Load power sources for peak efficiency

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When attempting to load a power source, you might instinctively make the load resistance equal to the Thevenin-equivalent power-supply resistance. This approach, however, yields only 50% efficiency. There's a better way.

A derivation of the maximum efficiency for the simplified circuit shown in **Fig 1** leads to the trivial solution of infinite load resistance. A more realistic power source **(Fig 2)**, however, constantly dissipates power through R_P.

Now you can find a unique, finite value for R_L. Consider the circuit shown in **Fig 3.** With all resistances represented in terms of the series resistance R_S (ie, $R_P=\alpha R_s$ and $R_L=\beta R_s$),

$$\text{OUTPUT POWER} = P_{OUT} = \left(\frac{V}{R_s}\right)^2 \frac{\alpha^2\beta\, R_s}{(\alpha+\beta+\alpha\beta)^2} \quad (1)$$

$$\text{INPUT POWER} = P_{IN} = \frac{V^2\,(\alpha+\beta)}{R_s\,(\alpha+\beta+\alpha\beta)} \quad (2)$$

$$\text{EFFICIENCY} = \frac{P_{OUT}}{P_{IN}} \times 100\% = \frac{100\%}{\frac{1}{\beta}+\frac{2}{\alpha}+1+\frac{\beta}{\alpha^2}+\frac{\beta}{\alpha}} \quad (3)$$

$$f(\beta) = \frac{1}{\beta}+\frac{2}{\alpha}+1+\frac{\beta}{\alpha^2}+\frac{\beta}{\alpha} \quad (4)$$

$$\frac{df(\beta)}{d\beta} = -\frac{1}{\beta^2}+\frac{2}{\alpha^2}+\frac{1}{\alpha} = 0. \quad (5)$$

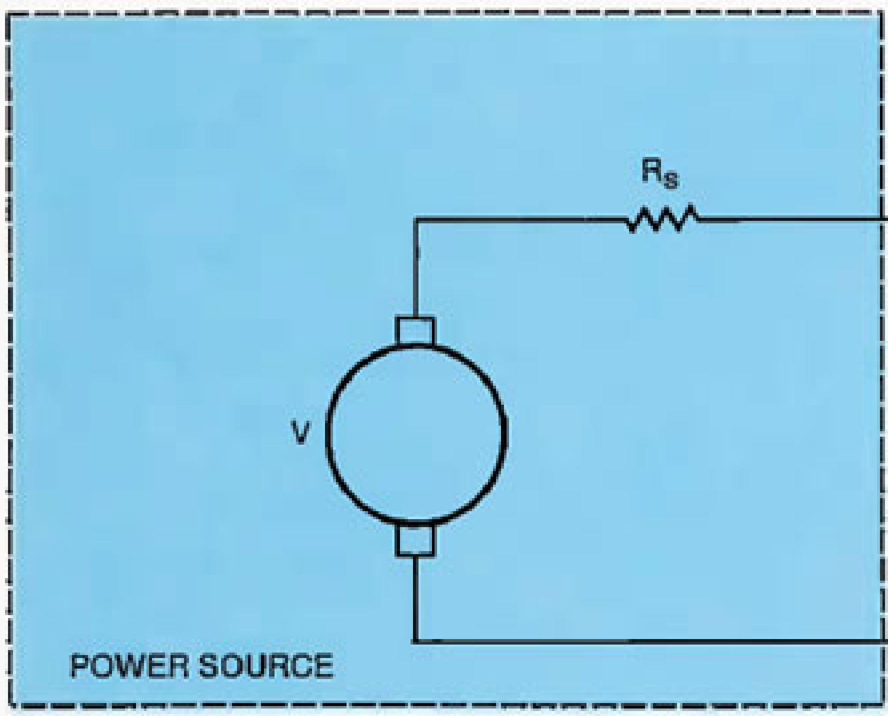

Fig 1—A simplified power-source model *produces a trivial solution for load resistance.*

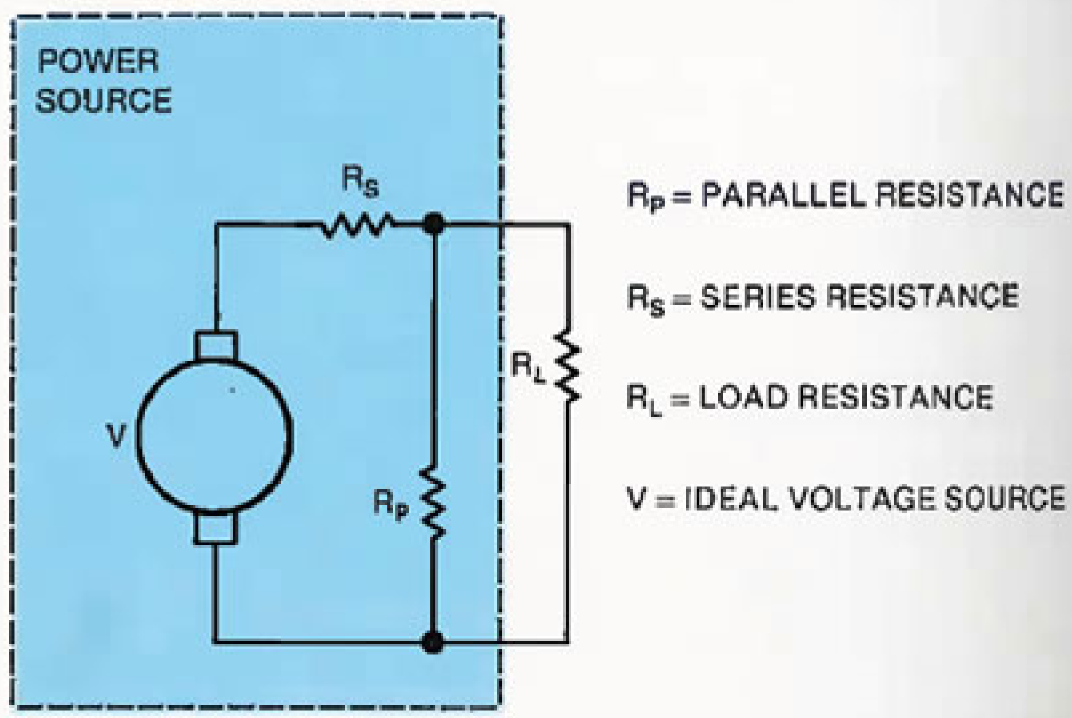

Fig 2—A more realistic power-source model *constantly dissipates power through R_P.*

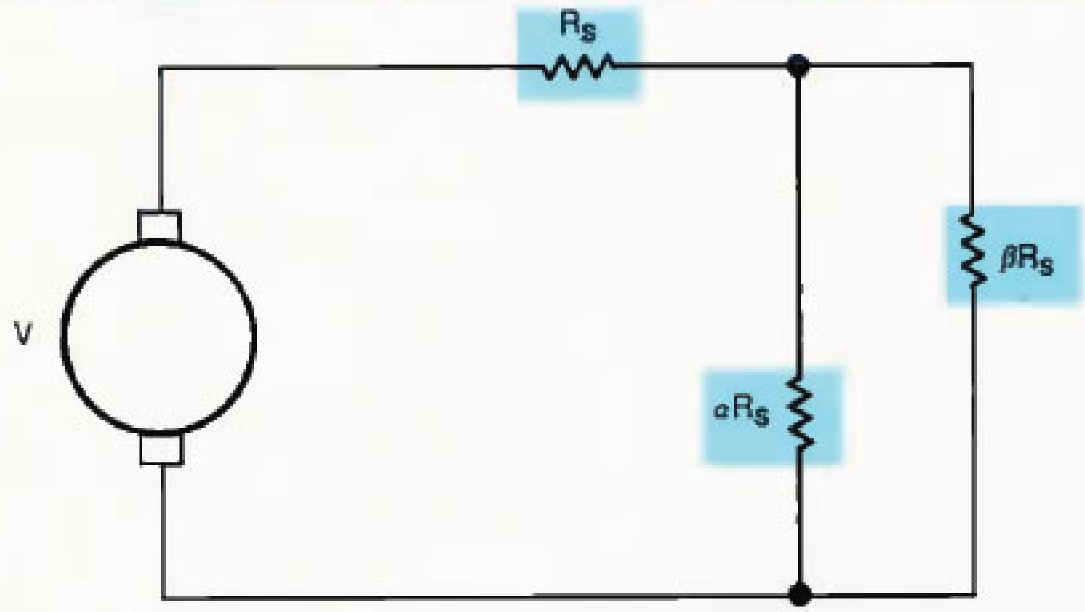

Fig 3—Determination of optimum loading *starts with a representation of all resistances in terms of the series resistance.*

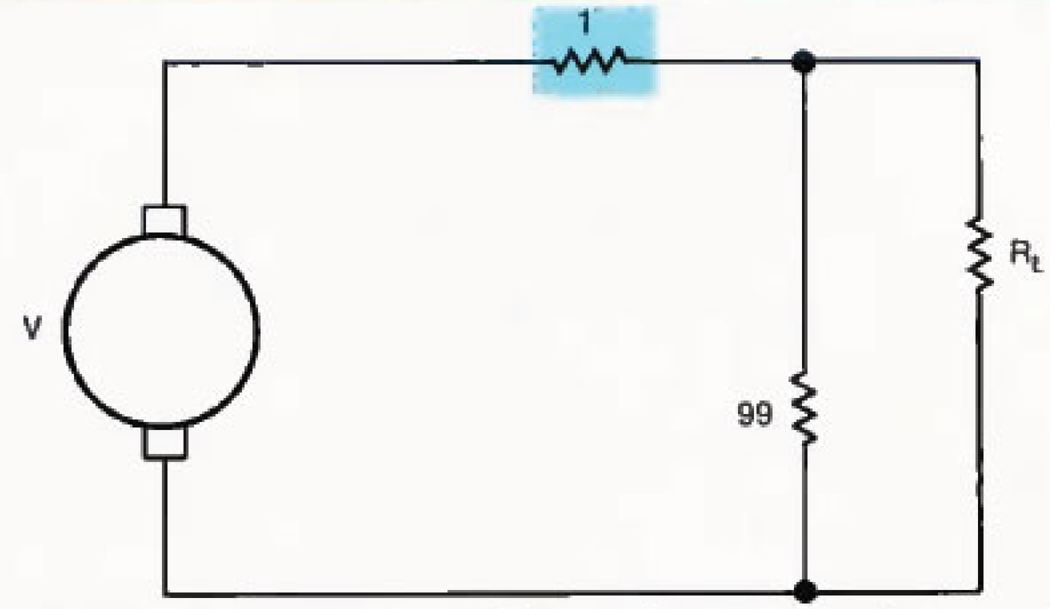

Fig 4—The sought-after series resistance *is the open-loop resistance; closed-loop resistance can appear to be several orders of magnitude less than the actual resistance.*

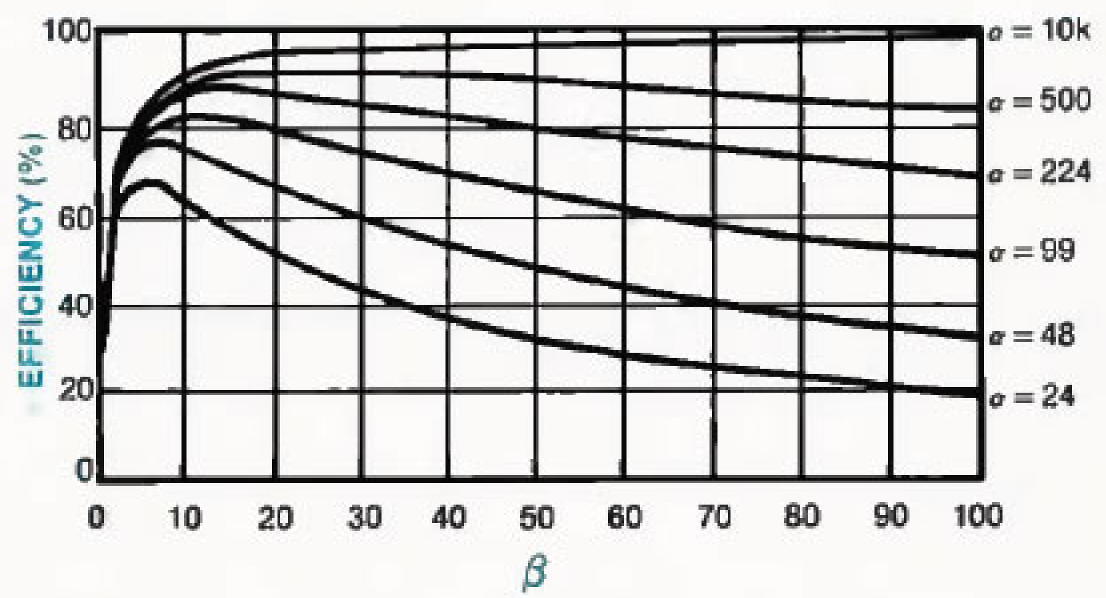

Fig 5—Compare *loading efficiency to load resistance for a range of losses.*

Thus, the efficiency reaches a maximum value for the value of β where f(β) has a minimum. This relationship implies that for maximum efficiency,

$$b = \frac{\alpha}{\sqrt{1+\alpha}}. \quad (6)$$

As an example, calculate the value of R_L that provides the maximum efficiency for the circuit shown in **Fig 4:**

$$R_s = 1\,\Omega$$

$$\alpha = \frac{99}{1} = 99$$

$$\therefore\ \beta = \frac{99}{\sqrt{1+99}} = 9.9$$

MAX EFFICIENCY

$$= \frac{100\%}{\frac{1}{9.9} + \frac{2}{9.9} + 1 + \frac{9.9}{(99)^2} + \frac{9.9}{99}}$$

$$= 82\%$$

$$R_L = \beta R_S = (9.9)(1) = 9.9\,\Omega.$$

Note that when determining R_S, you are seeking the open-loop resistance. In a closed loop, the power-source series resistance can appear to be several orders of magnitude less than the actual resistance. As illustrated in **Fig 5**, the larger you make α (the ratio of parallel resistance to series resistance) the more slowly the efficiency varies with respect to β. Reducing α—increasing parallel losses—lowers the peak efficiency.

Consider the case where α=24. The maximum efficiency obtained would be 67% when R_L=4.8R. If you reduce R_L from 4.8R to 2R, the efficiency drops to 59%. However, when you increase α to 10k, the maximum efficiency obtained is 98% when R_L=100R. R_L can now vary by ±75% and change the efficiency by no more than −2.1%. **EDN**

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