COUPLING OF TRANSIENT RADIATED FIELDS INTO LINES

by

R. J. Mohr
AIL, a division of CUTLER-HAMMER
Deer Park, New York

Summary

A convenient analysis technique is derived for predicting the response of lines to transient radiated electromagnetic fields such as the electromagnetic pulse (EMP) associated with nuclear detonations. Simplifying assumptions, valid in the frequency range covered by the significant components of EMP, allow solution by means of a simple equivalent circuit. The circuit includes the effects of line losses and angle of incidence of the arriving field. Agreement with previously published results is shown.

Introduction

The intense EMP resulting from a nuclear detonation can induce extremely large currents in lines. Accurate prediction of the characteristics of such currents is vital to gage the necessary protective design of equipment terminating the lines. Literature has been published on solutions to this problem.[1,2] Unfortunately, the techniques , while precise, are discouragingly complicated.

In this paper the response of an infinite line in free space to a transient radiated electromagnetic field will be examined. The results of this important case can readily be extended to cover lines of finite length and to include the effects of proximity to reflecting/absorbing mediums, such as earth.

The problem will first be approached in a heuristic manner. Assuming the line has a characteristic impedance, Z_o, independent of frequency, the idealized response to a step function electromagnetic wave will be found. Examination of the result will suggest a simple equivalent circuit representation for the problem. However, it will be recognized that its validity is critically dependent on the assumed behavior of Z_o. Armed with an insight into the problem, a more rigorous approach can then be more easily followed to investigate the solution more closely. At critical points in the development, comparisons with the simplified approach will show good agreement; at a key point, Z_o will be found to be readily expressible in terms of known line parameters. Detailed study of these parameters, over the frequency range significant in EMP analysis, will show that our earlier assumptions were justified with only slight refinement of the equivalent circuit. The effect of proximity to earth will then be shown. Finally, comparisons with previously published results will verify the model.

Simplified Analysis

Consider the case, in Figure 1, of oblique incidence of a step function electromagnetic field on an infinite line. For reference purposes, the wavefront is assumed to have crossed the origin, x = 0, at time, t = 0. The situation is shown a short time, t_1, later when the wavefront has progressed along the line to $x_1 = ct_1/\sin\theta$. Figure 2 shows the equivalent circuit in the incremental vicinity around x_1. The current that will be caused to flow in the incremental section of line due to the electric field at that section is:

$$\Delta I_{x_1} = \frac{E_i \cos\theta}{2\,Z_o}\,\Delta x \qquad (1)$$

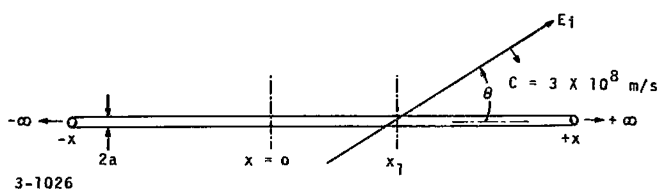

Figure 1. Infinite Line in Free Space Subjected to Obliquely Incident Wavefront

Neglecting attenuation for the moment, this current increment will propagate in both directions along the line at the velocity of light. The incremental current will arrive at x = 0, at $t_2 = t_1 + t_1/\sin\theta$. At that time a number, $ct_1/(\sin\theta)\,\Delta x$, of such increments will have arrived at x = 0 from the positive x direction. Simultaneously, similar current increments will have been arriving at x = 0 proceeding from the negative x direction. At $t = t_2$ a number, $ct_2/(1 - \sin\theta)\,\Delta x$, of such positively traveling increments will have arrived at x = 0. At $t = t_2$ the total current at x = 0 is then:

$$I_o(t_2) = \frac{E_i \cos\theta\,\Delta x}{2\,Z_o}\left[\frac{ct_1}{\sin\theta\,\Delta x} + \frac{ct_2}{(1-\sin\theta)\,\Delta x}\right]$$

$$= \frac{E_i\,ct_2}{Z_o \cos\theta} \qquad (2)$$

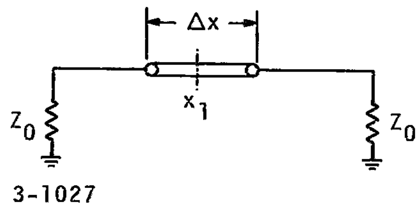

Figure 2. Equivalent Circuit of Figure 1 at x_1

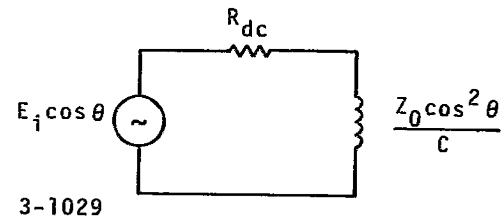

Figure 4. Equivalent Circuit Giving Response of Figure 4

Figure 3 is a graphical representation of equation 2. Figure 3 also indicates the effect of dc resistance, R_{dc} (ohms/m), in the line. It is seen that the response is initially a current ramp which then tops out at $t = t_m$ when I R_{dc} is equal to the tangential component of the incident electric field.

The corner plot response of Figure 3 is relatively idealized; our intuition would lead us to expect the current to top out more gradually as indicated by the dashed line in Figure 3. This resembles the response of a series L-R circuit to a step voltage input. The equivalent circuit so suggested is shown in Figure 4.

The result (Figure 4) is very simple. However, the value of Z_o is not known; additionally, it was implicitly assumed in the analysis that Z_o is independent of frequency. A more rigorious approach to investigate these areas follows.

Fourier Analysis Approach

Consider again the situation shown in Figure 1; however, assume that the incident field is sinusoidal at radian frequency ω, and having wavelength λ. The tangential component of electric field at any point along the line is:

$$E_x(\omega) = E_i(\omega) \cos\theta \; e^{-\gamma_2 x} \tag{3}$$

where

$$\gamma_2 = j\frac{2\pi}{\lambda} \sin\theta$$

The differential current caused to flow at any point, x, on the line due to the tangential electric field at that point is:

$$d\,[I_x(\omega)]_x = \frac{E_i(\omega) \cos\theta \; e^{-\gamma_2 x} \, dx}{2\,Z_o} \tag{4}$$

This current element propagates in both directions along the line away from x. At the instant the current element of equation 4 is being generated, the current at x = 0 due to the current element previously generated at point x is

$$d\,[I_o(\omega)]_x = \begin{cases} d\,[I_x(\omega)]_x \; e^{-\gamma_1 x} & \text{for } x > 0 \\ d\,[I_x(\omega)]_x \; e^{\gamma_1 x} & \text{for } x < 0 \end{cases}$$

where

$\gamma_1 = \alpha + j\beta$ = the propagation constant of the line

The total current at x = 0 is obtained by summing, at x = 0, the current increments generated over the entire line as follows:

$$I_o(\omega) = \frac{E_i(\omega)\cos\theta}{2\,Z_o}\left[\int_{-\infty}^{0} e^{(\gamma_1 - \gamma_2)x}\,dx + \int_{0}^{\infty} e^{-(\gamma_1 + \gamma_2)x}\,dx\right] = \frac{E_i(\omega)\cos\theta}{Z_o}\;\frac{\gamma_1}{\gamma_1^2 - \gamma_2^2} \tag{5}$$

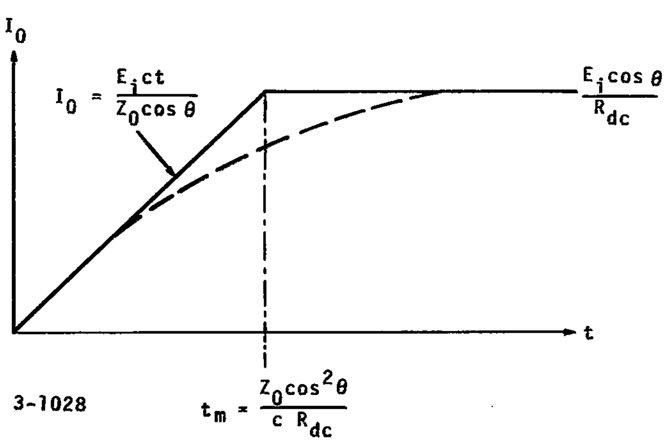

Figure 3. Response of Infinite Line in Free Space to a Step Function Electric Field

Let us investigate this result briefly. For a lossless line, $\gamma_1 = j\,2\pi/\lambda$, and equation 5 reduces to:

$$I_o(\omega) = \frac{E_i(\omega)}{\gamma_1 Z_o \cos\theta} = \frac{E_i(\omega)\,C}{j\omega Z_o \cos\theta} \qquad (6)$$

From inspection of equation 6, its LaPlace transform is written as:

$$I_o(S) = \frac{\mathcal{L}(E_i)C}{S\,Z_o \cos\theta} \qquad (7)$$

Hence, for a step function input of magnitude E_i,

$$I_o(t) = \frac{E_i\,c\,t}{Z_o \cos\theta} \qquad (8)$$

For the lossless case, the result is seen to be identical with that obtained in the previous section.

Returning to our development, examination of equation 5 suggests an equivalent circuit representation for the field problem; that is, it is of the form:

$$I = \frac{V}{Z}$$

where

$$Z = Z_o \frac{\gamma_1^2 - \gamma_2^2}{\gamma_1} \qquad (9)$$

and

$$V = E_i \cos\theta \qquad (10)$$

Such a representation will be convenient for solution when the excitation is a complex function of time, if Z is a well-behaved function of frequency. Let us examine this.

Consider first the case of broadside incidence where the solution is well known[3]. For this case, $\theta = 0$, so that $\gamma_2 = 0$. Then, rearranging equation 5:

$$\frac{E_i(\omega)}{I_o(\omega)} = Z_o\,\gamma_1 \qquad (11)$$

For convenience, $Z_o\gamma$ will be replaced by Z_F. Equation 9 can then be rewritten as:

$$Z = Z_F \frac{\gamma_1^2 - \gamma_2^2}{\gamma_1^2} \qquad (12)$$

The parameter Z_F will be defined as the field impedance of the line, since it relates the current caused to flow in the line to the incident field which causes the current. The field impedance of a line consists of two basic parts[3]; an external impedance, Z_{ext}, and an internal impedance, Z_{int}, such that:

$$Z_F = Z_{ext} + Z_{int}$$

where

$$Z_{ext} = R_{ext} + j\,X_{ext}$$

$$\doteq \pi\omega 10^{-7} + j\frac{\omega\mu}{2\pi}\ln\frac{\lambda}{2\pi a} \quad \text{for } a << \frac{\lambda}{2\pi}$$

$$Z_{int} = R_{int} + j\,X_{int}$$

$$\doteq \begin{cases} \dfrac{1}{\pi a^2 \sigma_c} + j\dfrac{\omega\mu}{8\pi} & \text{(at low frequencies)} \\[2ex] \dfrac{1}{2\pi a}\sqrt{\dfrac{\omega\mu}{2\sigma_c}} + \dfrac{j}{2\pi a}\sqrt{\dfrac{\omega\mu}{2\sigma_c}} & \text{(at high frequencies)} \end{cases}$$

μ = permeability of free space = $4\pi 10^{-7}$ H/m

σ_c = conductivity of line = 5.8×10^7 mho/m for copper

The limitation on the expression for Z_{ext} is not severe. For example, for a cable radius a = 5×10^{-3} m (0.5 cm), the expression is valid to at least 500 MHz. At low frequencies, R_{int} is simply R_{dc}; at high frequencies, R_{int} is R_{dc} modified by skin effect.

Figure 5 is a frequency plot of the component parts of Z_F for a range of line sizes. If attention is limited to the frequency range critical in EMP analysis, that is, below the EMP's 40 dB/decade break

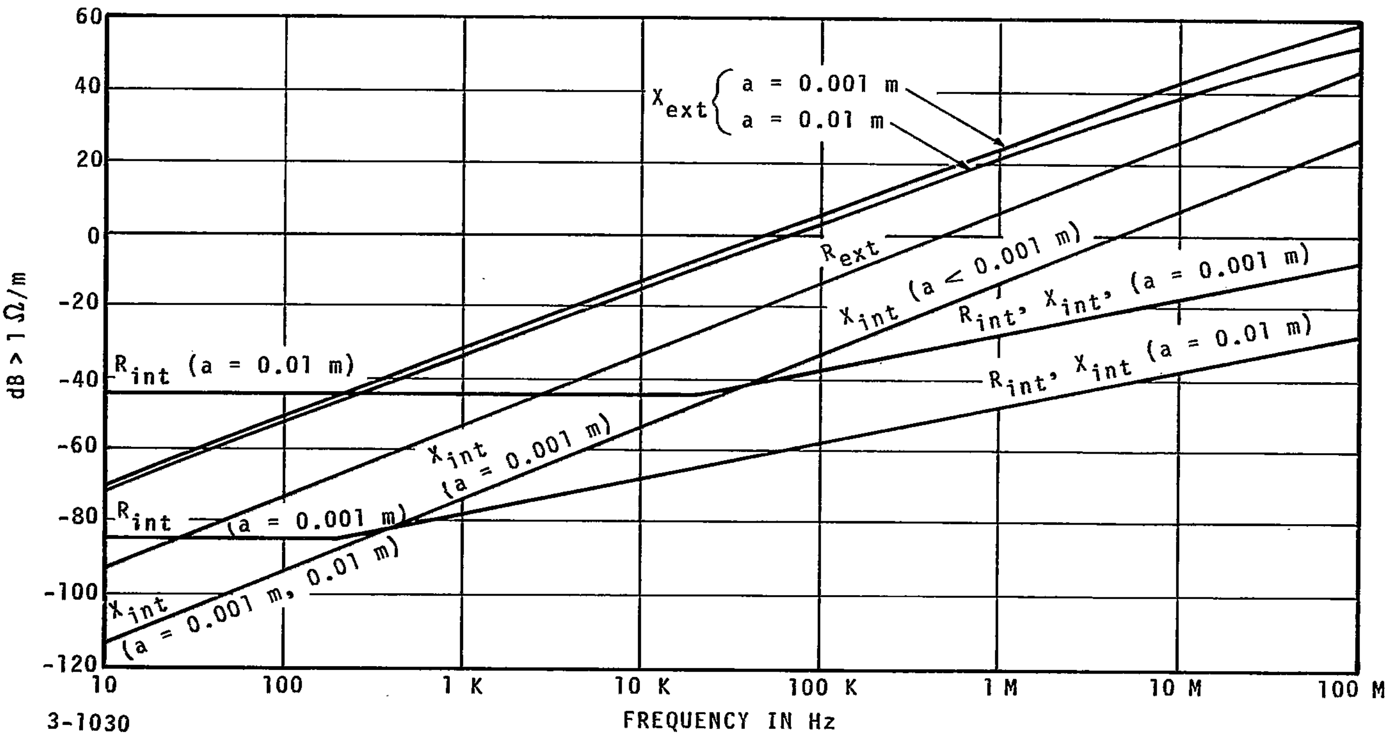

Figure 5. Component Parts of Field Impedance, Z_F

(typically about 5 MHz), several general observations are possible:

- At low frequencies, R_{int} dominates and is independent of frequency
- At high frequencies, X_{ext} dominates and behaves as an inductor whose inductance slowly decreases with increasing frequency
- R_{ext} varies directly with frequency and is of the order of 0.1 X_{ext}

Based on the first observation, at low frequencies, $Z_F = R_{dc}$.

From the second observation, replacement of X_{ext} with a fixed inductor should not cause any great error. To be conservative, the inductive part of Z_F will be determined at the highest frequency of interest, where the inductance is minimum, that is, at 5 MHz.

The approximately constant ratio of R_{ext} to X_{ext} will be a convenience in evaluating $\left(\lambda_1^2 - \lambda_2^2\right)/\lambda_1^2$ at high frequencies.

Therefore, in accordance with observations 1 and 2, evaluating X_{ext} at 5 MHz to determine L, and assuming a = 0.5 cm,

$$Z_F = R_{dc} + j\,\omega\,1.5 \times 10^{-6}$$

Consider the behavior of $\left(\gamma_1^2 - \gamma_2^2\right)/\gamma_1^2$. The propagation constant, γ_1, of the line, including the effect of its distributed resistance, can be written in the form[4]:

$$\gamma_1 = \alpha + j\beta = \frac{2\pi}{\lambda}(v + j\,u)$$

where

$$u = \cosh \phi$$

$$v = \sinh \phi$$

$$\phi = \frac{1}{2} \sinh^{-1}\left(\frac{R_F}{\omega L_F}\right)$$

Therefore;

$$\frac{\gamma_1^2 - \gamma_2^2}{\gamma_1^2} = \frac{\frac{v^2}{u^2} + j\,2\,\frac{v}{u} - 1 + \frac{\sin^2\theta}{u^2}}{\frac{v^2}{u^2} + j\,2\,\frac{v}{u} - 1} \qquad (13)$$

The behavior at low and high frequencies is as follows:

At Low Frequencies:

$$\text{As } f \longrightarrow 0,\ u^2 \longrightarrow v^2 >> 1$$

Hence equation 13 becomes:

$$\frac{\gamma_1^2 - \gamma_2^2}{\gamma_1^2} \longrightarrow \frac{j\,2\,vu + \sin^2\theta}{j\,2\,vu} \doteq 1$$

Therefore, since at low frequencies, $Z_F = R_{dc}$, equation 12 reduces to:

$$Z = \frac{\gamma_1^2 - \gamma_2^2}{\gamma_1^2} Z_F \doteq R_{dc} \qquad (14)$$

At High Frequencies:

From Figure 5:

$$\frac{R_F}{\omega L_F} \doteq \frac{R_{ext}}{X_{ext}} \doteq 0.1$$

Therefore, $\emptyset \doteq 0.05$ and $v \doteq 0.05$, $u \doteq 1$. Hence, equation 13 becomes:

$$\frac{\gamma_1^2 - \gamma_2^2}{\gamma_1^2} \doteq \frac{\cos^2\theta - j\,0.1}{1 - j\,0.1} = \frac{\cos^2\theta + 0.01}{1.01} - \frac{j\,0.1\,\sin^2\theta}{1.01}$$

Since at high frequencies $Z_F \doteq j\,\omega L_{ext}$, then equation 12 reduces to:

$$Z = Z_F \frac{\gamma_1^2 - \gamma_2^2}{\gamma_1^2} \doteq j\omega L_{ext} \frac{\cos^2\theta + 0.01}{1.01} + \frac{0.1}{1.01}\,\omega L_{ext}\,\sin^2\theta \qquad (15)$$

The impedance is seen to consist of an inductive reactance part and a resistive part that varies directly with frequency. In keeping with our approach of obtaining a simple, conservative solution the resistive part of equation 15 will be dropped. Then, at high frequencies:

$$Z \doteq j\omega L_{ext} \frac{\cos^2\theta + 0.01}{1.01} \doteq j\omega L_{ext}\,(\cos^2\theta + 0.01) \qquad (16)$$

The final equivalent circuit is shown in Figure 6. Note that, for θ close to 90 degrees, equation 16 reduces to:

$$Z = j\omega L_{ext} \cos^2\theta$$

This is of the form predicted from our elementary considerations as indicated in Figure 4.

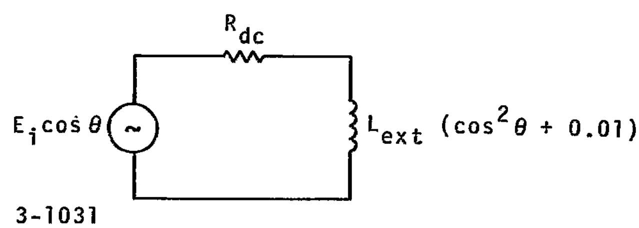

Figure 6. Equivalent Circuit of Infinite Line in Free Space Subjected to Incident Electromagnetic Field

Proximity to Earth

When the line is in proximity to earth, account must be taken of reflections due to the earth and any effects on the propagation characteristics of the line. The case where the line is laying on earth will be considered here. This case is relatively simple since the incident and reflected fields can be considered to arrive simultaneously; it is complicated by the fact that the line is at the interface of two very different mediums. It will be seen, by comparison of the end results with published results, that a fair estimate is to assume that proximity to earth does not affect Z_F, but that the velocity of wave propagation is decreased by the factor, $\sqrt{\epsilon_r}$, where ϵ_r is the relative dielectric constant of earth, typically >>1. Based on this:

$$\gamma_1 = \frac{2\pi\sqrt{\epsilon_r}}{\lambda}(v + ju) \qquad (17)$$

and

$$\frac{\gamma_1^2 - \gamma_2^2}{\gamma_1^2} = \frac{\frac{v^2}{u^2} + 2\,j\,\frac{v}{u} - 1 + \frac{\sin^2\theta}{\epsilon_r u^2}}{\frac{v^2}{u^2} + 2\,j\,\frac{v}{u} - 1} \qquad (18)$$

As in the free-space development:

At Low Frequencies:

$$\frac{\gamma_1^2 - \gamma_2^2}{\gamma_1^2} \doteq 1$$

and $Z \doteq R_{dc}$

At High Frequencies:

$$\frac{\gamma_1^2 - \gamma_2^2}{\gamma_1^2} \doteq 1 - \frac{\sin^2\theta}{1.01\,\epsilon_r} - j\,\frac{0.1\,\sin^2\theta}{1.01\,\epsilon_r}$$

Therefore,

$$Z = j\omega L_{ext}\left[1 - \frac{\sin^2\theta}{1.01\,\epsilon_r}\right] + \frac{\omega L_{ext}\,0.1\,\sin^2\theta}{1.01\,\epsilon_r}$$

Dropping the resistive part of Z, as in the free-space development,

$$Z = j\omega L_{ext}\left[1 - \frac{\sin^2\theta}{1.01\,\epsilon_r}\right] \doteq j\omega L_{ext}\left[1 - \frac{\sin^2\theta}{\epsilon_r}\right] \qquad (19)$$

Hence, the complete equation for the impedance of the equivalent circuit is:

$$Z = R_{dc} + j\omega L_{ext}\,(1 - \frac{\sin^2\theta}{\epsilon_r}) \qquad (20)$$

It should be noted that in using the equivalent circuit, the generator voltage as given by equation 10 is not applicable. A graphical solution is most expedient as shown in the following paragraph.

Comparison with Other Published Results

Infinite Line in Free Space. Figure 7 shows our equivalent circuit of the cable analyzed by Bates and Hawley. Figure 8 shows the predicted impulse spectrum for this circuit. Comparison with the results of Bates and Hawley[2] (Figure 4 of their paper) shows good agreement at all frequencies for $0 \le \theta \le 45$ degrees, and good agreement at low frequencies for all values of θ. For θ approaching 90 degrees, their high-frequency spectrum falls off much more slowly. This apparently is due to the deletion of R_{ext} in their expression for Z_{bb}, which is analogous otherwise to Z_F.

Infinite Line at Air/Earth Interface. In considering an obliquely incident wavefront arriving at earth, it is necessary to consider not only its horizontal, but also its vertical electric field components.[5] Each component, interacting with the intrinsic impedance of earth, will cause a horizontal component to appear at the air/earth interface as follows:

$$E_x(\omega) = E_i(\omega)\cos\theta\,\epsilon^{-\gamma_2 x}\,\frac{2\,Z_i}{Z_o + Z_i} + E_i(\omega)\sin\theta\,\epsilon^{-\gamma_2 x}\,\frac{Z_i}{Z_o} \qquad (21)$$

where

Z_o = intrinsic impedance of free space $= \sqrt{\frac{\mu}{\epsilon}}$

Z_i = intrinsic impedance of earth $\doteq \sqrt{\frac{j\omega\mu}{\sigma_e + j\omega\epsilon\epsilon_r}}$

μ = permeability of earth and of free space $= 4\pi \times 10^{-7}$ H/m

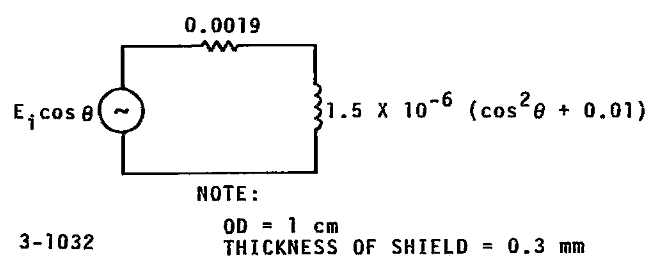

Figure 7. Equivalent Circuit of Copper Cable Shield in Free Space

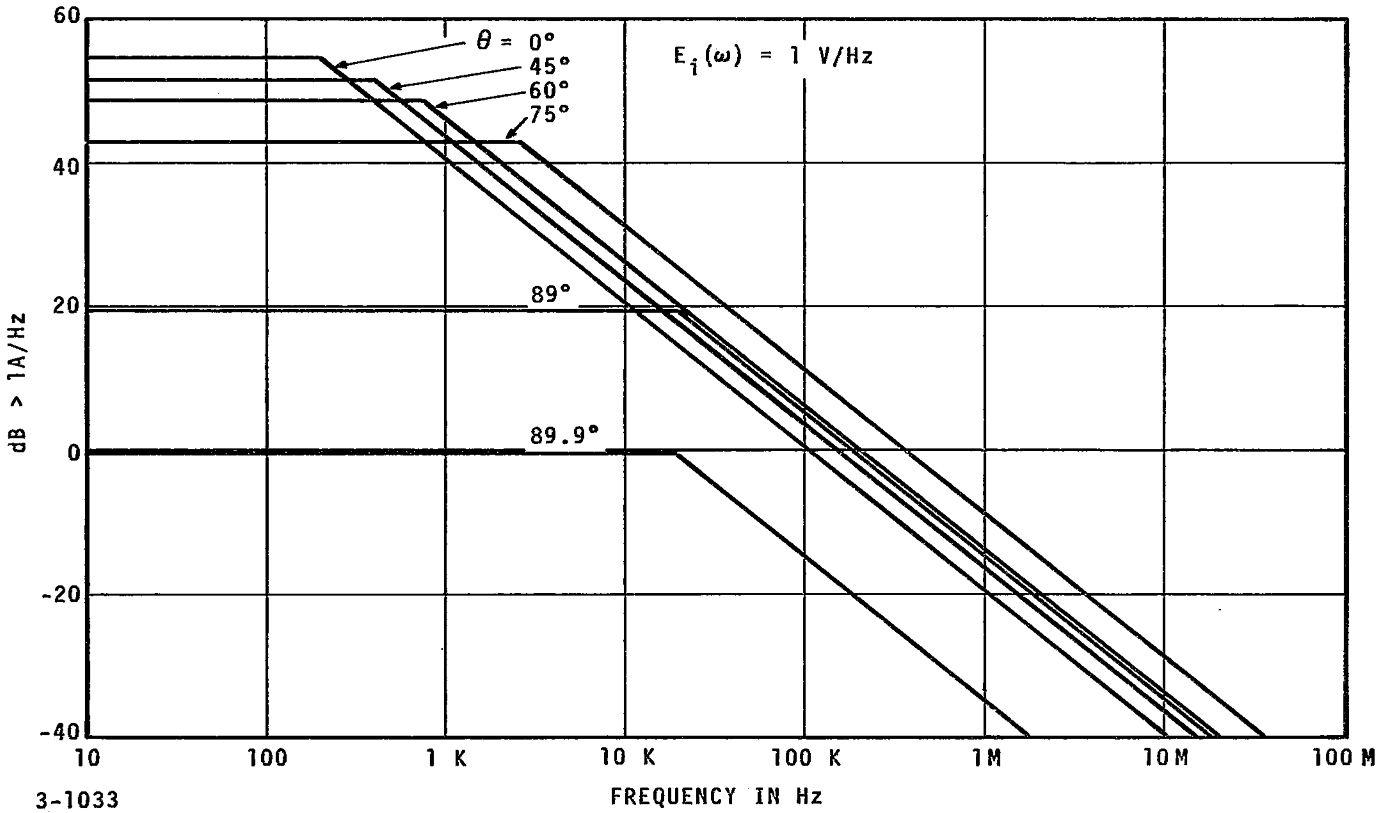

Figure 8. Impulse Response of Equivalent Circuit of Figure 7

ϵ = permittivity of free space = $\dfrac{1}{36\pi \times 10^9}$

ϵ_r = relative dielectric constant of earth

σ_e = conductivity of earth

The first expression on the right of equation 21 is the horizontal component of field existing at the interface due to the horizontal component of the incident wave; the second expression on the right is the horizontal component of field existing at the interface due to the vertical component of the incident wave.

Figure 9 is a corner plot of equation 21 for an impulse field, for $\theta = 0$ and $\theta = 90$ degrees, using $\sigma_e = 10^{-3}$ mho/m and $\epsilon_r = 15$ as nominal parameters for earth. Figure 10 shows the resultant response of the impedance represented by equation 20, to the spectra of Figure 9.

The cases analyzed above can be compared with the approximately similar cases analyzed by Bates and Hawley[2]. They analyzed the cases for the cable buried 1 m and for $\theta = 0$ and $\theta = 89$ degrees. Their results (their Figure 7) for $\theta = 0$ degree are very similar to those obtained here for $\theta = 0$ degree. Their results for $\theta = 89$ degrees, when compared with those obtained here for $\theta = 90$ degrees, show a slightly higher level at low frequencies and a more rapid falloff at high frequencies. The difference is, most likely, due to the slight difference in the cases being compared.

Conclusions

From an elementary approach, the form of a simple equivalent circuit (Figure 4) for analyzing the response of a line to a transient electromagnetic field was inferred. A more rigorous analysis refined the circuit and showed that it was valid, providing the frequency spectrum was suitably bounded. Comparison of predictions, using these equivalent circuits (Figure 6 and equation 20), with previously published results showed sufficiently good agreement to justify the use of the equivalent circuit solution for most EMP engineering predictions.

Acknowledgement

The author thanks Mr. E. W. Karpen of AIL for helpful review and comments on the paper.

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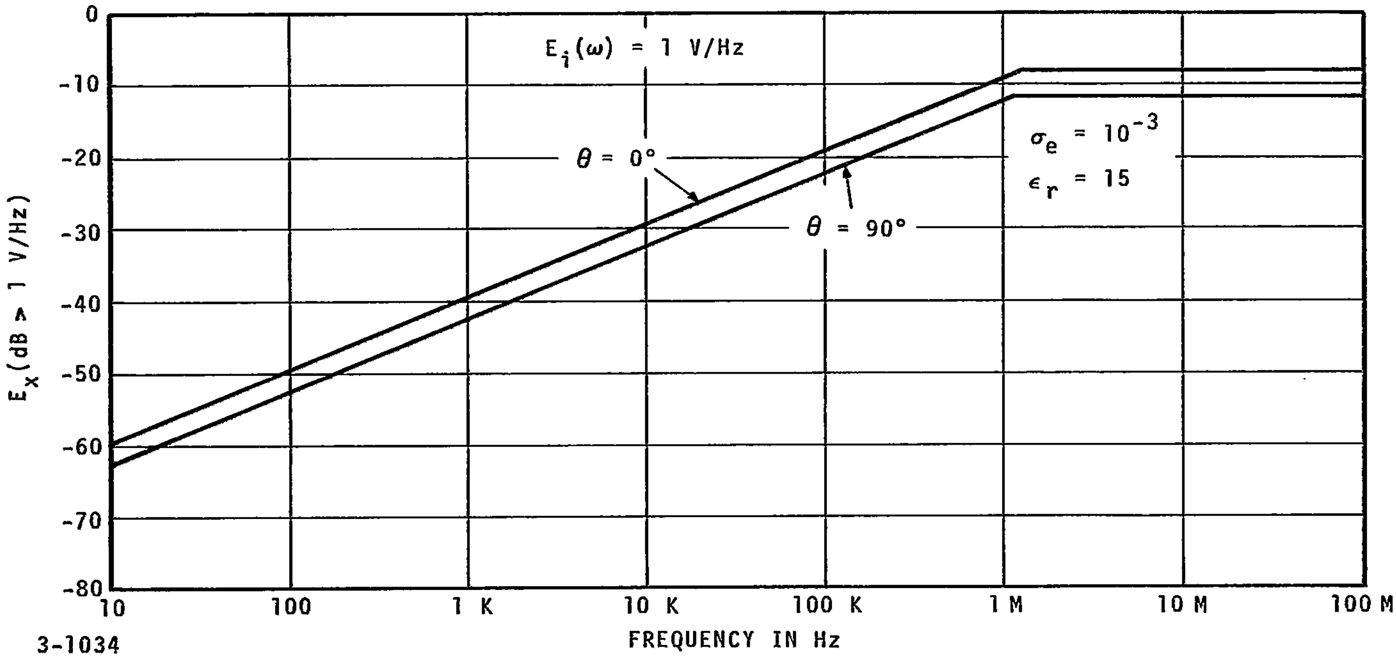

Figure 9. Tangential Electric Field at Air-Earth Interface

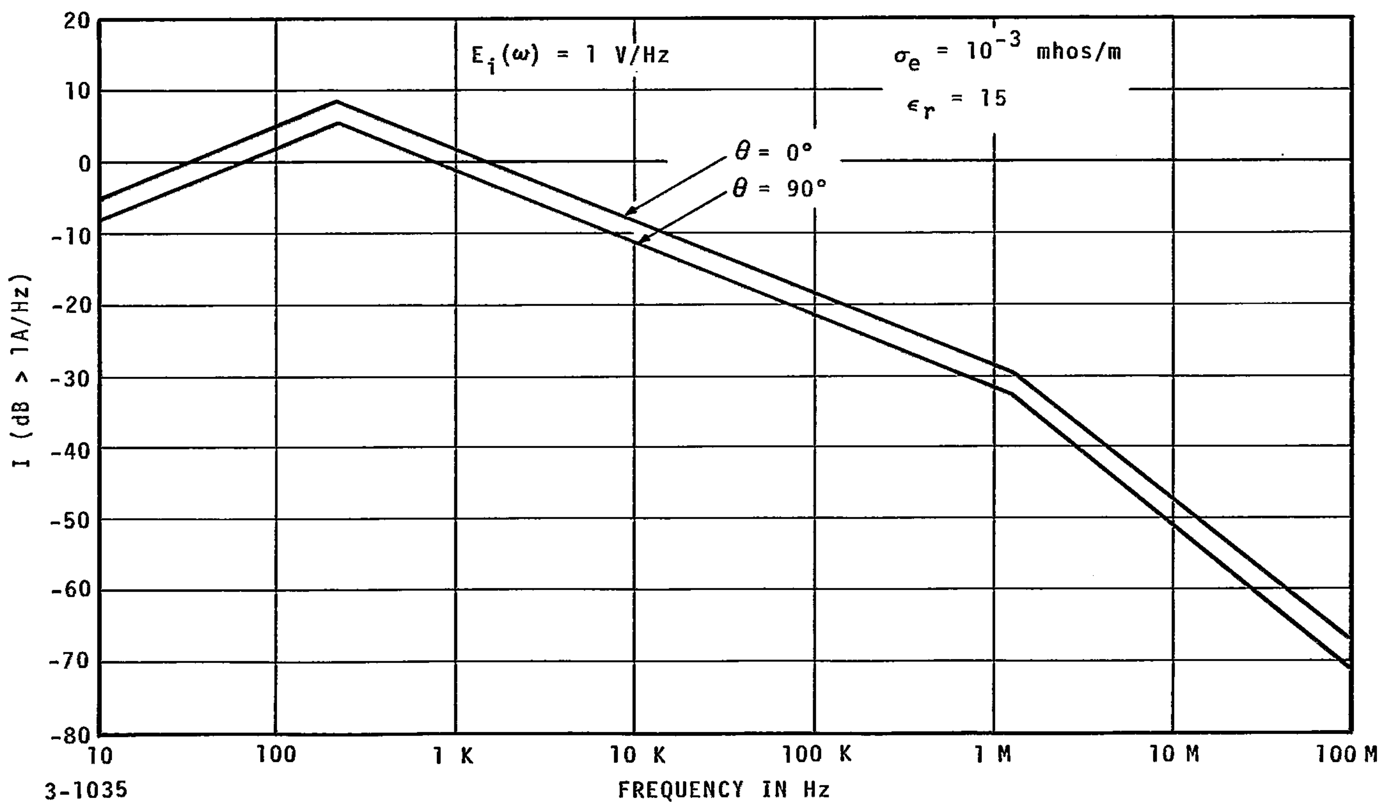

Figure 10. Radiated Impulse Response of Line Laying on Earth