Advanced Nonlinear Device Characterization Utilizing New Nonlinear Vector Network Analyzer and X-parameters

presented by:

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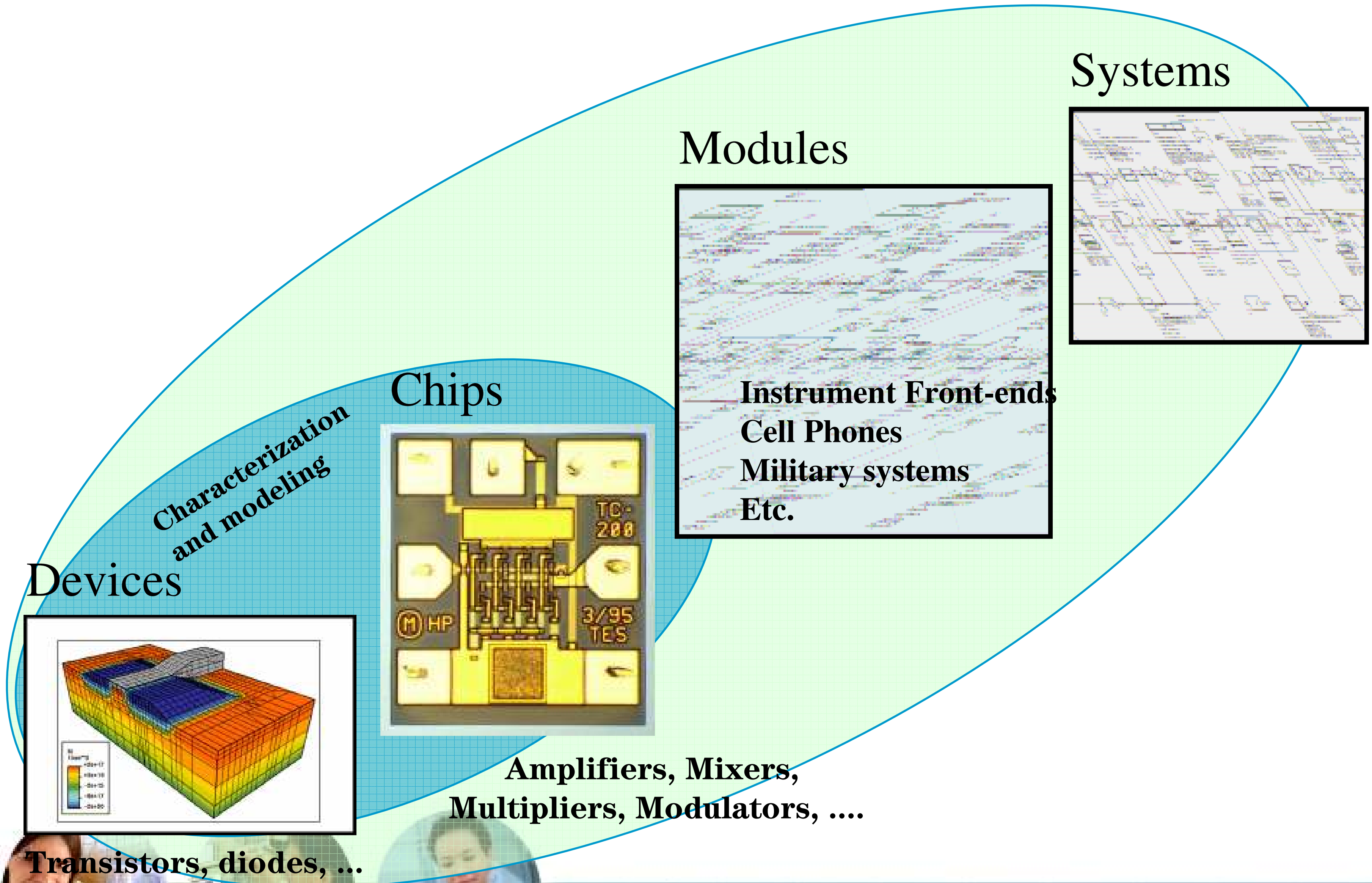
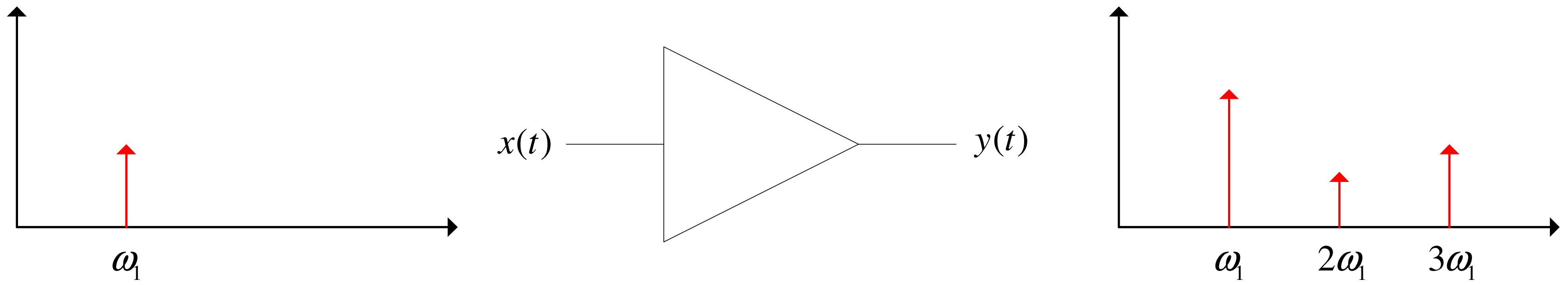
Presentation Outline

- ✓ Device Characteristics (Linear and Nonlinear)
- ✓ NVNA Hardware (The need for phase)
- ✓ NVNA Error Correction
- ✓ NVNA Measurements
    - Component Characterization
    - Multi-Envelope Domain
    - X-Parameters

Nonlinear Hierarchy from Device to System



Nonlinearities



$$x(t) = Ae^{j(w_0 t + \phi_0)}$$

$$\begin{aligned} y(t) &= a_0 + b_0 e^{j\theta_{b0}} x(t) \\ &+ c_0 e^{j\theta_{c0}} x(t)^2 \\ &+ d_0 e^{j\theta_{d0}} x(t)^3 \end{aligned}$$

$$\begin{aligned} y(t) &= a_0 + b_0 e^{j\theta_{b0}} \left[Ae^{j(\omega_0 t + \phi_0)} \right] \\ &+ c_0 e^{j\theta_{c0}} \left[A^2 e^{j2(\omega_0 t + \phi_0)} \right] \\ &+ d_0 e^{j\theta_{d0}} \left[A^3 e^{j3(\omega_0 t + \phi_0)} \right] \end{aligned}$$

The Need for Phase

Cross-Frequency Phase

Notice that each frequency component has an associated static phase shift. Each frequency component has a phase relationship to each other.

$$
\begin{aligned}
y = a_0 &+ b_0 e^{j\theta_{b0}} \left[A e^{j(\omega_0 t + \phi_0)} \right] \\
&+ c_0 e^{j\theta_{c0}} \left[A^2 e^{j2(\omega_0 t + \phi_0)} \right] \\
&+ d_0 e^{j\theta_{d0}} \left[A^3 e^{j3(\omega_0 t + \phi_0)} \right]
\end{aligned}
$$

Why Measure This?

If we can measure the absolute amplitude and cross-frequency phase we have knowledge of the nonlinear behavior such that we can:

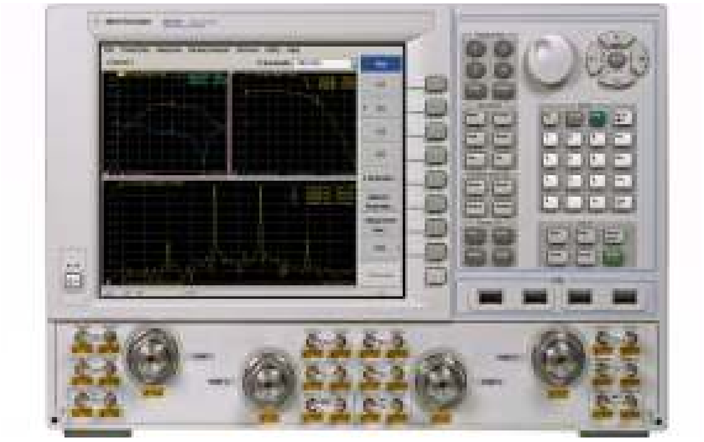
- Convert to time domain waveforms (eg: scope mode).
- Measure phase relationships between harmonics.
- Generate model coefficients.
- Measure frequency multipliers.
- Etc…..

NVNA Hardware

Take a standard PNA-X and add nonlinear measurement capabilities

Agilent's N5242A premier performance microwave network analyzer offers the highest performance, plus:

- 2- and 4-port versions
- Built-in second source and internal combiner for fast, convenient measurement setups
- Spectrally pure sources (-60 dBc)
- Internal modulators and pulse generators for fast, simplified pulse measurements
- Flexibility and configurability
- Large touch screen display with intuitive user interface

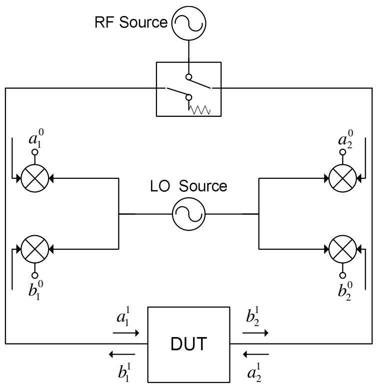


Measuring Unratioed Measurements on PNA-X

Unratioed Measurements – Amplitude

Works fine.

Ever tried to measure phase across frequency on an unratioed measurement?



Sweep 1

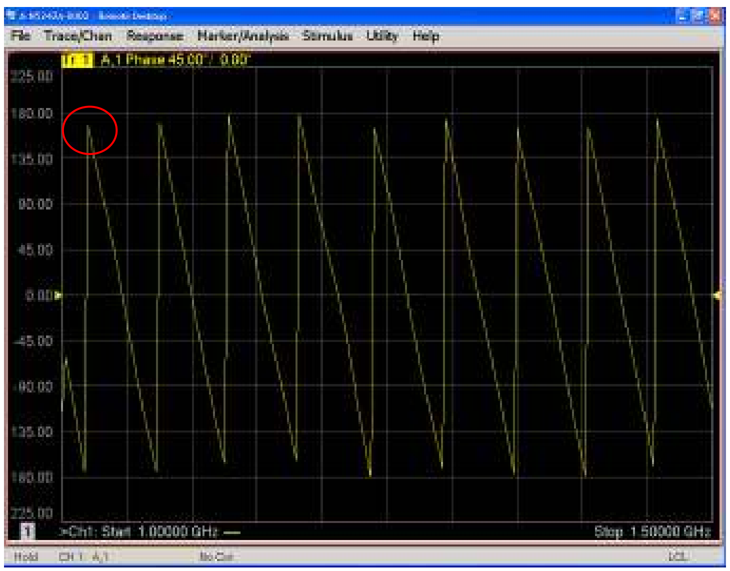
Sweep 2

Measuring Unratioed Measurements on PNA-X

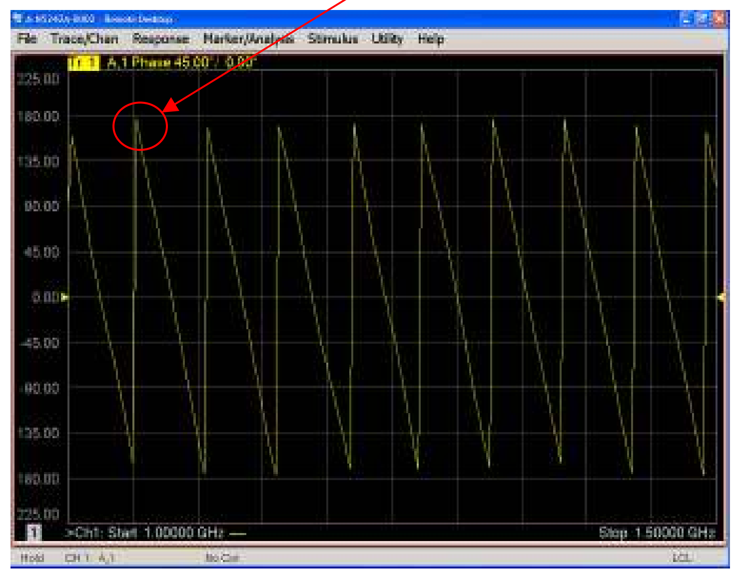
Unratioed Measurements – Phase

Phase response changes from sweep to sweep. As the LO is swept the LO phase from each frequency step from sweep to sweep is not consistent. This prevents measurement of the cross-frequency phase of the frequency spectra.

Phase Shifted



Sweep 1

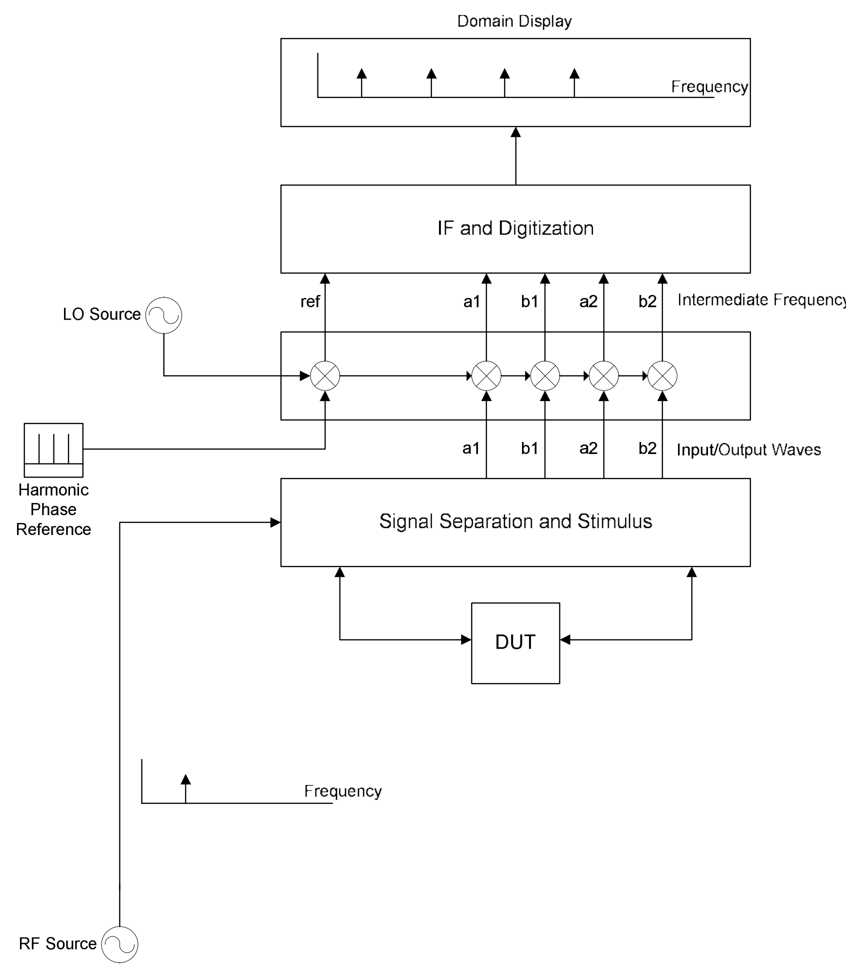


Sweep 2

NVNA Hardware Configuration

Generate Static Phase

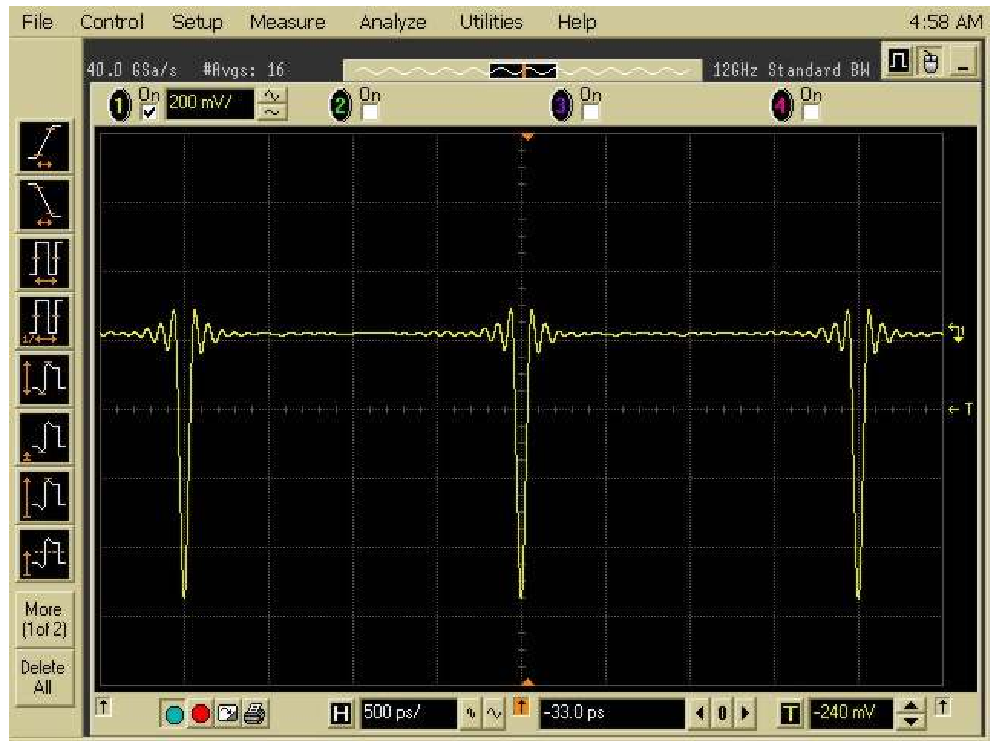
- Since we are using a mixer based VNA the LO phase will change as we sweep frequency. This means that we cannot directly measure the phase across frequency using unratioed (a1, b1) measurements.
- Instead…ratio (a1/ref, b2/ref) against a device that has a constant phase relationship versus frequency. A harmonic phase reference generates all the frequency spectrum simultaneously.
- The harmonic phase reference frequency grid and measurement frequency grid are the same (although they do not have to be generally). For example, to measure a maximum of 5 harmonics from the device (1, 2, 3, 4, 5 GHz) you would place phase reference frequencies at 1, 2, 3, 4, 5 GHz.



NVNA Hardware Configuration

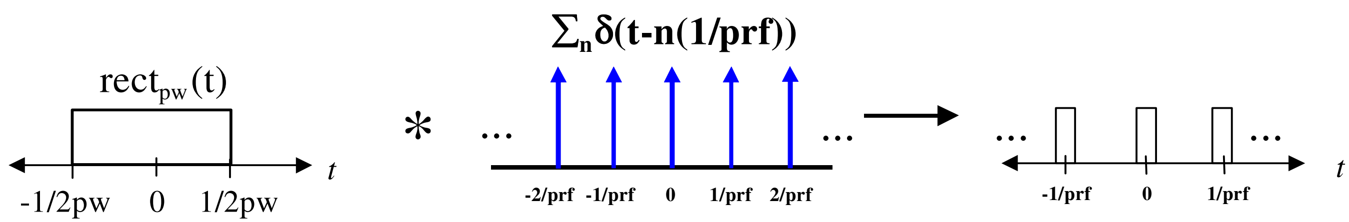
Phase Reference

- One phase reference is used to maintain a static cross-frequency phase relationship.
- A second phase reference standard is used to calibrate the cross-frequency phase at the device plane.
- The phase reference generates a time domain impulse. Fourier theory illustrates that a repetitive impulse in time generates a spectra of frequency content related to the pulse repetition frequency (PRF) and pulse width (PW).
- The cross-frequency phase relationship remains static.



Mathematical Representation of Pulsed DC Signal

$$y(t) = (rect_{pw}(t)) * shah_{\frac{1}{prf}}(t)$$

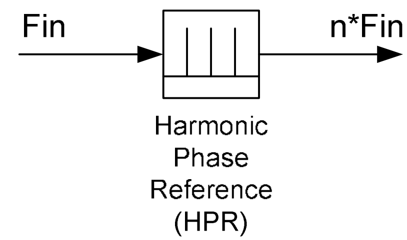


$$Y(s) = (pw \cdot sinc(pw \cdot s)) \cdot (prf \cdot shah(prf \cdot s))$$

$$Y(s) = (pw \cdot sinc(pw \cdot s)) \cdot (prf \cdot shah(prf \cdot s))$$

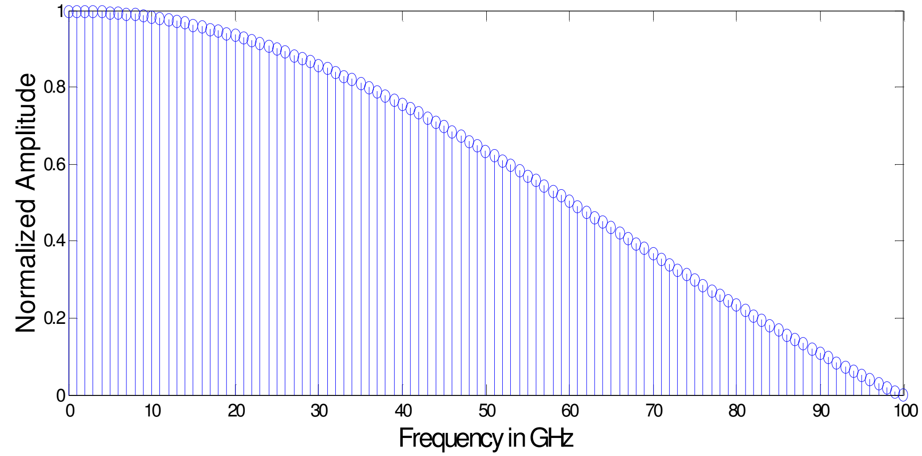
$$Y(s) = DutyCycle \cdot sinc(pw \cdot s) \cdot shah(prf \cdot s)$$

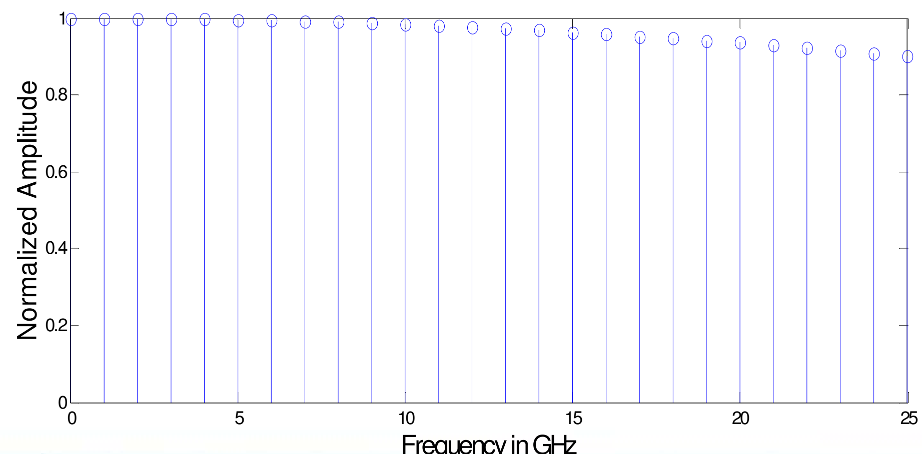
Frequency Domain Representation of Pulsed DC Signal



Frequency Response

- Drive phase reference with a Fin frequency.
- Get n*Fin at the output of the phase reference.
- Example:
    - Want to stimulate DUT with 1 GHz input stimulus and measure harmonic responses at 1, 2, 3, 4, 5 GHz.
    - Fin = 1 GHz
- Can practically use frequency spacings less than 1 MHz





NVNA Hardware Configuration

If we were to isolate a few of the frequencies from the phase reference we would see that the phase relationship remains constant versus input drive frequency and power.

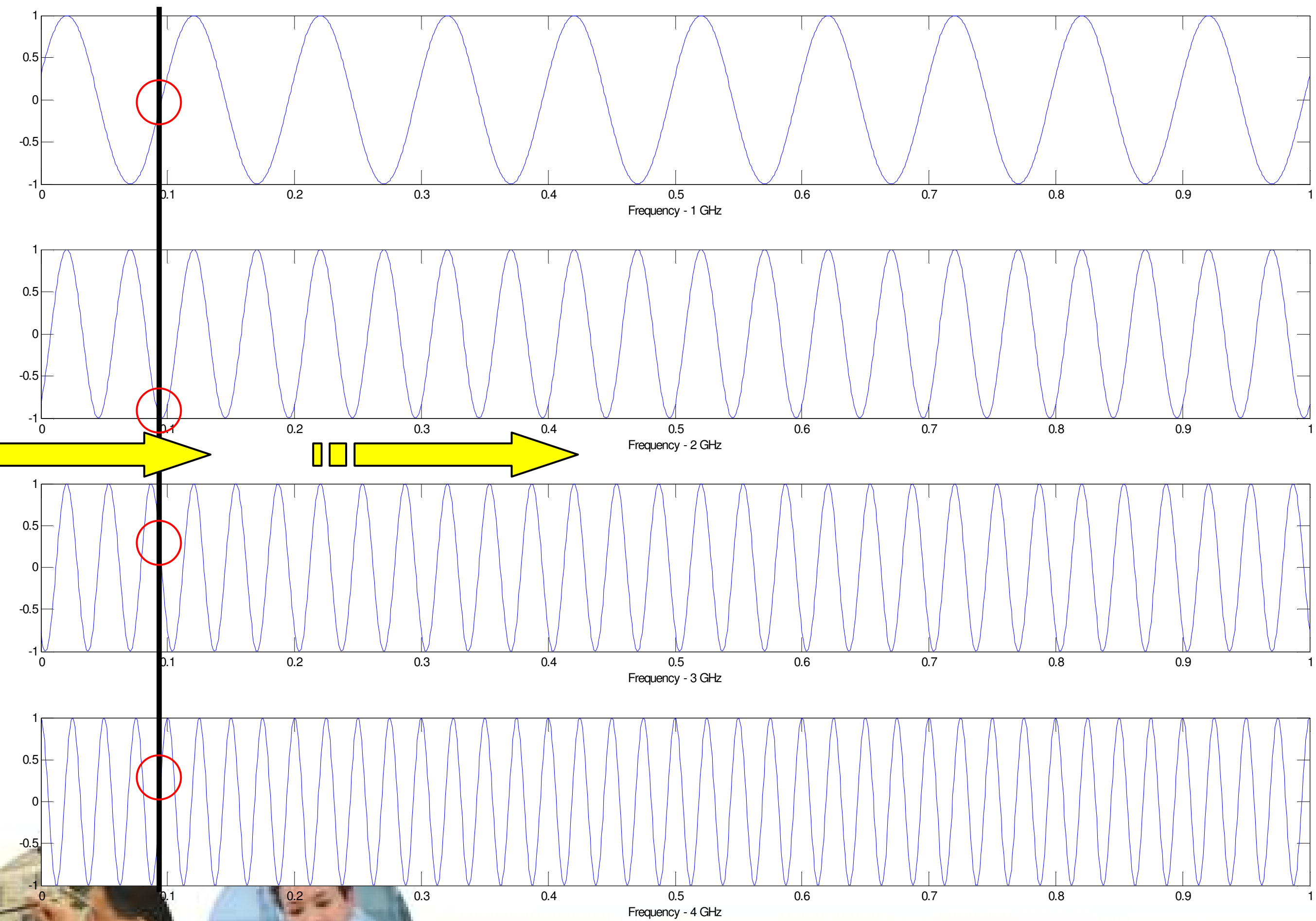
$\cos(\omega_0 t + \phi_0)$ 1 GHz

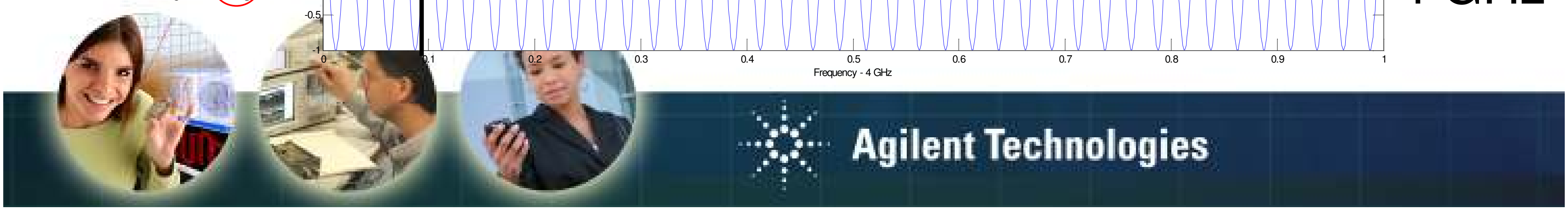
$\cos(2\omega_0 t + \phi_1)$ 2 GHz

Phase relationship between frequencies remains the same

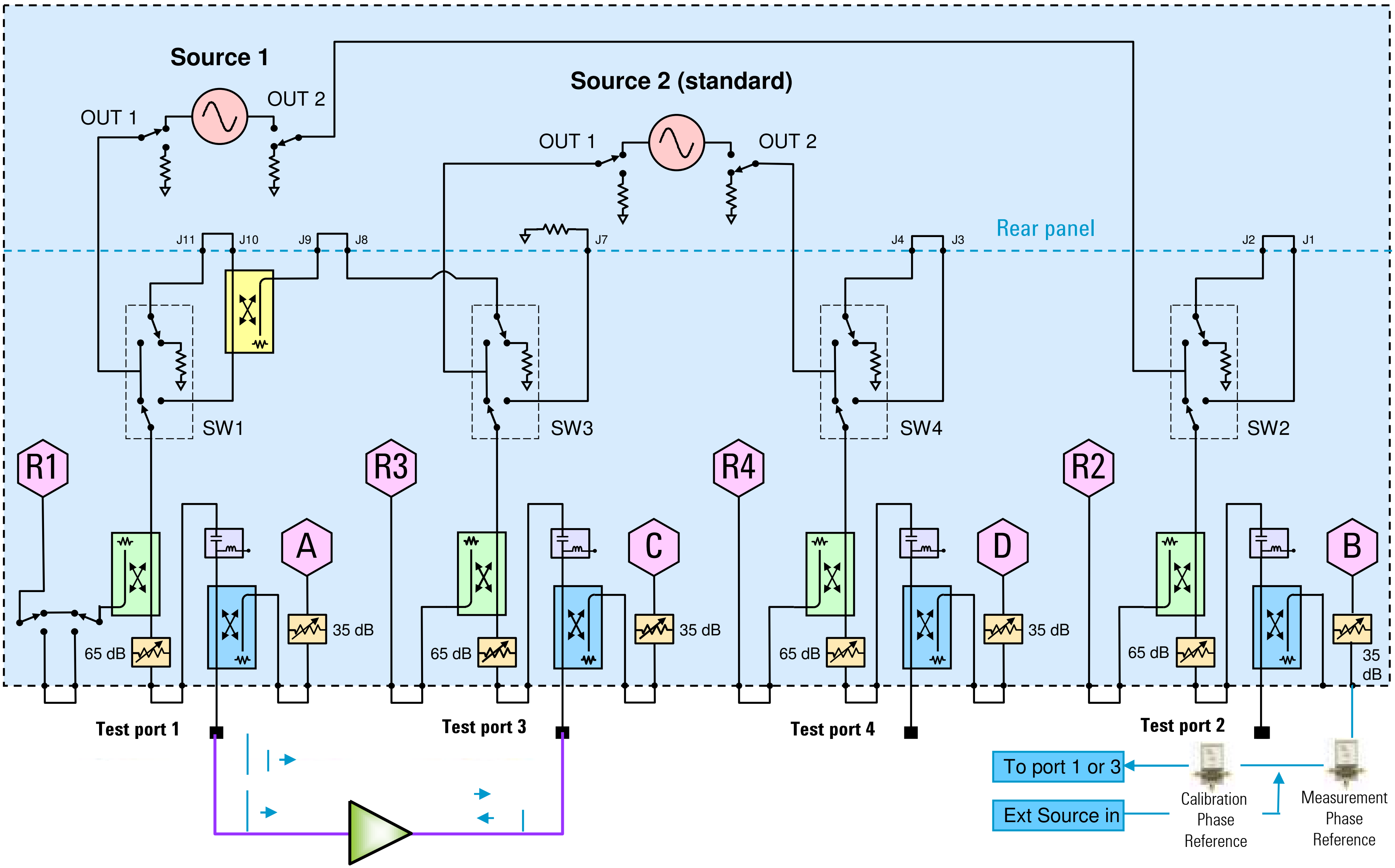
$\cos(3\omega_0 t + \phi_2)$ 3 GHz

$\cos(4\omega_0 t + \phi_3)$ 4 GHz

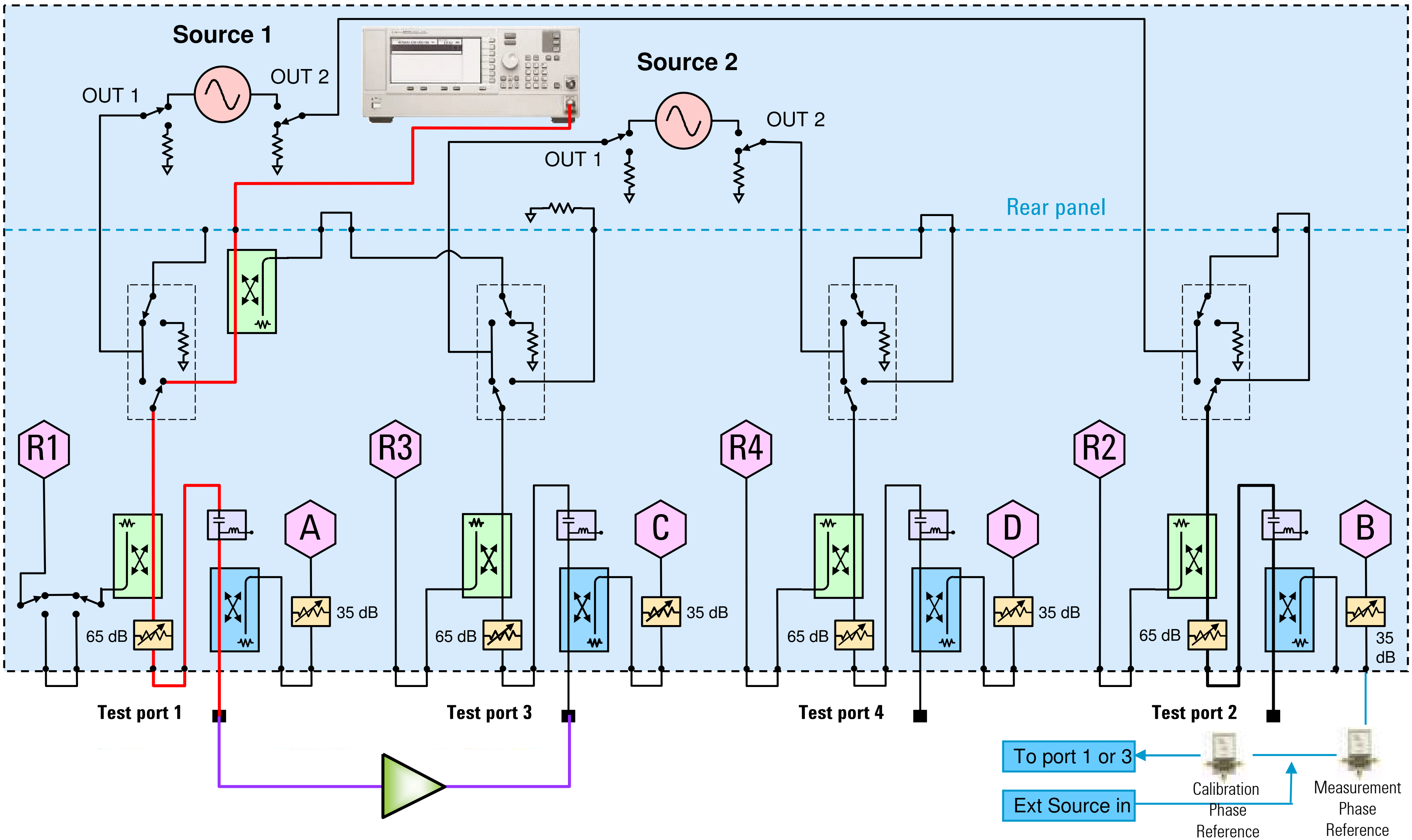




Example NVNA Configuration #1
Using external source for phase reference drive



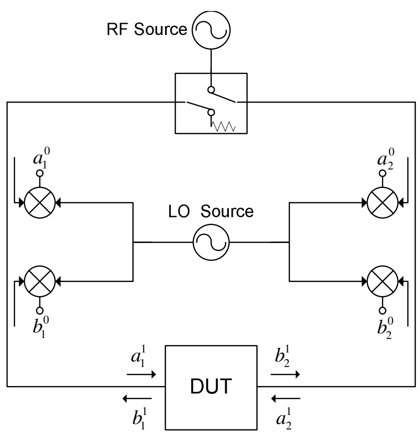
Example NVNA Configuration #2
Multi-Tone, SCMM, DC/RF, Calibrated receiver mode



NVNA Error Correction Algorithms

Generalized VNA HW

- **The input and output waves from a two port device are measured. Systematic measurement hardware errors prevent accurate measurements of the device.**
- **Calibration and error correction provide the means to get an accurate representation of the device characteristics.**



$a_1 \rightarrow$ Incident voltage traveling wave

$b_1 \rightarrow$ Reflected voltage traveling wave

$\sqrt{\mathrm{Z_o}} \rightarrow$ Normalization term

$\mathrm{V_1} \rightarrow$ Voltage applied to port 1 of device

$\mathrm{I_1} \rightarrow$ Current applied to port 1 of device

$$\mathrm{V_1} = \sqrt{\mathrm{Z_o}}\,[a_1 + b_1] \rightarrow \text{Incident voltage wave } + \text{ Reflected voltage wave}$$

$$\mathrm{I_1} = \frac{[a_1 - b_1]}{\sqrt{\mathrm{Z_o}}} \rightarrow \text{Incident current wave } - \text{ Reflected current wave}$$

Therefore (in units of $\sqrt{\text{Watts}}$),

$$a_1 = \frac{1}{2\sqrt{\mathrm{Z_o}}}[\mathrm{V_1} + \mathrm{I_1 Z_o}] \qquad \text{Incident Power} = |a_1|^2$$

$$b_1 = \frac{1}{2\sqrt{\mathrm{Z_o}}}[\mathrm{V_1} - \mathrm{I_1 Z_o}] \qquad \text{Reflected Power} = |b_1|^2$$

Therefore (in units of Volts),

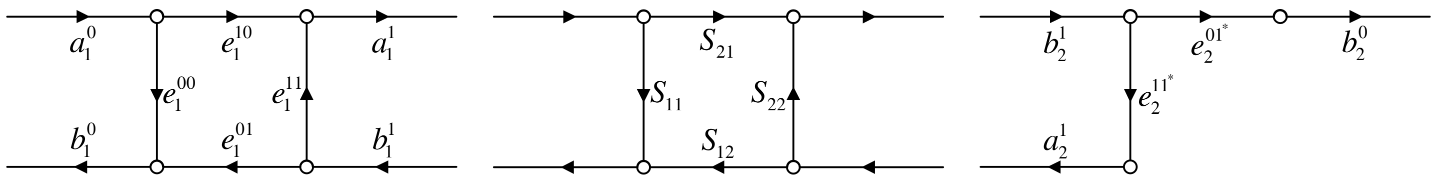
$$a_1 = \frac{1}{2}[\mathrm{V_1} + \mathrm{I_1 Z_o}] \qquad \text{Incident Power} = \frac{|a_1|^2}{Z_o}$$

$$b_1 = \frac{1}{2}[\mathrm{V_1} - \mathrm{I_1 Z_o}] \qquad \text{Reflected Power} = \frac{|b_1|^2}{Z_o}$$

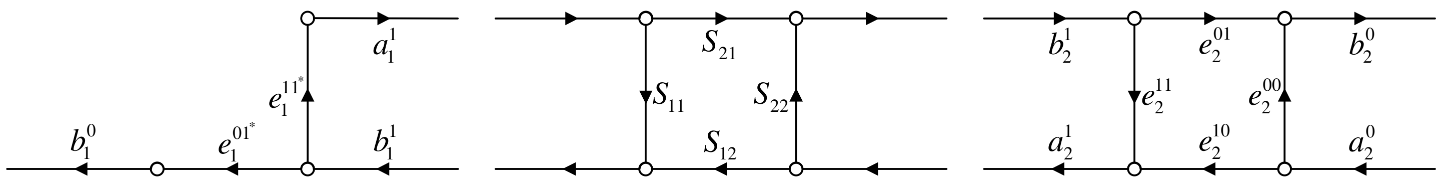
NVNA Error Correction Algorithms

12 Term Error Model

- **A 12 term error model is often used to eliminate systematic measurement errors. Assume crosstalk negligible**



Forward Direction



Reverse Direction

NVNA Error Correction Algorithms

12 Term Error Model - Terminology

- **Common terminology used today**

$$e_1^{00} = \text{Port 1 Directivity}(dp_1)$$
$$e_1^{11} = \text{Port 1 Source Match}(smp_1)$$
$$e_2^{11^*} = \text{Forward Load Match}(lm_{fwd})$$
$$e_1^{10}e_1^{01} = \text{Forward Reflection Tracking}(rt_{fwd})$$
$$e_1^{10}e_2^{01^*} = \text{Forward Transmission Tracking}(tt_{fwd})$$

Port 1

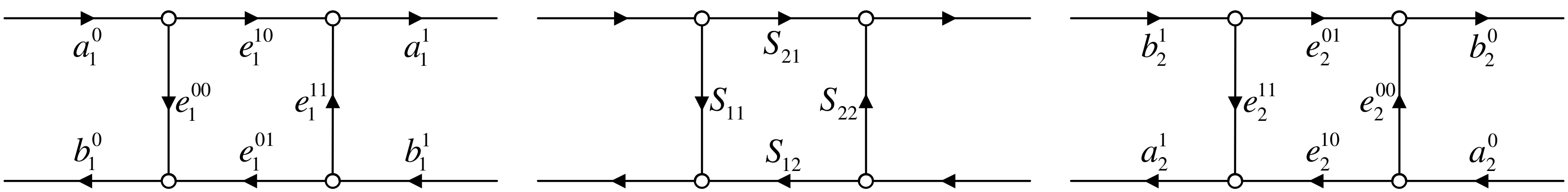
$$e_2^{00} = \text{Port 2 Directivity}(dp_2)$$
$$e_2^{11} = \text{Port 2 Source Match}(smp_2)$$
$$e_1^{11^*} = \text{Reverse Load Match}(lm_{rev})$$
$$e_2^{10}e_2^{01} = \text{Reverse Reflection Tracking}(rt_{rev})$$
$$e_2^{10}e_1^{01^*} = \text{Reverse Transmission Tracking}(tt_{rev})$$

Port 2

NVNA Error Correction Algorithms

Generalized 8 Term Error Model

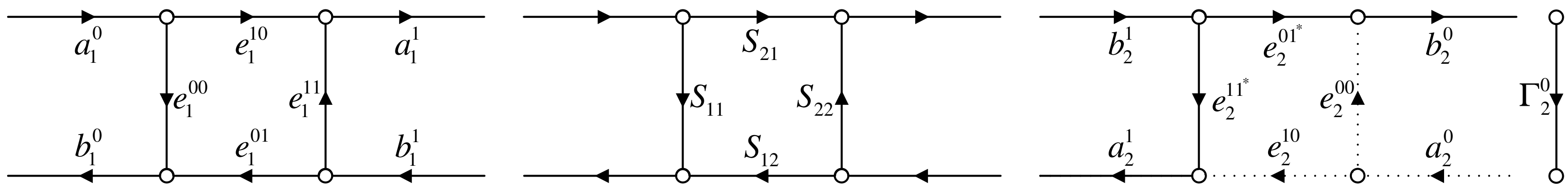
- **The 8 term model accounts for changes in the match of the source and load by either measuring all the ‘a’ and ‘b’ waves or by calculating the ‘a’ waves from match coefficients (like delta match).**



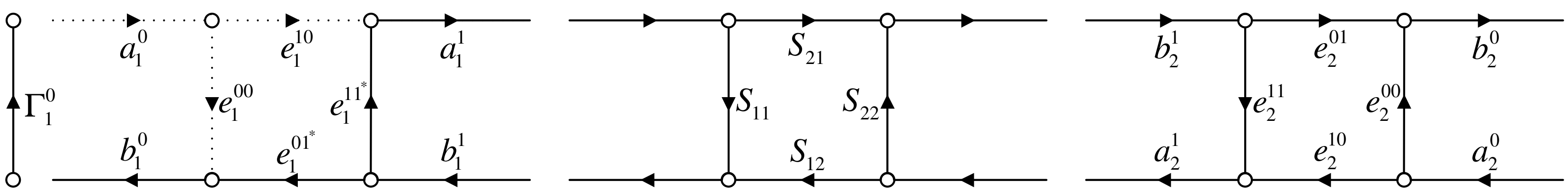
NVNA Error Correction Algorithms

Conversion of 12 Term Model to 8 Term Model

- **To utilize the standard vector calibration algorithms a conversion is done to generate the 8 term model from the 12 term model.**



Forward Direction



Reverse Direction

NVNA Error Correction Algorithms

Conversion Equations

- **The conversion relationship equations to map the 12 term coefficients to the 8 term coefficients**

$$e_1^{00} = dp_1$$

$$e_1^{11} = e_1^{11^*} - \frac{e_1^{10} e_1^{01} \Gamma_1^0}{1 - e_1^{00} \Gamma_1^0} = lm_{rev} - \frac{rt_{fwd} \Gamma_1^0}{1 - dp_1 \Gamma_1^0} = smp_1$$

$$e_1^{10} e_1^{01} = rt_{fwd}$$

$$e_1^{10} e_2^{01} = e_1^{10} e_2^{01^*} \left[1 - e_2^{00} \Gamma_2^0 \right] = tt_{fwd} \left[1 - dp_2 \Gamma_2^0 \right]$$

$$\Gamma_1^0 = \frac{a_1^0}{b_1^0}$$

Port 1

$$e_2^{00} = dp_2$$

$$e_2^{11} = e_2^{11^*} - \frac{e_2^{10} e_2^{01} \Gamma_2^0}{1 - e_2^{00} \Gamma_2^0} = lm_{fwd} - \frac{rt_{rev} \Gamma_2^0}{1 - dp_2 \Gamma_2^0} = smp_2$$

$$e_2^{10} e_2^{01} = rt_{rev}$$

$$e_2^{10} e_1^{01} = e_2^{10} e_1^{01^*} \left[1 - e_1^{00} \Gamma_1^0 \right] = tt_{rev} \left[1 - dp_1 \Gamma_1^0 \right]$$

$$\Gamma_2^0 = \frac{a_2^0}{b_2^0}$$

Port 2

NVNA Error Correction Algorithms

Conversion Equations

- **Instead of calculating the gamma terms we can instead directly calculate the 8 term model tracking coefficients from the 12 term coefficients.**

$$\Gamma_2^0 = \frac{lm_{fwd} - smp_2}{rt_{rev} + dp_2\left[lm_{fwd} - smp_2\right]}$$

$$\Gamma_1^0 = \frac{lm_{rev} - smp_1}{rt_{fwd} + dp_1\left[lm_{rev} - smp_1\right]}$$

$$e_1^{10} e_2^{01} = tt_{fwd}\left[1 - dp_2\Gamma_2^0\right]$$

$$e_2^{10} e_1^{01} = tt_{rev}\left[1 - dp_1\Gamma_1^0\right]$$

$$e_1^{10} e_2^{01} = tt_{fwd}\left[1 - dp_2\frac{lm_{fwd} - smp_2}{rt_{rev} + dp_2\left[lm_{fwd} - smp_2\right]}\right]$$

$$e_2^{10} e_1^{01} = tt_{rev}\left[1 - dp_1\frac{lm_{rev} - smp_1}{rt_{fwd} + dp_1\left[lm_{rev} - smp_1\right]}\right]$$

Port 1

Port 2

NVNA Error Correction Algorithms

8 Term Model Coefficients

- **We now have the 8 term model coefficients…however we need to isolate the terms to relate the amplitude and cross-frequency phase.**

$$e_1^{00}$$
$$e_1^{11}$$
$$e_1^{10}e_1^{01}$$
$$e_1^{10}e_2^{01}$$

Port 1

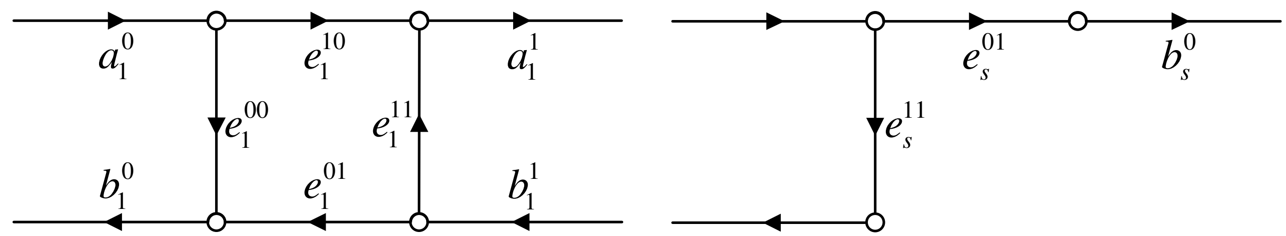
$$e_2^{00}$$
$$e_2^{11}$$
$$e_2^{10}e_2^{01}$$
$$e_2^{10}e_1^{01}$$

Port 2

NVNA Error Correction Algorithms

Isolating Coefficients - Amplitude

- **Error model of VNA port and amplitude (power sensor and meter) calibration device. This isolates the amplitude of one of the tracking coefficients.**



$$a_1^1 = a_1^0 e_1^{10} + b_1^1 e_1^{11}$$

$$b_1^0 = a_1^0 e_1^{00} + b_1^1 e_1^{01}$$

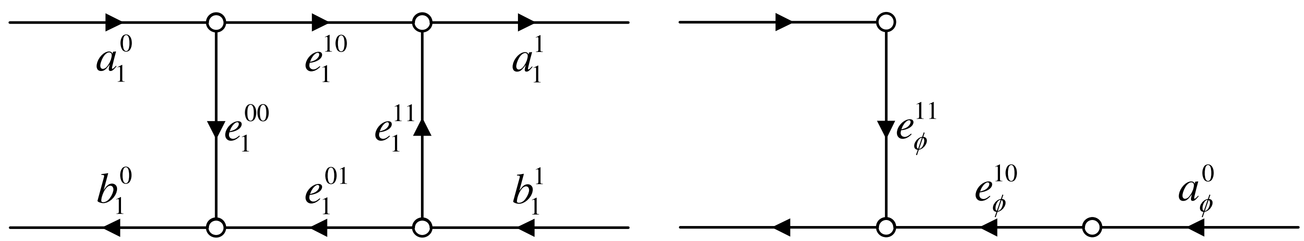
$$a_1^1 = \frac{1}{e_1^{01}}\left[b_1^0 e_1^{11} + a_1^0 \left[e_1^{10} e_1^{01} - e_1^{00} e_1^{11} \right] \right]$$

$$\left| e_1^{01} \right|^2 = \frac{\left| b_1^0 e_1^{11} + a_1^0 \left[e_1^{10} e_1^{01} - e_1^{00} e_1^{11} \right] \right|^2}{\left| a_1^1 \right|^2} \rightarrow \text{The power meter returns the power of } a_1^1 = \left| a_1^1 \right|^2$$

NVNA Error Correction Algorithms

Isolating Coefficients - Phase

- **Error model of VNA port and phase reference (harmonic comb generator) calibration device. This isolates the phase of one of the tracking coefficients by relating the phase (cross-frequency phase) at all frequencies.**



$$b_1^0 = \frac{a_\phi^0 e_\phi^{10} e_1^{01} + a_1^0 e_1^{00}\left[1 - e_1^{11} e_\phi^{11}\right] + a_1^0 e_1^{10} e_\phi^{11} e_1^{01}}{1 - e_1^{11} e_\phi^{11}} = \frac{a_\phi^0 e_\phi^{10} e_1^{01} + a_1^0 e_1^{00} - a_1^0 e_\phi^{11}\left[e_1^{00} e_1^{11} - e_1^{10} e_1^{01}\right]}{1 - e_1^{11} e_\phi^{11}}$$

$$e_1^{01} = \frac{b_1^0\left[1 - e_1^{11} e_\phi^{11}\right] - a_1^0 e_1^{00} + a_1^0 e_\phi^{11}\left[e_1^{00} e_1^{11} - e_1^{10} e_1^{01}\right]}{a_\phi^0 e_\phi^{10}}$$

$$\phi\left(e_1^{01}\right) = \phi\left(\frac{b_1^0\left[1 - e_1^{11} e_\phi^{11}\right] - a_1^0\left[e_1^{00} - e_\phi^{11}\left[e_1^{00} e_1^{11} - e_1^{10} e_1^{01}\right]\right]}{a_\phi^0 e_\phi^{10}}\right) \rightarrow \text{The phase reference term } a_\phi^0 e_\phi^{10} \text{ is known}$$

NVNA Error Correction Algorithms

Isolating the Rest of the Coefficients in the 8 Term Model

- **We now have the 8 term model coefficients…however we need to isolate the terms to relate the amplitude and cross-frequency phase.**

e_1^{00} e_1^{11} $e_1^{10}e_1^{01}$ $e_1^{10}e_2^{01}$

e_2^{00} e_2^{11} $e_2^{10}e_2^{01}$ $e_2^{10}e_1^{01}$

Terms already isolated

Terms to isolate

Calculation path

e_1^{00}

e_1^{11}

e_1^{01}

Isolate amplitude and cross-frequency phase using power sensor and phase reference

$$e_1^{10} = \frac{e_1^{10}e_1^{01}}{e_1^{01}}$$

Port 1

e_2^{00}

e_2^{11}

$$e_2^{01} = \frac{e_2^{10}e_2^{01}}{e_2^{10}}$$

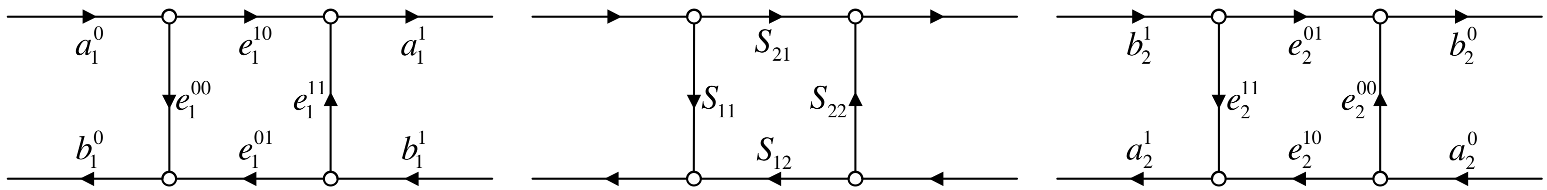
$$e_2^{10} = \frac{e_2^{10}e_1^{01}}{e_1^{01}}$$

Port 2

NVNA Error Correction Algorithms

Error Correction Matrix

- We now have isolated all the error coefficients in the 8 term model and can now relate the uncorrected waves to the corrected wave of the DUT. Notice each 'R' term is multiplied by e_1^{01} and e_2^{01} which provide the cross-frequency phase relationship between the uncorrected and corrected 'a' and 'b' waves.



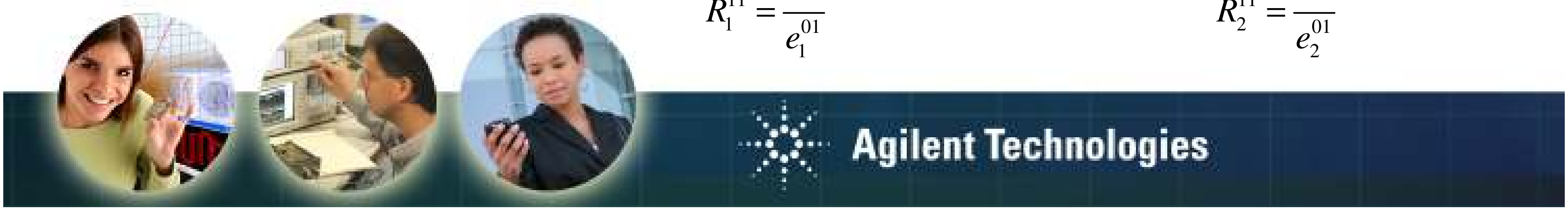
$$\begin{bmatrix} a_1^1 \\ b_1^1 \\ a_2^1 \\ b_2^1 \end{bmatrix} = \begin{bmatrix} R_1^{00} & R_1^{01} & 0 & 0 \\ R_1^{10} & R_1^{11} & 0 & 0 \\ 0 & 0 & R_2^{00} & R_2^{01} \\ 0 & 0 & R_2^{10} & R_2^{11} \end{bmatrix} \begin{bmatrix} a_1^0 \\ b_1^0 \\ a_2^0 \\ b_2^0 \end{bmatrix}$$

$$R_1^{00} = \frac{1}{e_1^{01}}\left[e_1^{10}e_1^{01} - e_1^{00}e_1^{11}\right] \qquad R_2^{00} = \frac{1}{e_2^{01}}\left[e_2^{10}e_2^{01} - e_2^{00}e_2^{11}\right]$$

$$R_1^{01} = \frac{1}{e_1^{01}}\left[e_1^{11}\right] \qquad R_2^{01} = \frac{1}{e_2^{01}}\left[e_2^{11}\right]$$

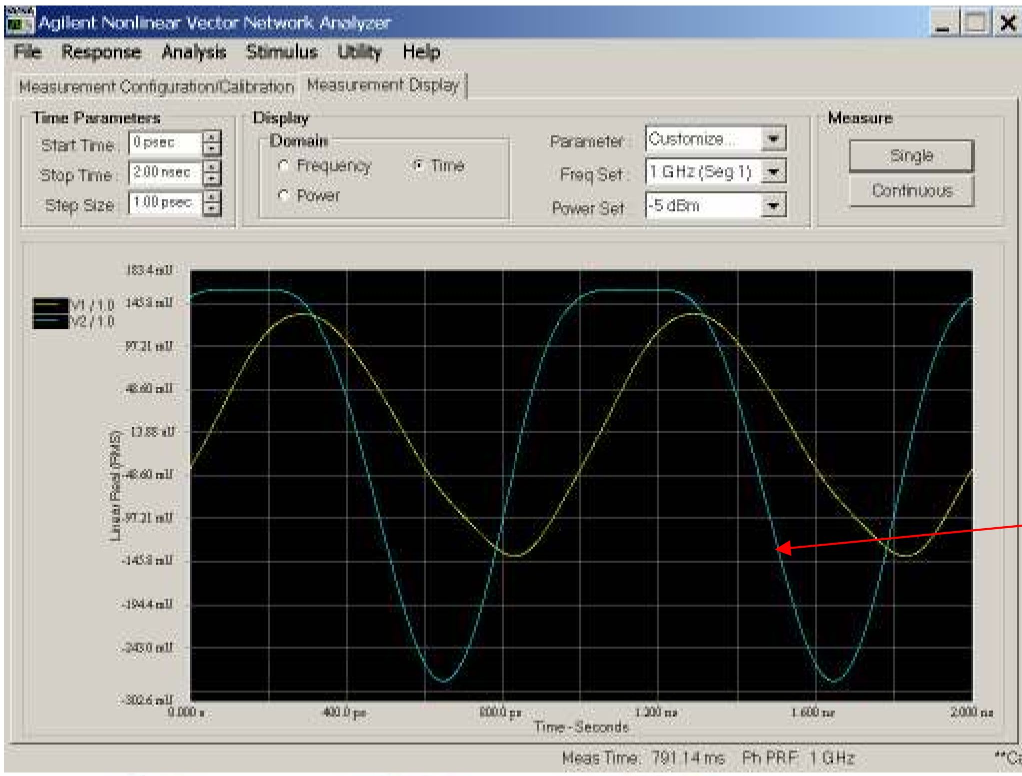
$$R_1^{10} = \frac{1}{e_1^{01}}\left[-e_1^{00}\right] \qquad R_2^{10} = \frac{1}{e_2^{01}}\left[-e_2^{00}\right]$$

$$R_1^{11} = \frac{1}{e_1^{01}} \qquad R_2^{11} = \frac{1}{e_2^{01}}$$



NVNA Applications (What does it do?)

Time domain oscilloscope measurements with vector error correction applied



View time domain (and frequency domain) waveforms (similar to an oscilloscope) but with vector correction applied (measurement plane at DUT terminals)

Vector corrected time domain voltages (and currents) from device

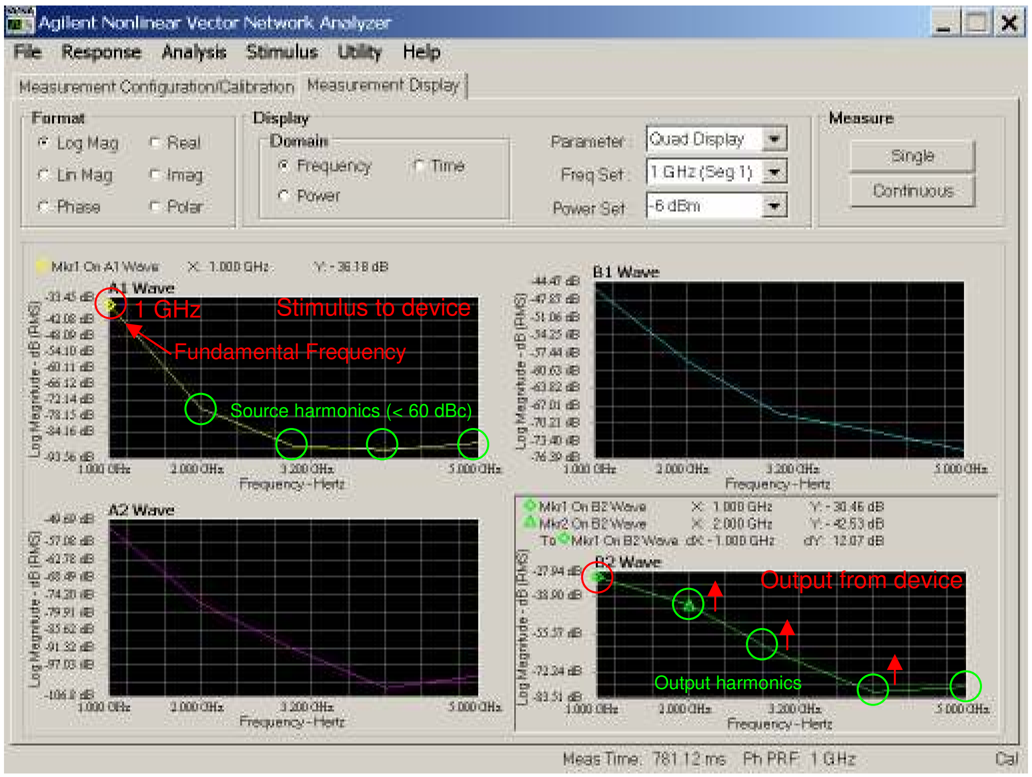
NVNA Applications (What does it do?)

Measure amplitude and cross-frequency phase of frequencies to/from device with vector error correction applied

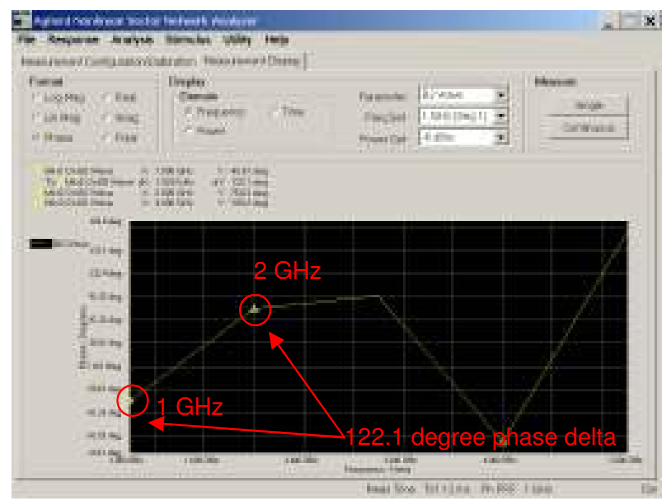
View absolute amplitude and phase relationship between frequencies to/from a device with vector correction applied (measurement plane at DUT terminals)

Useful to analyze/design high efficiency amplifiers such as class E/F

Can also measure frequency multipliers



Input output frequencies at device terminals



Phase relationship between frequencies at output of device

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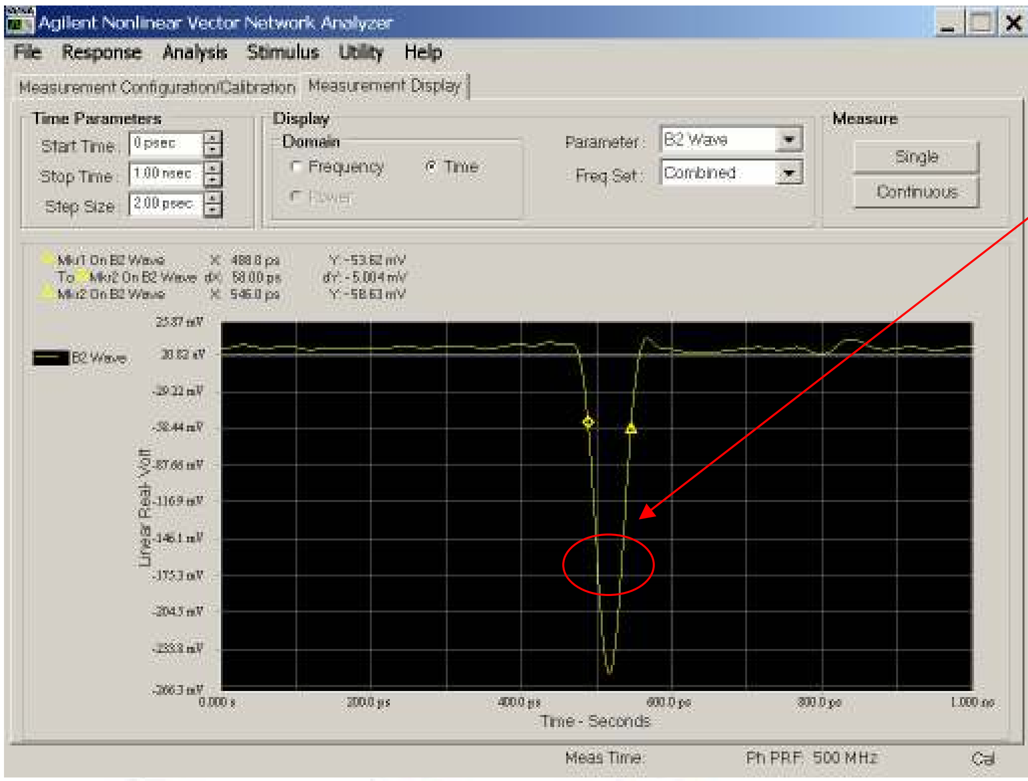
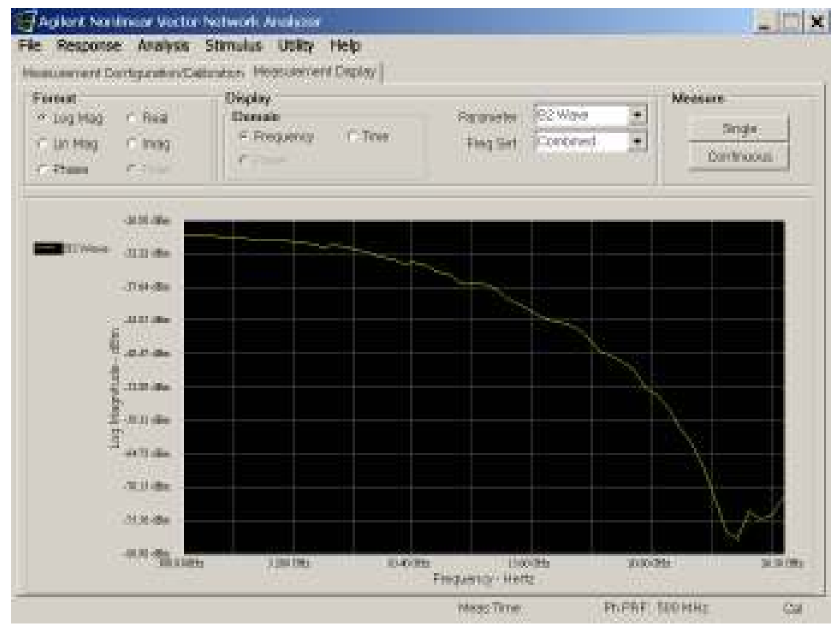
NVNA Applications (What does it do?)

Measurement of narrow (fast) DC pulses with vector error correction applied

View time domain (and frequency domain) representations of narrow DC pulses with vector correction applied (measurement plane at DUT terminals)

Less than 50 ps



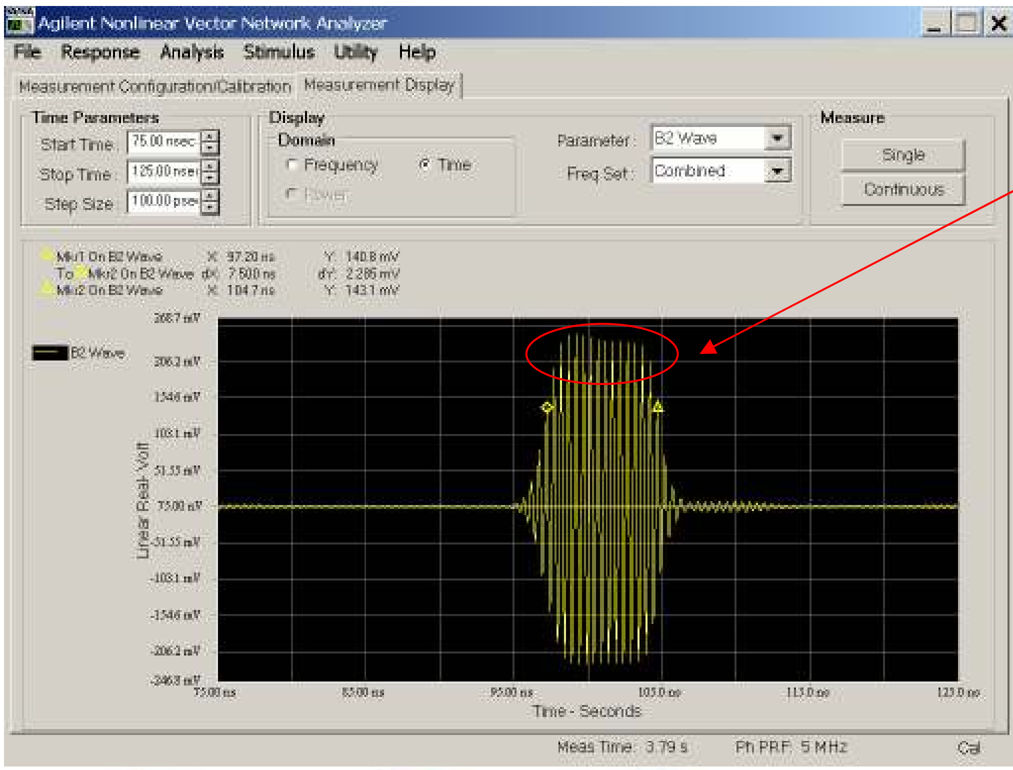


NVNA Applications (What does it do?)

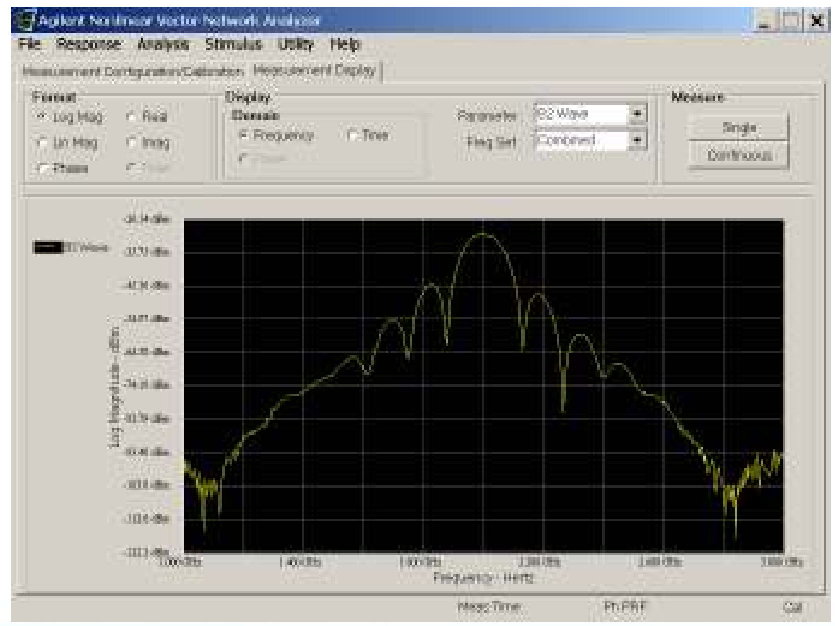
Measurement of narrow (fast) RF pulses with vector error correction applied

View time domain (and frequency domain) representations of narrow RF pulses with vector correction applied (measurement plane at DUT terminals)

Using wideband mode (resolution ~ 1/BW ~ 1/26 GHz ~ 40 ps)



Example: 10 ns pulse width at a 2 GHz carrier frequency. Limited by external source not NVNA. Can measure down into the picosecond pulse widths



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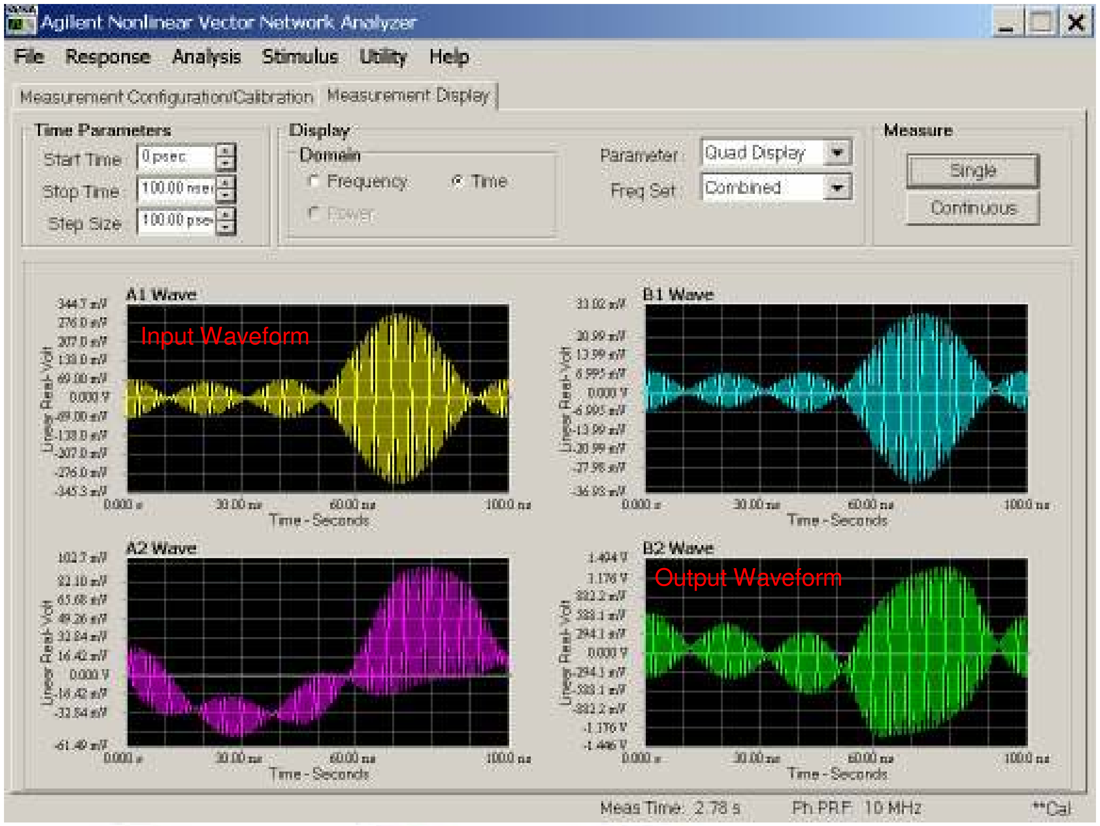
NVNA Applications (What does it do?)

Measurements of multi-tone stimulus/response with vector error correction applied



View time and frequency domain representations of a multi-tone stimulus to/from a device with vector correction applied (measurement plane at DUT terminals)

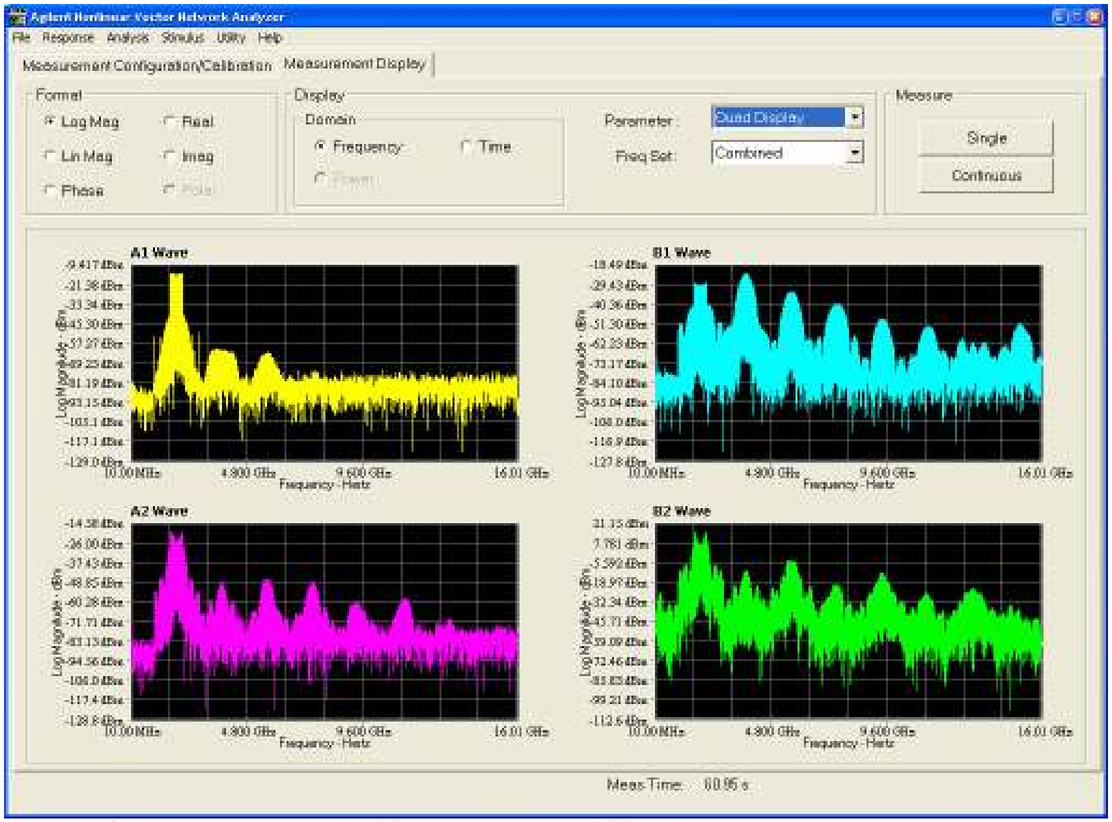
Stimulus is 5 frequencies spaced 10 MHz apart centered at 1 GHz measuring all spectrum to 20 GHz

Measure amplitude AND PHASE of intermodulation products

Generated using external source (PSG/ESG/MXG) using NVNA and vector calibrated receiver

NVNA Applications (What does it do?)

Calibrated measurements of multi-tone stimulus/response with narrow tone spacing



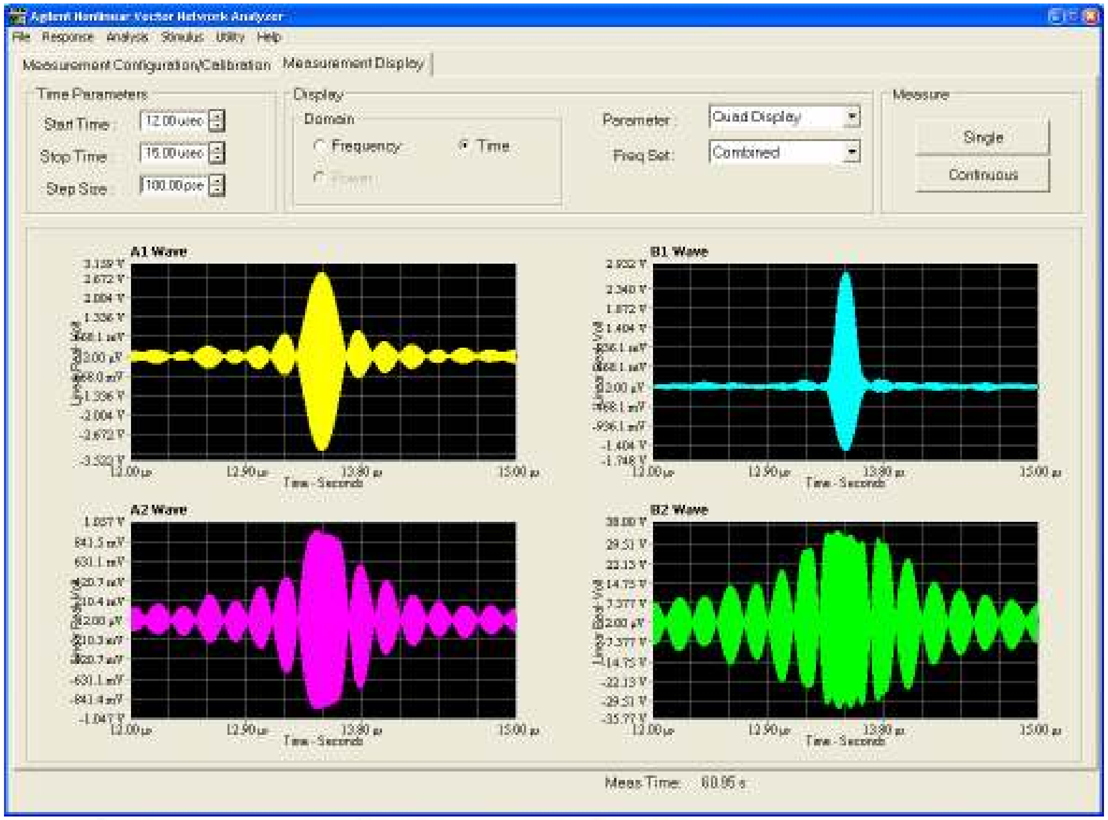
View time and frequency domain representations of a multi-tone stimulus to/from a device with vector correction applied (measurement plane at DUT terminals)

Stimulus is 64 frequencies spaced ~80 kHz apart centered at 2 GHz. The NVNA is measuring harmonics to 16 GHz (8th harmonic)

Multi-tone often used to mimic more complex modulation (i.e. CDMA) by matching complementary cumulative distribution function (CCDF). Multi-tone can be measured very accurately.

NVNA Applications (What does it do?)

Calibrated measurements of multi-tone stimulus/response with narrow tone spacing

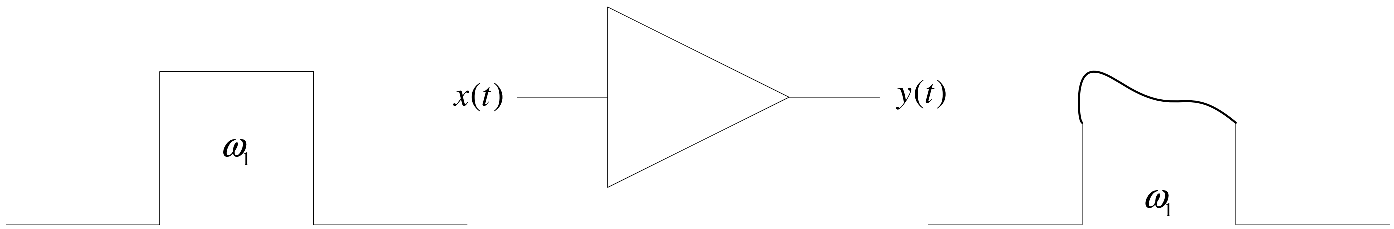


View time and frequency domain representations of a multi-tone stimulus to/from a device with vector correction applied (measurement plane at DUT terminals)

Stimulus is 64 frequencies spaced ~80 kHz apart centered at 2 GHz. The NVNA is measuring harmonics to 16 GHz (8th harmonic)

Multi-Envelope Domain

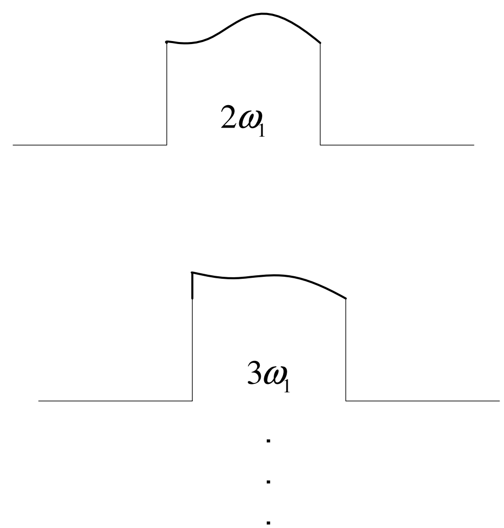
Memory Effects in Nonlinear Devices



$$x(t) = |A_1| e^{j\theta_1} e^{-j\omega_1 t}$$

Single frequency pulse with fixed phase and amplitude versus time

- Can measure envelope of the fundamental and harmonics with NVNA error correction applied. Use to analyze memory effects in nonlinear devices.
- Get vector corrected amplitude and phase of envelope.
- Use to measure and analyze memory effects in nonlinear devices.

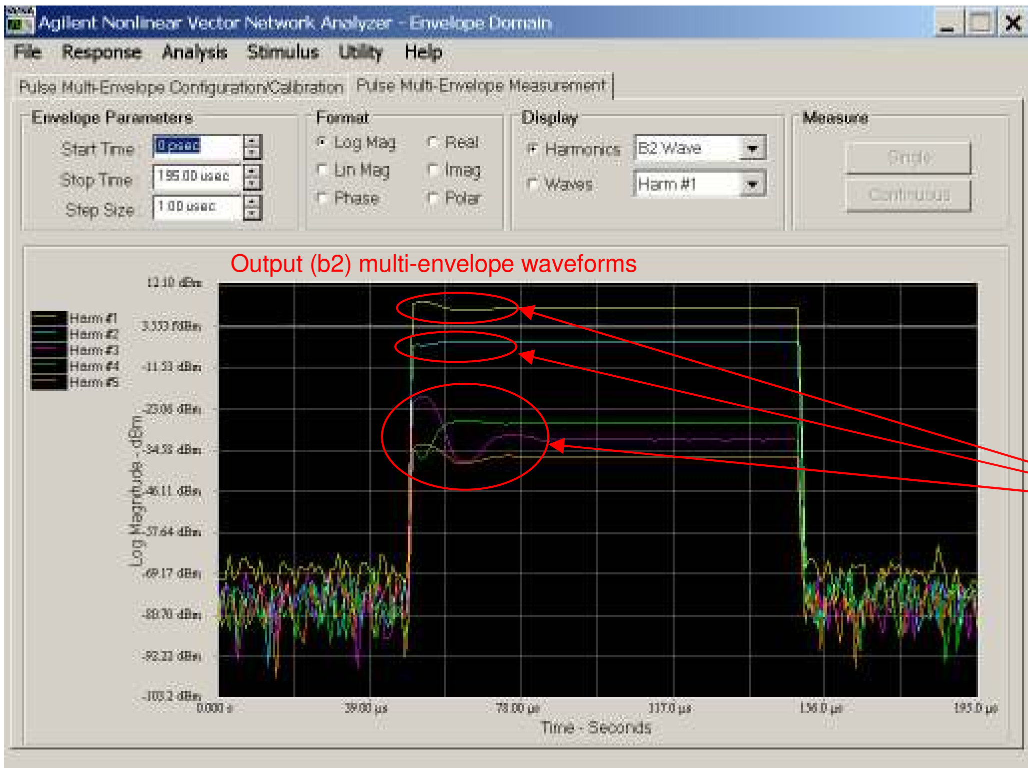


$$y(t) = \sum_{n=-\infty}^{\infty} |B_n(t)| e^{j\phi_n(t)} e^{-j\Omega_n t}$$

Multiple frequencies envelopes with time varying phase and amplitude

NVNA Applications (What does it do?)

Measure memory effects in nonlinear devices with vector error correction applied



View and analyze dynamic memory signatures using the vector error corrected envelope amplitude and phase at the fundamental and harmonics with a pulsed (RF/DC) stimulus

Each harmonic has a unique time varying envelope signature

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NVNA Applications (What does it do?)

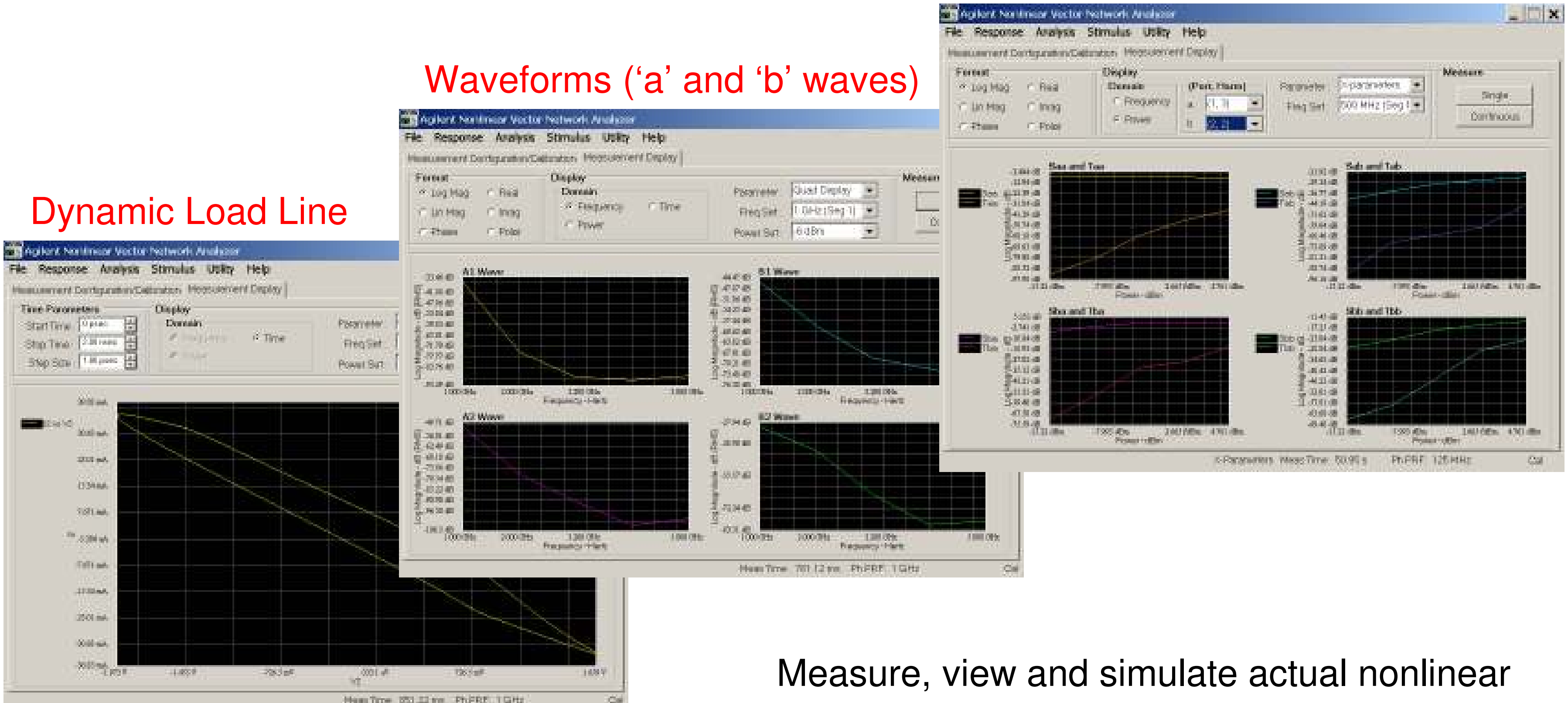
Measure modeling coefficients and other nonlinear device parameters

… More

X-parameters

Waveforms (‘a’ and ‘b’ waves)

Dynamic Load Line



Measure, view and simulate actual nonlinear data from your device

X-Parameters:

A New Paradigm for
Interoperable Measurement, Modeling,
and Simulation of *Nonlinear*
Microwave and RF Components

S-parameters: linear measurement, modeling, & simulation

- Easy to measure at high frequencies
  - measure voltage traveling waves with a (linear) vector network analyzer (VNA)
  - don't need shorts/opens which can cause devices to oscillate or self-destruct
- Relate to familiar measurements (gain, loss, reflection coefficient ...)
- Can cascade S-parameters of multiple devices to predict system performance
- Can import and use S-parameter files in electronic-simulation tools (e.g. ADS)
- BUT: No harmonics, No distortion, No nonlinearities, …
  Invalid for nonlinear devices excited by large signals, despite *ad hoc* attempts

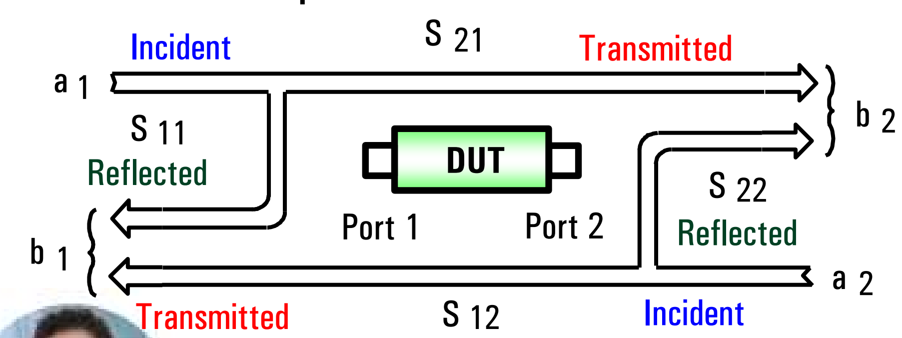
Linear Simulation:
Matrix Multiplication

S-parameters

$$b_1 = S_{11}a_1 + S_{12}a_2$$

$$b_2 = S_{21}a_1 + S_{22}a_2$$

Measure with linear VNA:
Small amplitude sinusoids



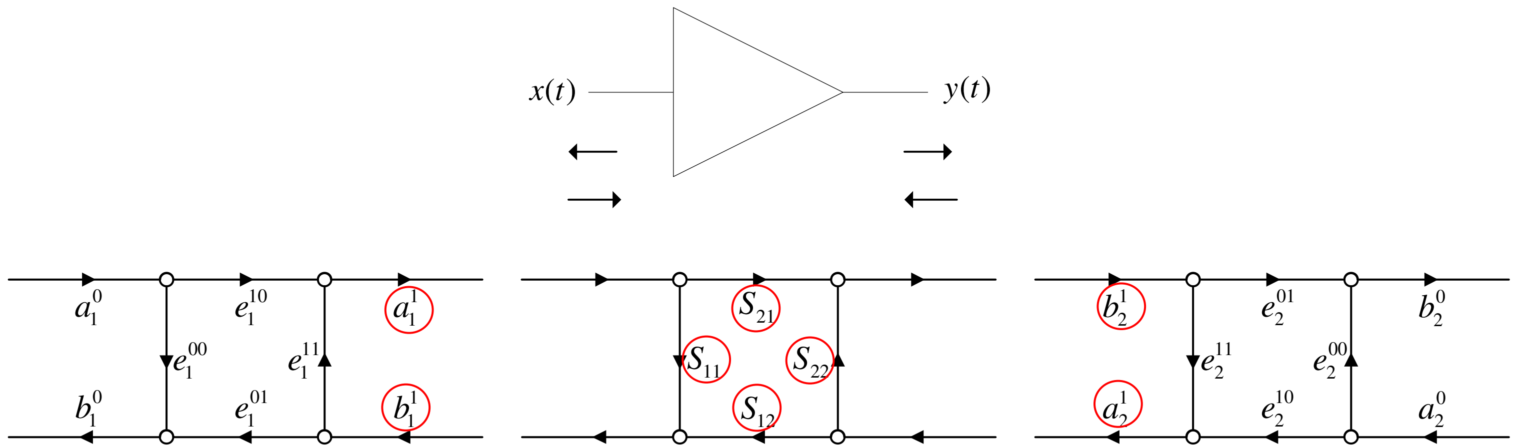
Model Parameters:
Simple algebra

$$S_{ij} = \left.\frac{b_i}{a_j}\right|_{\substack{a_k = 0 \\ k \neq j}}$$

Scattering Parameters – Linear Systems

Linear Describing Parameters

- **Linear S-parameters by definition require that the S-parameters of the device do not change during measurement.**



$$b_1 = S_{11}a_1 + S_{12}a_2$$
$$b_2 = S_{21}a_1 + S_{22}a_2$$

$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$

S-Parameter Definition

To solve VNA's traditionally use a forward and reverse sweep (2 port error correction).

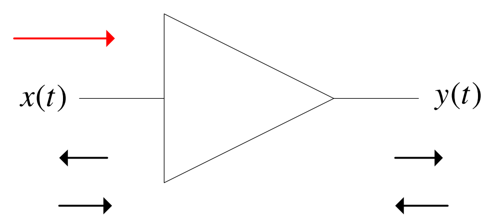
Scattering Parameters – Linear Systems

Linear Describing Parameters

- **If the S-parameters change when sweeping in the forward and reverse directions when performing 2 port error correction then by definition the resulting computation of the S-parameters becomes invalid.**

$$b_1 = S_{11}a_1 + S_{12}a_2$$
$$b_2 = S_{21}a_1 + S_{22}a_2$$

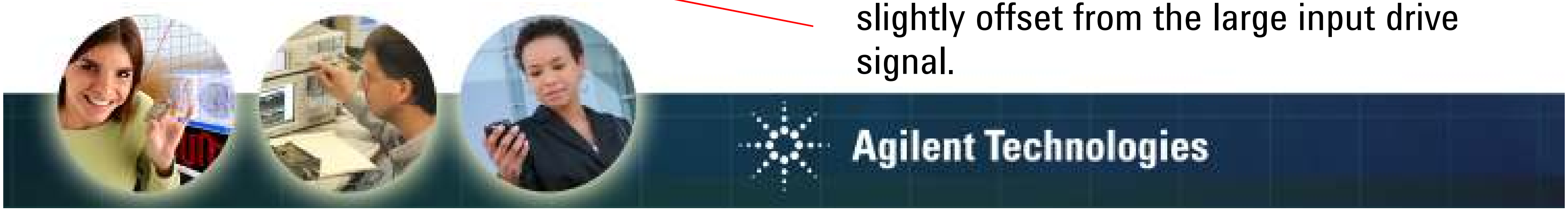
$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$



$$\begin{bmatrix} b_1^f & b_1^r \\ b_2^f & b_2^r \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} a_1^f & a_1^r \\ a_2^f & a_2^r \end{bmatrix}$$

Hot S22

This is often why customers are asking for Hot S22 because the match is changing versus input drive power and frequency (Nonlinear phenomena). Hot S22 traditionally measured at a frequency slightly offset from the large input drive signal.



X-parameters the nonlinear Paradigm:

Have the potential to revolutionize the Characterization, Design, and Modeling of nonlinear components and systems

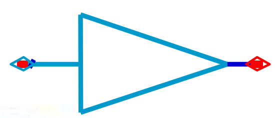
X-parameters are the mathematically correct extension of S-parameters to large-signal conditions.

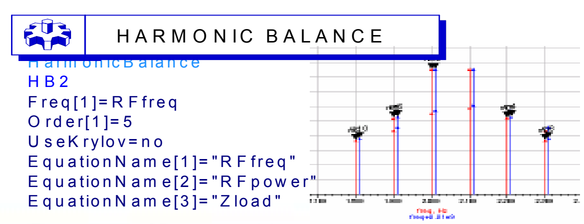
- Measurement based, device independent, identifiable from a simple set of automated NVNA measurements
- Fully nonlinear (Magnitude *and* phase of distortion)
- Cascadable (correct behavior in even highly mismatched environment)
- Extremely accurate for high-frequency, distributed nonlinear devices

NVNA:
Measure device X-parms

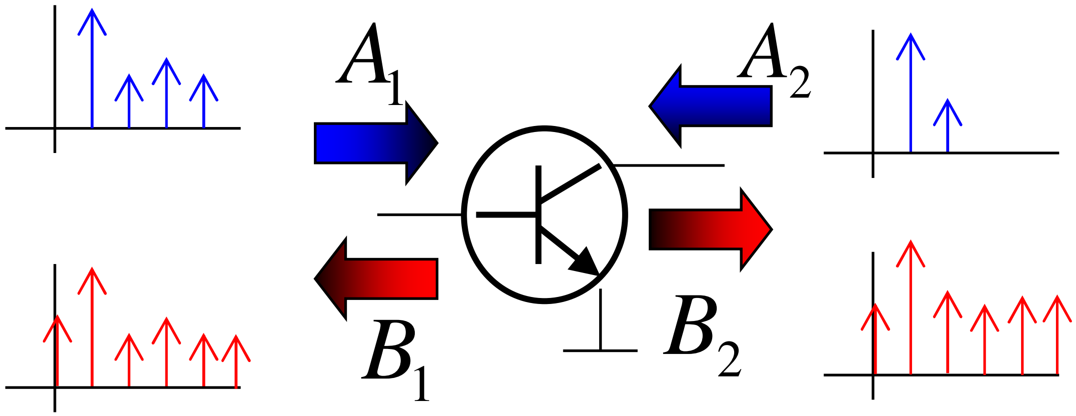
PHD component :
Simulate using X-parms

ADS:
Design using X-parms





X-parameters come from the Poly-Harmonic Distortion (PHD) Framework

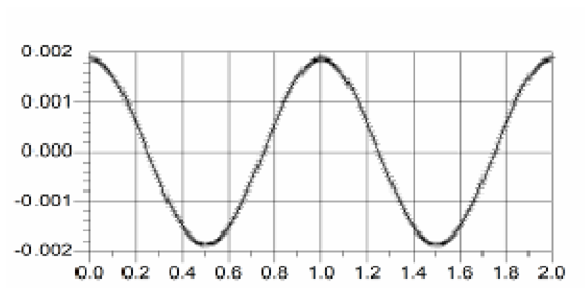


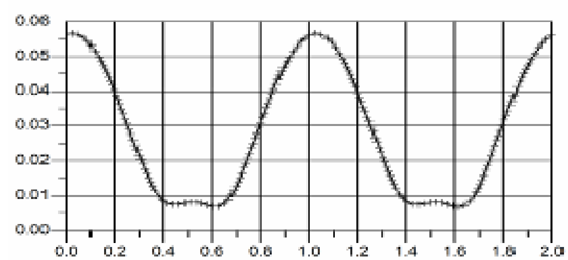
$$B_{1k} = F_{1k}(DC, A_{11}, A_{12}, \ldots, A_{21}, A_{22}, \ldots)$$

$$B_{2k} = F_{2k}(DC, A_{11}, A_{12}, \ldots, A_{21}, A_{22}, \ldots)$$

Harmonic (or carrier) Index

Port Index





Unifies
S-parameters
Load-Pull,
Time-domain
load-pull

Data and Model formulation in Frequency (Envelope) Domain

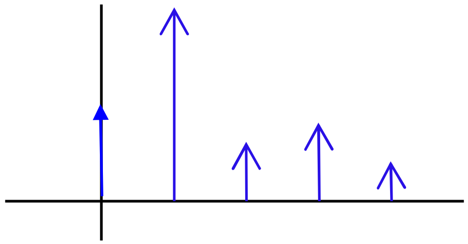
Magnitude and phase enables complete time-domain input-output waveforms

X-parameters: Systematic Approximations to NL Mapping

Trade measurement time, size, accuracy for speed, practicality

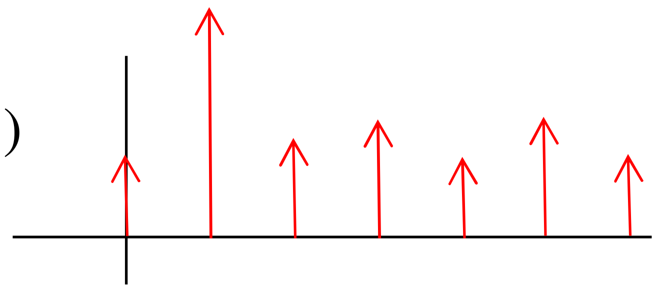
Incident

Scattered

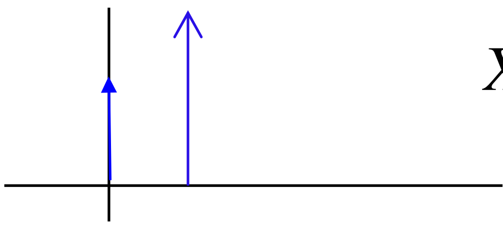


$$B_k(DC, A_1, A_2, A_3, \ldots)$$

Multi-variate NL map

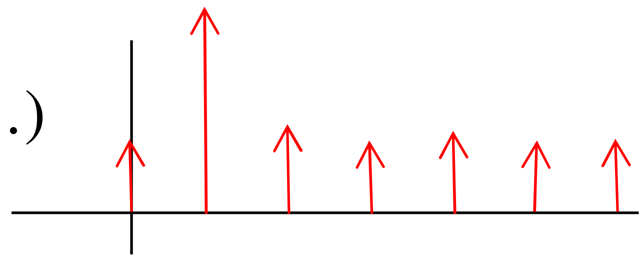


$$\approx$$

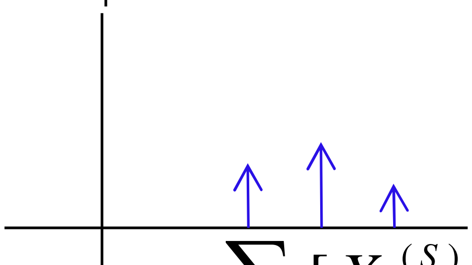


$$X_k^{(F)}(DC, A_1, 0, 0, 0, \ldots)$$

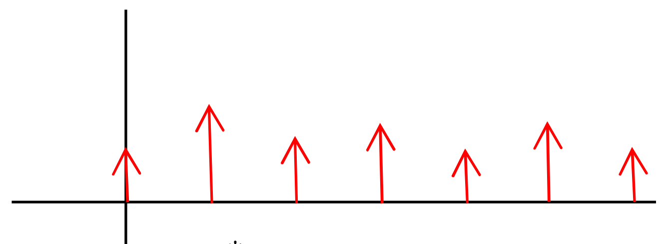
Simpler NL map



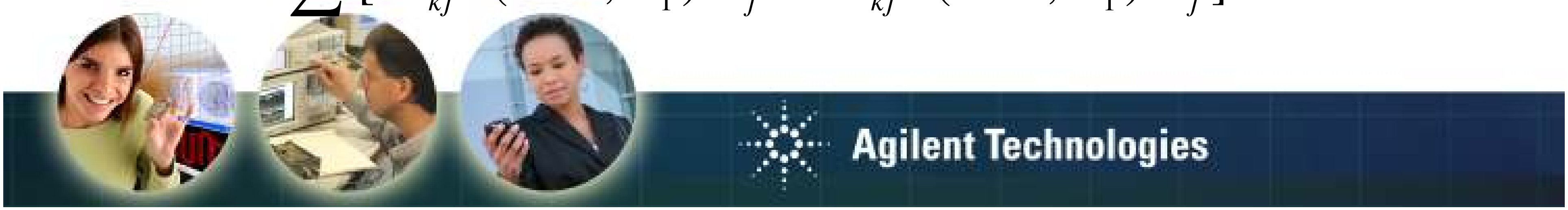
$$+$$



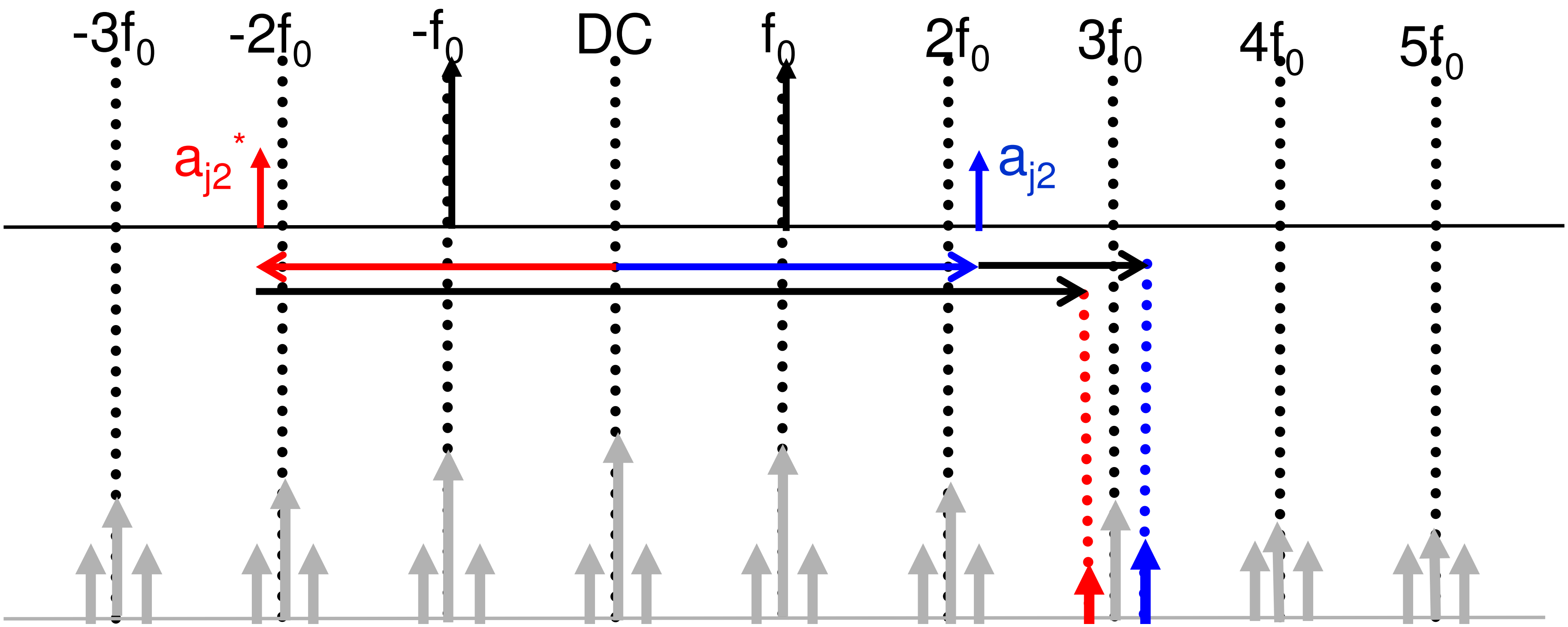
Linear non-analytic map



$$\sum [X_{kj}^{(S)}(DC, A_1)A_j + X_{kj}^{(T)}(DC, A_1)A_j^*]$$



X-parameter Experiment Design & Identification



$X^{(T)}_{i3,j2} \cdot a^{*}_{j2}$ $\quad$ $X^{(S)}_{i3,j2} \cdot a_{j2}$

$5f_0 - f_1$ $\quad$ $f_0 + f_1$

There are *two contributions* at each harmonic

From different orders in NL conductance of driven system

$$B_{e,f} = X^{(F)}_{ef}(|A_{11}|)P^{f} + \sum_{g,h} X^{(S)}_{ef,gh}(|A_{11}|)P^{f-h} \cdot a_{gh} + \sum_{g,h} X^{(T)}_{ef,gh}(|A_{11}|)\ P^{f+h} \cdot a^{*}_{gh} \qquad P = e^{j\varphi(A_{11})}$$

Scattering Parameters – Nonlinear Systems
X-parameters

$$b_{ij} = X_{ij}^{(F)}(|a_{11}|)P^{j} + \sum_{k,l\neq(1,1)} \left(X_{ij,kl}^{(S)}(|a_{11}|)P^{j-l} \cdot a_{kl} + X_{ij,kl}^{(T)}(|a_{11}|) \; P^{j+l} \cdot a_{kl}^{*} \right)$$

Definitions

- **i = output port index**
- **j = output frequency index**
- **k = input port index**
- **l = input frequency index**

Description

- **The X-parameters provide a mapping of the input and output frequencies to one another.**

X-Parameters Collapse to S-Parameters in Linear Systems

By definition, $P = \frac{a_1}{|a_1|}$

$$b_{ij} = X_{ij}^{(F)}(|a_{11}|)P^{j} + \sum_{k,l \neq (1,1)} \left(X_{ij,kl}^{(S)}(|a_{11}|)P^{j-l} \cdot a_{kl} + X_{ij,kl}^{(T)}(|a_{11}|) \; P^{j+l} \cdot a_{kl}^{*} \right)$$

For small $|a_{11}|$ (linear), X^T terms go to 0.
Cross-frequency terms also go to 0

$$b_{ij} = X_{ij}^{(F)}(|a_{11}|)P^{j} + \sum_{\substack{l=j \\ k,l \neq (1,1)}} \left(X_{ij,kl}^{(S)} P^{j-l} \cdot a_{kl} \right)$$

Consider fundamental frequency (j = 1). Harmonic index is no longer needed.

$$b_{i} = X_{i}^{(F)}(|a_{11}|)P + \sum_{k \neq 1} \left(X_{ik}^{(S)} \cdot a_{k} \right)$$

Assume 2 port (i and k = 1 -> 2)

$$b_1 = X_{1}^{(F)}(|a_{11}|)P + X_{12}^{(S)} \cdot a_2$$
$$b_2 = X_{2}^{(F)}(|a_{11}|)P + X_{22}^{(S)} \cdot a_2$$

For small $|a_{11}|$ (linear), X^F terms go to $S_{i1} \cdot |a_{11}|$, and X^S terms are equal to linear S parameters

$$b_1 = S_{11}|a_1|P + S_{12}a_2$$
$$b_2 = S_{21}|a_1|P + S_{22}a_2$$

$$b_1 = S_{11}a_1 + S_{12}a_2$$
$$b_2 = S_{21}a_1 + S_{22}a_2$$

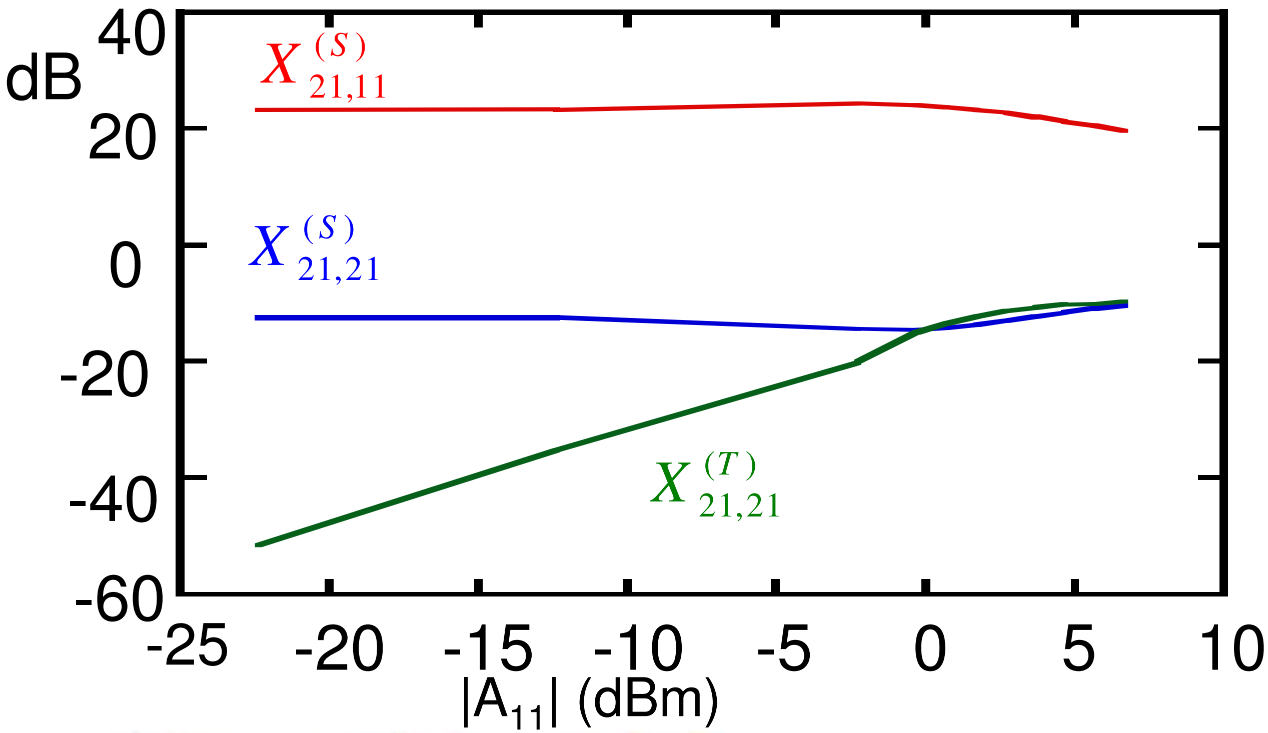
Definitions

- **i = output port index**
- **j = output frequency index**
- **k = input port index**
- **l = input frequency index**

Example: Fundamental Component

$$B_{21}(|A_{11}|) = X_{21}^{(F)}(|A_{11}|)P + X_{21,21}^{(S)}(|A_{11}|)A_{21} + X_{21,21}^{(T)}(|A_{11}|)P^{2}A_{21}^{*}$$

$$B_{21}(|A_{11}|) = X_{21,11}^{(S)}(|A_{11}|)A_{11} + X_{21,21}^{(S)}(|A_{11}|)A_{21} + X_{21,21}^{(T)}(|A_{11}|)P^{2}A_{21}^{*}$$



$$X_{21,11}^{(S)}(|A_{11}|) \underset{|A_{11}|\to 0}{\longrightarrow} s_{21}$$

$$X_{21,21}^{(S)}(|A_{11}|) \underset{|A_{11}|\to 0}{\longrightarrow} s_{22}$$

$$X_{21,21}^{(T)}(|A_{11}|) \underset{|A_{11}|\to 0}{\longrightarrow} 0$$

Reduces to (linear) S-parameters in the appropriate limit

X-parameter Experiment Design & Identification

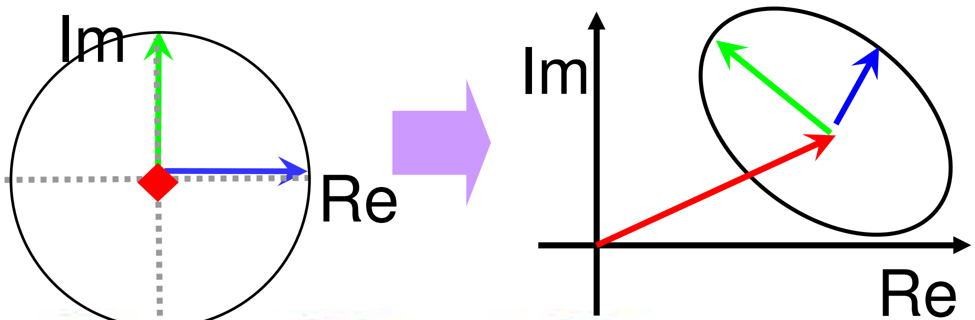
Ideal Experiment Design

E.g. functions for B_{pm} (port p, harmonic m)

$$\underbrace{B_{pm} = X^{(F)}_{pm}(|A_{11}|)P^{m}} + \underbrace{X^{(S)}_{pm,qn}(|A_{11}|)P^{m-n}A_{qn} + X^{(T)}_{pm,qn}(|A_{11}|)P^{m+n}A^{*}_{qn}}$$

Perform 3 independent experiments with fixed A_{11} using orthogonal phases of A_{21}

input A_{qn} output B_{pm}



$$B^{(0)}_{pm} = X^{(F)}_{pm}(|A_{11}|)P^{m}$$

$$B^{(1)}_{pm} = X^{(F)}_{pm}(|A_{11}|)P^{m} + X^{(S)}_{pm,qn}(|A_{11}|)P^{m-n}A^{(1)}_{qn} + X^{(T)}_{pm,qn}(|A_{11}|)P^{m+n}A^{(1)}_{qn}$$

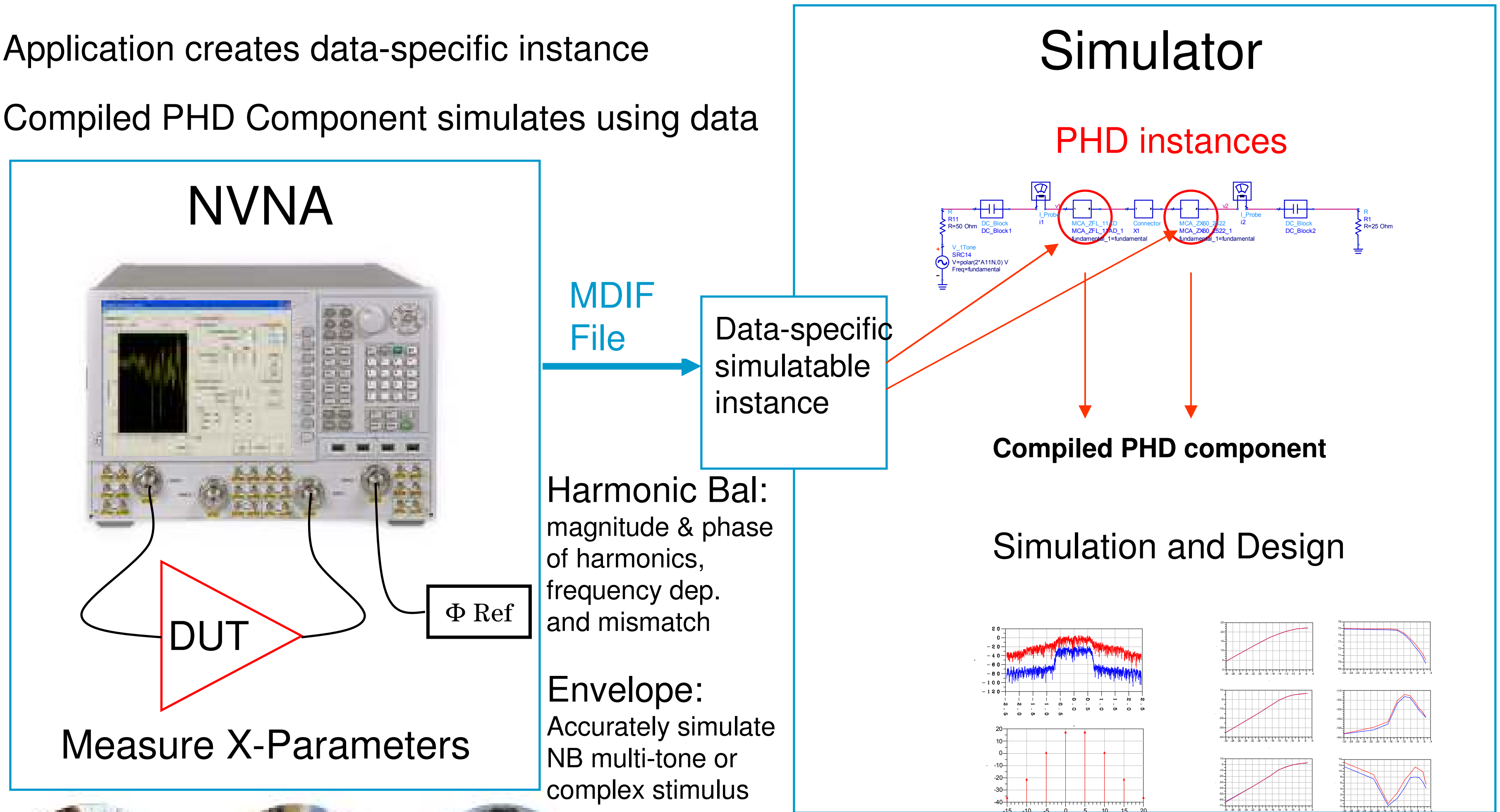
$$B^{(2)}_{pm} = X^{(F)}_{pm}(|A_{11}|)P^{m} + X^{(S)}_{pm,qn}(|A_{11}|)P^{m-n}A^{(2)}_{qn} + X^{(T)}_{pm,qn}(|A_{11}|)P^{m+n}A^{(2)*}_{qn}$$

X-parameter Application Flow

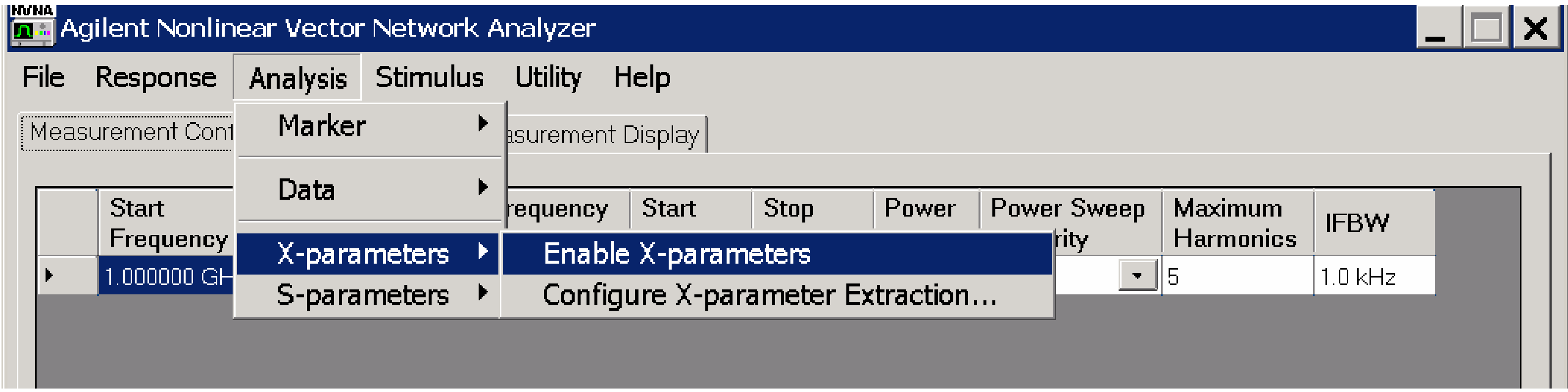
Automated DUT X-params measured on NVNA

Application creates data-specific instance

Compiled PHD Component simulates using data



Measurement on the NVNA

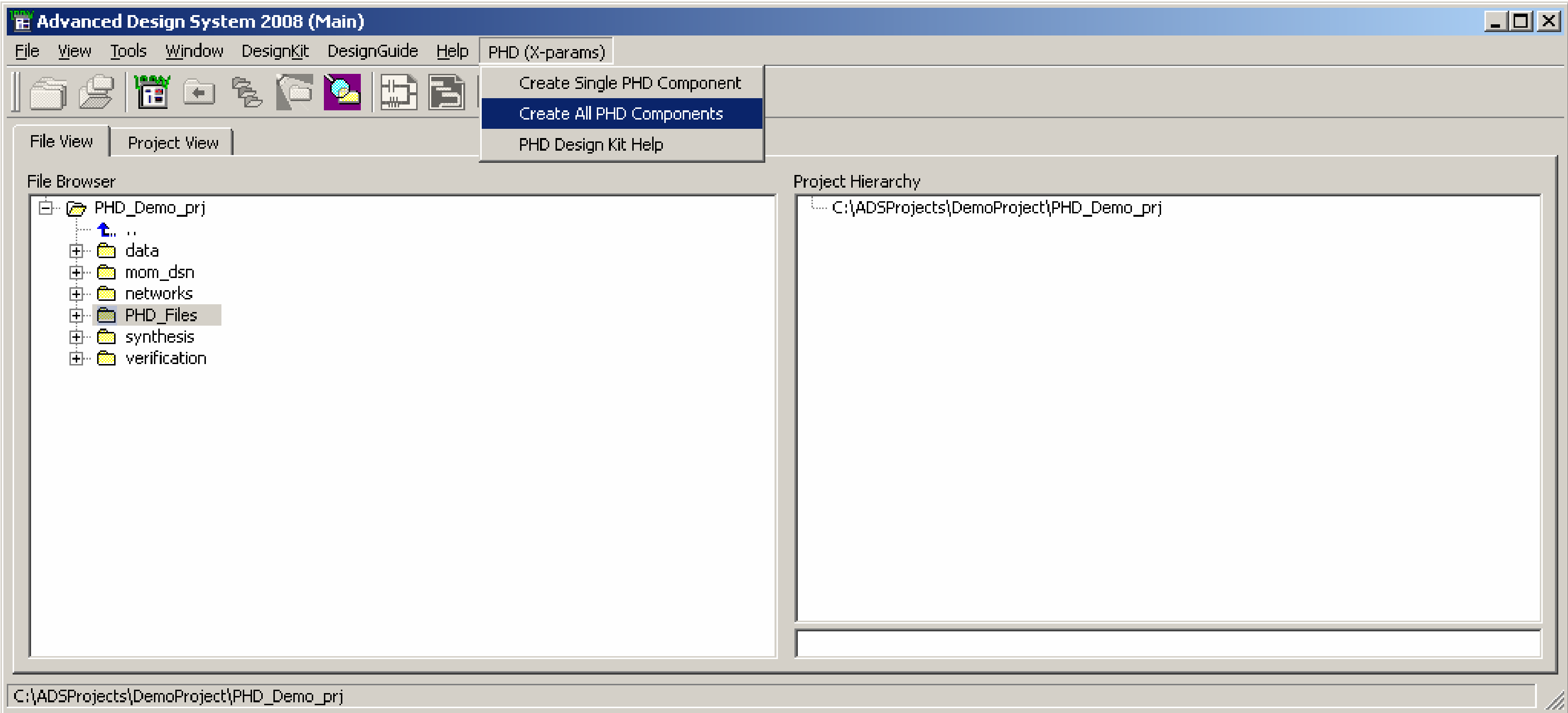


Switching from general measurements to X-parameter measurements is as simple as selecting “Enable X-parameters”

Measuring X-parameters for 5 harmonics at 5 fundamental frequencies with 15 power points each (75 operating points) can take less than 5 minutes

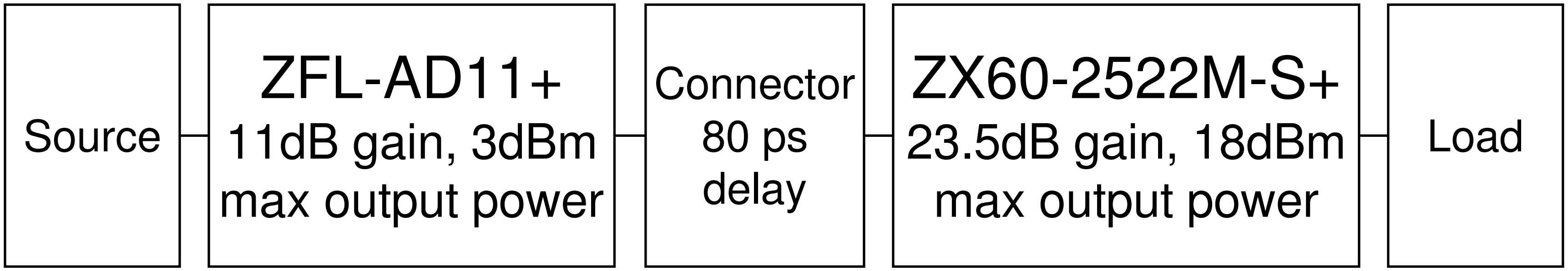
DC bias information can be measured using external instruments (controlled by the NVNA) and included in the data

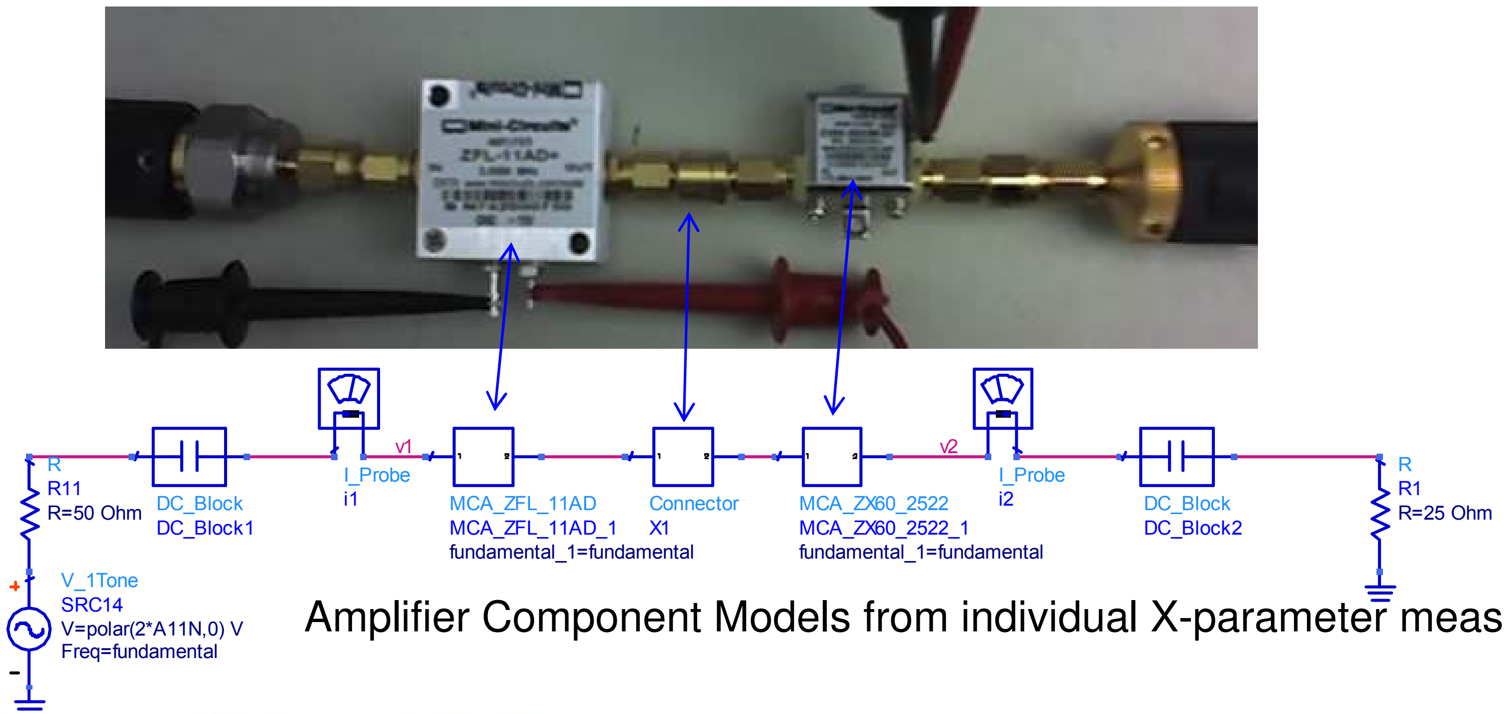
PHD Design Kit



The MDIF file containing measured X-parameters is imported into ADS by the PHD Design Kit, creating a component that can be used in Harmonic Balance or Envelope simulations.

Measurement-based nonlinear design with X-parameters



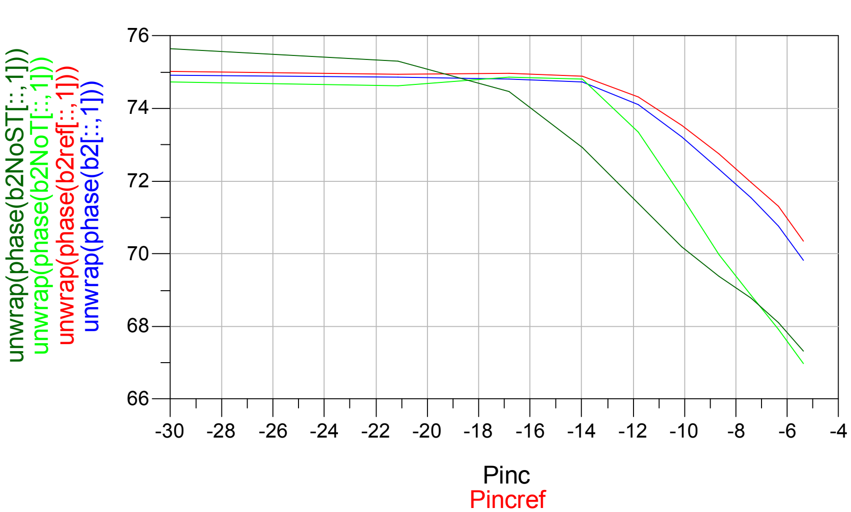


Amplifier Component Models from individual X-parameter measurements

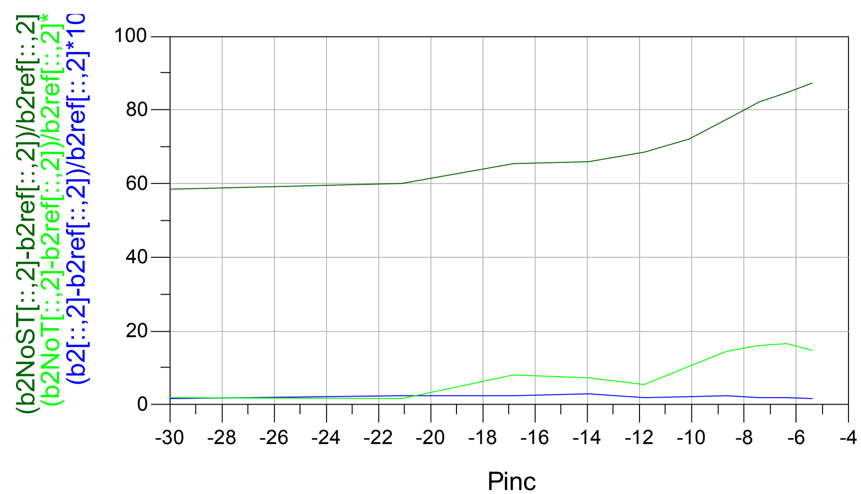
Results
Cascaded Simulation vs. Measurement

Red: Cascade Measurement
Blue: Cascaded X-parameter Simulation
Light Green: Cascaded Simulation, No $X^{(T)}$ terms
Dark Green: Cascaded Models, No $X^{(S)}$ or $X^{(T)}$ terms

Fundamental Phase



Second Harmonic % Error



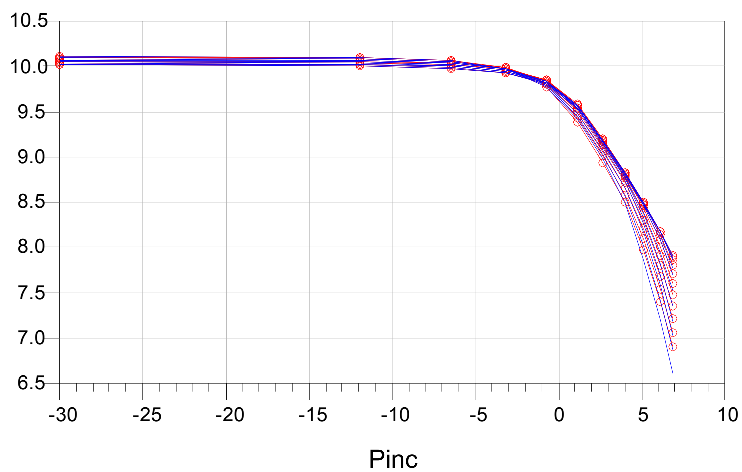
Large-mismatch capability (load-pull)

Full Nonlinear Dependence on both A1 & A2 [13]

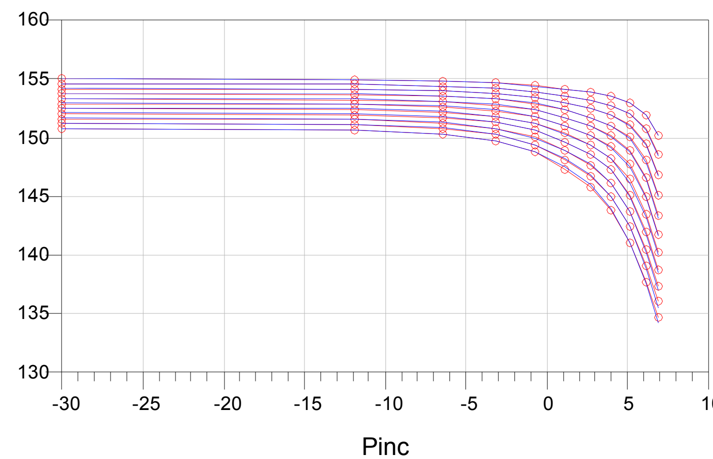
$$B_{ik} = X_{ik}^{(F)}(A_{11}, A_{21}, 0, 0, \ldots) + \begin{array}{c}\text{Terms } \textit{linear} \text{ in the} \\ \text{remaining components}\end{array}$$

Mismatched Loads: $0 \le |\Gamma| \le 1$

Fundamental Gain

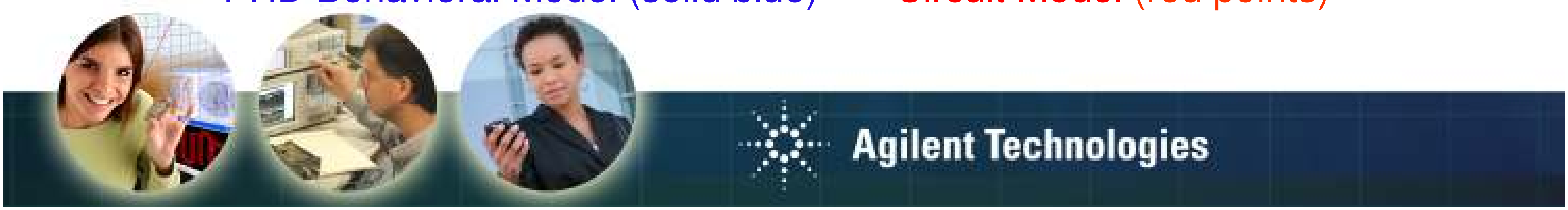


Fundamental Phase



Fundamental frequency: 4 GHz

PHD Behavioral Model (solid blue) Circuit Model (red points)

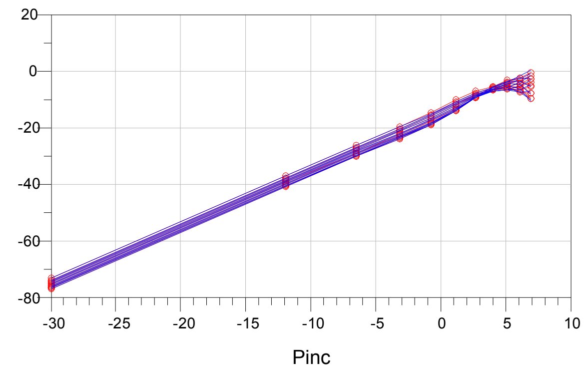


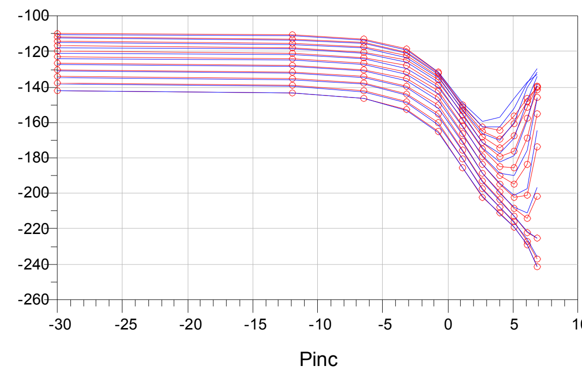
Large-mismatch capability (load-pull)

Mismatched Loads: $0 \le |\Gamma| \le 1$

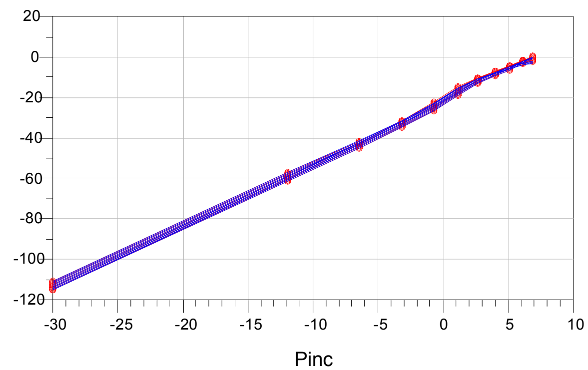
Magnitude | Phase

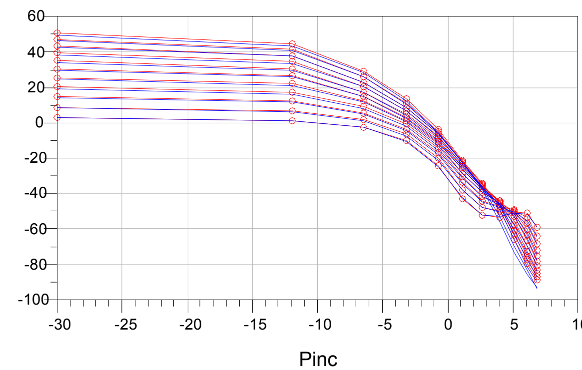
Second Harmonic





Third Harmonic





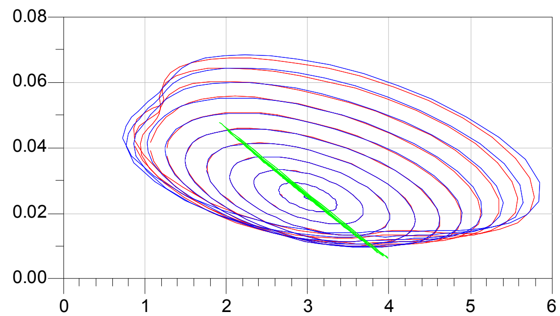
Fundamental frequency: 4 GHz

PHD Behavioral Model (solid blue) Circuit Model (red points)

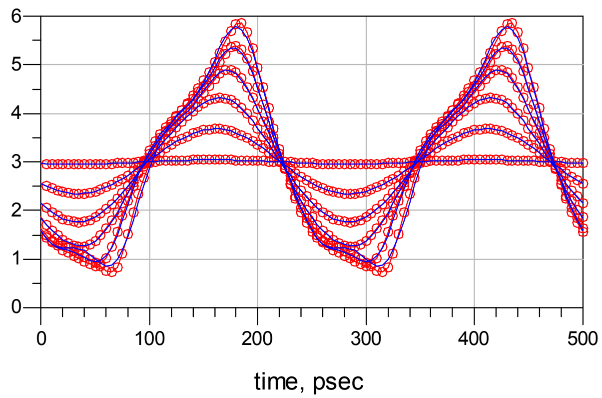
Large-mismatch capability (load-pull)

Mismatched Loads: $0 \leq |\Gamma| \leq 1$

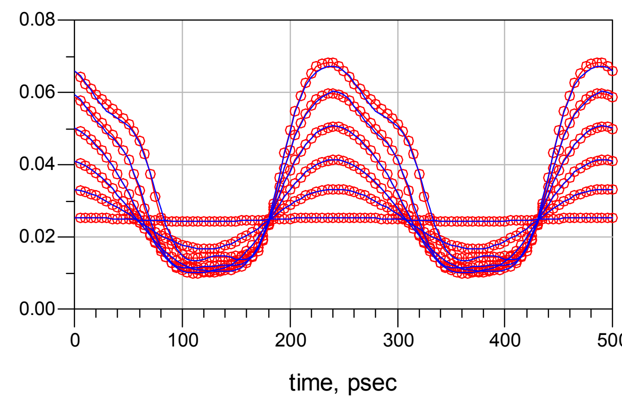
Dynamic Load-lines (green for matched case)



Port 2 Voltage



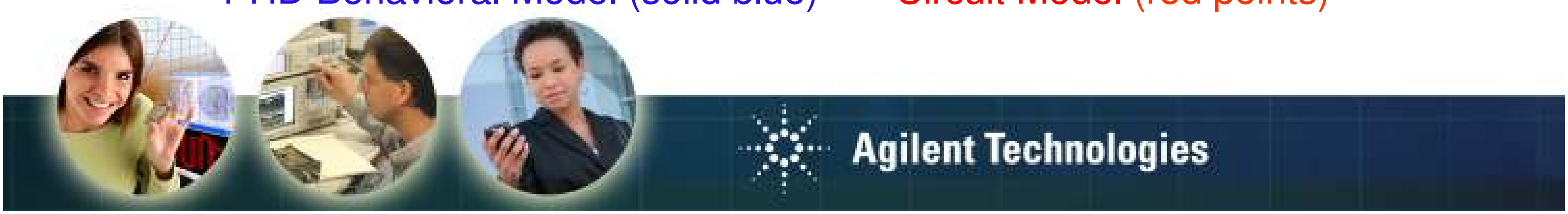
Port 2 Current



Fundamental frequency: 4 GHz

PHD Behavioral Model (solid blue)

Circuit Model (red points)



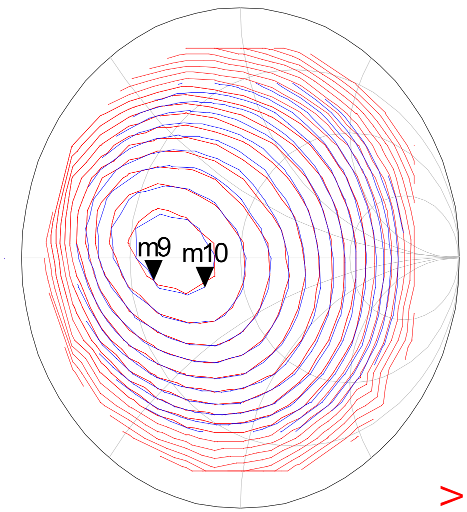
PHD-Simulated vs Load-Pull Measured Contours & Waveforms from *Load-dependent X-parameters* (WJ transistor)

Red: Load-pull data

Blue: PHD model simulated

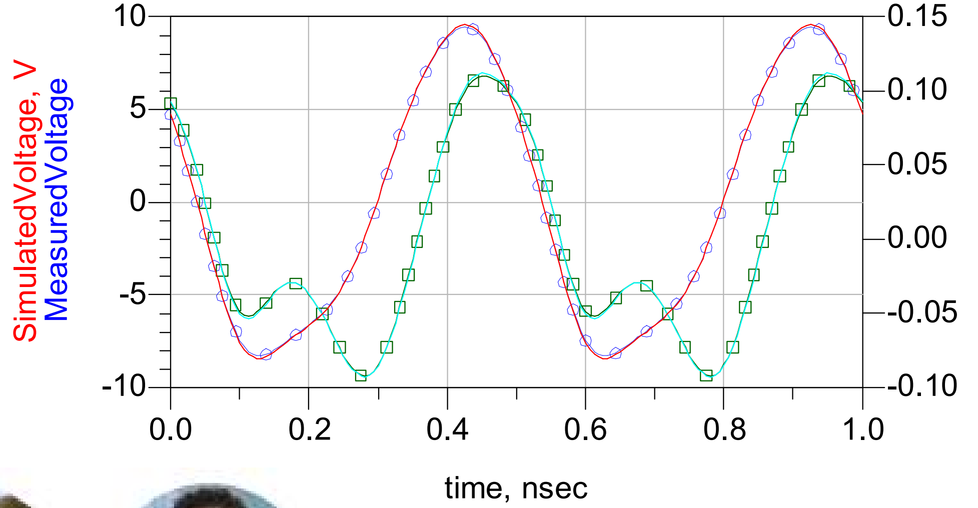
Fundamental Gamma = 0.383+j*0.31

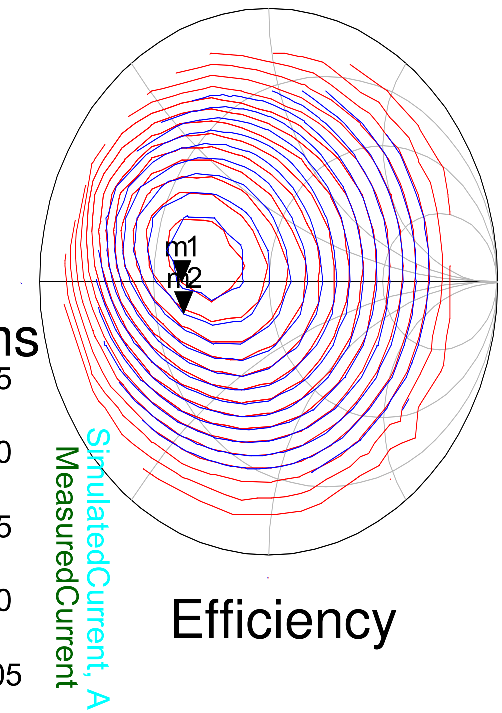
WJ Transistor



Power Delivered

Measured & Simulated Waveforms

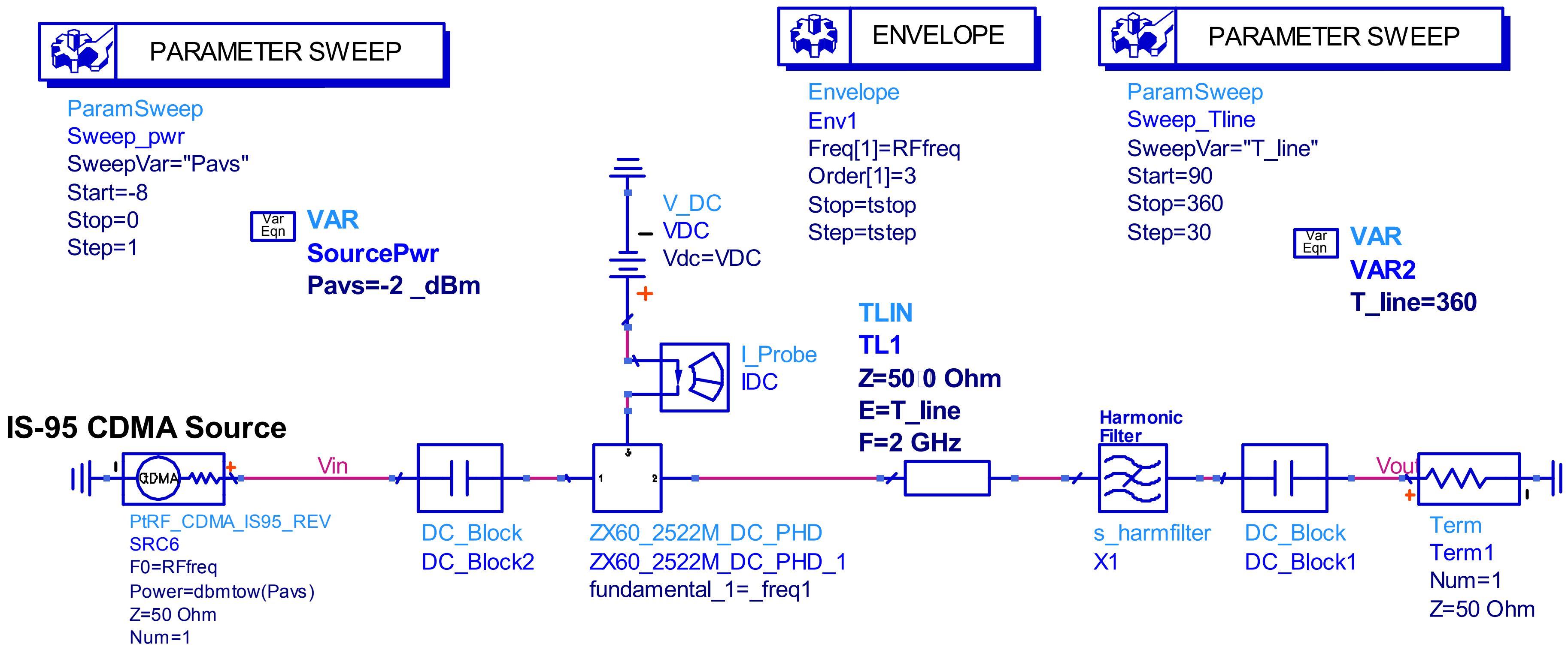




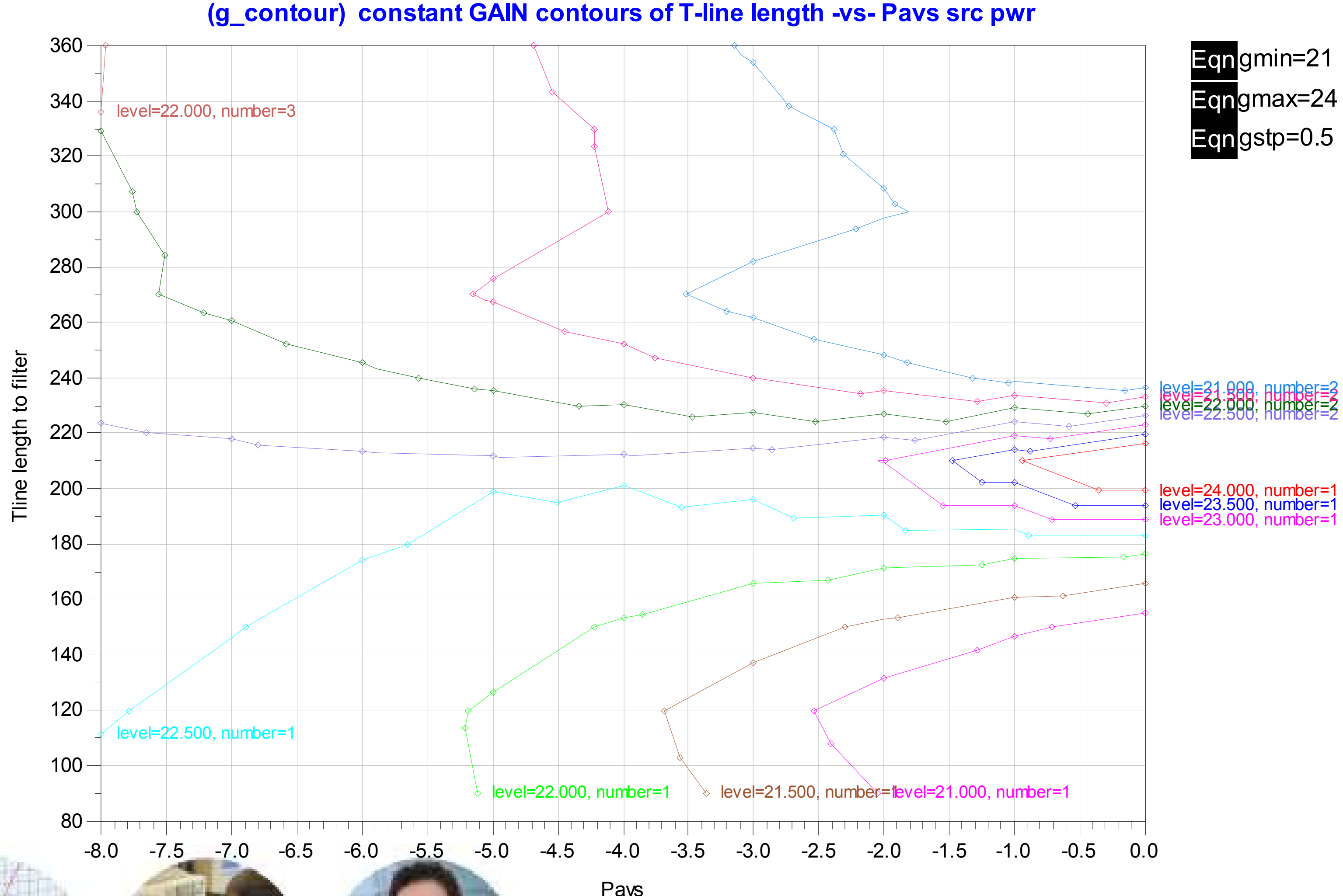
Efficiency

Amplifier with bias; standard compliant wideband modulation source;

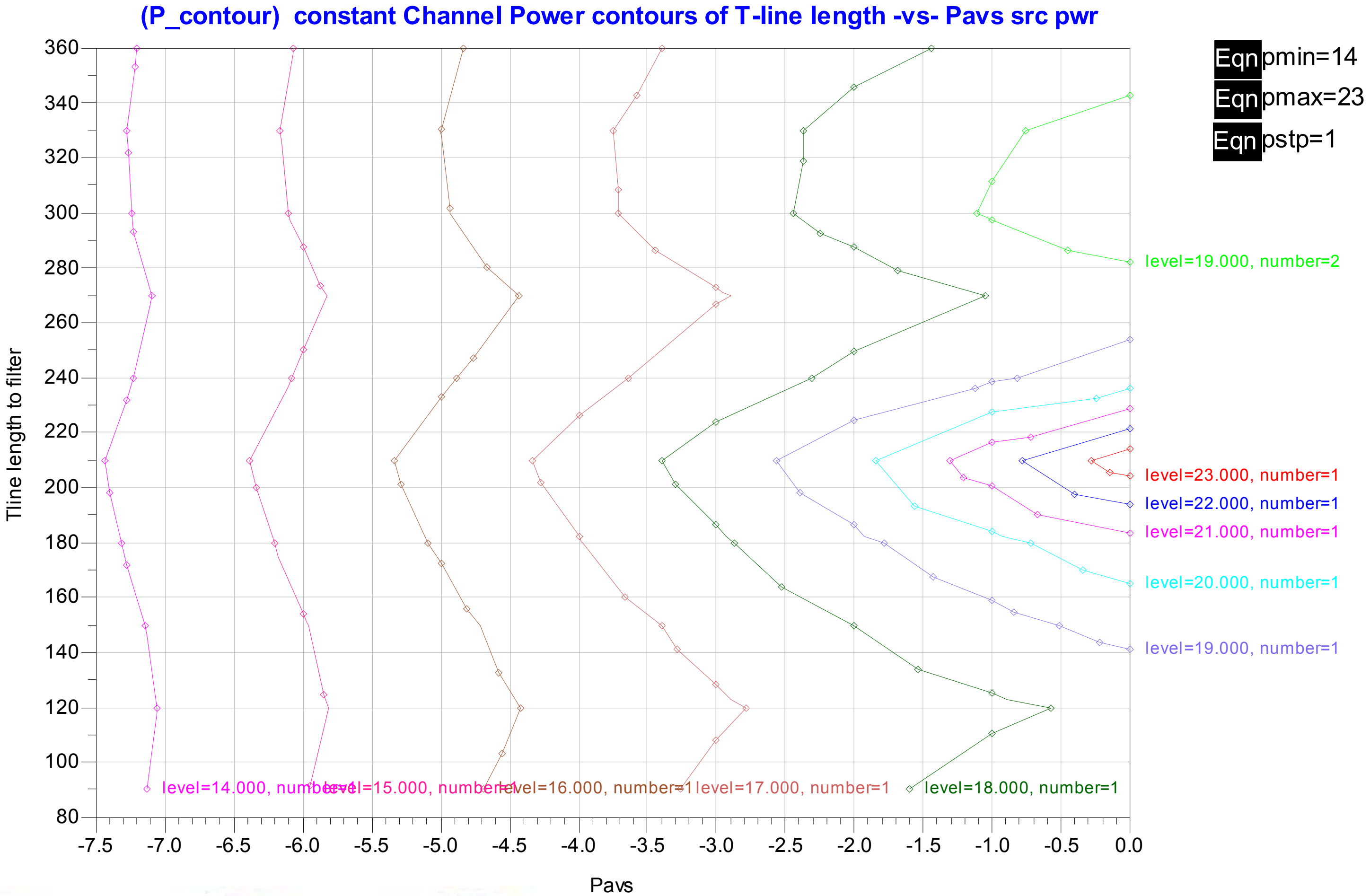
Parametric sweep of A) source power (dBm), & B) electrical distance (degrees) from output of amp to 2nd harmonic notch filter [fundamental = 1GHz]



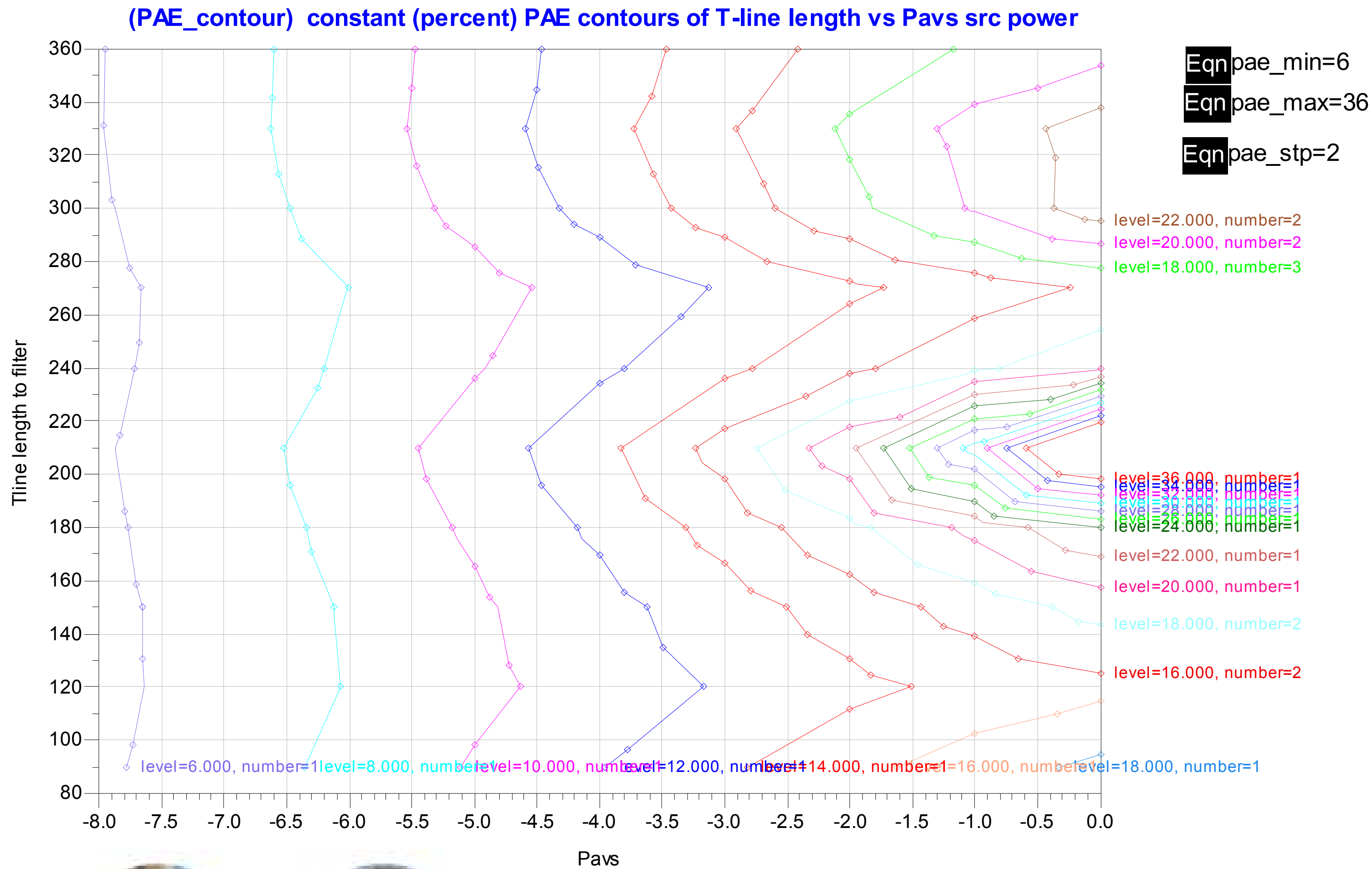
Gain in dB
+21 to +24 in 0.5 dB steps

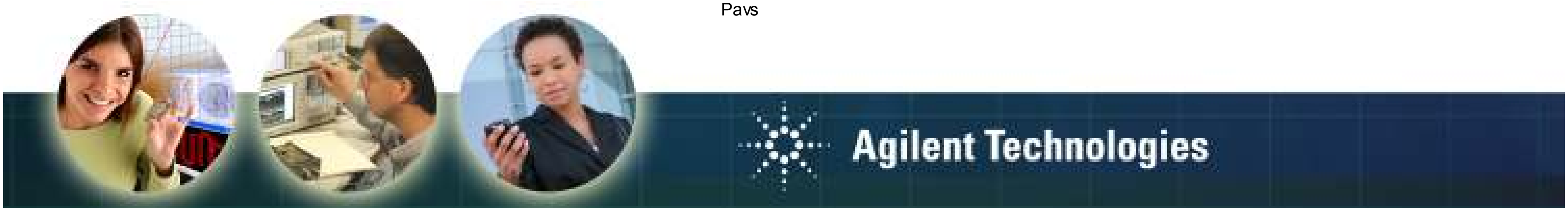


Power delivered (dBm)
+14 to +23 in 1 dB steps



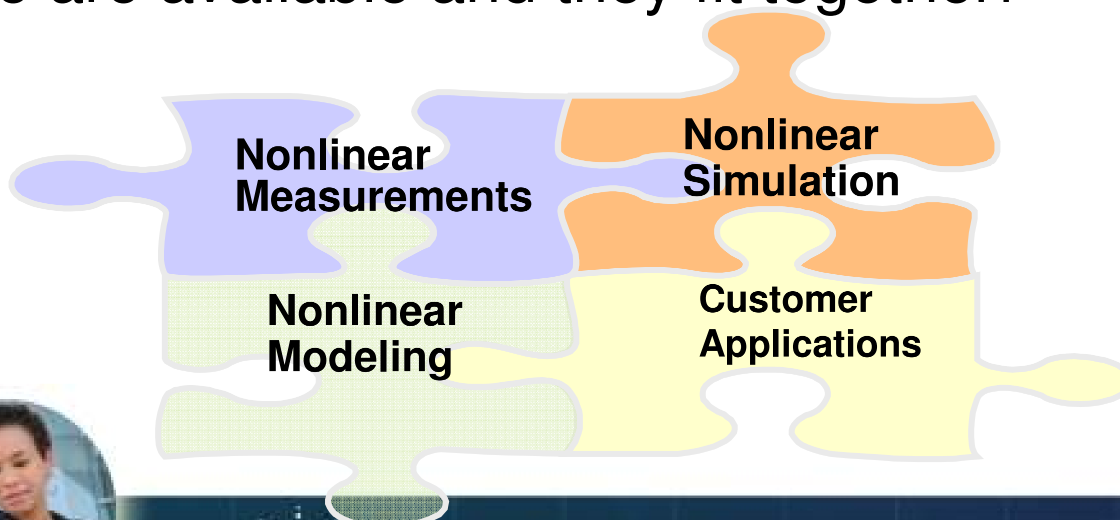
Power Added Efficiency (%)
6% to 36% in 2% steps





Summary

- X-parameters are a mathematically correct superset of S-parameters for nonlinear devices under large-signal conditions
  - *Rigorously derived from general PHD theory*; flexible, practical, powerful
- X-parameters can be accurately measured by automated set of experiments on the new Agilent NVNA instrument
- Together with the PHD component, measured X-parameters can be used in ADS to design nonlinear circuits
- All pieces of the puzzle are available and they fit together!



Summary

New Nonlinear Vector Network Analyzer based on a standard PNA-X

New phase calibration standard

Vector (amplitude/phase) corrected nonlinear measurements from 10 MHz to 26.5 GHz

Calibrated absolute amplitude and relative phase (cross-frequency relative phase) of measured spectra traceable to standards lab

26 GHz of vector corrected bandwidth for time domain waveforms of voltages and currents of DUT

Multi-Envelope domain measurements for measurement and analysis of memory effects

X-parameters: Extension of Scattering parameters into the nonlinear region providing unique insight into nonlinear DUT behavior

X-parameter extraction into ADS PHD block for nonlinear simulation and design