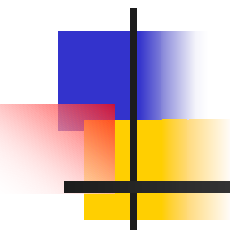


Oscillator Oddities: The art of oscillator design, and its impact on system performance



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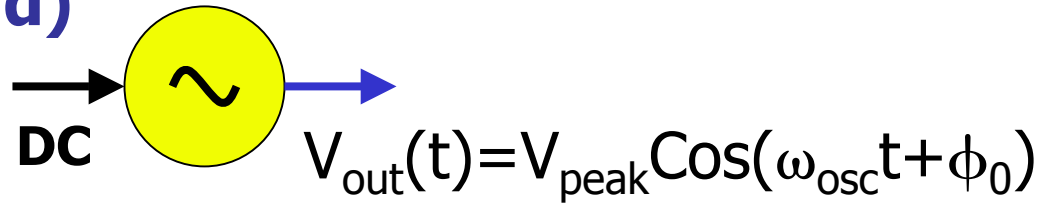
Oscillator Design Concepts and Specifications - Topics

“While oscillators have been around for a long time, oscillator design is complex and sometimes still mystifying.”

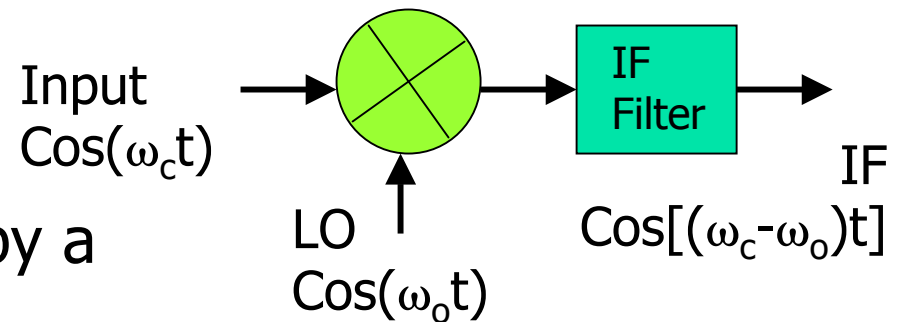
- **Time**
- **Oscillator Design Basics**
- **Oscillator Specifications**
 - **Frequency Stability**
 - **Spurious & Parasitic Oscillations**
 - **Pulling (VSWR effects)**
 - **Phase Noise**
- **Effects of Phase Noise**
- **Phase Noise & Error Probability**
- **Jitter -**

Time: Oscillators are Timing Devices

(Cycles / Second)



- DC Input – AC Output
- Uses
 - Timing
 - Synchronization
 - Frequency Translation of Information
 - Modulated Carrier
 - Local Oscillator
- Frequency is determined by a Resonant Circuit
- Oscillator Configurations
 - Negative Resistance Oscillator
 - Feedback Oscillator -





Timing in Seconds. What is a Second?

- Standard Interval (SI) unit of time: **Second**
- Prior to 1967 unit of time was based on astronomical observations
 - Second was "1/31,556,925.9747 of the tropical year..."
- Redefined, in October, 1967, at the XIII General Conference of Weights and Measures.
 - Second is "9,192,631,770 transitions between the two hyperfine levels of the ground state of the cesium atom 133."
 - Hyperfine levels are when the Electrons & nucleus magnetic moments align in parallel or anti-parallel -

Accuracy of Time Measurements

Time Period	Clock/Milestone	Accuracy Per Day
4th millennium B.C.	Day & night divided into 12 equal hours	
Up to 1280 A.D.	Sundials, water clocks	~1 h
~1280 A.D.	Mechanical clock invented- assembly time for prayer was first regular use	~30 min
14th century	Clock making becomes a major industry Hour divided into minutes and seconds	~15 min
~1345	Clock time used to regulate people's lives (work hours)	
15th century	Time's impact on science becomes significant	~2 min
16th century	(Galileo times physical events, e.g., free-fall) First pendulum clock (Huygens)	~1 min
1656	Temperature compensated pendulum clocks	~10 s
18th century	Electrically driven free-pendulum clocks	1 s
19th century	Wrist watches become widely available	10-1 s
~1910 to 1920	Electrically driven tuning forks	10-3 s
1920 to 1934	Quartz crystal clocks (and watches)	10-5 s
1949 to 1955	Atomic clocks	10-9 s
1955 to 1967	Cesium Atomic Clock at the National Physical Laboratory	a second in 300 years
1967 to present	Cesium clocks measure frequency with an accuracy of from 2 to 3 parts in 10^{14} 2 nanoseconds per day	one second in 1,400,000 years

Navigation Drove Accurate Timing

- Principal motivator in man's search for better clocks
- Latitude
 - Even in ancient times, measured by observing the stars' positions
- Longitude, the problem became one of timing
 - Earth makes one revolution in 24 hours
 - Can be determined from the time difference between local time (which was determined from the sun's position) and the time at the Greenwich meridian (which was determined by a clock):
 - **Longitude in degrees = (360 degrees/24 hours) x t in hours.**
 - In 1714, the British government offered a reward of 20,000 pounds to the first person to produce a clock that allowed the determination of a ship's longitude to 30 nautical miles at the end of a six week voyage (i.e., a clock accuracy of three seconds per day).
 - Englishman John Harrison won the competition in 1735 for his chronometer invention, a **spring-driven clock**
 - The moving parts are controlled and counterbalanced by springs so that, unlike a pendulum clock, **H1 is independent of the direction of gravity.** -



The First Oscillator

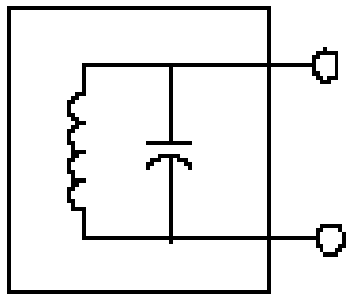
- First radio transmitter used a spark between two nodes to generate RF.
- Generated a large range of frequencies
- Only suitable to send coded messages i.e. Morse code
- Now illegal because of the large bandwidth used.
- Still causes interference in cars. -

Oscillator Design Basics

- Negative Resistance Oscillators
- Feedback Oscillators

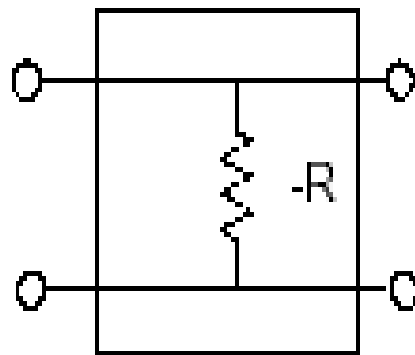
Negative Oscillators Basic Configuration

Resonator
Circuit



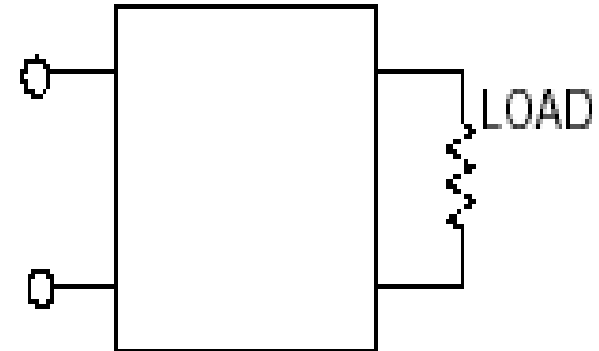
YIG Resonator or
Varactor Tuned
Circuit

Active
Circuit



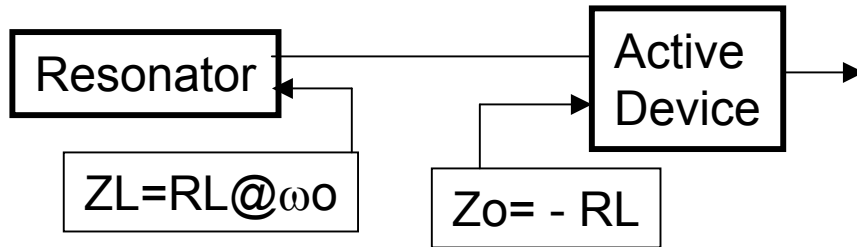
Transistor, Tunnel
Diode, Gunn
Diode, etc.

Output
Network



Passive Matching
Ckt & Buffer
Amplifier -

Theory of Negative Resistance Oscillators



Reflection coefficient

$$\rho := \frac{V_r}{V_i}$$

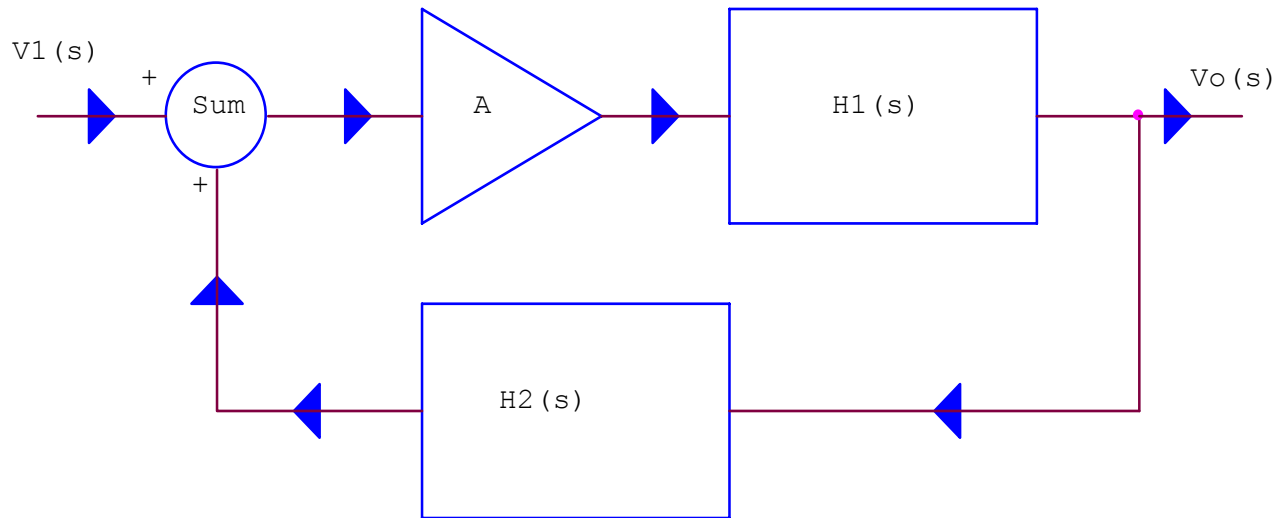
$$\rho := \frac{(Z_L - Z_o)}{Z_L + Z_o}$$

Resonator is a One port network

- at Resonance (F_o)
 - Z_L is real only at the resonant frequency ($Z_L(F_o)$)
 - $Z_L(F_o) = -Z_o$
 - Result: Reflected voltage without an incident voltage (oscillates)
- An Emitter Follower is a classic negative resistance device
- Technique used at microwave frequencies
 - Spacing between components often precludes the establishment of a well defined feedback path. -

Feedback Oscillators (Two port networks)

Feedback Model.



$$(V1 + V0 * H2) * A * H1 = V0$$
$$V1 * A * H1 = V0(1 - A * H1 * H2)$$

$$\frac{V0}{V1} := \frac{(A \cdot H1(s))}{1 - A \cdot H1(s) \cdot H2(s)}$$

- **$A * H1(s) * H2(s) = \text{open loop gain} = AL(s)$** -

Barkhausen Criteria

- Barkhausen criteria for a feedback oscillator

- open loop gain = 1
- open loop phase = 0

- $|A \cdot H_1(s) \cdot H_2(s)| = |AL(s)| = 1$

- $\text{Angle}(A \cdot H_1(s) \cdot H_2(s)) = 0$

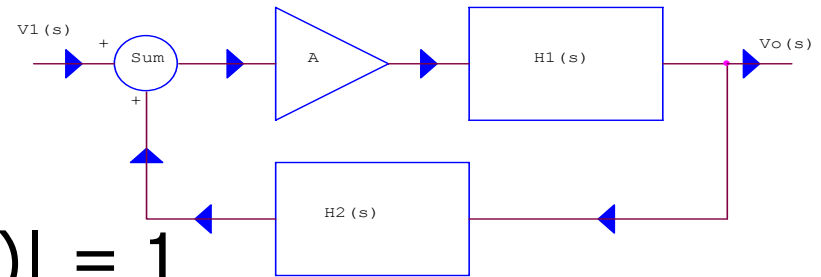
- $s = \omega_o$ (for sinusoidal signals)

- $\text{Re } AL(\omega_o) = 1$

- $\text{Im } AL(\omega_o) = 0$

- Transfer function blows up (Output with no Input)

- V_o is finite when $V_1 = 0$ -

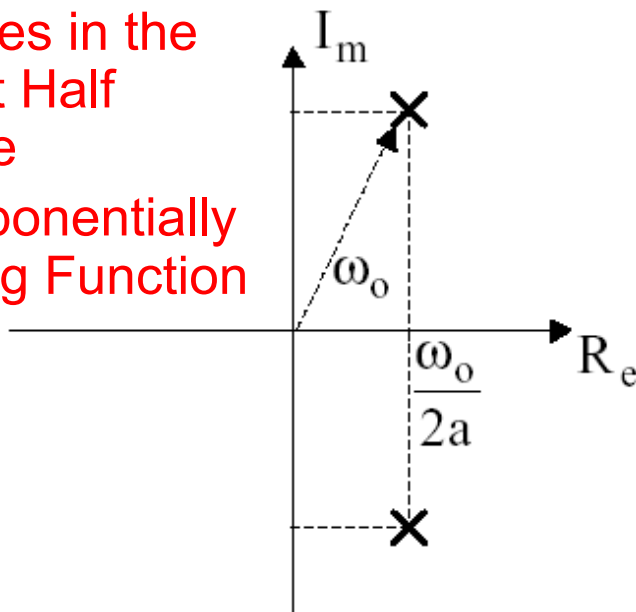


$$\frac{V_o}{V_1} := \frac{(A \cdot H_1(s))}{1 - A \cdot H_1(s) \cdot H_2(s)}$$

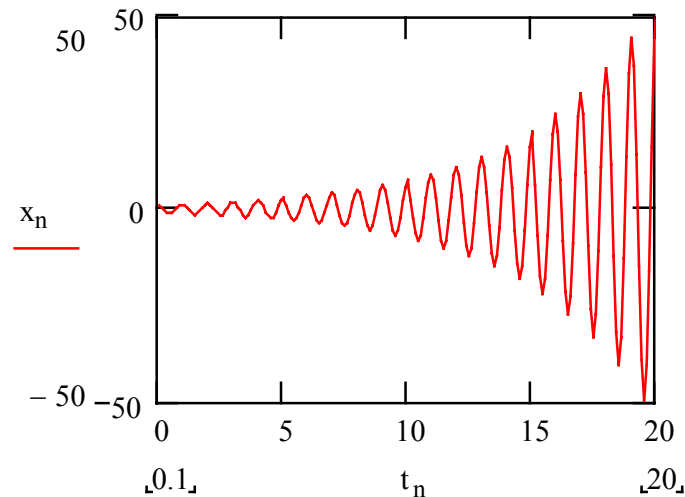
Designing conditions for Start-Up

- To start an oscillator it must be triggered
 - Trigger mechanism: Noise or a Turn-On transient
- Open loop gain must be greater than unity
- Phase is zero degrees (exponentially rising function)

- Poles in the Right Half Plane
- Exponentially Rising Function

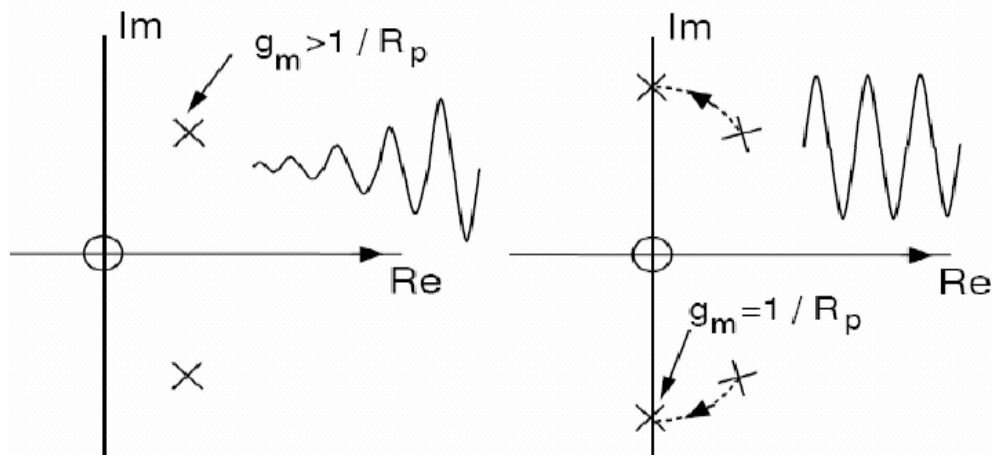


$$x_n := e^{\alpha \cdot t_n} \cdot \cos(2 \cdot \pi \cdot \omega \cdot t_n)$$



$$\alpha = \text{Real Part of } A \cdot H1(s) \cdot H2(s), \quad \alpha > 1$$

Amplitude Stabilization



- As amplitude increases Gain decreases the effective g_m (transconductance gain) is reduced
- Poles move toward the Imaginary axis
- Oscillation amplitude stabilizes when the poles are on the imaginary axis
- Self correcting feedback (variable g_m) maintains the poles on the axis and stabilizes the amplitude -

Oscillator Amplitude is Not Random

$$I_e := I_{ES} \cdot \left(e^{\frac{V_{be}}{V_t}} - 1 \right) \quad V_t := \frac{(k \cdot T)}{q}$$

$$V_{be} = V_{be_{DC}} + V_{be_{AC}}$$

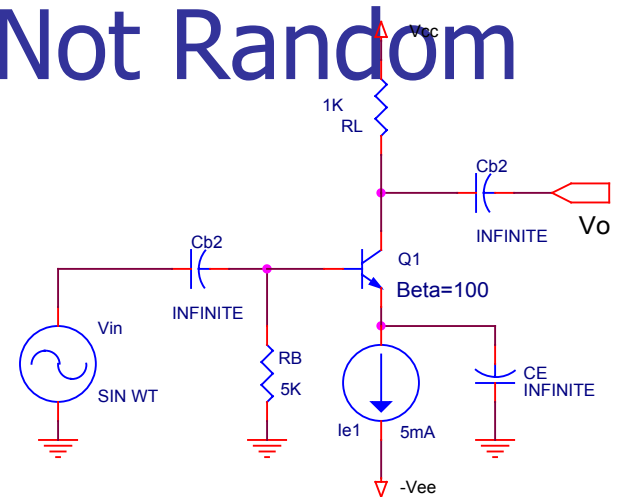
$$i_C := I_{ES} \cdot e^{V_{BE} \cdot \left(\frac{q}{k \cdot T} \right)} \cdot e^{V_1 \cdot \left(\frac{q}{k \cdot T} \right) \cdot \cos(\omega \cdot t)}$$

$V_1 = V_p =$ Peak value of AC component

$$x := \frac{V_1}{V_t}$$

$$i_C := I_{ES} \cdot e^{V_{BE} \cdot \left(\frac{q}{k \cdot T} \right)} \cdot e^{(x) \cdot \cos(\omega \cdot t)}$$

$$I_c := \alpha \cdot I_{eq} \cdot \left(e^{x \cdot \cos(\omega \cdot t)} \right)$$



$$q := 1.6021773310^{-19} \cdot \text{coul}$$

$$T := 298 \cdot \text{K}$$

$$k := 1.38065810^{-23} \cdot \frac{\text{joule}}{\text{K}}$$

$$V_t = 0.026V$$

q = charge of an electron
 T = Temperature in degrees Kelvin
 K = Boltzman's Constant

Fourier Expansion of Collector Current

$$I_c := \alpha \cdot I_{eq} \cdot \left(e^{x \cdot \cos(\omega \cdot t)} \right)$$

$e^{x \cos(\omega t)}$ has a Fourier expansion = $I_0(x) + 2 \sum I_n(x) \cos(n\omega t)$
 Σ is from 1 to ∞

$I_n(x)$ is a modified Bessel function of the first kind of order n and argument x

$$I_n := \frac{1}{2 \cdot \pi} \cdot \int_{-\pi}^{\pi} e^{x \cdot \cos(\theta)} \cdot \cos(n \cdot \theta) d\theta$$

- $I_c \approx I_{eq} \cdot (I_0(x) + 2 \sum I_n(x) \cos(n\omega t))$
- **Note: I_0 modifies the DC Current -**

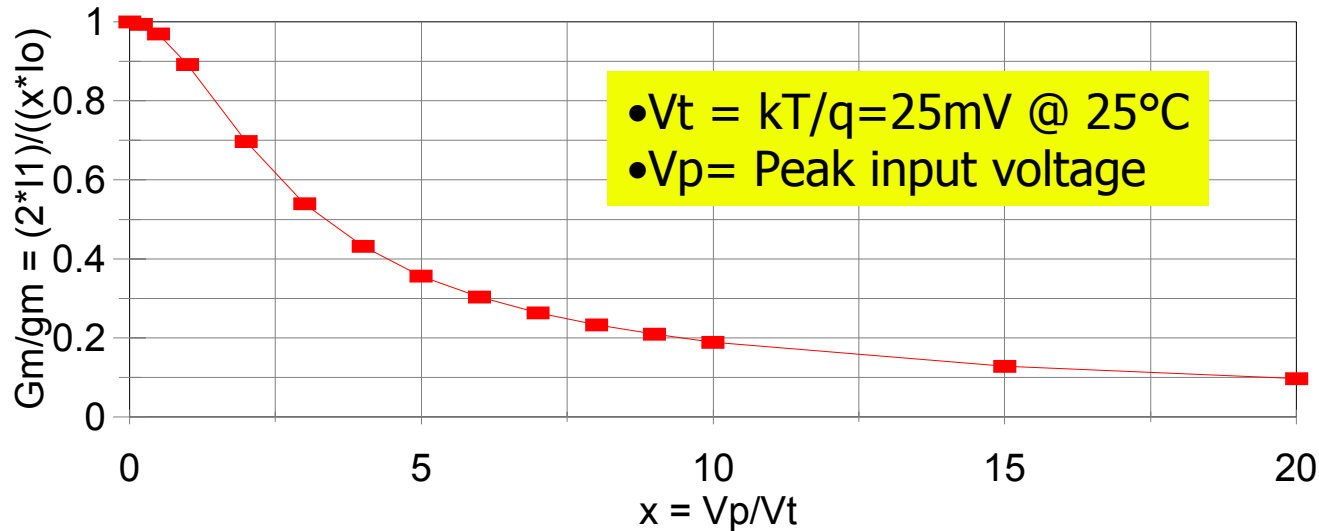


Large Signal Transconductance Gain - G_m

- Large signal transconductance (Fundamental)
 $\text{gain} = G_m = [g_m * ((2 * I_1) / I_0)] / x$
- I_0 & I_1 are zero order & 1st Order Bessel Functions of the first kind with argument $x = V_p / V_t$
- V_p is the Peak voltage of the sinusoidal signal at the Base-Emitter junction [$V_p \sin(2 \pi F_o t)$]
- $V_t = kT/q$
 - k = Boltzman's Constant
 - T = Temperature in Degrees Kelvin
 - q = Charge on an electron
- g_m = Small signal transconductance gain
 - $g_m = I_{eq} / V_t$ (I_{eq} = Quiescent emitter current) -

Large Signal vs Small Signal Gain as a Function of x

Normalized Transconductance Gain vs x
 G_m/g_m vs $x=V_p/V_t$



x = Vp/Vt	Gm/gm (2*I1)/(x*I0)
0	1
0.2	0.995
0.5	0.97
1	0.893
2	0.698
3	0.54
4	0.432
5	0.357
6	0.304
7	0.264
8	0.234
9	0.21
10	0.19
15	0.129
20	0.0975

- ∴ Large signal transconductance gain = $G_m = [g_m * ((2 * I_1) / I_0)] / x$
- Gain compression ratio for increasing signal = G_m/g_m
 - $G_m/g_m = [((2 * I_1) / I_0)] / x = [((2 * I_1) / I_0)] / (V_p / V_t)$
 - Self Correcting Amplitude: V_p (Signal) goes up, gain G_m goes down,
- Note that the Output signal amplitude is a function of $I_{eq} = g_m * V_t$
- Higher I_{eq} higher output amplitude -

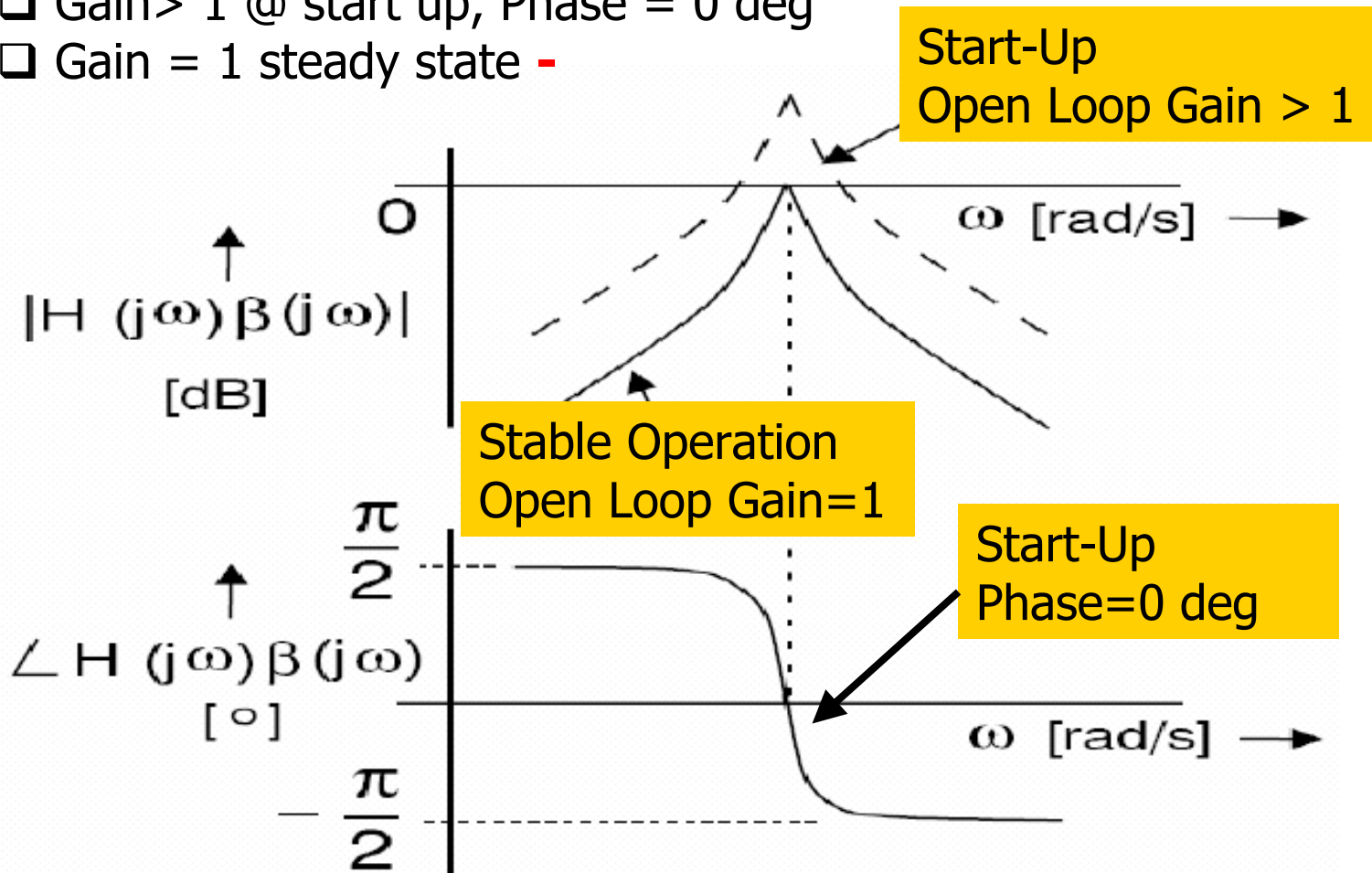
Table A-3 Tabulation of $2I_n(x)/I_0(x)$ vs. x for $n = 1, 2, 3, 4, 5$

x	$2I_1(x)/I_0(x)$	$2I_2(x)/I_0(x)$	$2I_3(x)/I_0(x)$	$2I_4(x)/I_0(x)$	$2I_5(x)/I_0(x)$
0.0	0.0	0.0	0.0	0.0	0.0
0.5	0.4850	0.0600	0.0050	0.0003	0.0000
1.0	0.8928	0.2144	0.0350	0.0043	0.0004
1.5	1.1923	0.4103	0.0981	0.0179	0.0026
2.0	1.3955	0.6045	0.1866	0.0445	0.0086
2.5	1.5300	0.7760	0.2884	0.0839	0.0200
3.0	1.6200	0.9200	0.3933	0.1335	0.0374
3.5	1.6822	1.0387	0.4951	0.1900	0.0607
4.0	1.7270	1.1365	0.5906	0.2506	0.0893
4.5	1.7607	1.2175	0.6785	0.3129	0.1222
5.0	1.768	1.2853	0.7585	0.3751	0.1584
5.5	1.8076	1.3427	0.8311	0.4360	0.1970
6.0	1.8247	1.3918	0.8969	0.4949	0.2370
6.5	1.8390	1.4342	0.9564	0.5513	0.2779
7.0	1.8511	1.4711	1.0104	0.6050	0.3189
7.5	1.8615	1.5036	1.0595	0.6560	0.3598
8.0	1.8705	1.5324	1.1043	0.7042	0.4001
8.5	1.8784	1.5580	1.1452	0.7497	0.4396
9.0	1.8854	1.5810	1.1827	0.7926	0.4782
9.5	1.8916	1.6018	1.2172	0.8330	0.5157
10.0	1.8972	1.6206	1.2490	0.8712	0.5520
10.5	1.9022	1.6377	1.2784	0.9072	0.5872
11.0	1.9068	1.6533	1.3056	0.9412	0.6211
11.5	1.9110	1.6677	1.3309	0.9733	0.6538
12.0	1.9148	1.6809	1.3545	1.0036	0.6854
12.5	1.9183	1.6931	1.3765	1.0324	0.7157
13.0	1.9215	1.7044	1.3970	1.0596	0.7450
13.5	1.9244	1.7149	1.4163	1.0854	0.7731
14.0	1.9272	1.7247	1.4344	1.1099	0.8002
14.5	1.9298	1.7338	1.4515	1.1332	0.8262
15.0	1.9321	1.7424	1.4675	1.1554	0.8513
15.5	1.9344	1.7504	1.4827	1.1765	0.8754
16.0	1.9365	1.7579	1.4970	1.1966	0.8987
16.5	1.9384	1.7650	1.5105	1.2158	0.9211
17.0	1.9403	1.7717	1.5234	1.2341	0.9426
17.5	1.9420	1.7781	1.5356	1.2516	0.9634
18.0	1.9436	1.7840	1.5472	1.2683	0.9835
18.5	1.9452	1.7897	1.5582	1.2843	1.0028
19.0	1.9466	1.7951	1.5687	1.2997	1.0215
19.5	1.9480	1.8002	1.5788	1.3144	1.0395
20.0	1.9493	1.8051	1.5883	1.3286	1.0569

Tabulation of the Bessel Function Solution

Bode Diagram is Used to Assess Oscillation Conditions

- Gain > 1 @ start up, Phase = 0 deg
- Gain = 1 steady state -



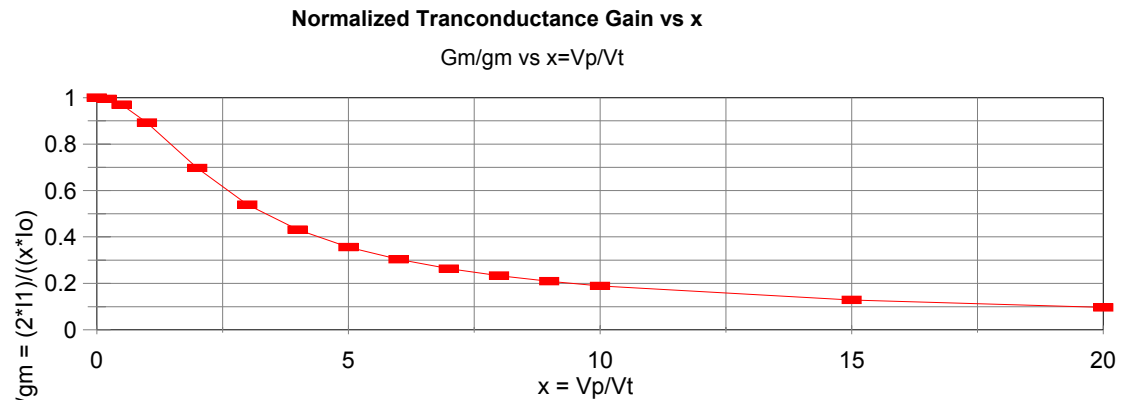
$$H(j\omega)\beta(j\omega) = \text{Open Loop Gain}$$

Oscillator Amplitude

Open Loop Gain (Small Signal (g_m)) > 1

Oscillation level increases until the open gain = 1 (@ G_m)

- Look at the graph G_m/g_m & find x
- $X = V_p/V_t$
- Calculate V_p
- $V_p * G_m =$ Peak value of Collector current (I_{cp})
- Oscillator Output Voltage = $I_{cp} * R_L * \cos(\omega t)$
- Oscillator harmonics are the higher order Bessel functions attenuated by the rejection of the collector resonant circuit

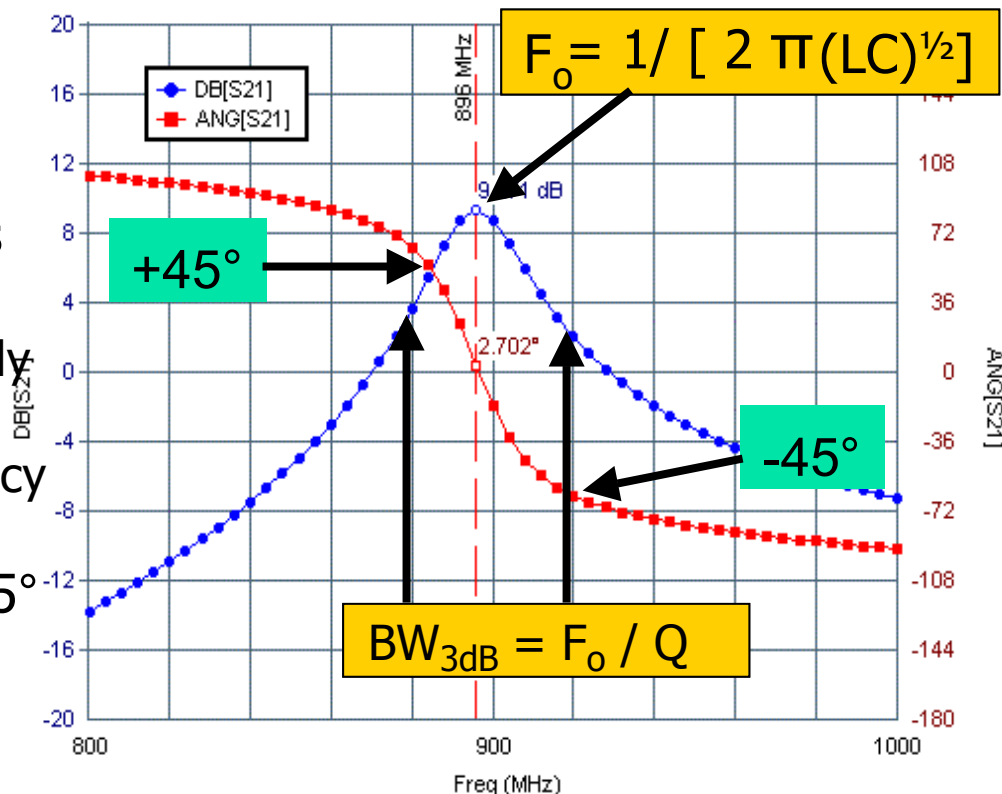


Oscillator Specifications

Frequency Stability

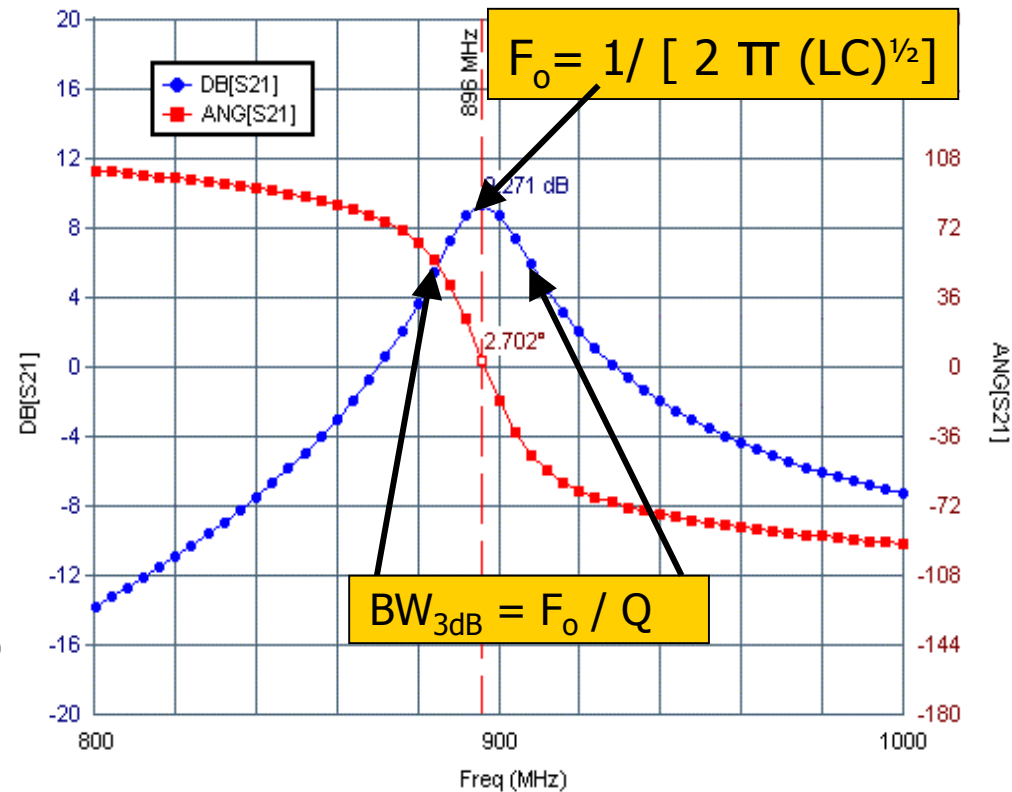
Conditions for Oscillation

- Sufficient gain in the 3 dB bandwidth (Open Loop Gain > 1)
- Components around the loop are real (Resistive, Zero Phase)
- Circuit oscillates at resonance $\omega_0 = 1/(LC)^{1/2} = 2*\pi* F_0$
- **Coarse frequency** of oscillation is determined by the resonant frequency - **Amplitude**
- **Fine Frequency** of oscillation is determined by **PHASE**
 - Loop phase shifts automatically compensated
 - Phase changes forces frequency off of F_0
- 3 dB bandwidth provides $\pm 45^\circ$ compensating phase -



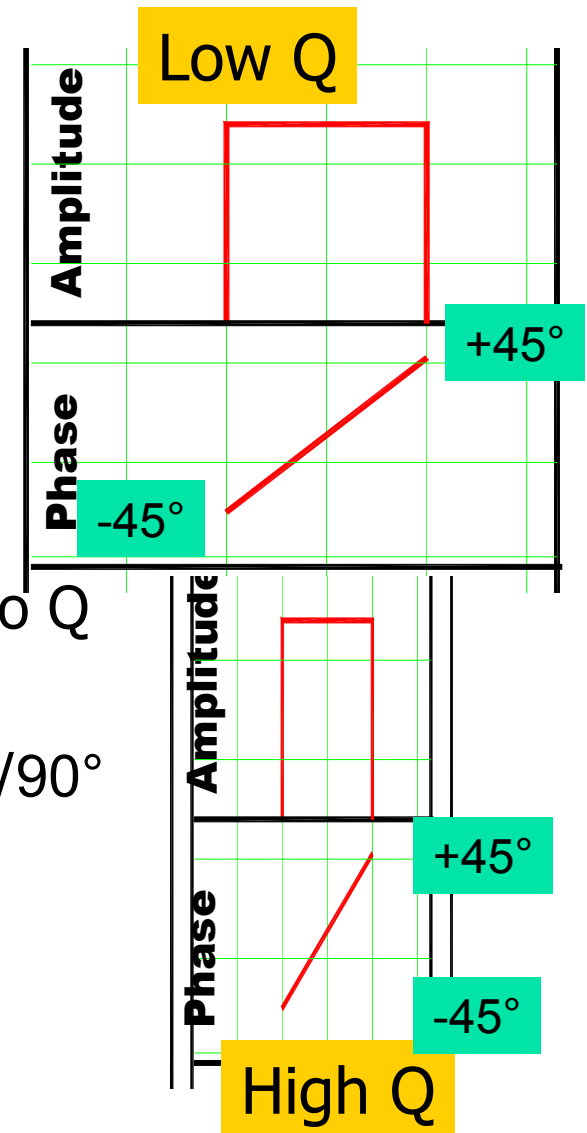
Factors Affecting Oscillator Stability

- Stability of the Resonator
- Q of the resonator
- Causes of Oscillator Frequency Drift
 - Change in resonant frequency
 - Change of Open Loop Phase



Parasitic Phase Shifts vs Frequency Stability

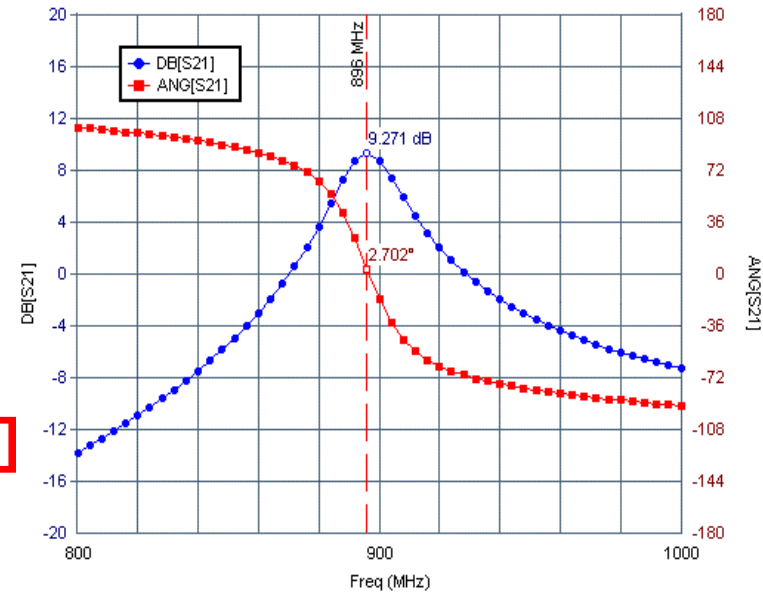
- $Q = F_0/BW_{3dB} \rightarrow BW_{3dB} = F_0/Q$
- 1 Pole Resonant Circuit
 - 3 dB bandwidth shifts $\pm 45^\circ$
- Phase change $(\Delta\phi/Hz) \approx 90^\circ / BW_{3dB}$



- Frequency stability vs Phase is proportional to Q
 - Loop Self Corrects Phase Variations
 - $\Delta F_0 / \Delta\phi \approx BW_{3dB} / 90^\circ \text{ (Hz/Deg)} = [F_0/Q]/90^\circ$
 - **$\Delta F_0 / \Delta\phi \approx F_0 / (Q * 90^\circ) \text{ (Hz/Deg)}$**
 - Higher Q Smaller $\Delta F_0 / \Delta\phi$ (phase)
- Parasitic Phase shifts have less effect on frequency in Higher Q circuits -

Frequency Stability – Resonator Dependent

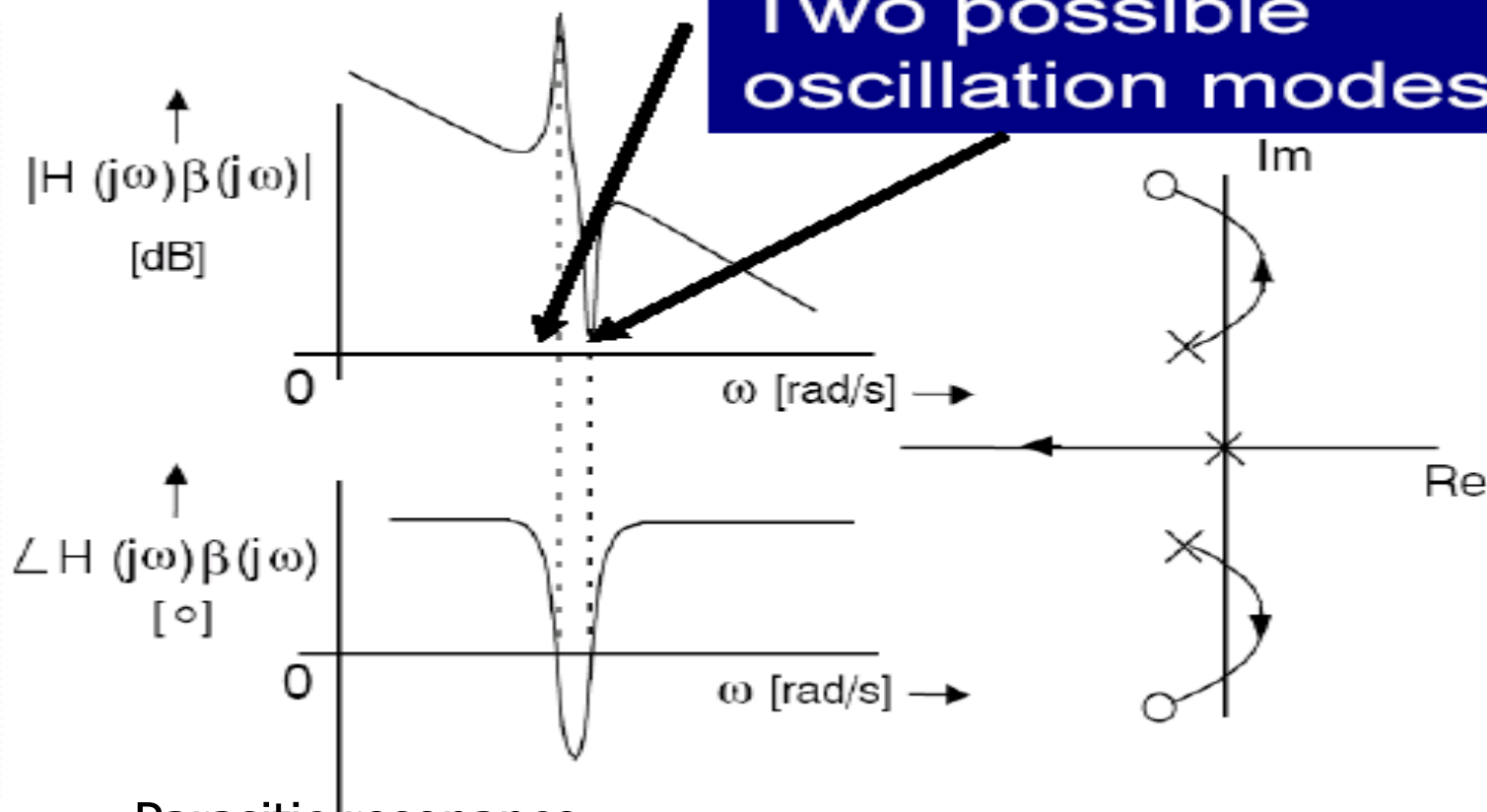
- Center Frequency Resonator (Fo)
- Q of the Resonator
 - Phase Stability (A function of $Q = F_0 / BW_{3dB}$)
- $\Delta F_0 / \odot \phi \text{ (Hz/Deg)} \approx F_0 / [90^\circ Q]$



	Q	Q	Stability
	Min	Max	PPM/C
LC Resonators:	50	150	100
Cavity resonators	500	1000	10
Dielectric resonators:	2,000	10000	1
SAW devices:	300	10000	0.1
Crystals	50000	1000000	0.01

Parasitic Oscillations

Two possible oscillation modes



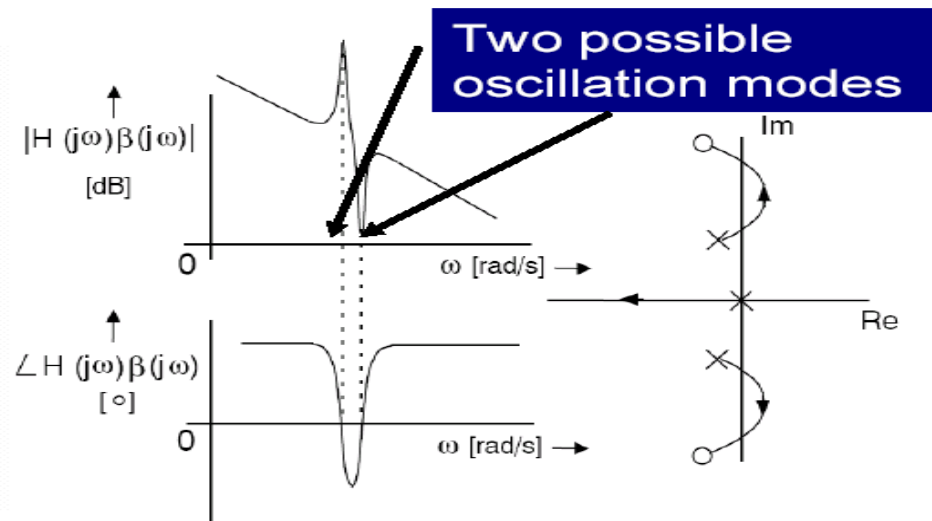
- Parasitic resonance
- Gain > 1 when the phase = 0 Degree
- Poles are in the Right half plane
- Stabilization at 2 points on the imaginary axis
- **Cure: Add a Filter**
 - **Lower the gain at the parasitic frequency -**

Multiple Oscillations

When multiple oscillation conditions coexist multi-oscillation can be present.

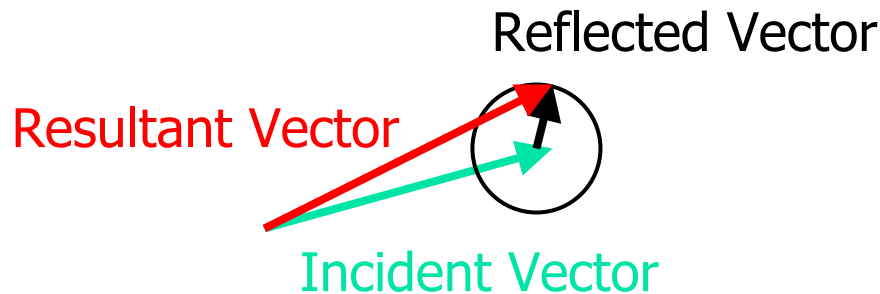
- Usually distorts the desired periodic signal.
- Signal can be useless for most applications

□ Discontinuities in the transfer function must be eliminated -

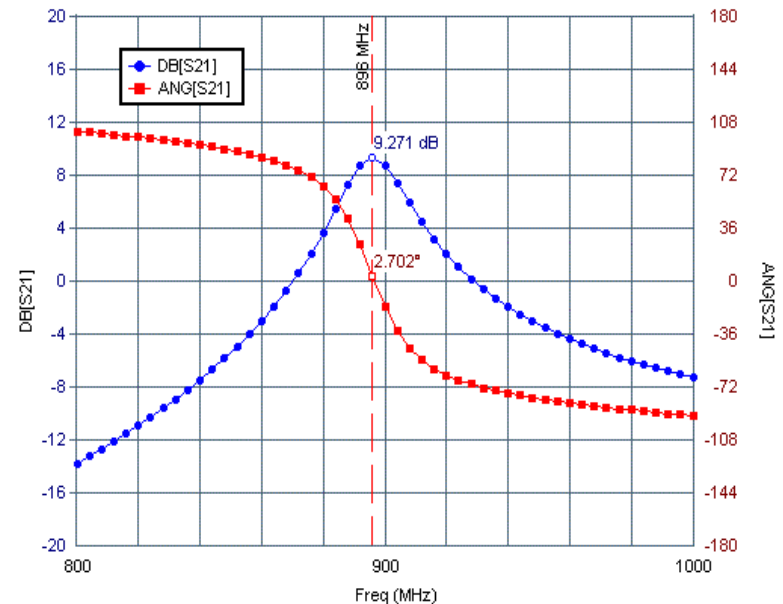


Pulling (VSWR effects)

Change in frequency due to variations in VSWR: Amplitude and/or phase



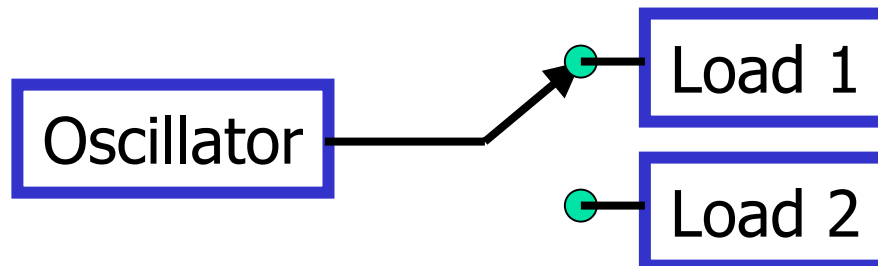
Note that the reflected vector changes the phase of the vector in the oscillator feedback loop



Changing phase moves the frequency within the resonator bandwidth -

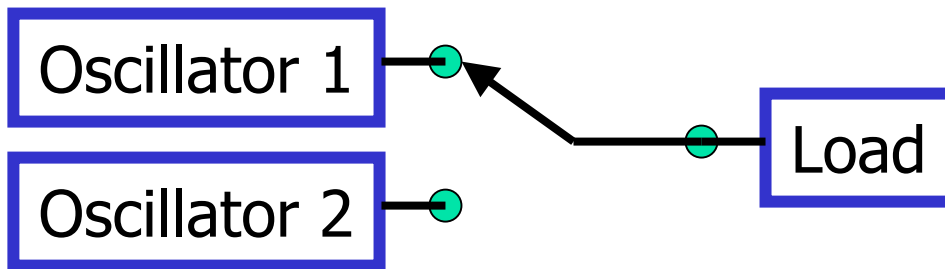
Switching Oscillator Loads

- During switching the VSRW dynamically changes
- Frequency of oscillation could change
- Phase Locked Oscillators
 - Oscillator Frequency changes faster than the loop corrects
 - An oscillator can lose phase lock
- Sufficient isolation during dynamic conditions is required -



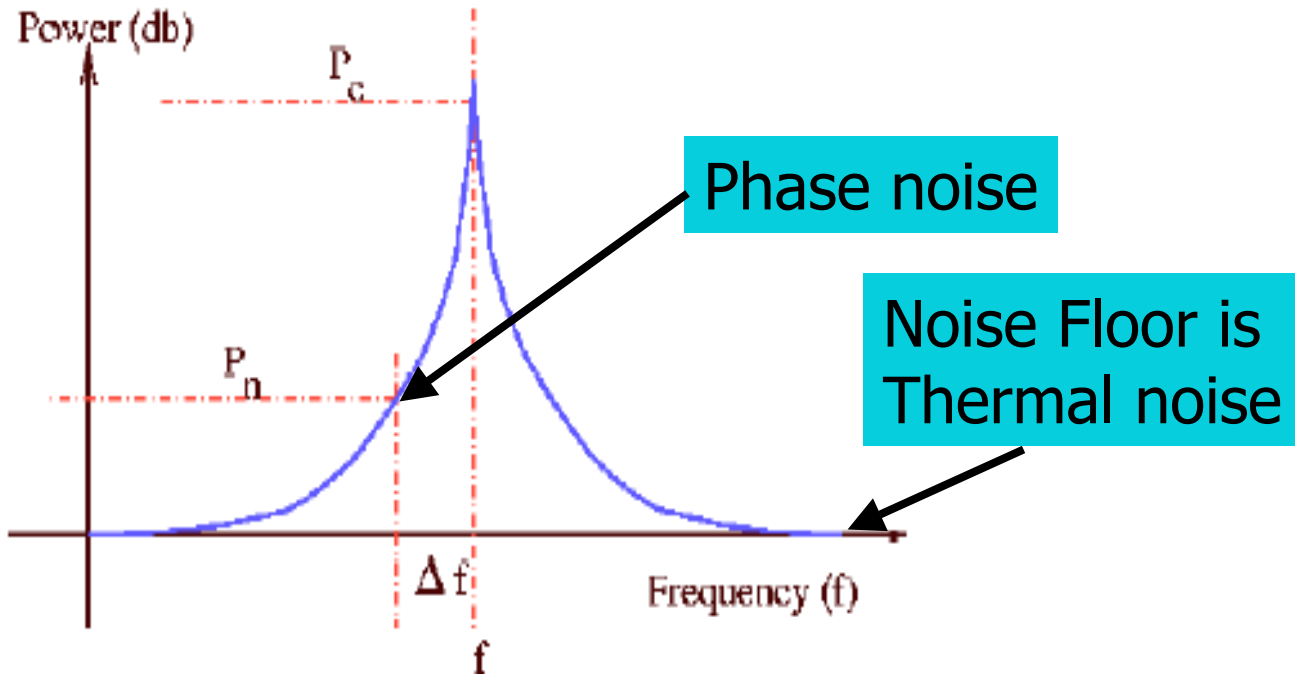
Switched Oscillators

- Load Changes effect oscillator frequency
- Oscillator frequency will not settle until the switching transient is over
- Suggested solutions
 - The used of terminated switches are encouraged
 - Very good isolation between the oscillator and the switch -



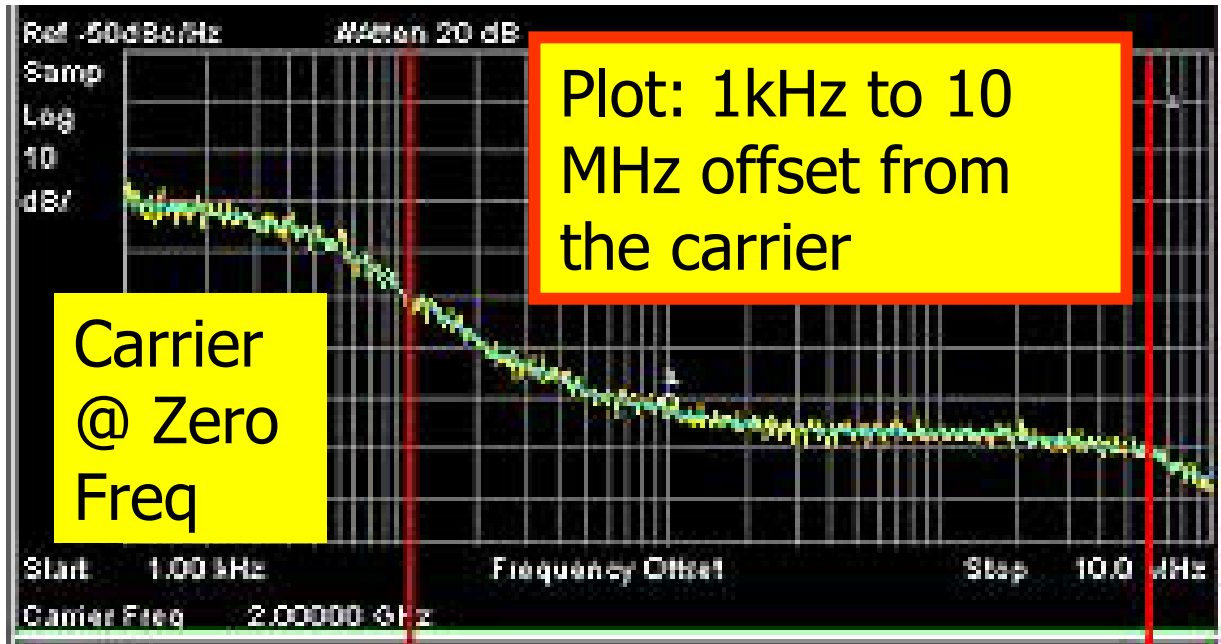
Phase Noise

Phase Noise Spectral Density Function



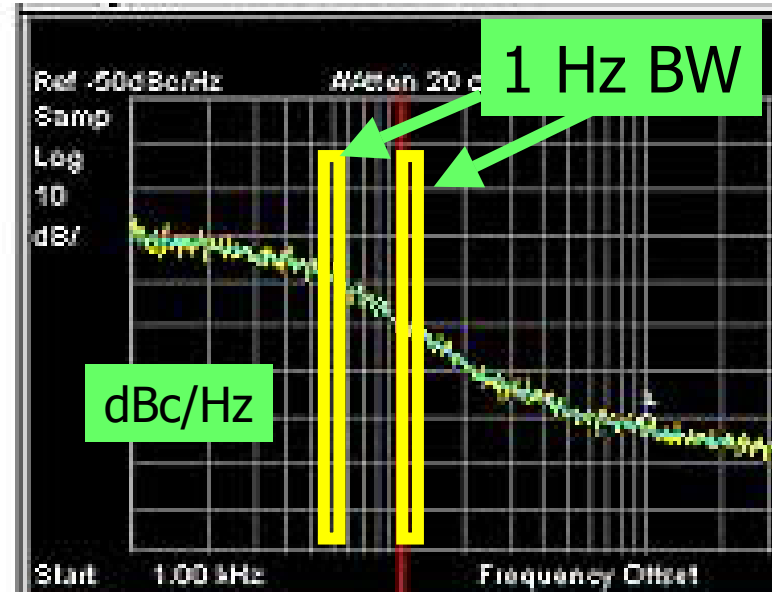
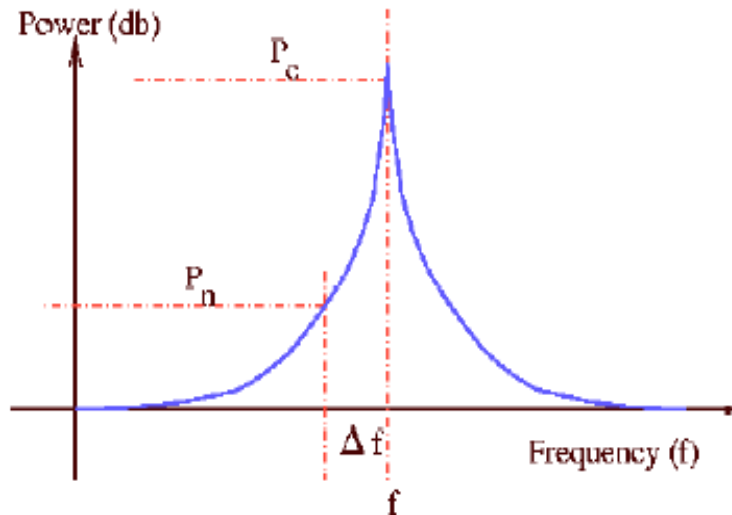
- Phase Noise ($\Delta\Phi_{\text{RMS}}$) is phase (frequency) modulation of the carrier
- Measure of the Sources Spectral Purity
- Close to the carrier Phase Noise is dominant
- Far from the carrier Thermal Noise is dominant -

Phase Noise Spectral Density Function Single Side Band RMS Close to the Carrier



- Carrier is translated to zero frequency to create a carrier null
- Eliminates Spectrum Analyzer Phase Noise -

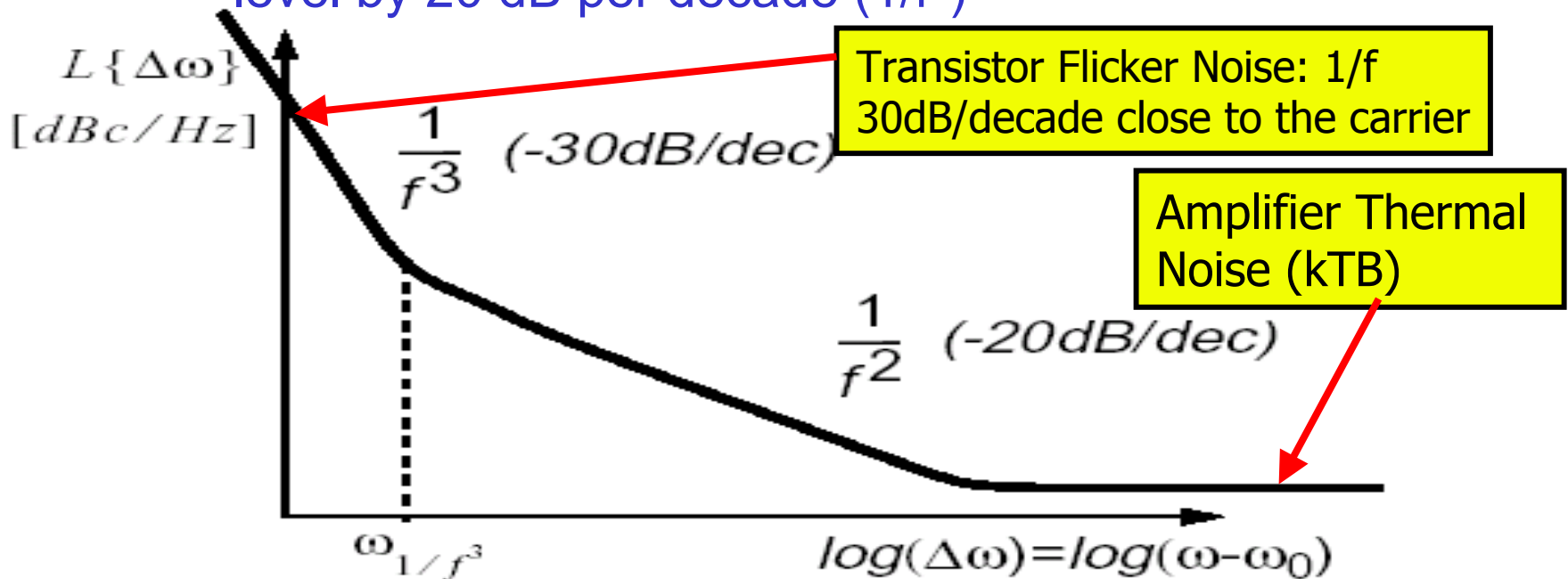
Oscillator Phase Noise



- Each Noise Resolution bandwidth (1Hz in this case) is represented as a modulating signal (Narrow Band FM)
 - Offset from the carrier determines the modulating frequency, F_m
 - sidebands (dBc) = $20 \cdot \text{Log}_{10}(\beta/2)$ for $\beta = \text{small} (<1)$
 - β (modulation index) is phase noise in Radians
- Total Phase noise is the integrated sum over the band of interest -

Power Spectrum of Phase Noise

- ❑ $\beta = \Delta F / F_m$
- ❑ $\beta = \text{small } (<1)$ sidebands (dBc) = $20 \cdot \text{Log}_{10}(\beta/2)$
- ❑ dBc = $20 \cdot \text{Log}_{10}([\Delta F / F_m] / 2)$
- ❑ dBc = $20 \cdot \text{Log}_{10}(\Delta F) - 20 \cdot \text{Log}_{10}(F_m) - 6\text{dB}$
- ❑ If the driving noise spectral density function is constant, ΔF is constant
- ❑ The offset frequency $f = F_m$ decreases the side band level by 20 dB per decade ($1/f^2$) -



Oscillator Phase Noise Characteristics

Phase noise can be estimated by a simplified version of Leeson's equation:

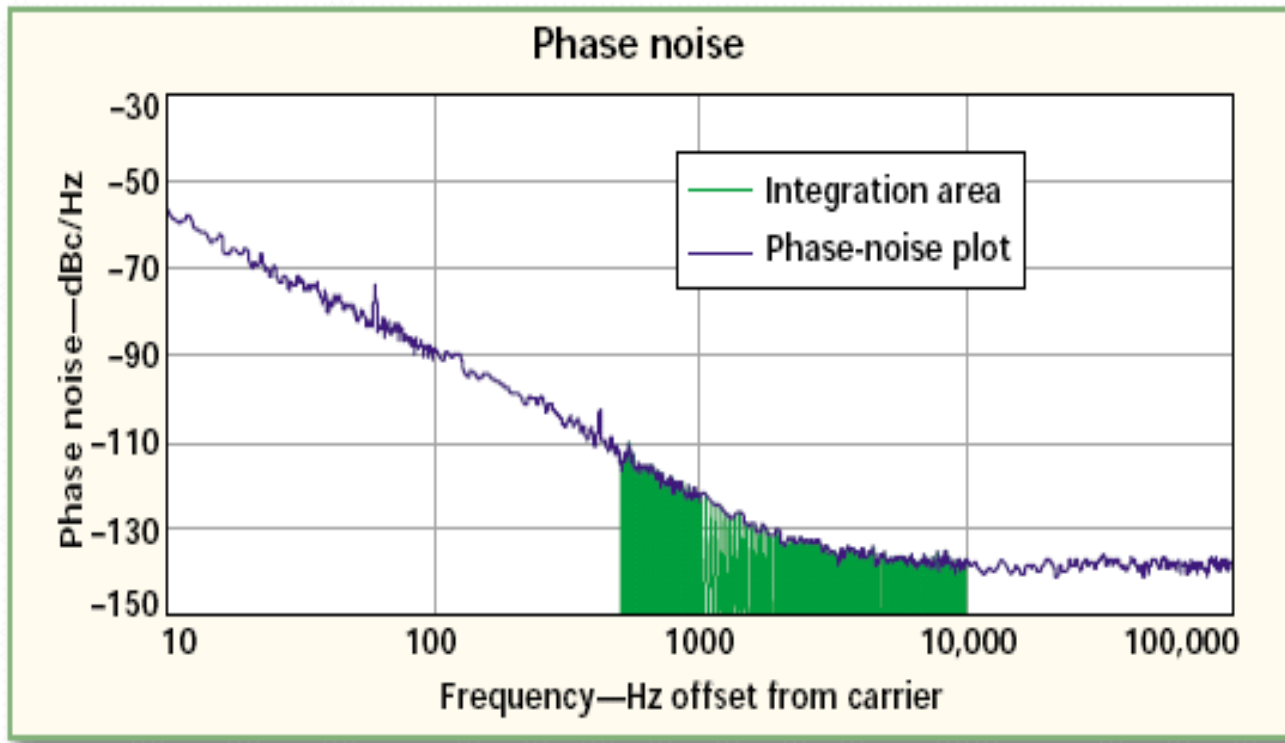
- Qu of the circuit
- P_{sig} is RF Power
- Center Frequency (ω_0)
- Offset from the carrier ($\Delta\omega$)
- K is boltzman's constant
- T is temperature in degree Kelvin
- F is the noise factor of the oscillator amplifier

$$10 \log \left[\frac{2kTF}{P_{sig}} \left(\frac{\omega_0}{2Q_u \Delta\omega} \right)^2 \right]$$

- This is noise power (single sideband) to carrier power ratio normalized to a 1 Hz bandwidth. Units are dBc/Hz.
- Phase noise multiplication effects
 - 20 Log(N) where N is Multiplication Factor -

Phase Noise in Degrees RMS

Total Phase Noise in Degrees RMS is the Integrated phase noise over the band of interest -

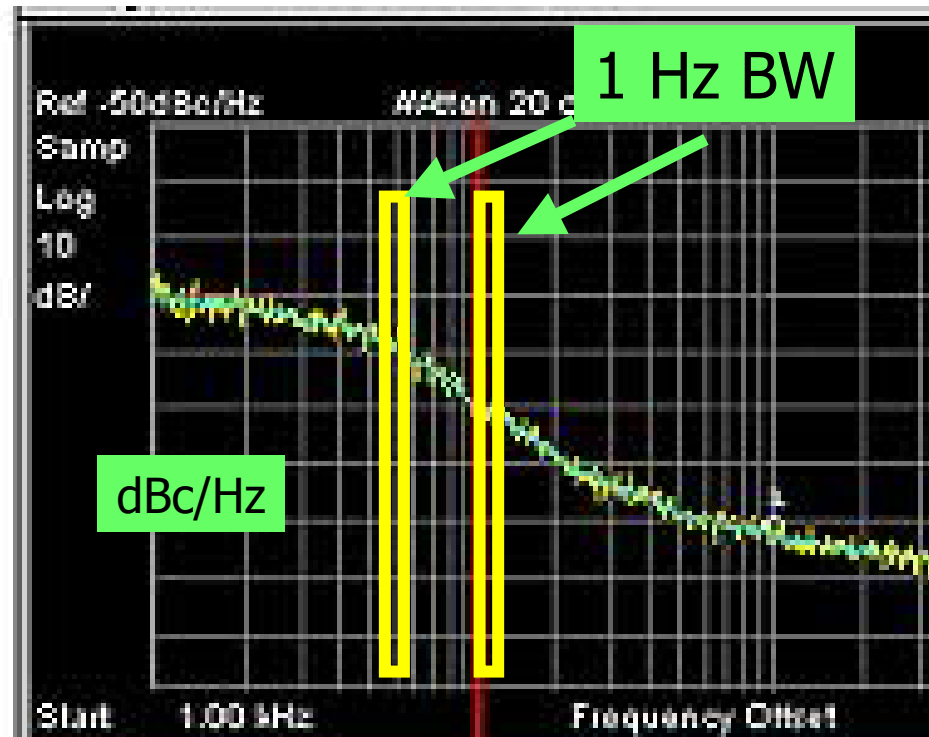
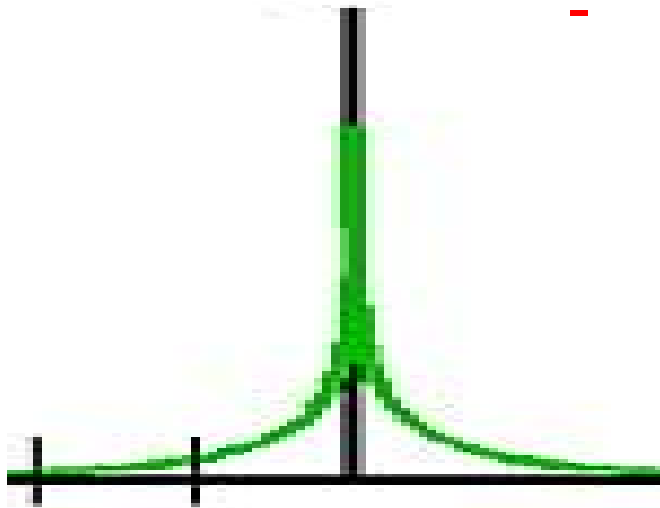


- Band of interest 500Hz to 10kHz

Total RMS Phase Noise

- Each 1 Hz bandwidth (dBc/Hz) is the result of narrow band modulation ($\beta < 0.5$)
- Convert SSB (dBc/Hz) to Degrees RMS ($\Delta\Phi_{\text{RMS}}$)
- Total Phase Noise

$$\beta_{\text{Total}} := \sqrt{(\beta_1)^2 + (\beta_2)^2 + (\beta_3)^2}$$

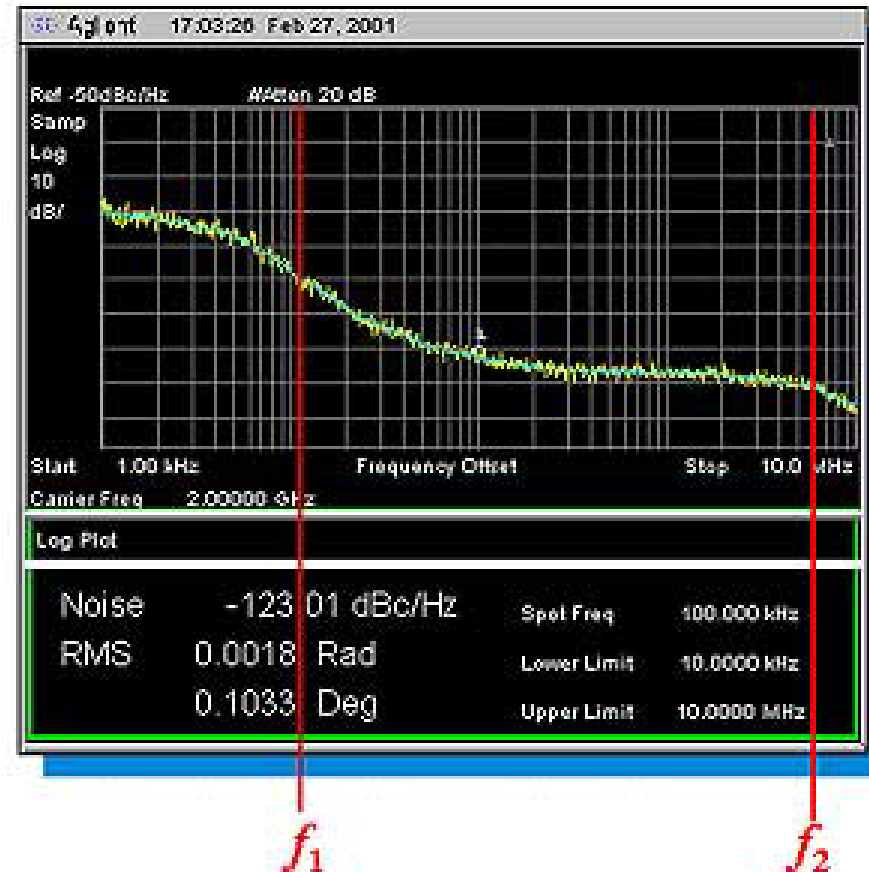


RMS Phase Noise Integration Limits

- Sum ONLY over Applicable Frequencies
- Typically 1/50 Symbol Rate to 1 Symbol Rate (f_1 to f_2)
- Integrate in segments ≤ 1 decade

$$(\Delta\Phi_{\text{RMS}})_{\text{Total}} := \sqrt{[(\Delta\Phi_{\text{RMS}})_1]^2 + [(\Delta\Phi_{\text{RMS}})_2]^2 + [(\Delta\Phi_{\text{RMS}})_3]^2}$$

- Beware of the number of poles in the loop (-20dB/decade/pole)
- $\Delta\Phi_{\text{RMS}}$ is the Root Mean Square (1 Standard Deviation, 1σ) -





Effects of Phase Noise

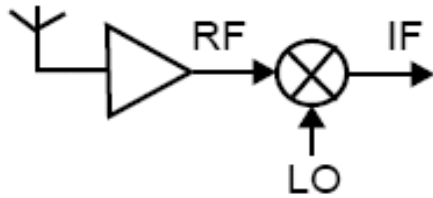
- Thermal Noise is signal dependent
 - Higher signal: Higher S/N
- **Phase Noise is not signal level dependent**
 - Effects system operation at all signal levels
- Low Phase Noise must be designed into the Oscillator
 - Higher levels of noise near the carrier
 - Cannot be eliminated by filtering
 - Limiting has no effect (LO in a mixer) -



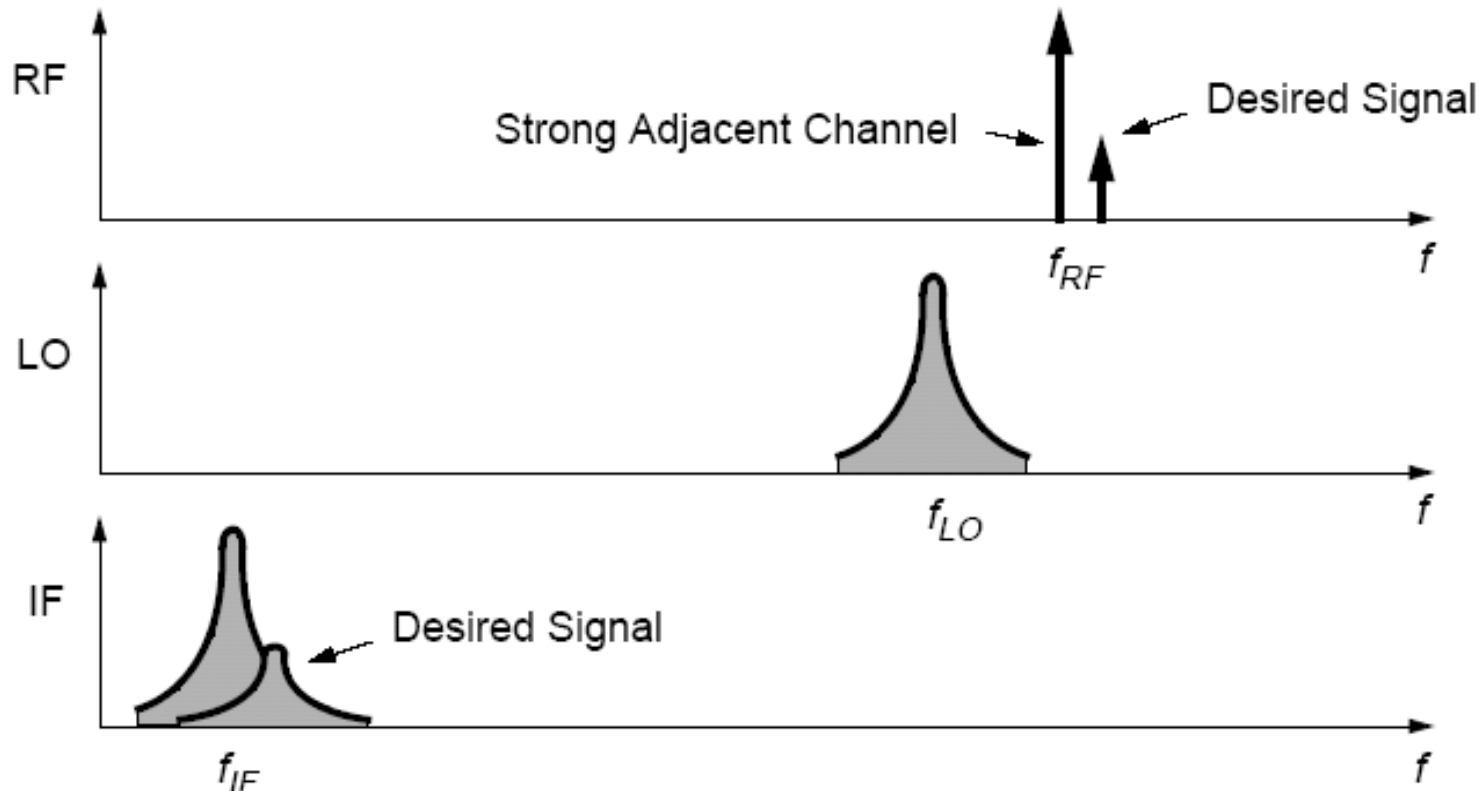
System Problems Due to Phase Noise

- Limits receivers' dynamic range
 - Effects channel spacing
 - Limits Doppler radar performance
- Causes bit errors in digital communication systems
 - Phase Errors
 - Limits synchronization accuracy – Jitter -

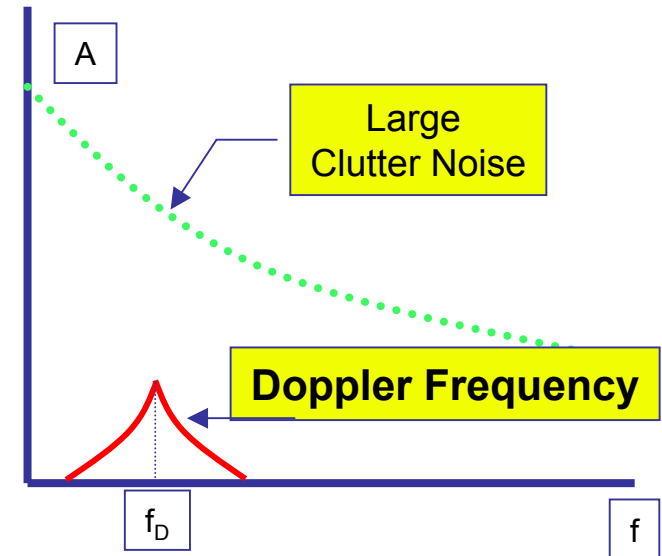
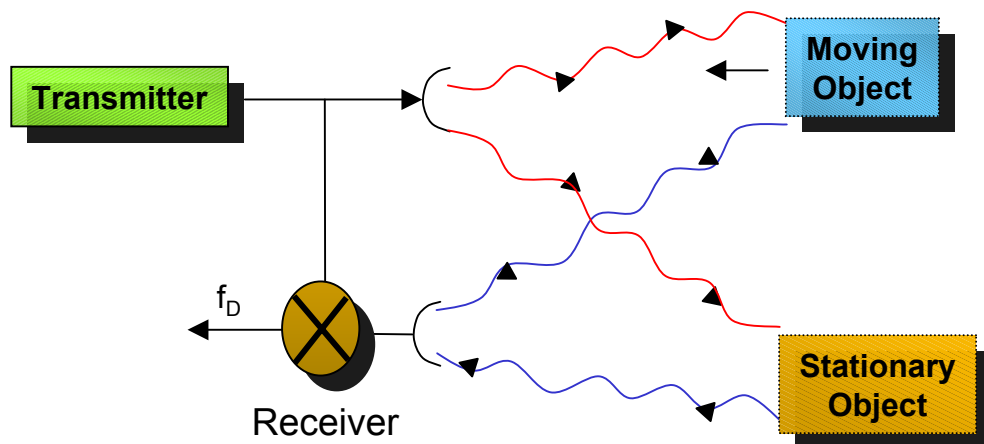
Phase Noise Effects in RF Applications



- Desired signal close to the carrier is buried under the phase noise of an adjacent carrier -

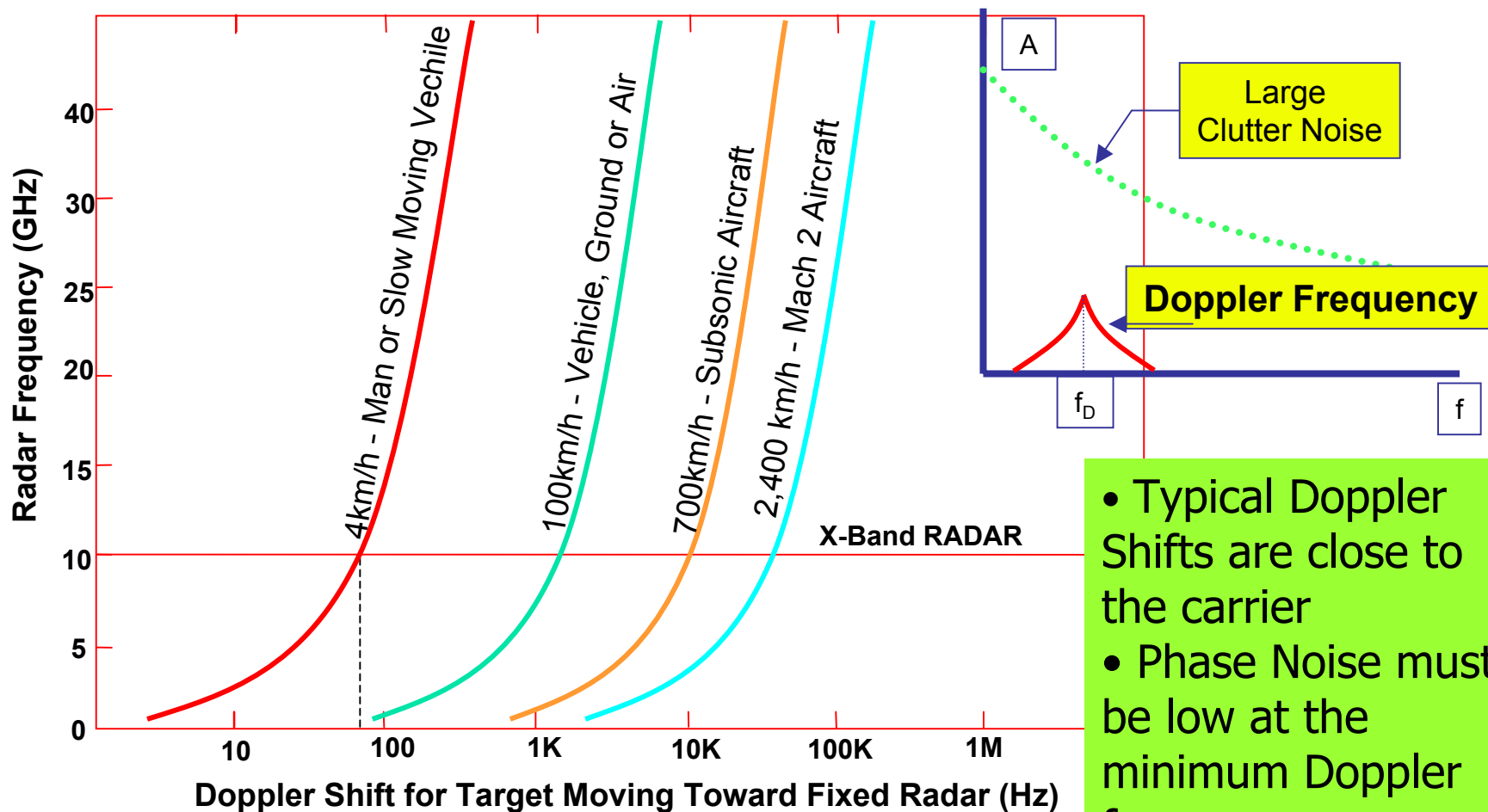


Effect of Noise in Doppler Radar System



- RADAR return is Doppler-shifted from the moving target + large stationary (clutter) signal
- Phase noise on the clutter could mask the target signal -

Typical Doppler Shifts

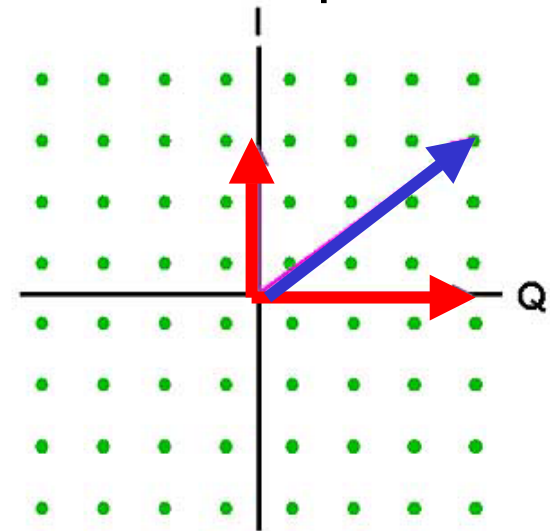
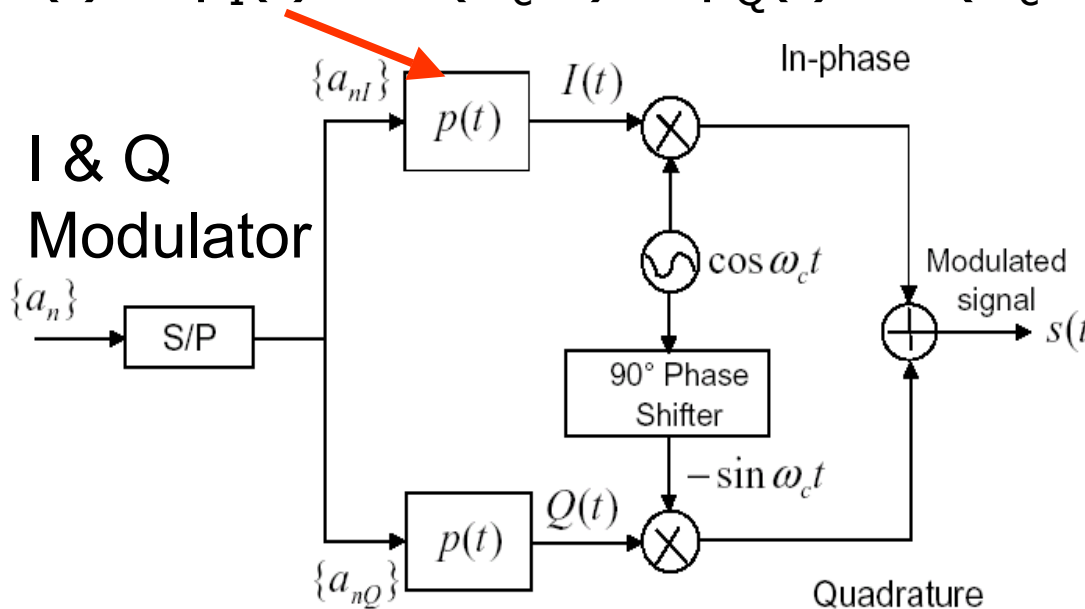


- Typical Doppler Shifts are close to the carrier
- Phase Noise must be low at the minimum Doppler frequency -

QAM Modulation

- Quadrature signals (QPSK) have discrete Amplitudes
 - I & Q Vector Phase ($0^\circ / 180^\circ$ & $90^\circ / 270^\circ$)
 - $p_I(t)$ & $p_Q(t)$ = Discrete (Binary) Amplitude Steps
- Resultant vectors points to a constellation of points

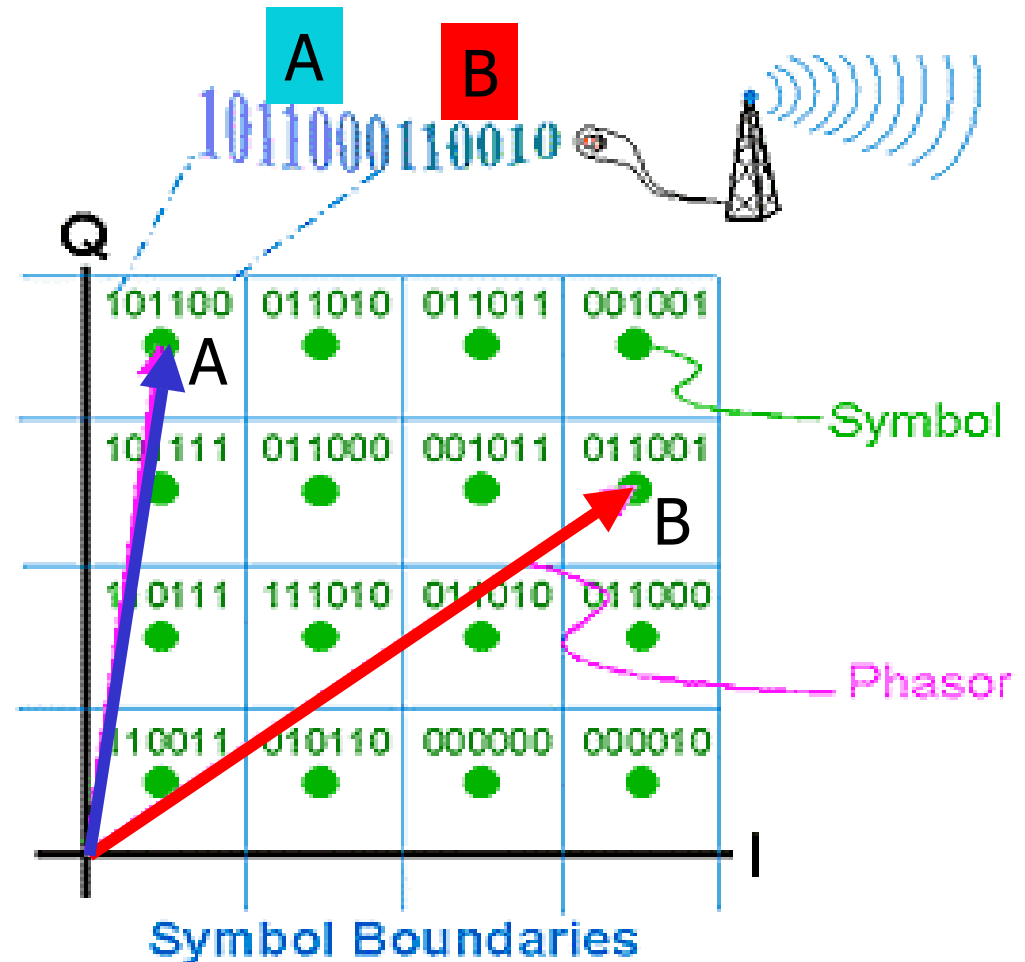
$$S(t) = p_I(t) * \cos(\omega_c * t) + p_Q(t) * \sin(\omega_c * t)$$



Carrier Vector is the summation of the I & Q vectors -

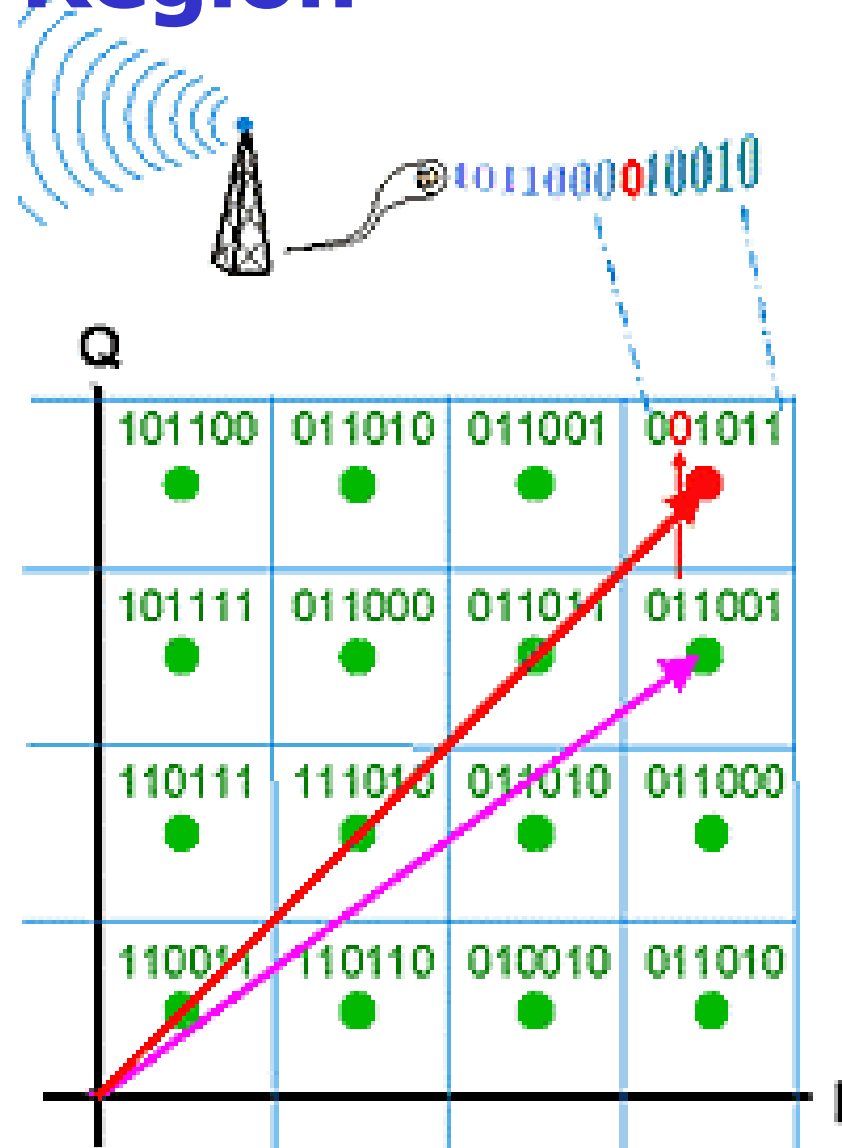
64-QAM Modulation (six bit code 2^6)

- Each Vector position points to a 6 bit code -



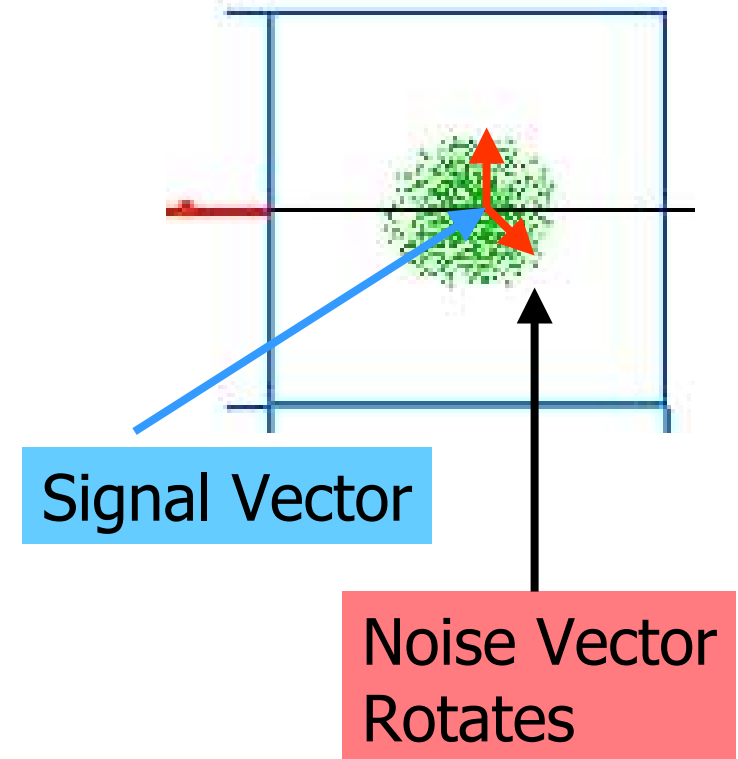
QAM Decision Region

- ❑ Lines between the constellation points are the threshold levels
- ❑ Signals residing in the square are assumed to reside at the discrete vector location.
- ❑ Codes are usually selected so a wrong threshold decision is only a 1 bit error -



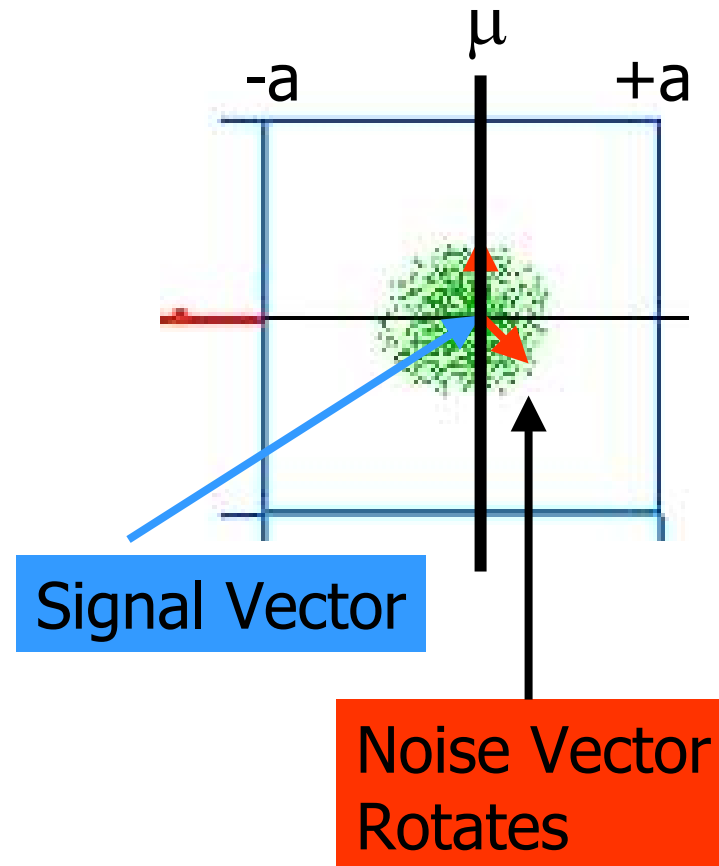
Threshold Boundary

- ❑ Bit Error: Received Vector Falls Outside Boundary
- ❑ Amplitude Vector without deterministic errors (Blue)
- ❑ Add noise vector (Red)
 - ❑ Random Phase (Rotates 360°)
 - ❑ Gaussian Amplitude Distribution -



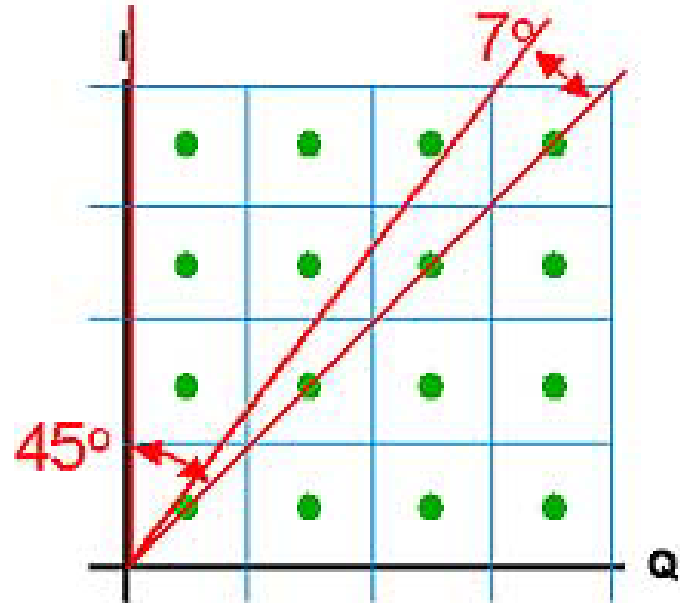
Standard Deviation & RMS Noise

- $\sigma=1 \leftrightarrow \rightarrow$ RMS Noise
- μ is the ideal signal point
- Error Probability = number of σ from μ to "a" (>0)
- Error Probability >0
- Example
 - $P(a=|4\sigma|)$ Bit Error = 6.3×10^{-5}



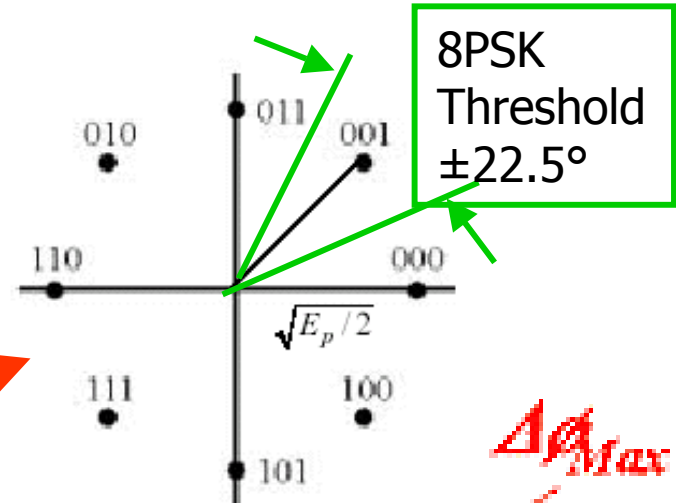
QAM Geometric Effects

- ❑ Maximum angle error is dependent on Symbol Location
- ❑ Outer Symbols Tolerate the least angle error
 - ❑ Outer symbol error = 7°
 - ❑ Inner symbol error = 45°
- ❑ Allowable Error Window is smaller for More Complex Modulation -

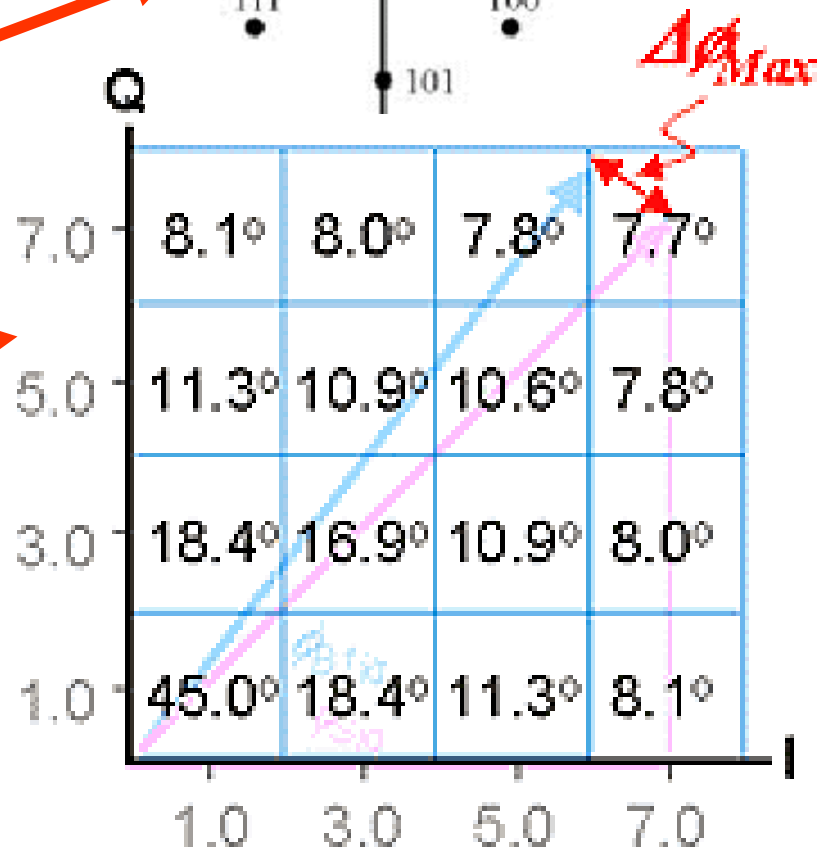


Modulation	Error
• 2QAM	90.0°
• 4QAM	45.0°
• 16QAM	16.9°
• 32AM	10.9°
• 64QAM	7.7°
• 128QAM	5.1°

System Phase Noise

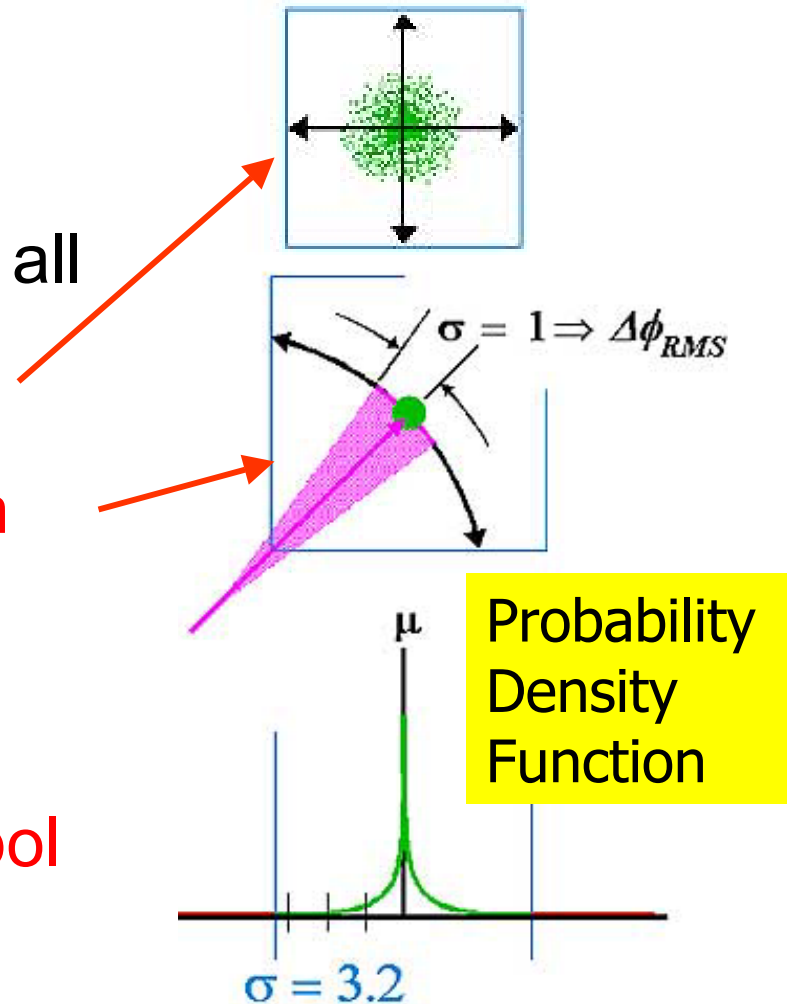


- ❑ Constant Amplitude Modulation (e.g. 8PSK)
 - ❑ Phase Noise threshold is constant ($\pm 22.5^\circ$)
- ❑ QAM Modulation
 - ❑ Allowable Phase Noise is a function of Bit Position
 - ❑ QPSK is $\pm 45^\circ$
 - ❑ 16QAM is $\pm 16.9^\circ$
 - ❑ 64QAM is $\pm 7.7^\circ$



Phase Noise Effects in Digital Applications

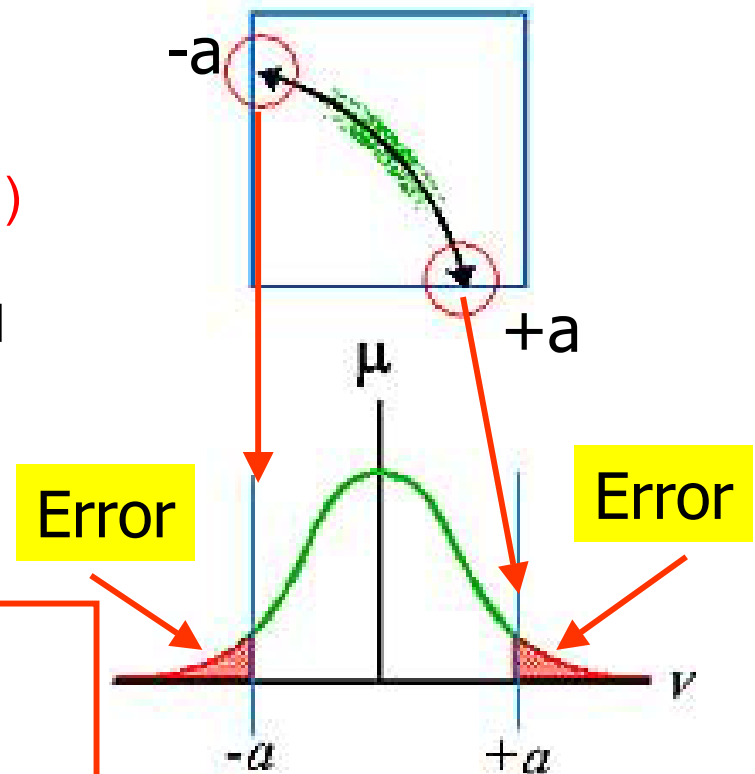
- Thermal Noise: Random in all directions
 - Relevant at Low Power
- **Phase Noise: Random on the Angular Axis**
 - **Independent of Signal Power**
- Errors occur on Both Symbol Boundaries -



Phase Noise & Error Probability

- Probability Density Function (pdf)
 - μ = Average angle
 - σ Standard Deviation
- **RMS value of Phase Noise ($\Delta\Phi_{\text{RMS}}$) = 1σ (Standard Deviation)**
- Probability of Error (BER) is related to the number of σ 's to the boundary

■ $P(> 1\sigma) = .318$	68.2%
■ $P(> 2\sigma) = .046$	95.4%
■ $P(> 3\sigma) = 2.7 \times 10^{-3}$	99.73%
■ $P(> 4\sigma) = 6.3 \times 10^{-5}$	99.9937%
■ $P(> 5\sigma) = 5.7 \times 10^{-7}$	99.999943%

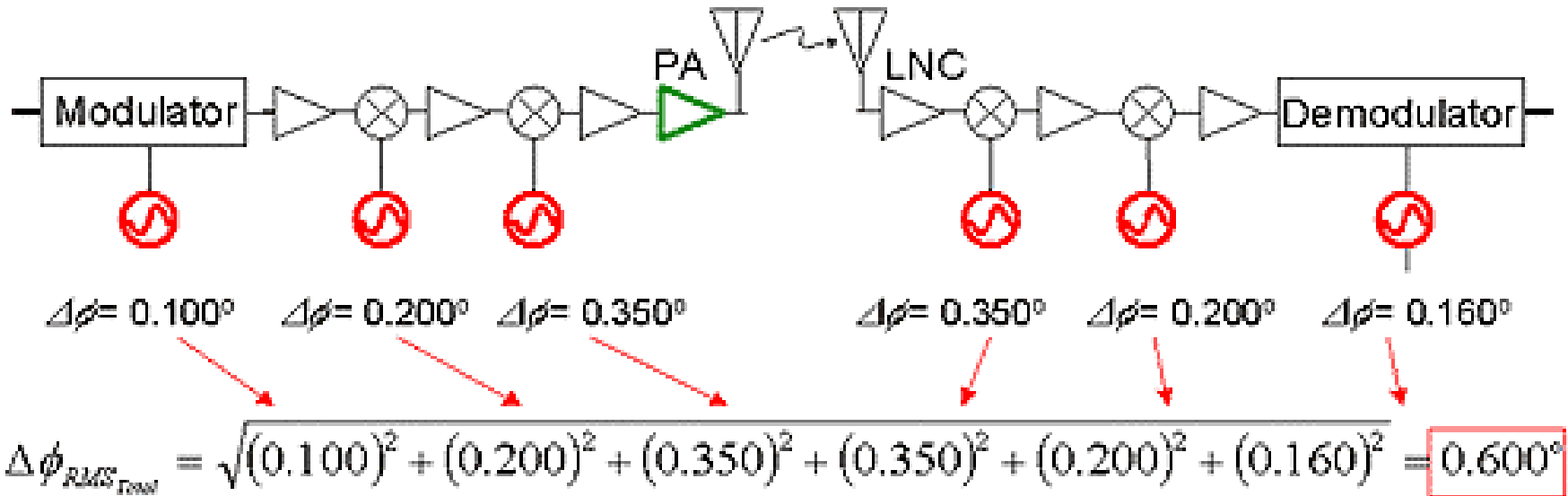


Note: Typical Bit Error Rate are 1 part per million
 $\therefore 5\sigma$ is normal -

Phase Noise Allocation Budget

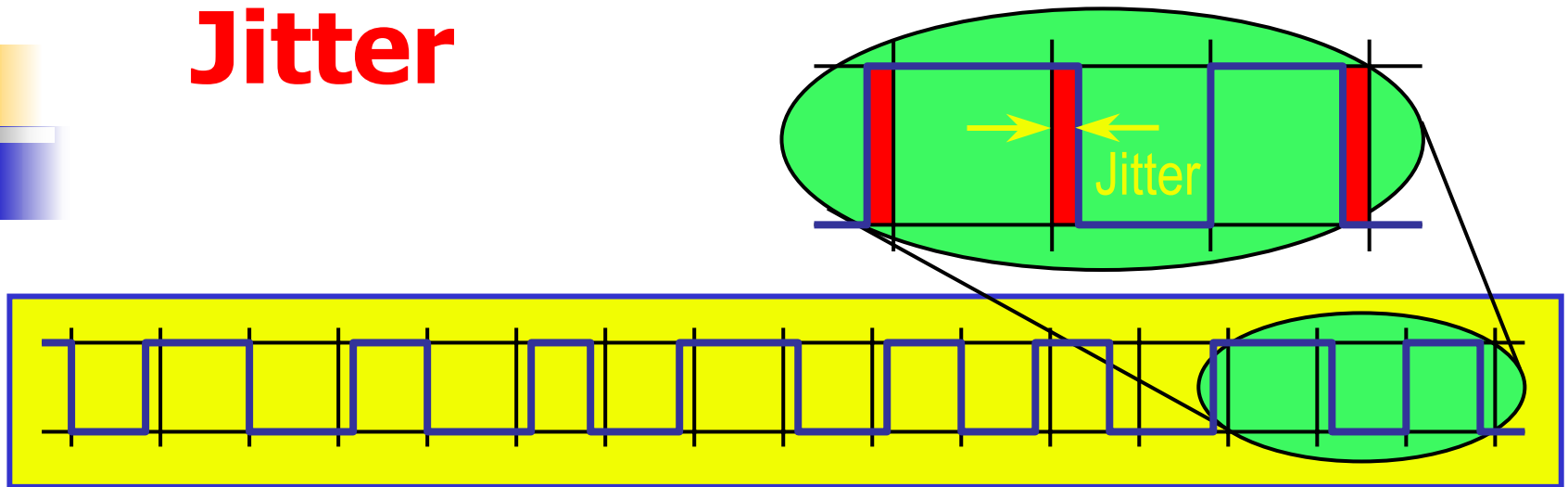
Transmitter

Receiver



- 1σ phase noise is 0.6°
- Add thermal noise under small signal conditions
- Add deterministic phase errors, e.g. Group delay distortion, Power amplifier compression, etc.
 - Deterministic errors effects the initial pointing of the vectors -

Jitter



- Jitter is an undesired fluctuation in the timing of events
 - Modeled as a “noise in time”

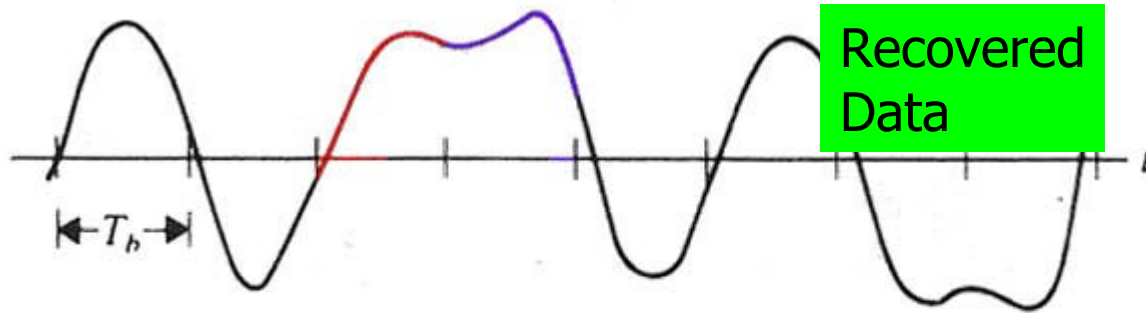
$$v_j(t) = v(t + j(t))$$

- Jitter is the Time-domain equivalent of phase noise

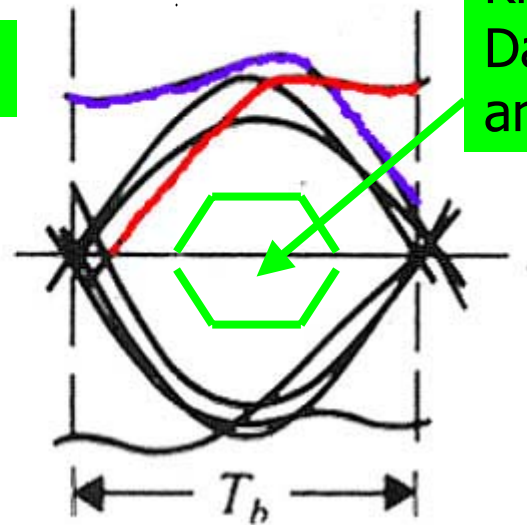
$$j(t) = \phi(t)T / 2\pi$$

- Jitter is caused by
 - phase noise on a clock
 - Thermal noise on a threshold -

Constructing an Eye Diagram

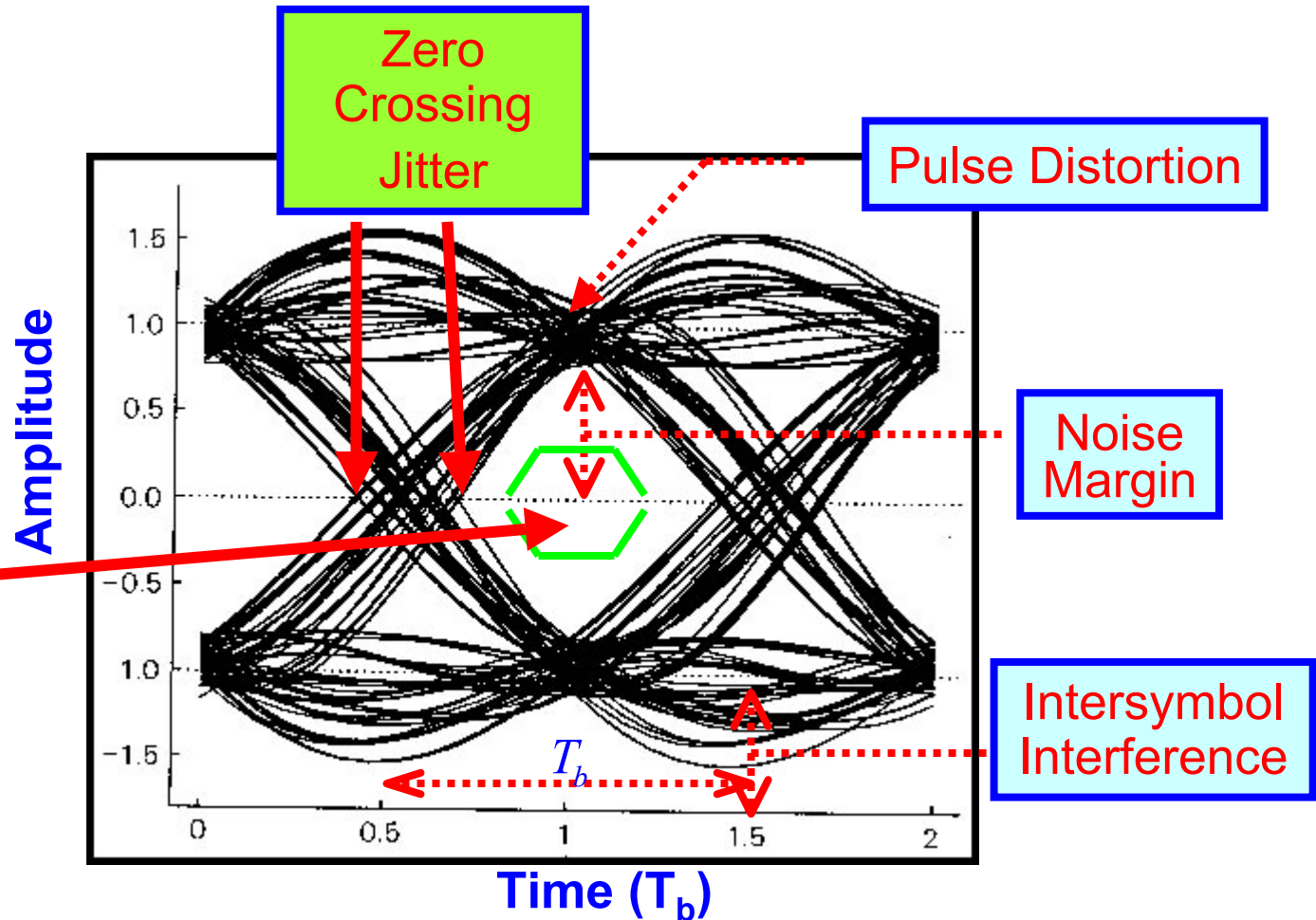


Fold Data "1"s & "0"s Overlap



Eye Diagrams

- Inside Trapezoid is the area of acceptable Sampling Errors both in time and threshold voltage
- Optimum Sampling is in the center of the "Eye" -



Conclusion

- Oscillator parameters are predictable but complex
- Completed 1st step in de-mystifying oscillators
- Suggestions for related lectures and comments are welcome
 - hhausman@miteq.com