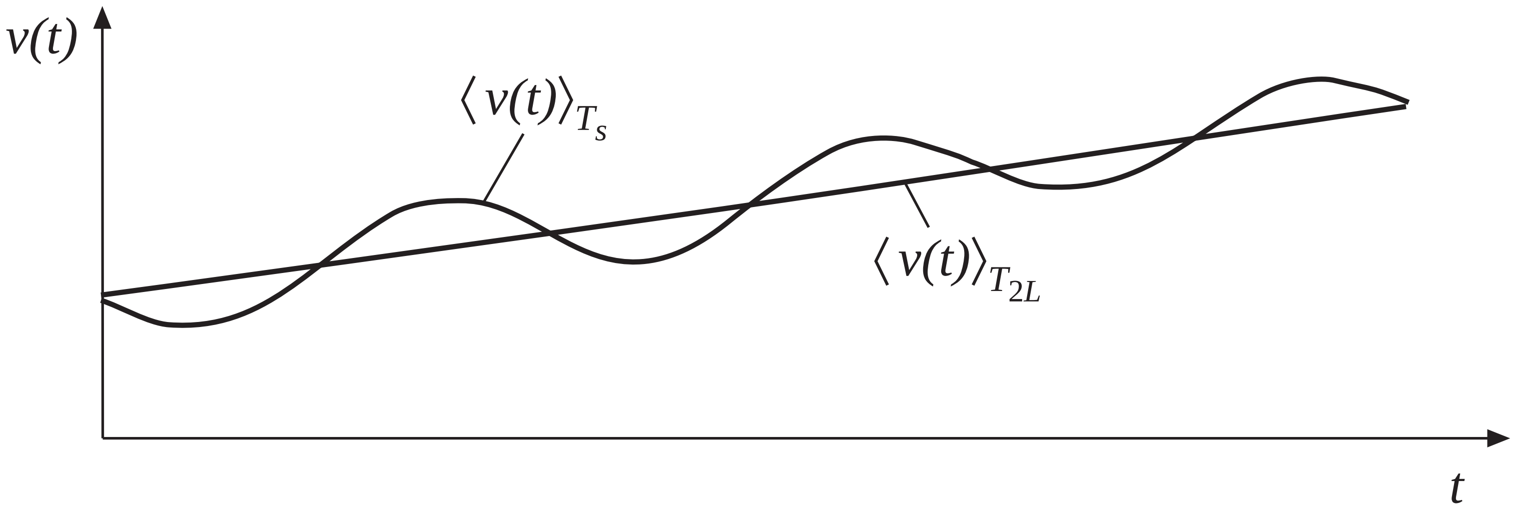
which contains a constant term plus a second-harmonic term

Separation of power source into its constant and time-varying components



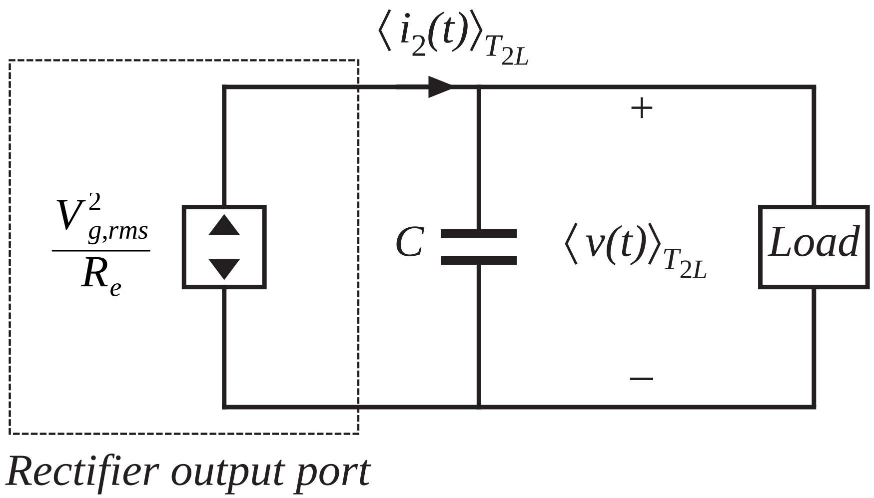
The second-harmonic variation in power leads to second-harmonic variations in the output voltage and current

Removal of even harmonics via averaging



$$T_{2L} = \frac{1}{2}\,\frac{2\pi}{\omega} = \frac{\pi}{\omega}$$

Resulting averaged model



Time invariant model

Power source is nonlinear

Perturbation and linearization

The averaged model predicts that the rectifier output current is

$$\left\langle i_2(t) \right\rangle_{T_{2L}} = \frac{\left\langle p(t) \right\rangle_{T_{2L}}}{\left\langle v(t) \right\rangle_{T_{2L}}} = \frac{v_{g,rms}^2(t)}{R_e(v_{control}(t))\,\left\langle v(t) \right\rangle_{T_{2L}}}$$

$$= f\left(v_{g,rms}(t),\, \left\langle v(t) \right\rangle_{T_{2L}},\, v_{control}(t)) \right)$$

Let

$$\left\langle v(t) \right\rangle_{T_{2L}} = V + \hat{v}(t)$$

$$\left\langle i_2(t) \right\rangle_{T_{2L}} = I_2 + \hat{i}_2(t)$$

$$v_{g,rms} = V_{g,rms} + \hat{v}_{g,rms}(t)$$

$$v_{control}(t) = V_{control} + \hat{v}_{control}(t)$$

with

$$V >> \left| \hat{v}(t) \right|$$

$$I_2 >> \left| \hat{i}_2(t) \right|$$

$$V_{g,rms} >> \left| \hat{v}_{g,rms}(t) \right|$$

$$V_{control} >> \left| \hat{v}_{control}(t) \right|$$

Linearized result

$$I_2 + \hat{i}_2(t) = g_2 \hat{v}_{g,rms}(t) + j_2 \hat{v}(t) - \frac{\hat{v}_{control}(t)}{r_2}$$

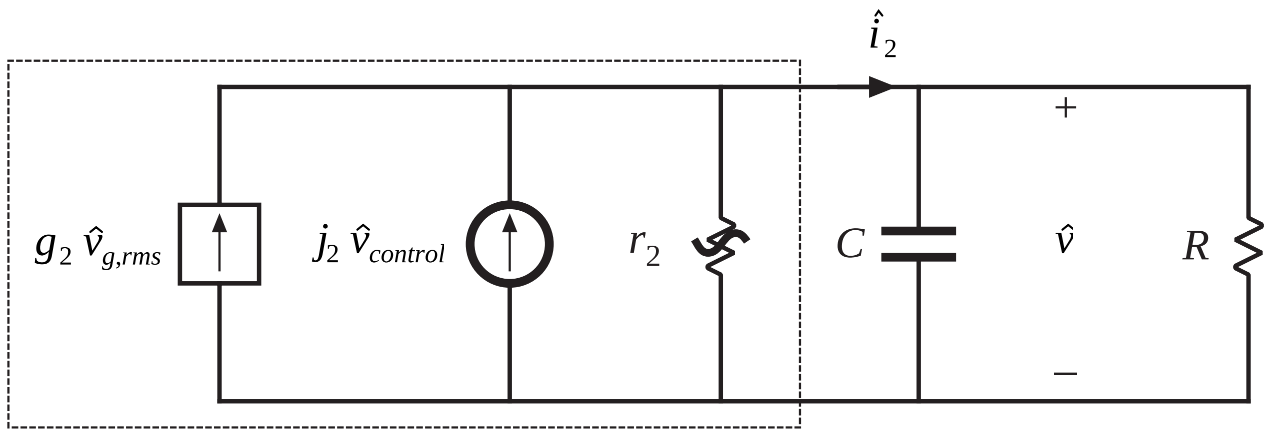
where

$$g_2 = \left. \frac{df\left(v_{g,rms},\, V,\, V_{control}\right)}{dv_{g,rms}} \right|_{v_{g,rms} = V_{g,rms}} = \frac{2}{R_e(V_{control})} \frac{V_{g,rms}}{V}$$

$$\left(-\frac{1}{r_2}\right) = \left. \frac{df\left(V_{g,rms},\, \left\langle v \right\rangle_{T_{2L}},\, V_{control}\right)}{d\left\langle v \right\rangle_{T_{2L}}} \right|_{\left\langle v \right\rangle_{T_{2L}} = V} = -\frac{I_2}{V}$$

$$j_2 = \left. \frac{df\left(V_{g,rms},\, V,\, v_{control}\right)}{dv_{control}} \right|_{v_{control} = V_{control}} = -\frac{V_{g,rms}^2}{V R_e^2(V_{control})} \left. \frac{dR_e(v_{control})}{dv_{control}} \right|_{v_{control} = V_{control}}$$

Small-signal equivalent circuit



Rectifier output port

Predicted transfer functions

Control-to-output

$$\frac{\hat{v}(s)}{\hat{v}_{control}(s)} = j_2\, R||r_2\, \frac{1}{1 + sC\, R||r_2}$$

Line-to-output

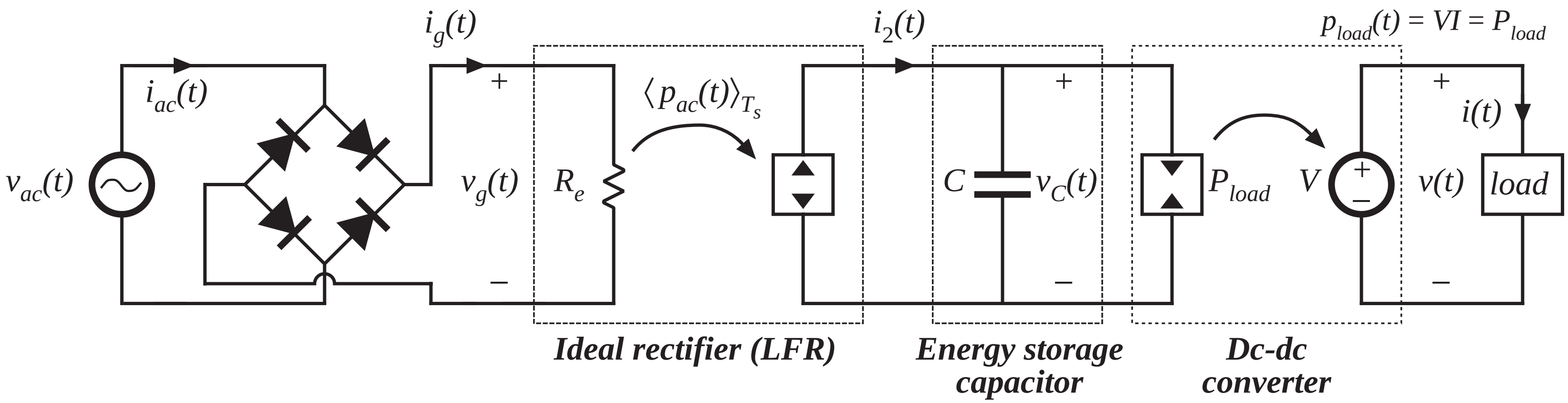
$$\frac{\hat{v}(s)}{\hat{v}_{g,rms}(s)} = g_2\, R||r_2\, \frac{1}{1 + sC\, R||r_2}$$

Model parameters

Table 18.1 Small-signal model parameters for several types of rectifier control schemes

Controller type	g_2	j_2	r_2
Average current control with feedforward, Fig. 18.9	0	$\dfrac{P_{av}}{VV_{control}}$	$\dfrac{V^2}{P_{av}}$
Current-programmed control, Fig. 18.10	$\dfrac{2P_{av}}{VV_{g,rms}}$	$\dfrac{P_{av}}{VV_{control}}$	$\dfrac{V^2}{P_{av}}$
Nonlinear-carrier charge control of boost rectifier, Fig. 18.14	$\dfrac{2P_{av}}{VV_{g,rms}}$	$\dfrac{P_{av}}{VV_{control}}$	$\dfrac{V^2}{2P_{av}}$
Boost with hysteretic control, Fig. 18.13(b)	$\dfrac{2P_{av}}{VV_{g,rms}}$	$\dfrac{P_{av}}{VT_{on}}$	$\dfrac{V^2}{P_{av}}$
DCM buck–boost, flyback, SEPIC, or Cuk converters	$\dfrac{2P_{av}}{VV_{g,rms}}$	$\dfrac{2P_{av}}{VD}$	$\dfrac{V^2}{P_{av}}$

Constant power load



Rectifier and dc-dc converter operate with same average power

Incremental resistance R of constant power load is negative, and is

$$R = -\frac{V^2}{P_{av}}$$

which is equal in magnitude and opposite in polarity to rectifier incremental output resistance r_2 for all controllers except NLC

Transfer functions with constant power load

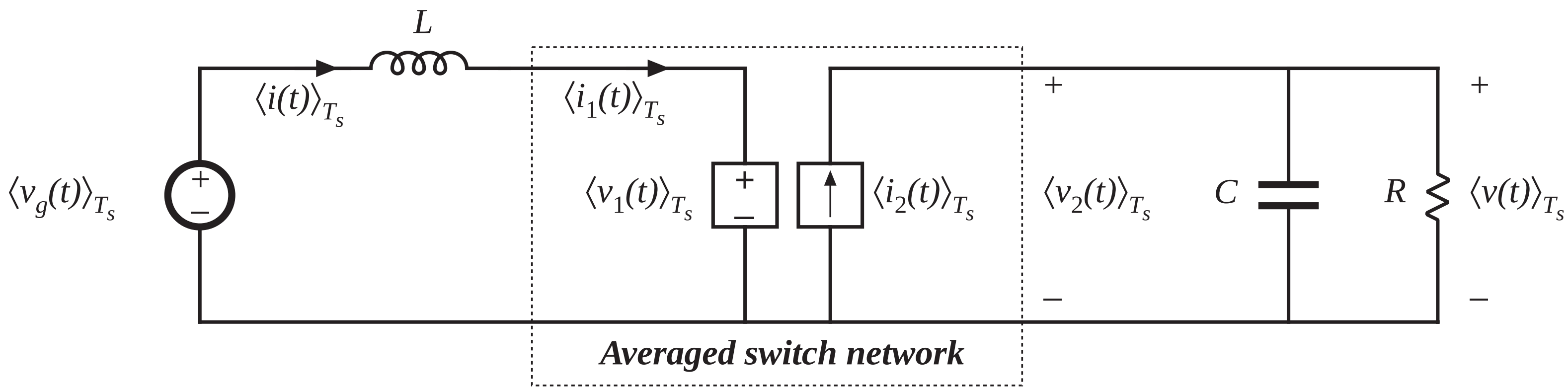
When $r_2 = -R$, the parallel combination $r_2 \parallel R$ becomes equal to zero. The small-signal transfer functions then reduce to

$$\frac{\hat{v}(s)}{\hat{v}_{control}(s)} = \frac{j_2}{sC}$$

$$\frac{\hat{v}(s)}{\hat{v}_{g,rms}(s)} = \frac{g_2}{sC}$$

18.3.2 Modeling the inner wide-bandwidth average current controller

Averaged (but not linearized) boost converter model, Fig. 7.42:



In Chapter 7, we perturbed and linearized using the assumptions

$$\left\langle v_g(t)\right\rangle_{T_s} = V_g + \hat{v}_g(t)$$

$$d(t) = D + \hat{d}(t) \quad \Rightarrow d'(t) = D' - \hat{d}(t)$$

$$\left\langle i(t)\right\rangle_{T_s} = \left\langle i_1(t)\right\rangle_{T_s} = I + \hat{i}(t)$$

$$\left\langle v(t)\right\rangle_{T_s} = \left\langle v_2(t)\right\rangle_{T_s} = V + \hat{v}(t)$$

$$\left\langle v_1(t)\right\rangle_{T_s} = V_1 + \hat{v}_1(t)$$

$$\left\langle i_2(t)\right\rangle_{T_s} = I_2 + \hat{i}_2(t)$$

Problem: variations in v_g, i_1 , and d are not small.

So we are faced with the design of a control system that exhibits significant nonlinear time-varying behavior.

Linearizing the equations of the boost rectifier

When the rectifier operates near steady-state, it is true that

$$\left\langle v(t) \right\rangle_{T_s} = V + \hat{v}(t)$$

with

$$\left| \hat{v}(t) \right| << \left| V \right|$$

In the special case of the boost rectifier, this is sufficient to linearize the equations of the average current controller.

The boost converter average inductor voltage is

$$L \frac{d\left\langle i_g(t) \right\rangle_{T_s}}{dt} = \left\langle v_g(t) \right\rangle_{T_s} - d'(t)V - d'(t)\hat{v}(t)$$

substitute:

$$L \frac{d\left\langle i_g(t) \right\rangle_{T_s}}{dt} = \left\langle v_g(t) \right\rangle_{T_s} - d'(t)V - d'(t)\hat{v}(t)$$

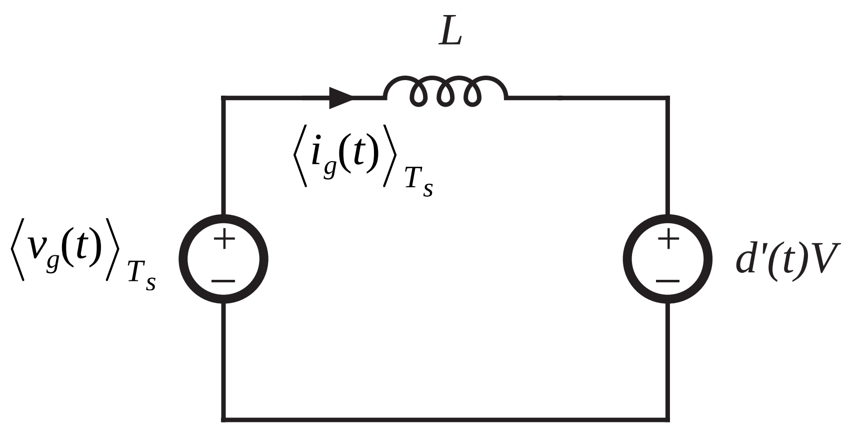
Linearized boost rectifier model

$$L\frac{d\left\langle i_g(t)\right\rangle_{T_s}}{dt} = \left\langle v_g(t)\right\rangle_{T_s} - d'(t)V - d'(t)\hat{v}(t)$$

The nonlinear term is much smaller than the linear ac term. Hence, it can be discarded to obtain

$$L\frac{d\left\langle i_g(t)\right\rangle_{T_s}}{dt} = \left\langle v_g(t)\right\rangle_{T_s} - d'(t)V$$

Equivalent circuit:



$$\frac{i_g(s)}{d(s)} = \frac{V}{sL}$$

The quasi-static approximation

The above approach is not sufficient to linearize the equations needed to design the rectifier averaged current controllers of buck-boost, Cuk, SEPIC, and other converter topologies. These are truly nonlinear time-varying systems.

An approximate approach that is sometimes used in these cases: the *quasi-static approximation*

Assume that the ac line variations are much slower than the converter dynamics, so that the rectifier always operates near equilibrium. The quiescent operating point changes slowly along the input sinusoid, and we can find the slowly-varying "equilibrium" duty ratio as in Section 18.1.

The converter small-signal transfer functions derived in Chapters 7 and 8 are evaluated, using the time-varying operating point. The poles, zeroes, and gains vary slowly as the operating point varies. An average current controller is designed, that has a positive phase margin at each operating point.

Quasi-static approximation: discussion

In the literature, several authors have reported success using this method

Should be valid provided that the converter dynamics are suffieiently fast, such that the converter always operates near the assumed operating points

No good condition on system parameters, which can justify the approximation, is presently known for the basic converter topologies

It is well-understood in the field of control systems that, when the converter dynamics are not sufficiently fast, then the quasi-static approximation yields neither necessary nor sufficient conditions for stability. Such behavior can be observed in rectifier systems. Worst-case analysis to prove stability should employ simulations.