Fundamentals of the Plane Electromagnetic Shield

Presented to the Electromagnetic Compatibility Society
of the Long Island Section of the IEEE

March 11, 2008

At Telephonics Corporation 815 Broad Hollow Road, Farmingdale NY

By

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Foreword

This presentation is based on the presentation: “Schelkunoff’s Approach to Shielding;” given by Richard Mohr to the IEEE TC-4 Committee at the 2007 IEEE International Electromagnetic Compatibility Symposium in Oahu Hawaii, July 2007. Portions of the presentations are extracted from his EMC training seminar: “Getting Your Product Into EMC Compliance”.

Purpose

Review basics of the plane electromagnetic shield as developed by Schelkunoff in a way that will provide insight into his equations and so answer questions on their meaning and use, and to serve as a foundation for further investigations.

Outline

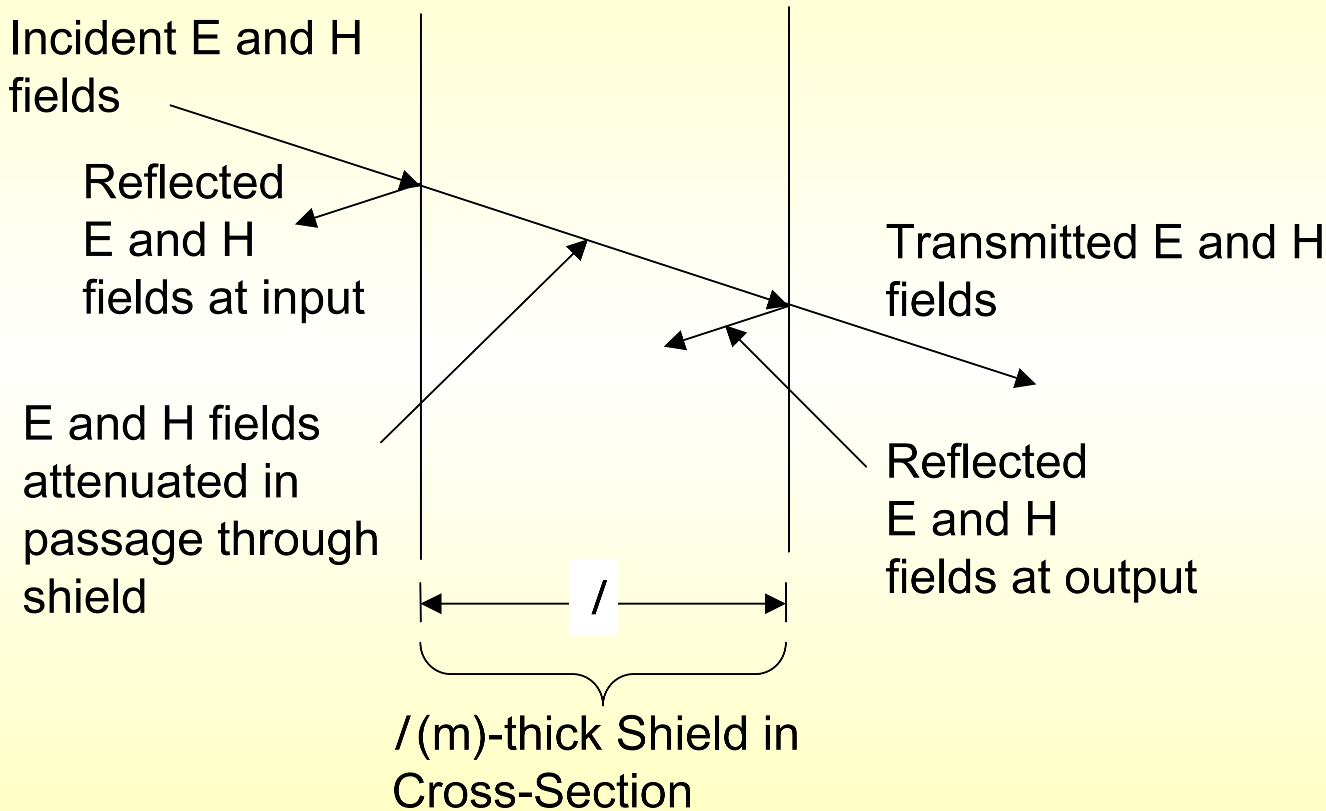
- Overview of electromagnetic shielding
- Shield action, reflection and absorption
- Schelkunoff's approach to the electromagnetic shield
- Transmission line analogy of shield
- Basic parameters in shielding analysis
- Derivation of shielding equations from multiple re-reflection analysis
- Reduction of shielding equations to conventional R-A-B shield formulation
  - Review of R, A, B factors with sample values
  - Limiting cases: high loss, low loss including lump element model in low loss case
- Summary
- Conclusions

Overview of Electromagnetic Shielding

1. Electromagnetic shields are the top level of EMI control
   a) Prevent radiated emissions from enclosure
   b) Prevent entry of radiated fields
   c) Terminate shields of shielded interfaces
   d) Drain EMI from interface filters
2. Factors affecting achievable shielding effectiveness
   a) Basic material of shield: its conductivity and permeability
   b) Thickness of shield
   c) Characteristics of electromagnetic field
      i. Far field
      ii. Near field: low impedance, high impedance
   d) Enclosure size and shape
   e) Seams, apertures, penetrations

<u>This talk will address Items 2 a), b), c)</u>

Shield Action
Reflection and Absorption

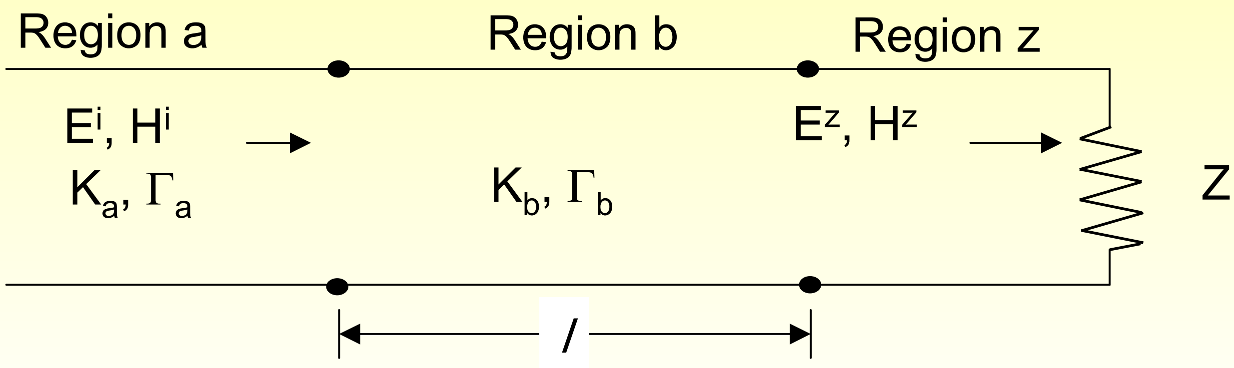


Shield acts by reflection and absorption

Schelkunoff’s Approach to Electromagnetic Shielding

- Schelkunoff saw the analogy of shield action with a transmission line
- He employed transmission line models to develop equations for a plane electromagnetic shield.
- He extended his model to cover shielding effectiveness of elementary enclosures.
- His approaches have since served as the starting point for much follow-on efforts

Transmission Line Analogy



E^i and H^i are analogous to voltage and current, respectively, on a transmission line

E^z and H^z are analogous to the terminal voltage and current, respectively, at Z

K_a, Γ_a are wave impedance and propagation constant of media at input to shield

K_b, Γ_b are wave impedance and propagation constant of shield

Z is the impedance at the output of the shield

Shield is modeled as a lossy line section within a transmission line

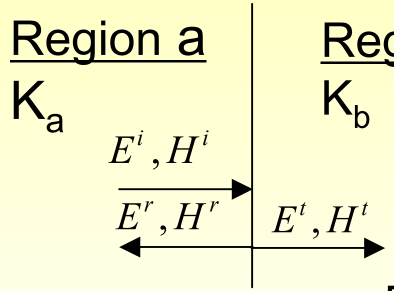
Basic Parameters In Shielding

-Symbol	Description	Value in free space	Value in Copper at 1 MHz
ε	Permitivity	$8.85*10^{-12}$F/m	$8.85*10^{-12}$F/m
μ	Permeability	$4\pi*10^{-7}$H/m	$4\pi*10^{-7}$H/m
g	Conductivity	0	$5.8*10^{7}$S/m
K	Wave impedance	120π Ohms/square	$(j\omega\mu/g)^{1/2}$ Ohms/square = $3.690*10^{-4}(j)^{1/2}$ Ohms/square
Γ	Propagation constant	$j2\pi/\lambda$ radians/meter	$(j\omega\mu g)^{1/2}$ Nepers/meter = $2.140*10^{4}(j)^{1/2}$ Nepers/m
α	Attenuation constant	0	$15.13(f\mu_r g_r)^{1/2}$Nepers/m = $1.513*10^{4}$ Nepers/m
Δ (=$1/\alpha$)	Skin depth	-	$6.61*10^{-2}/ (f\mu_r g_r)^{1/2}$ m = $6.609*10^{-5}$ m

Multiple Re-Reflection Model

- Schelkunoff arrived at his shielding equations by two different routes:
  - Transmission line solution
  - Multiple re-reflection solution
- The multiple reflection solution provides valuable insight and is repeated here

EM Refresher



H-field reflection coefficient in Region a at interface with Region b:

$$q_{H,a-b} = \frac{H^r}{H^i} = \frac{K_a - K_b}{K_a + K_b}$$

E-field reflection coefficient in Region a at interface with Region b:

$$q_{E,a-b} = \frac{E^r}{E^i} = \frac{K_b - K_a}{K_a + K_b} (= -q_{H,a-b})$$

H-field transfer coefficient from Region a to Region b

$$p_{H,a-b} = \frac{H^t}{H^i} = 1 + q_{H,a-b} = \frac{2K_a}{K_a + K_b}$$

E- field transfer coefficient from Region a to Region b

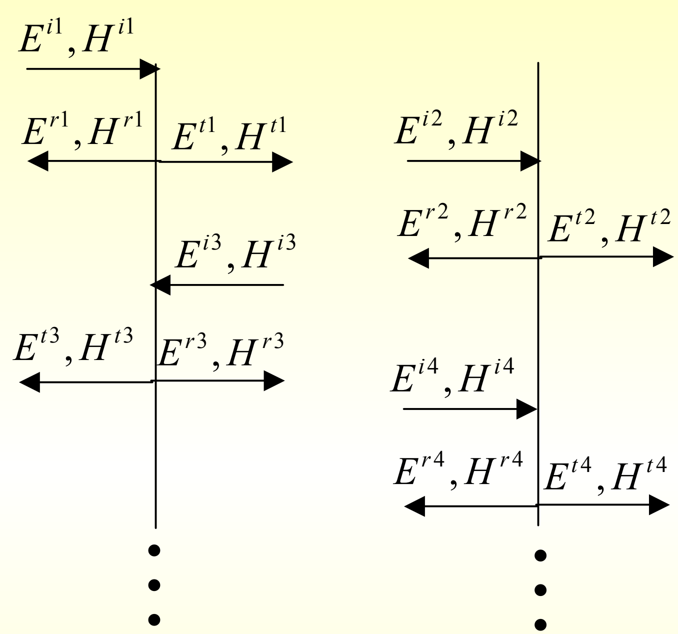
$$p_{E,a-b} = \frac{E^t}{E^i} = 1 + q_{E,a-b} = \frac{2K_b}{K_a + K_b}$$

Field transfer across a length, *l*, of Region b:

$$\frac{E^l}{E^i} = \frac{H^l}{H^i} = e^{-\Gamma_b l}$$

Re-Reflection Development

Region a Region b Region Z



Net output is:

$$H^{z} = H^{t2} + H^{t4} + H^{t6} + \ldots, \text{ and } E^{z} = E^{t2} + E^{t4} + E^{t6} + \ldots$$

The re-reflected components:

- *Increase the transmission through the shield*
- *Decrease net input reflection*
- *Increase absorption loss over the one-way absorption loss*

Solution by Re-reflection Model

$$\frac{H^{t2}}{H^{i1}} = p_{H,1}e^{-\Gamma_b l}p_{H,2} = p_H e^{-\Gamma_b l}$$

$$\frac{H^{t4}}{H^{i1}} = p_{H,1}e^{-\Gamma_b l}q_{H,2}e^{-\Gamma_b l}q_{H,1}e^{-\Gamma_b l}p_{H,2} = p_H e^{-\Gamma_b l}q_H e^{-2\Gamma_b l}$$

$$\frac{H^{t6}}{H^{i1}} = p_{H,1}e^{-\Gamma_b l}q_{H,2}e^{-\Gamma_b l}q_{H,1}e^{-\Gamma_b l}q_{H,2}e^{-\Gamma_b l}q_{H,1}e^{-\Gamma_b l}p_{H,2} = p_H e^{-\Gamma_b l}\left(q_H e^{-2\Gamma_b l}\right)^2$$

$$\vdots$$

Note, in above, $p_H = p_{H1}p_{H2}$; $q_H = q_{H1}q_{H2}$

The components of the transmitted field are recognized as comprising a geometric series with first term, $p_H e^{-\Gamma_b l}$, and multiplying factor, $q_H e^{-2\Gamma_b l}$.

The sum of the infinite series is then, $\frac{H^Z}{H^{i1}} = T_H = \frac{p_H e^{-\Gamma_b l}}{1 - q_H e^{-2\Gamma_b l}}$

Similarly: $\frac{E^Z}{E^{i1}} = T_E = \frac{Z}{K_a} T_H$

Transmission Through Shield

Analysis results of transmission line model for shield

H - field shielding :

$$S_H = \frac{1}{T_H} = \frac{H^{i1}}{H^Z} = \frac{1 - q_H e^{-2\Gamma_b l}}{p_H e^{-\Gamma_b l}}$$

$$\text{Where}: q_H = \frac{(K_a - K_b)(Z - K_b)}{(K_a + K_b)(Z + K_b)},\ p_H = \frac{4K_a K_b}{(K_a + K_b)(Z + K_b)}$$

E - field shielding :

$$S_E = \frac{1}{T_E} = \frac{E^{i1}}{E^Z} = \frac{K_a}{Z} S_H$$

- *q_H is the product of the H-field reflection coefficients of the input and output interfaces of the shield section, when viewed from the shield section*
- *p_H is the product of the H-field transmission factors across the input and output interfaces of the shield section*

Conventional RAB Formulation

Shielding formulation when $Z = K_a$:

$$S(dB) = 20\log\left|\frac{H^i}{H^Z}\right| = 20\log\left|\frac{E^i}{E^Z}\right| = 20\log\left|\frac{1}{T}\right| = 20\log\left|\frac{1}{p}\right| + 20\log\left|e^{\Gamma_b l}\right| + 20\log\left|1 - q_H e^{-2\Gamma_b l}\right|$$

$$= R(dB) + A(dB) + B(dB)$$

$$\text{Where}: q = \frac{(k-1)^2}{(k+1)^2},\ p = \frac{4k}{(k+1)^2}$$

$$\text{and k} = \frac{K_a}{K_b}$$

Where:

T: Transmission factor across the shield $E^Z/E^i = H^z/H^i$

R (dB): Combined, one-pass, reflection losses at input and output interfaces

A (dB): Total one-pass absorption loss through the shield

B (dB): Factor to correct for multiple re-reflections within shield barrier

Reflection Loss Factor, R

The Reflection Factor, R, is the combined input and output 1-pass reflection loss

$$R = 20\log\left|\frac{1}{p}\right| = 20\log\frac{|k+1|^2}{4|k|}$$

$$\text{For } k = \frac{K_a}{K_b} >> 1 \text{ (typically)}, R \cong 20\log\frac{|k|}{4}$$

$$\text{For copper at 1 MHz, } R = 20\log\frac{1}{4(9.788)*10^{-7}} = 108.14dB$$

For shields with thickness greater than about 1 skin depth, the net reflection loss will be nearly as great as R.

For shields with thickness less than about 1 skin depth, the net reflection loss will be reduced below R, because of multiple reflections; this is included in correction factor, B.

Absorption Loss Factor, A

Absorption loss, A, is determined by the real part, α (Nepers/m), of the propagation constant, $\Gamma=\alpha+j\Beta = \alpha+j\alpha$, in the shield. In a shield of thickness, l,:

$$A(dB) = 8.686\alpha l(dB)$$

For copper at 1 MHz,

$A=1.513*10^4$ Nepers/m=3.336 dB/mil

For thin shields (less than about 1 skin-depth), the net absorption loss, while small, will be larger than the 1-pass loss, A, due to the relatively significant amplitudes of the multiple re-reflections; this is included in the correction factor B.

Correction Factor B

$$B(dB) = 20\log\left|1 - \frac{(k-1)^2}{(k+1)^2}e^{-2\Gamma l}\right|$$

$$\text{With } k >> 1, \frac{(k-1)^2}{(k+1)^2} \cong 1$$

$$\text{Then, } B(dB) \cong 20\log\left|1 - e^{-2\Gamma l}\right|$$

With a thick shield, say 2 skin-depths thick, the exponential factor in the expression goes to 0.018 and the factor within the absolute value brackets reduces to about 0.982 (so, B=-0.158 dB).

With a very thin shield, the factor within the absolute value brackets reduces to $|2\Gamma l|$. So with a copper shield with thickness of, say, 1.724×10^{-8} m (2.609×10^{-4} skin depths at 1 MHz), B= -62.64 dB.
Note, the shield thickness chosen is much less than a skin depth and its DC resistance is 1 Ohm/square.

With large absorption loss, the correction factor is small;
With small absorption loss, the correction factor is large

Low Loss and High Loss Cases

Table 2 High loss/low loss summary at 1 MHz

	Thickness (Skin Depths) (Note 1)	Thickness (mils)	R (dB)	A (dB)	B (dB)	Net S (dB)
Thick shield	2	5.21	108.14	17.37	-0.158	125.36
Thin Shield	$2.609*10^{-4}$	$6.793*10^{-4}$ (Note 2)	108.14	0.0023	-62.64	45.50

Notes:

1. At 1 MHz, 1 skin-depth $=6.609*10^{-5}$ m= 2.604 mils
2. Sheet resistance at this thickness: $6.793*10^{-4}$ mils = $1.24*10^{-8}$ m is $R_S=1/lg=1/(1.24*10^{-8}*5.8*10^{7}=1$ Ohm/Square

Reduction to Low-Loss Case

H - field shielding :

$$T_H = \frac{H^Z}{H^i} = \frac{p_H e^{-\Gamma_b l}}{1 - q_H e^{-2\Gamma_b l}} = \frac{p_H}{e^{\Gamma_b l} - q_H e^{-\Gamma_b l}}$$

$$= \frac{4K_a K_b}{(K_a + K_b)(Z + K_b)} * \frac{1}{e^{\Gamma l} - \frac{(K_a - K_b)(Z - K_b)}{(K_a + K_b)(Z + K_b)} e^{-\Gamma_b l}} = \frac{4K_a K_b}{(K_a + K_b)(Z + K_b) e^{\Gamma_b l} - (K_a - K_b)(Z - K_b) e^{-\Gamma_b l}}$$

For low loss case, $\Gamma_b l << 1,\ e^{\Gamma_b l} \cong 1 + \Gamma_b l$. Note that $\Gamma_b l / K_b = R_S$, the DC resistance of the shield in Ohms/sq; also, typically, $K_b << K_a, Z$, then :

$$T_H = \frac{4}{2 + \frac{2Z}{K_a} + \frac{2Z}{R_S} + \frac{K_b^2}{R_S K_a}} \cong \frac{2}{1 + \frac{Z}{K_a R_S}(K_a + R_S)} \quad \text{also,}\ T_E = \frac{2}{1 + \frac{K_a}{Z R_S}(Z + R_S)}$$

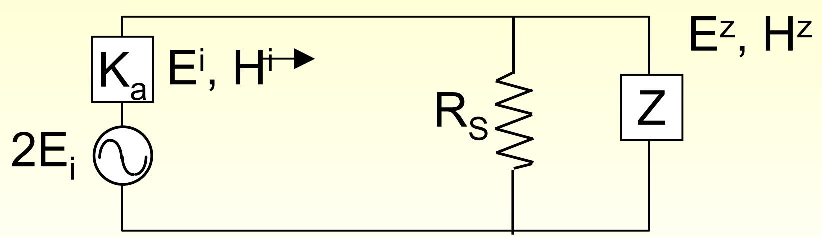
With $Z = K_a, T_H = T_E = \dfrac{2}{2 + \dfrac{K_a}{R_s}}$

Example :

$K_a = 377, R_b = 1, \therefore T_H = T_E = \dfrac{2}{2 + 377} = 0.00528 = 45.5dB$ (compare with Table 2)

Low Loss Lumped Element Model

Shield is approximated by its DC resistance.



<u>H-Field Shielding</u>

$$T_H = \frac{H^Z}{H^i} = \frac{\dfrac{2E_i}{K_a + \dfrac{R_S Z}{R_b + Z}} \dfrac{R_S Z}{R_S + Z} \dfrac{1}{Z}}{E \big/ K_a}$$

$$= \frac{2}{1 + \dfrac{Z}{K_a R_S}(K_a + R_S)}$$

<u>E-field Shielding</u>

$$T_E = \frac{E^Z}{E^i} = \frac{\dfrac{2E_i}{K_a + \dfrac{R_S Z}{R_S + Z}} \dfrac{R_S Z}{R_S + Z}}{E_i}$$

$$= \frac{2}{1 + \dfrac{K_a}{Z R_S}(Z + R_S)}$$

This is an exact model for the low loss case on previous slide

Summary

- Background for the shielding equations was reviewed
- General shielding equation was derived from the multiple re-reflection model
- General equation rearranged into familiar R-A-B form
- The R-A-B parameters were reviewed and sample values were presented
- The high-loss case was shown to be reducible to R+A
- For the low-loss case the general equation reduced to that for a lump element model

Conclusions

- The general shielding equations are derivable from the multiple re-reflection model
- The correction factor, B, corrects for multiple re-reflections in the shield
- In high-loss case, the total shielding is the sum of the single-pass input and output reflection losses, and the single-pass transmission line attenuation
- In low-loss case the shielding equations reduce to a lumped model with the shield replaced by its DC resistance

Bibliography

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2. R. B. Schulz, et al, “Shielding Theory and Practice,” IEEE Trans. On EMC, vol 30, Issue 3, August 1988, pp187-201. This is a very comprehensive ant detail treatment of the Schelkunoff approach to shielding.

SPEAKER BIOGRAPHY

Richard Mohr has over 30 years in his specialization in Electromagnetic Compatibility (EMC). In 1984 he formed R. J. Mohr Associates, Inc. and is its President and Chief Consultant. The company provides design support to client companies in EMC and in related areas. He has published widely on design, prediction, and test techniques in EMC disciplines. His classic papers in the 60's on cross coupling between open and shielded lines over a ground plane provided the basis for follow-on efforts in this important case ever since. His unique model for explaining the shielding action of wire shields is used almost universally in analyses and in training seminars. The current presentation is extracted from his training seminar "Getting Your Product Into EMC Compliance".

He is a Professional Engineer registered in NY State, and a NARTE-Certified EMC Engineer. The IEEE has recognized his contributions in EMC modeling and design with several awards, including the prestigious Stoddart Award for his work in cabling. In 1996, he was elected to the grade Fellow of the IEEE, and cited by the Board of Directors of the IEEE "For the advancement of practical models for application in the electromagnetic compatibility design of electronic equipment". In 2004 the EMC Society of the IEEE elevated him to the grade of Honorary Life Member.