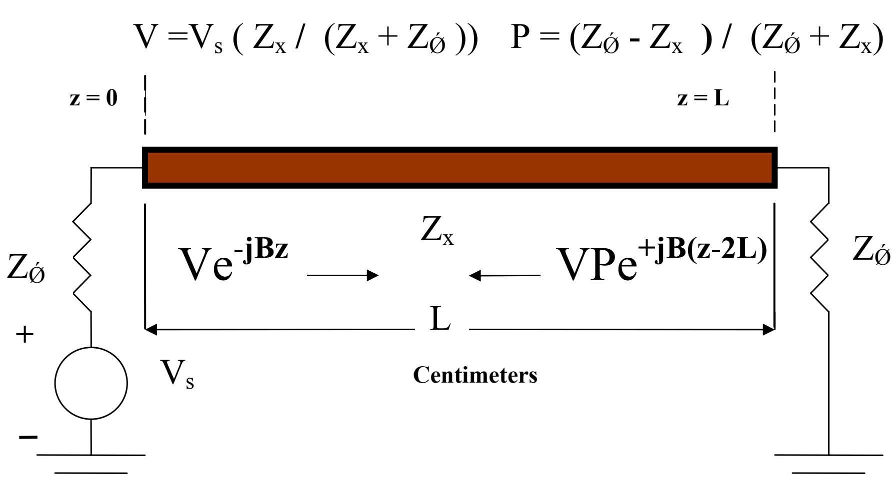
In a coupler made up of parallel coupled lines there is a phase relationship between the through port and the coupled port. The electrical phase of the coupled port is +90 degrees when the through port is referenced as 0 degrees. This +90 degree phase occurs at all frequencies for which the coupler has a good match. Parallel line couplers which have this characteristic are designed symmetrically with an odd number of sections.

It is true that these couplers have 90 degree electrical length lines at the center frequency of operation, but this has nothing to do with the 90 degree phase relationship between coupled and through ports. It does have everything to do with getting maximum coupling amplitude at the center frequency. Whatever creates this phase shift must then be independent of frequency.

The explanation for this can summed up in one sentence but would take pages of derivation to prove it. The explanation is the following sentence. **"At a coupled port equal forward and reverse propagating waves combine 0 degree and 180 degree phase components generated by mode reflection coefficients to produce a 90 degree phase shift independent of frequency."** This statement now needs to be put into the correct context with transmission line theory along with even and odd mode analysis. It should now be clear that variation of line length will not "tune" the relative phase shift of the coupled port. It is only the even and odd mode impedances that can do this. This frequency independent 90 degree phase shift is a very useful property, especially for passive broadband designs.

Transmission Line Theory

A transmission line of length L with a characteristic impedance Zx is terminate at both ends with load and source impedances $Z_{\acute{\varnothing}}$ along with a source voltage Vs shown in **Fig. 1**. The forward and reverse propagating waves are shown in complex exponential form. Initial voltage division at the input produces voltage V and the reflection coefficient P determines the voltage amplitude of the reverse propagating wave.



λ = wavelength
B = 2π / λ = wave propagation constant

Fig. 1

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Transmission Line Theory

There are multiple reflections on this transmission line and the reflection coefficient at both ends is P because the terminating impedances are both $Z_Ø$. This infinite series of reflections can be analyzed by the following equations. The infinite series equations can be put into a closed form using mathematical identities. The net result is that the initial forward and reverse reflected voltage components are scaled by the constant factor $1/(1- P^2e^{-jB(2L)})$ in a closed form expression.

$Ve^{-jBz} + VPe^{+jB(z-2L)} + VP^2e^{-jB(z+2L)} + VP^3e^{+jB(z-4L)} + VP^4e^{-jB(z+4L)} + VP^5e^{+jB(z-6L)} \ldots$ = Sum total of voltage reflections at the input.

$$= (Ve^{-jBz} + VPe^{+jB(z-2L)})(1 + \sum_{N=1}^{N=\infty} (P^2e^{-jB(2L)})^N)$$

$$= (Ve^{-jBz} + VPe^{+jB(z-2L)})(1- P^2e^{-jB(2L)})^{-1}$$ Closed form expression.

When z = 0 the voltage at the input is given and when z = L, the voltage at the other end of the line away from the source is given.

$20\log(|P|)$ = Return loss of a terminated port.

$\frac{1+ |P|}{1 - |P|}$ = VSWR of terminated port.

The terminating impedances could also be connecting transmission lines with characteristic impedances and terminations equal to $Z_Ø$. The result would be the same but in a slightly different form as follows. $V_s/2$ = Incident voltage wave from source. The transmitted wave amplitude into the Z_x characteristic impedance line would then be $(V_s/2)(2 Z_x / (Z_x + Z_Ø)) = V_s(Z_x / (Z_x + Z_Ø))$ which is the original incident voltage divider result.

These results can now be applied to coupled lines using even and odd mode analysis. Each mode will first be analyzed independently using transmission line theory and then the mode voltages will be combined to produce the total coupled line response.

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Coupled Line Even and Odd Mode Analysis

A set of coupled lines is shown in **Fig. 2** along with corresponding equations. Coupled transmission lines have two modes of wave propagation and they can each be analyzed separately as an even or an odd mode propagating wave down the transmission lines.

$V_{iO} = V_O \left(Z_O / (Z_O + Z_{\text{Ø}}) \right)$
$V_{iE} = V_E \left(Z_E / (Z_E + Z_{\text{Ø}}) \right)$
$V = V_{iE} + V_{iO}$

$P_O = (Z_{\text{Ø}} - Z_O) / (Z_{\text{Ø}} + Z_O)$
$P_E = (Z_{\text{Ø}} - Z_E) / (Z_{\text{Ø}} + Z_E)$
$P_E + P_O = 0 \quad B = 2\pi / \lambda$

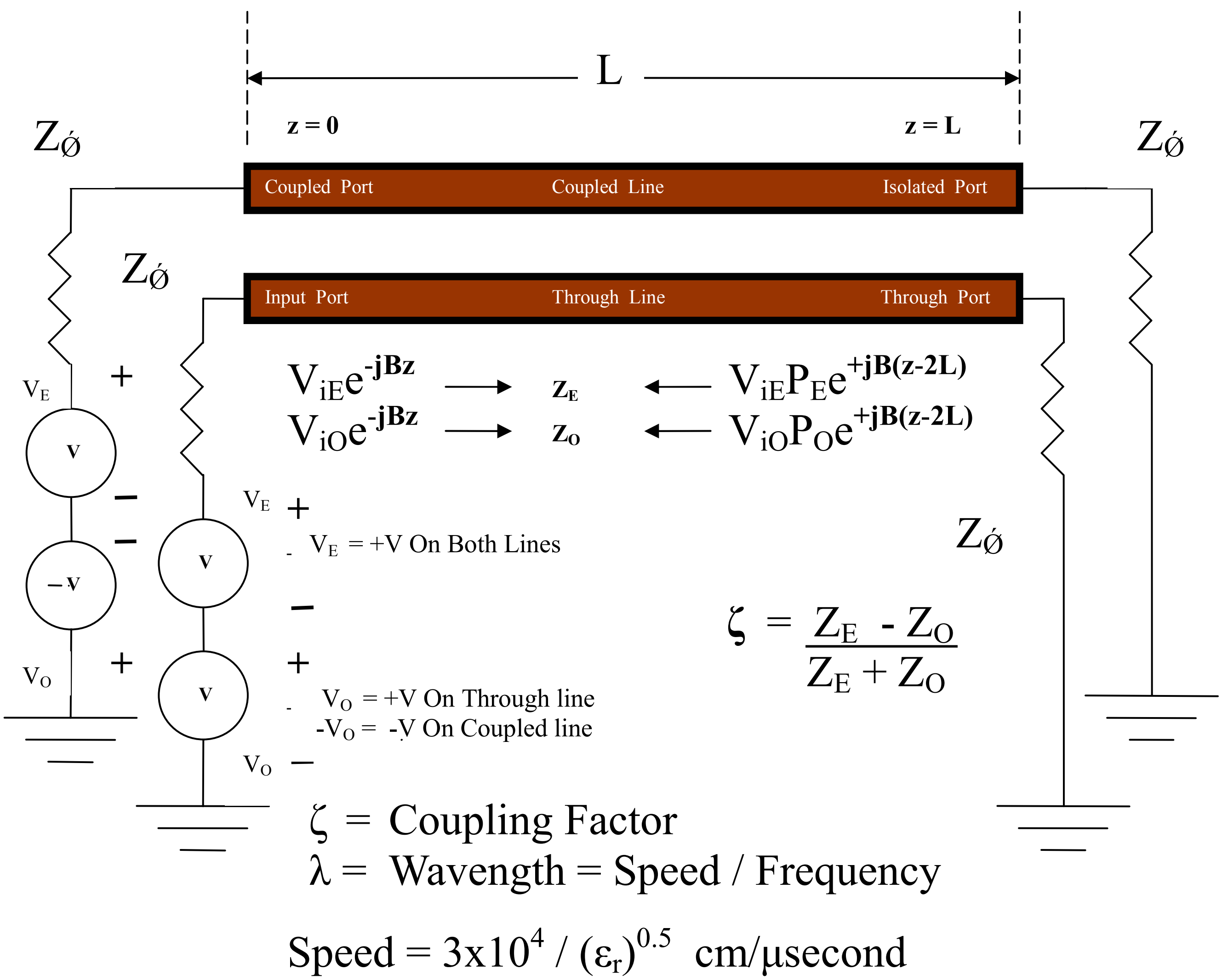

Fig. 2

Coupled Line Even and Odd Mode Analysis

A net input voltage of 2V is applied at the input port which is the sum of the even and odd mode voltages. The coupled port which in reality is without any voltage sources has a net applied voltage of 0V. This is represented by canceling even and odd voltage sources with the coupled port odd mode voltage source being 180 degrees out of phase with all the other voltage sources. This demonstrates clearly that the port boundary conditions force the even and odd mode incident voltages to be in phase at the input port and 180 degrees out of phase at the coupled port.

There are a number of important relationships between the two modes and the two propagating wave directions. These relationships are defined in terms of the even and odd mode impedances. When the terminating impedance $Z_{\acute{\emptyset}}$ are made equal to the geometric average of Z_E and Z_O then the following derivations apply.

$Z_{\acute{\emptyset}} = (Z_E Z_O)^{1/2}$ Terminations must be made equal to the geometric average of the even and odd mode impedances.

$P_E = -P_O$ Mode reflection coefficients have equal magnitudes and opposite signs.

$P_E < 0 \quad P_O > 0$

$| V_{iE}| - |V_{iO}| = | P_E || V_{iE}| + | P_O|| V_{iO}| = |V|| P_E| = |V|| P_O|$

$| V_{iE}| > |V_{iO}|$ The magnitude difference between the incident even and odd mode voltages is equal to the sum of the reflected mode components. A key relationship!

$|V_{iO}|(1+ P_O) = |V_{iE} |(1+ P_E)$ The magnitudes of total even and odd Mode voltages are equal at all ports.

$B_E = B_O = B$ Mode wave propagation constants are equal.

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Coupled Line Even and Odd Mode Analysis

These equations are all derived from the first relationship between the terminating impedances and the mode impedances. Using these relationships to analyze the coupled line voltages, it can be seen that the incident mode voltages are 180 degrees out of phase and the difference is positive and in phase with the mode voltages on the through line. This is because $|V_{iE}| > |V_{iO}|$. The forward traveling wave can now be seen as the sum of these two voltages which are 180 degrees out of phase to start with. At the other end of the line the voltages are reflected by their corresponding reflection coefficients. Only the even mode reflection coefficient is negative ($P_E = |P_E| \angle 180$) while the odd mode reflection coefficient is positive ($P_O = |P_O| \angle 0$) resulting in a 180 degree phase shift just to the reflected even mode component. This now puts both reflected modes in phase each other and each with an added 180 degree phase shift component. The forward and reverse traveling voltage waves change phase as they propagate down the transmission line. This line phase shift is equal to -Bz radians for the forward wave and for the reverse reflected wave +B(z-2L) radians. At z=0 the forward wave has undergone 0 radians or 0 degrees of line phase shift and it is in phase with the forward wave on the through line. The reverse reflected wave has undergone a total phase shift of $-2BL+\pi$ at z=0. The total line phase shift is -2BL radians and the phase shift from mode reflections and phase inversions is π radians or 180 degrees. These two wave voltages sum together to produce the coupled voltage at +90 phase relative to the through port. At z=L the through port line phase shift or total phase shift is –BL radians. From these results an equation can be written that demonstrates a 90 degree phase shift at the coupled port for any line length.

$$\frac{(-2BL + \pi)}{2} - (-BL) = +\frac{\pi}{2} = +90 \text{ degrees difference for all BL}$$

Net Coupled Port Phase — Through Port Phase

This equation demonstrates that for any line length BL the relative phase shift between the through port and the coupled port is +90 degrees independent line length L. The factor -2BL is the line phase shift for reverse reflected wave. The term π comes from the even mode negative reflection coefficient and the initial 180 degree relative phase of the odd mode. Vector addition of the 0 radians forward wave component and $(-2BL + \pi)$ radians reverse wave component at the coupled port produces a net port phase shift of $(-BL + \pi/2)$ radians. Subtracting the phase shift of –BL at the through port from the net coupled port phase shift leaves a phase difference of only $+\pi/2$ radians or +90 degree between these ports. In other words "**At a coupled port equal forward and reverse propagating waves combine 0 degree and 180 degree phase components generated by mode reflection coefficients to produce a 90 degree phase shift independent of frequency.**"

In order to complete and clarify this analysis, a general set of equations for coupled lines along with a graphical vector representation of them will be included. This should cover all the other important points regarding coupled lines. The following two equations are the general cases for determining voltages as a function of z anywhere along a single section coupled line or through line.

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Equations Derived from Even and Odd Mode Analysis

$$\frac{(V_{iE}e^{-jBz} + V_{iE}P_Ee^{+jB(z-2L)})}{(1- P_E^2e^{-jB(2L)})} - \frac{(V_{iO}e^{-jBz} + V_{iO}P_Oe^{+jB(z-2L)})}{(1- P_O^2e^{-jB(2L)})} = V_{Cline}$$

$$\frac{(V_{iE}e^{-jBz} + V_{iE}P_Ee^{+jB(z-2L)})}{(1- P_E^2e^{-jB(2L)})} + \frac{(V_{iO}e^{-jBz} + V_{iO}P_Oe^{+jB(z-2L)})}{(1- P_O^2e^{-jB(2L)})} = V_{TLine}$$

From these two equations the voltages at all 4 ports can be easily determined by selecting z = 0 or z = L along with the corresponding line for the given port. These equations are simply the transmission line equations split into even and odd modes with corresponding voltage polarities. Using the mode relationships given earlier and setting z = 0 on the coupled line makes $V_{Coupled} = V_{Cline}$.

$$\frac{(V|P_E|\, e^{-jB(0)} + V|P_E|e^{-jB(2L)+ j\pi})}{(1- |P_E|^2e^{-jB(2L)})} = V_{Coupled} \qquad |P_E| = |P_O|$$

Similarly setting z = L for the through line makes $V_{Through} = V_{TLine}$.

$$\frac{Ve^{-jBL} - V|P_E||P_E|e^{+jB(L-2L)}}{(1- |P_E|^2e^{-jB(2L)})} = \frac{Ve^{-jBL}(1- |P_E|^2)}{(1- |P_E|^2e^{-jB(2L)})} = V_{TLine} = V_{Through}$$

The denominator is the same for both therefore only the phase difference of the numerators needs to be looked at. The phase of the numerator of the coupled port is given by these equations.

$$TAN^{-1}(Sin(-2BL + \pi)/(1+ Cos(-2BL + \pi)) = \theta_{Coup-num}$$

$$\frac{(Sin(-2BL + \pi)}{(1+ Cos(-2BL + \pi))} = \frac{2(Sin(-BL + \pi/2))(Cos(-BL + \pi/2))}{(1+ 2Cos^2(-BL + \pi/2) - 1)}$$

$$= \frac{Sin(-BL + \pi/2)}{Cos(-BL + \pi/2)}$$

$$TAN^{-1}(Sin(-BL + \pi/2)/Cos(-BL + \pi/2)) = -BL + \pi/2 = \theta_{Coup-num}$$

The arguments can be simplified using trigonometric identities. The phase of the coupled port is $+\pi/2$ radians or +90 degrees more than the phase of the through port which is –BL radians not including the same phase offsets of the common denominators.

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Equations Derived from Even and Odd Mode Analysis

Rewriting these equations directly shows the $+\pi/2$ radian phase difference. These equations were written in this form to more clearly demonstrate what is actually happening electrically with the forward and reverse voltage wave amplitudes in the coupled line structure. Their equivalent conventional reduced forms are also shown. Reducing and simplifying these equations to conventional forms obscures the real interactions of these wave amplitudes.

Coupled Port

$$\frac{(2V|P_E|\mathrm{Cos}(-BL + \pi/2))e^{-jB(L)+ j\pi/2}}{(1- |P_E|^2 e^{-jB(2L)})} = \frac{j\zeta\ \mathrm{Sin}(BL)}{(1 - \zeta^2)^{0.5}(\mathrm{Cos}(BL)) +j\ \mathrm{Sin}(BL)}$$

Through Port

$$\frac{V(1- |P_E|^2)e^{-jBL}}{(1- |P_E|^2 e^{-jB(2L)})} = \frac{(1 - \zeta^2)^{0.5}}{(1 - \zeta^2)^{0.5}(\mathrm{Cos}(BL)) +j\ \mathrm{Sin}(BL)}$$

Coupling Factor

The term ζ is the Coupling Factor and is equal to the magnitude of the maximum coupling between the lines. The maximum coupling is written as maximum coupling = $\zeta_{dB} = 20\mathrm{Log}(\zeta)$ for $BL = \pi/2$. With maximum coupling comes minimum output at the through port resulting in a maximum insertion loss. This is also written as maximum insertion loss = $20\mathrm{Log}(1- \zeta^2)^{1/2}$ for $BL = \pi/2$. These results are demonstrated in the equations derived from the previous even and odd analysis. From these equation all the power entering and leaving the coupler can be accounted for with no power going to the isolated port. The Coupling Factor can be used to define this entirely at maximum mid band coupling. This analysis is only for single section couplers but it is useful for understanding other coupler configurations.

Multi-section couplers have electrical characteristics very similar to single section couplers as long as these couplers are symmetrical and have an odd number of sections. The central coupled line in a multi-section coupler has the highest coupling and also has the nearly the same even and odd mode impedance relationships as as a single section coupler. The coupled line sections add and subtract small amounts of coupling over frequency to produce a broadband equal ripple characteristics. For example the coupling at mid band of a 3 section coupler is $C_{mid\text{-}band} \approx \zeta_{Center} - 2\zeta_{Ends}$. This means that the coupling at mid band is reduced by twice the coupling value of the end lines when compared to a single section coupler thus producing a more broad band equal ripple response at less coupling. This approximation is accurate for coupling less than -10 dB. For a given coupling value this means the central line must have a higher coupling value to offset the effect of the outer coupled lines.

Isolated and Input Port Analysis

The analysis up to this point covers the electrical characterization of the coupled and through ports for single section coupled lines only The input and isolated ports also need investigation to complete this coupled line analysis. Using the earlier equations for coupled and through lines, the voltage at the isolated port can be determined by setting z = L and the input port voltage can also be determined by setting z = 0.

Isolated Port
Set z = L

$$\frac{V_{iE}\,(1+P_E)e^{-jBL}}{(1-P_E{}^2e^{-jB(2L)})} - \frac{V_{iO}\,(1+P_O)e^{-jBL}}{(1-P_O{}^2e^{-jB(2L)})} = V_{Cline} = V_{Isolated} = 0$$

Input Port
Set z = 0

$$\frac{V_{iE}\,(1+P_E e^{-jB(2L)})}{(1-P_E{}^2e^{-jB(2L)})} + \frac{V_{iO}(1+P_O e^{-jB(2L)})}{(1-P_O{}^2e^{-jB(2L)})} = V_{TLine} = V_{Input} = V$$

The total voltage at the isolated port is 0 volts and the total voltage at the input port is V volts which is half the applied voltage of 2V volts from the combined mode voltages at the Iinput port. This is because the input impedance is $Z_Ø$ ohms and is matched to the source impedance of $Z_Ø$ ohms dividing its' voltage by 2.

The incident even mode voltage at the isolated port is greater than the incident odd mode voltage, however the total voltage adds up to 0 volts because the even mode reflection coefficient is negative and the odd mode reflection coefficient is positive.

The sum of the incident and reflected voltages for both modes produces no net voltage at the isolated port. $V_{iE}(1+P_E) - V_{iO}(1+P_O) = 0$ volts. The denominators are equal $(1-P_E{}^2e^{-jB(2L)}) = (1-P_O{}^2e^{-jB(2L)})$ and can be factored out of this equation.

The input port as well as all other ports impedance are matched to $Z_Ø$ ohms and this means that there can be no net reflected voltage at the input port. The voltage relationship between the reflected modes heading back to the input port is the same as the incident mode voltages going to the isolated port scaled by the negative magnitude of the reflection coefficients $-|P_E|$ $(|P_E| = |P_O|)$. This means that there will no net voltage generated by the reflected mode at the input port just as there will be no net voltage generated by the incident mode at the isolated port when the effect of the reflection coefficients is summed into the net voltage in both cases.

In this particular ideal example the isolation which is the ratio of input port power to isolated port power and the input port return loss are both infinite because the net voltage at the isolated port is 0 Volts and the net reflected voltage back to the input port is also 0 volts 20Log(0) = ϖ. In a real coupler the input return loss and the isolation are not infinite and one parameter affects the other. An important measure of how well a coupler performs is the directivity which is $|ISOLATION_{dB}| - |COUPLING_{dB}| = DIRECTIVITY_{dB}$. This is the ratio of power going to the coupled port to the power going to the isolated port. Good directivity requires high isolation. To get high isolation means that the return loss must also be high and the transmission line phase shift of the even and odd modes must be equal. This requires equal path lengths and phase velocities.

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Propagation Delay and the Effects of Multiple Reflections

The phase information in the equations for through and coupled ports can be used to determine the signal propagation delays from the input port. The differential equation for propagation delay is written as

$$\text{Delay} = \tau = -\frac{\delta\theta}{\delta f}$$

Delay is the phase slope as a function of frequency and for a matched transmission line it is a straight line with a negative slope. Adding a minus sign makes this equal to time which can only be a positive value. The phase shift magnitude increases linearly with increasing frequency and constant slope resulting in a constant delay for a given length of transmission line. If the effect of multiple reflections is not included, the coupled port phase shift is $\theta_{Coup\text{-}num} = -BL + \pi/2$ radians and the through port phase is $\theta_{Through\text{-}num} = -BL$ radians. This does not include the common denominator phase shift term $\theta_{Com\text{-}den}$ which is the effect of multiple reflections. Since it is common to both ports, it does not affect the relative phase between the ports.

$$\theta_{Coupled} = \theta_{Coup\text{-}num} - \theta_{Com\text{-}den} \qquad \tau_{Coupled} = -\frac{\delta\theta}{\delta f}_{Coup\text{-}num} + \frac{\delta\theta_{Com\text{-}den}}{\delta f}$$

$$\theta_{Through} = \theta_{Through\text{-}num} - \theta_{Com\text{-}den} \qquad \tau_{Through} = -\frac{\delta\theta}{\delta f}_{Through\text{-}num} + \frac{\delta\theta_{Com\text{-}den}}{\delta f}$$

$$\tau_{Coupled} = \tau_{Through} = \frac{(\varepsilon_r)^{0.5}(L)}{3\text{x }10^4} + \frac{\delta\theta}{\delta f}_{Com\text{-}den} \ \mu\text{seconds}$$

The through port delay is equal to the coupled port delay even though they have a 90 degree ($\pi/2$ radians) phase difference!

Phase shift, amplitude and delay are modified slightly by the summation of voltages from even and odd mode multiple reflections. The term $1/(1 - P_E^2 e^{-jB(2L)}) = 1/(1 - P_O^2 e^{-jB(2L)})$ is the common denominator which modifies the magnitude and phase to account for the effect of these multiple voltage wave reflections upon the initial incident and reflected voltage waves. Every reflected voltage component is the same as the previous reflected voltage wave multiplied by the magnitude of the corresponding mode reflection coefficient. The magnitude, phase and delay of this particular term can be written as follows.

$$((1 - P_E^2\text{Cos}(-BL))^2 + (P_E^2\text{Sin}(-BL))^2)^{-1/2} = \text{Magnitude} \approx 1$$

$$-\text{Tan}^{-1}((P_E^2\text{Sin}(-BL))/(1 - P_E^2\text{Cos}(-BL))) = \text{Phase} \approx 0$$

$$\frac{\delta\{\text{Tan}^{-1}((P_E^2\text{Sin}(-BL))/(1 - P_E^2\text{Cos}(-BL)))\}}{\delta f} = \text{Delay} \approx 0$$

The approximations are accurate for small values of coupling which means $\zeta dB < -10$ dB and $|P_E| = |P_O| << 1$. Increased coupling results in a greater effect from multiple reflections on magnitude, phase and delay.

Large reflection coefficients also can increase the effect of multiple reflections as in the case of the Schiffmann Phase Shifter which creates 90 phase offsets relative to a given length of line. This phase shifter is a coupler with two of its' ports shorted together which makes $P_E = 1$ and $P_O = -1$. This also creates a strong delay effect.

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Coupled Line Even and Odd Mode Vector Analysis

The last and best way to illustrate how coupled lines really work is by numerical example using vector analysis **Figs. 3-5**. The following example will demonstrate how even and odd mode components interact using the initial forward and reflected waves.

Using the previous coupled line drawing example for ζ_{dB} = -3 dB.
Then Z_E = 120.71 Ω, Z_O = 20.71 Ω, P_E = -0.4142, P_O = 0.4142,
and $Z_{\emptyset} = (Z_E Z_O)^{1/2}$ = 50 Ω. At the input port setting V = 1 volt makes V_{iE} = 0.7071 volts and V_{iO} = 0.2929 volts. The total source voltage is then 2.0 volts and the total incident voltage drop at the input port is $V_{iE} + V_{iO}$ = 1 volt. At the coupled port the total incident voltage is the difference between these voltages which is $V_{iE} - V_{iO}$ = 0.4142 volts. An arbitrary electrical length of 60 degrees (π/3 radians = BL) is also used in the following sequence of vector diagrams. Other port vector magnitude values are given as follows.

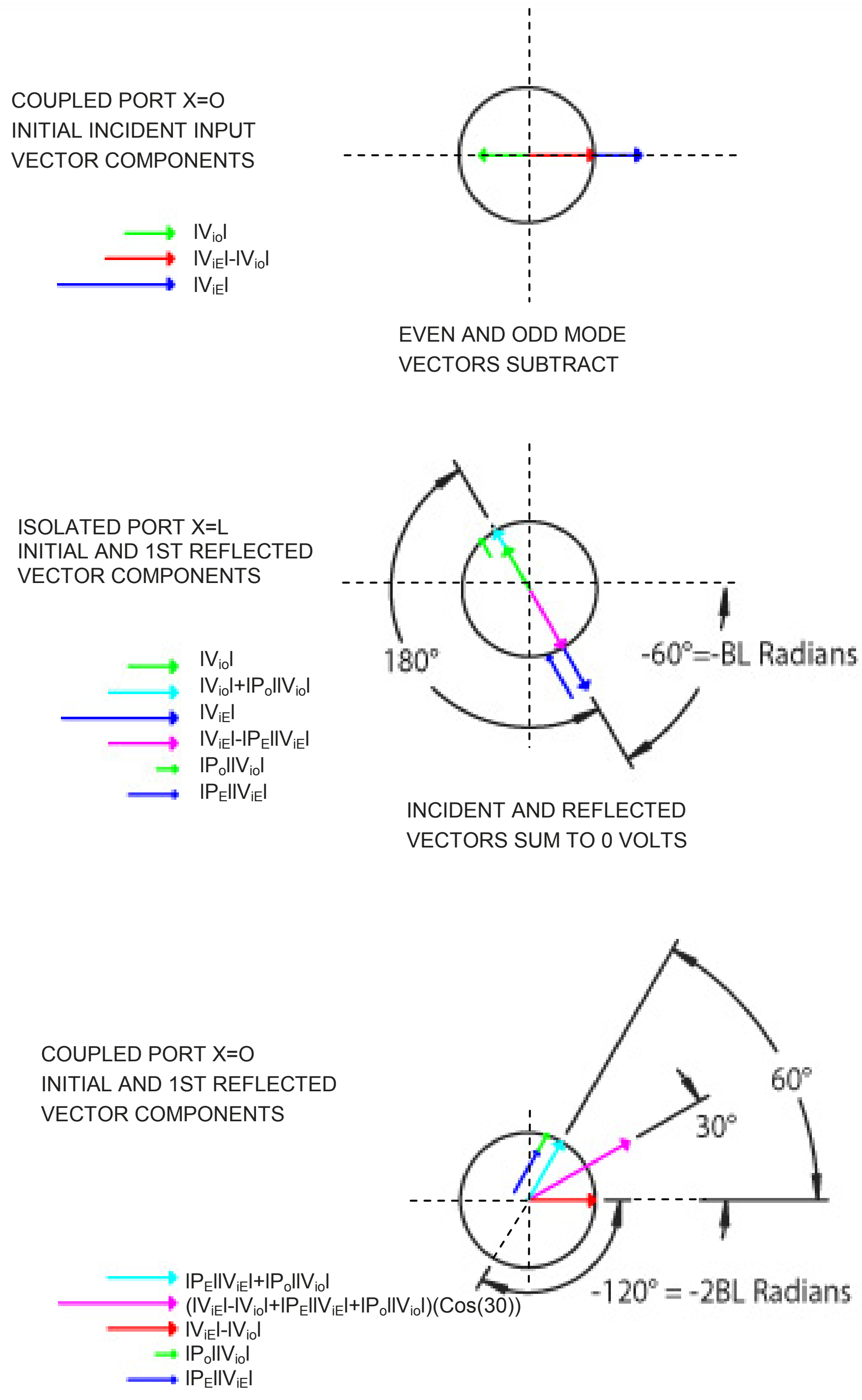
Coupled Port
$|P_E||V_{iE}| = 0.2929$
$|P_O||V_{iO}| = 0.1213$
$|P_E||V_{iE}| + |P_O||V_{iO}| = .4142$
$(|V_{iE}| - |V_{iO}| + |P_E||V_{iE}| + |P_O||V_{iO}|)(\text{Cos}(30)) = .7174$

Isolated Port
$|V_{iO}| + |P_O||V_{iO}| = |V_{iE}| - |P_E||V_{iE}| = .4142$

Through Port
$|V_{iO}| + |P_O||V_{iO}| + |V_{iE}| - |P_E||V_{iE}| = 0.8284$

Input Port
$|V_{iO}||P_O| + |P^2_O||V_{iO}| = |V_{iE}||P_E| - |P^2_E||V_{iE}| = 0.1715$

The vector components are generated sequentially starting with the initial incident voltages at the coupled and input ports. Next the 1st reflected components coming from the isolated and through ports are added in vectorially. These in turn generate the 2nd reflected components at the coupled and input ports. There are infinitely more reflections after this however the coupler characteristics are clearly illustrated by these 3 sets of voltage vector components.



EQUAL INCIDENT AND REFLECTED VECTORS ADD AND THE ANGLE BETWEEN THEM IS DIVIDED BY 2. AS L INCREASES BL RADIANS APPROACHES 90 DEGREES AND THIS ANGLE GOES TO 0 DEGREES RESULTING IN MAXIMUM COUPLED OUPUT.

Fig. 3

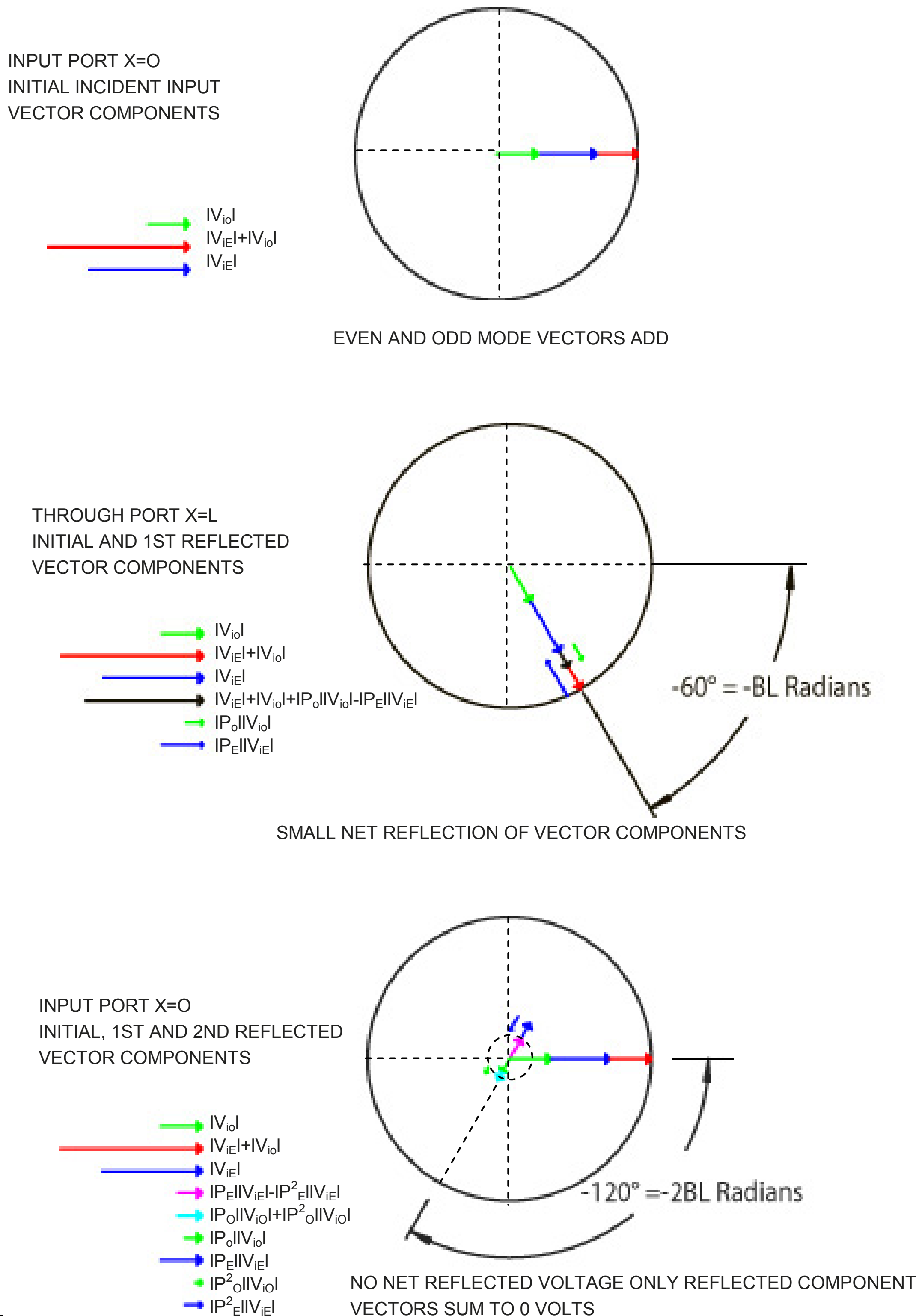
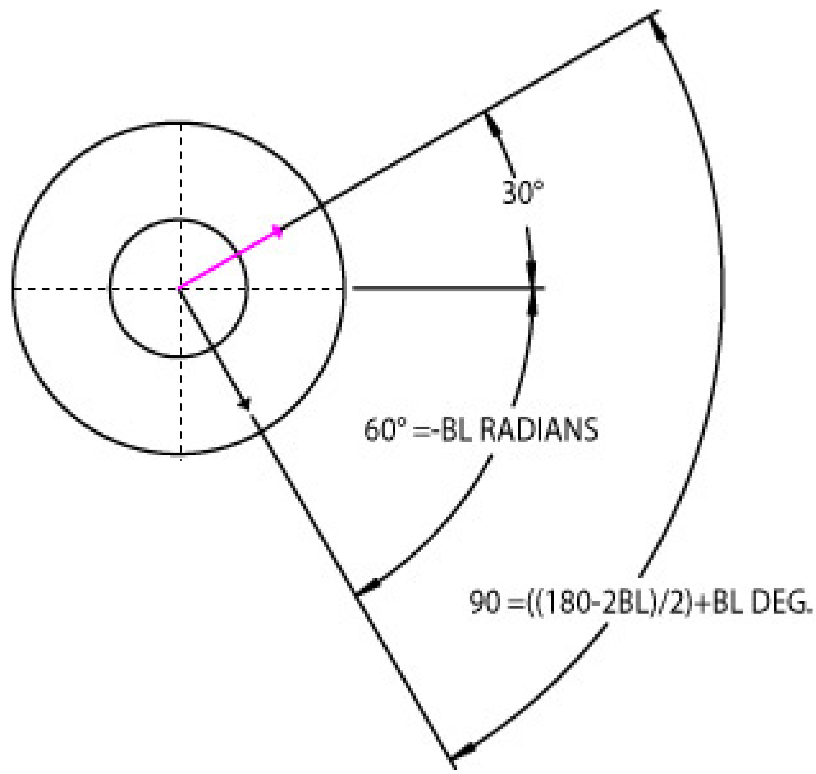

INPUT PORT X=O
INITIAL INCIDENT INPUT
VECTOR COMPONENTS
IVioI
IViEI+IVioI
IViEI
EVEN AND ODD MODE VECTORS ADD
THROUGH PORT X=L
INITIAL AND 1ST REFLECTED
VECTOR COMPONENTS
IVioI
IViEI+IVioI
IViEI
IViEI+IVioI+IPoIIVioI-IPEIIViEI
IPoIIVioI
IPEIIViEI
-60° = -BL Radians
SMALL NET REFLECTION OF VECTOR COMPONENTS
INPUT PORT X=O
INITIAL, 1ST AND 2ND REFLECTED
VECTOR COMPONENTS
IVioI
IViEI+IVioI
IViEI
IPEIIViEI-IP2EIIViEI
IPOIIViOI+IP2OIIViOI
IPoIIVioI
IPEIIViEI
IP2OIIViOI
IP2EIIViEI
-120° =-2BL Radians
NO NET REFLECTED VOLTAGE ONLY REFLECTED COMPONENT
VECTORS SUM TO 0 VOLTS

Fig. 4

THIS DEMONSTRATES THAT THE PHASE ANGLE BETWEEN THE COUPLED PORT AND THE THROUGH PORT IS ALWAYS INDEPENDENT OF BL OR L AND WILL ALWAYS BE 90 DEG.

COUPLED PORT X=O AND THROUGH PORT X=L NET INITIAL AND 1ST REFLECTED VECTOR COMPONENTS FOR EACH PORT. EACH SET OF REFLECTIONS STARTING FROM THE INPUT SIDE HAS THE SAME VECTOR RELATIONSHIPS AS THE INITIAL VECTOR COMPONENTS AND REFLECTIONS.



$(|V_{iE}|-|V_{io}|+|P_E||V_{iE}|+|P_o||V_{io}|)(Cos(30))$

$|V_{iE}|+|V_{io}|+|P_O||V_{iO}|-|P_E||V_{iE}|$

Fig. 5

This last vector diagram shows that for any L or BL the relative phase between the coupled port and the through port will always be 90 degrees only if the even and odd mode impedance relationships are correct. This also shows that because this is a 3dB coupler these two vectors will have equal magnitudes when all the reflections are added in and the electrical line length BL increases to 90 degrees.

Derivation of Coupler Scattering Matrix Parameters

A scattering matrix or S parameter matrix can be used to describe coupler performance with any passive termination. The S parameters within the matrix are measured using terminations which are equal to the characteristic impedance $Z_\text{ø}$ of the coupler. This means that all the ports will be matched. The S parameters are the reflected and transmitted voltage wave components divided by the $(Z_\text{ø})^{1/2}$ making them equal to the square root of power and proportional to voltage. The coupler S parameters can be derived using the following port numbering assignments. The input port is Port 1, the coupled port is Port 2 , the isolated port is Port 3 and the through port is Port 4. From these port assignments the following S parameter matrix is given as.

$$\begin{vmatrix} B1 \\ B2 \\ B3 \\ B4 \end{vmatrix} = \begin{vmatrix} S11 & S12 & S13 & S14 \\ S21 & S22 & S23 & S24 \\ S31 & S32 & S33 & S34 \\ S41 & S42 & S43 & S44 \end{vmatrix} \begin{vmatrix} A1 \\ A2 \\ A3 \\ A4 \end{vmatrix}$$

The port input waves are A1, A2, A3 and A4. These port input wave components can also be termination reflection coefficients which are not matched to $Z_\text{ø}$. A matched termination would be equal to 0. The port output waves are B1, B2, B3 and B4. The matched port condition for the S parameter measurement also means that S11= S22 = S33 = S44 = 0 signifying no reflections when all the port terminations are $Z_\text{ø}$. The ports which are isolated from each other have no transmitted energy between them and this means that S13=S31= S42 =S24= 0.

At this point it should be mentioned that all port terminations are seen as being connected through transmission lines with the characteristic impedance of $Z_\text{ø}$. From a mathematical point of view they can be considered infinitely short, however the actual physical matched coupler realization contains a finite length of single mode line with an impedance of $Z_\text{ø}$. As the coupled lines are bent gradually away from each other, a net impedance of $Z_\text{ø}$ must be maintained until the uncoupled single mode matched condition is reached. What this means is that the instantaneous reflections at the isolated ports which add up to 0 volts will continue to be 0 volts regardless of the termination connected to the port which is isolated. This is because the instantaneous reflection encounters a line impedance of $Z_\text{ø}$ and this instantly causes all voltage components to sum to 0 volts. The wave transmitted to the termination from this point will then have an amplitude of 0 volts regardless of the termination used at this isolated port.

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Derivation of Coupler Scattering Matrix Parameters

Since all the S parameters are proportional to voltage, those S parameters involving ratios can use coupling and through port voltage ratios derived earlier for the corresponding S parameter components.

Coupled Ports

$$S12= S21 = S34 = S43 = \frac{j\zeta \, Sin(BL)}{(1 - \zeta^2)^{0.5}(Cos(BL)) + j\, Sin(BL)}$$

Through Ports

$$S14= S41 = S32 = S23 = \frac{(1 - \zeta^2)^{0.5}}{(1 - \zeta^2)^{0.5}(Cos(BL)) + j\, Sin(BL)}$$

Setting $\zeta = (0.5)^{1/2} = 0.7071$ makes the coupling value -3dB at the coupler center frequency (BL= 90 degrees). The numerical S parameter values are S12= S21 = S34 = S43 = j.7071 and S14= S41 = S32 = S23 = .7071. This creates the following numerical S parameter matrix for a 3dB coupler.

$$\begin{vmatrix} B1 \\ B2 \\ B3 \\ B4 \end{vmatrix} = \begin{vmatrix} 0.00 & j.7071 & 0.00 & .7071 \\ j.7071 & 0.00 & .7071 & 0.001 \\ 0.00 & .7071 & 0.00 & j.7071 \\ .7071 & 0.00 & j.7071 & 0.00 \end{vmatrix} \begin{vmatrix} A1 \\ T_A B2 \\ T_B B3 \\ T_C B4 \end{vmatrix}$$

Many very interesting coupler properties can be demonstrated by evaluating this S parameter matrix using different port terminations. When all port terminations are matched ($\Gamma_A = \Gamma_B = \Gamma_C = 0$) this coupler divides the input port power $(A1)^2$ equally between coupled and through ports with a 90^o phase difference. This continues to be the case if just the isolated port termination ($\Gamma_B \neq 0$) is not matched because no power is delivered to this port. If the coupled port has a mismatch equal to the through port mismatch ($\Gamma_A = \Gamma_C \neq 0$, $\Gamma_B = 0$) then again this coupler divides the input port power $(A1)^2$ equally between coupled and through ports with a 90^o phase difference but this time some of the power is reflected back. This reflected power goes entirely to the matched isolated port where it is dissipated with no further reflections. The input port does not receive any reflected power because of 180^o vector cancellation created by two 90^o phase shifts within the coupler. Having just the coupled port mismatched as a pure resistance ($\Gamma_A \neq 0$) results in unequal power division and degraded input port match however the 90^o phase difference between the coupled and through ports is maintained. This is because there are no voltage wave components that have been reflected back through the coupler to the coupled port to change the phase of the original instantaneous incident voltage which has a 90^o phase difference. Mismatches at the coupled port and isolated port will significantly change the 90^o phase difference along with the creating unequal power division and a degrade input port match.

The reason some port mismatch conditions do not degrade coupler performance is because the characteristic symmetry of a 3dB coupler along with its' port isolation. This symmetry property is used in balanced amplifiers and mixers to significantly improve mismatches by canceling the reflections from these microwave components.

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Conclusion

A complete description of the fundamental characteristics of a pair of matched coupled lines has been given. It has just been described using words, mathematical formulas and step by step vector analysis. The important conclusions will be listed below in bullet form.

- Relative port phase shift is independent of coupler length.
- Phase shift is affected by mode impedances.
- Maximum coupling occurs at BL = $\pi/2$ radians = 90°
- Good port match and isolation requires $Z_{\varnothing} = (Z_E Z_O)^{1/2}$
- High isolation requires mode line phase shifts to be equal.
- Mode phase velocity and mode path length determine line phase shift of each mode.
- $|ISOLATION_{dB}| - |COUPLING_{dB}| = DIRECTIVITY_{dB}$.
- Delay = $\tau = - \delta \theta / \delta f$
- The coupled and through port paths have equal delay.
- Multiple reflections affect delay if they are large.
- Multi-section coupler line sections add and subtract coupling.
- Equal coupled and through port mismatches preserve 3dB coupler amplitude balance.
- Equal coupled and through port mismatches preserve 3dB coupler 90° port phase relationships.