$$Z = \frac{1}{j2\pi fC} \rightarrow C = \frac{1}{j2\pi fZ}$$

$$Z = j2\pi fL$$
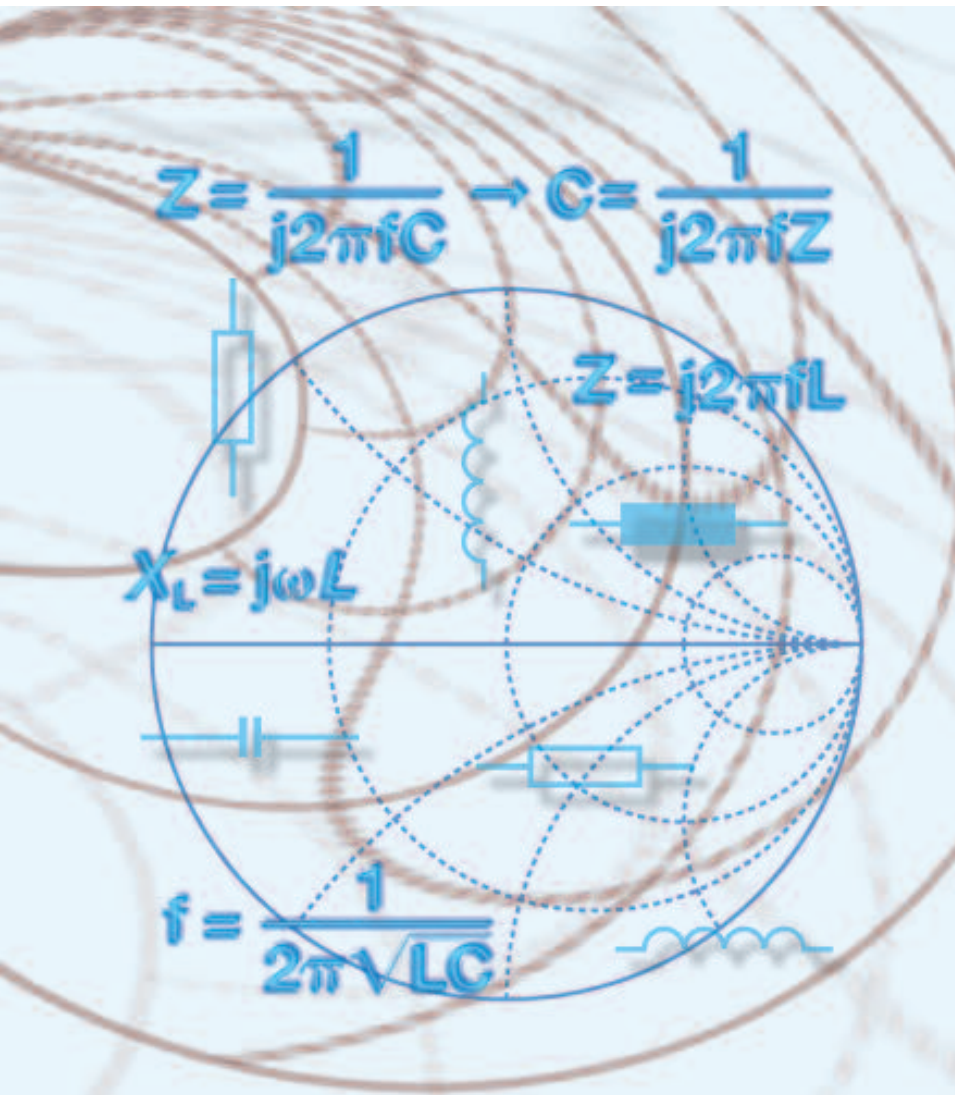
$$X_L = j\omega L$$

$$f = \frac{1}{2\pi\sqrt{LC}}$$

S-Parameters-Characteristics of Passive Components

This article describes a cost-effective method of determining the true characteristics of passive components using measurements of S-parameters. A more precise description of these component parameters can aid understanding of the arithmetic path, and the calculated values; from which equivalent circuit diagrams can be determined, both qualitatively and quantitatively, providing a basis for carrying out circuit simulations.

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Thomas Bluhm

S-Parameters - Characteristics of Passive Components

Understanding the true behaviour of passive components at frequencies above 100 MHz has become more crucial as the number of applications which operate in this frequency range increases. Under normal circumstances the component analysers and LCR equipment required for such measurements are seldom available and, where available, they are usually limited in their frequency range to a maximum of 3 GHz. However, the vectorial network analyser often found in HF development labs can provide a useful alternative. The following describes how, using such equipment, the "real life" parameters of resistors, inductors and capacitors can be determined through the application of measured S-parameters.

Daily Difficulties for Circuit Designers

Where passive components such as resistors, capacitors and inductors are required as part of a circuit design they are usually selected and applied based on their respective values. And should the designer be unsure exactly which nominal value should actually be used, the value can be determined quickly and simply with the use of a digital voltmeter or LCR meter. This works well up to the lower megahertz range, particularly in light of the wide range of instrumentation available today.
It becomes more difficult when

dealing with applications in the gigahertz range. Passive components then cease to behave as expected: capacitors become inductive, coils become capacitive, and even the value of resistors becomes frequency dependent. Consequently, verification of the actual component values becomes more difficult; with increasing frequency the range of suitable measurement equipment shrinks, and prices increase accordingly.
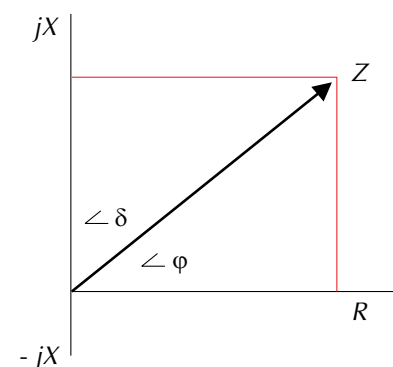

Fig. 1: The impedance of a component. The diagram comprises a real and imaginary element, described in cartesian form by $Z = R + jX$ or, in polar form by $Z = |Z| \times e^{-j\varphi}$.

Various HF Measurement Procedures

Where the characteristics of passive components are to be determined beyond the order of 100 MHz special RF-LCR equipment can be used - recently even up to 3 GHz. The advantages of such instruments are their comparatively large impedance measurement range spanning around 1 Ω

to 1 kΩ at 100 MHz, and the direct display of impedance, inductance, purity, capacitance, dissipation factor etc. Where measurements have to be made above 3 GHz, or money invested in a more generally applicable vector network analyser, this equipment can be used to determine the high frequency properties of passive components. Whereas the LCR meter computes the impedance directly from the measured current and voltage, the network generator permits only measurements of the S-parameters. Impedance and additional R, L and C characteristic properties can, however, also be determined from the reflection factor, S_{11}. Measurements are carried out ideally with the sample component grounded. Alternatively if, for example, the impedance of the sample is too low at the given frequency, or the connection to ground is of in-sufficient quality, the sample can also be tested in series with an ideal (or as close as possible) 50 Ω load resistor. The result must then be corrected by subtraction of the additionally measured 50 Ω.
The only requirements additional to the analyser are a spreadsheet application such as Excel, and a programme to represent results in a diagramm (e. g. Excel or Origin). Should models for circuit simulation be created, the demonstration version of PSpice is adequate. A limiting factor in the use of a network analyser is the reduced impedance measurement range of around 5 to 500 Ω due to the
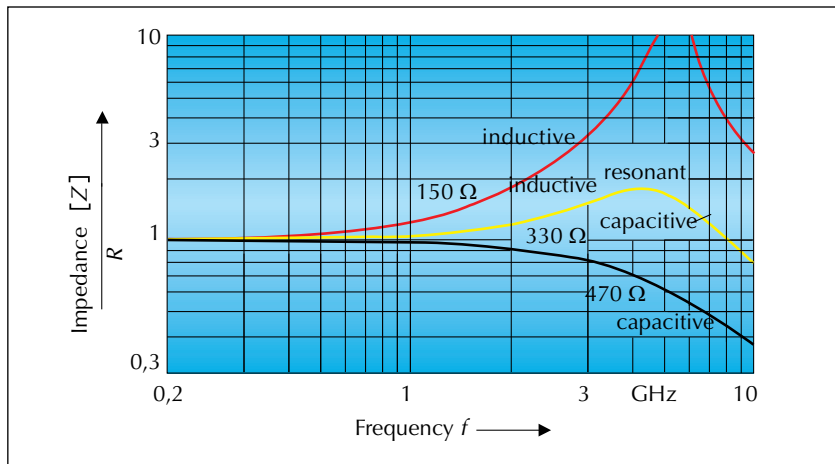
Fig. 2: Impedance characteristics of various MINI-MELF resistors related to frequency.

By way of example, Figure 2 shows the impedance characteristics of various MINI-MELF resistors. A rising plot indicates in this case the inductive behaviour of the resistor, a sinking characteristic indicates capacitive behaviour. The turning point (zero gradient) marks a resonance. Intersection of the plot with the Z/R = 1 plane does not indicate renewal of the true resistor; also here the impedance remains complex. (This can be illustrated in a three-dimensional representation. However, for the purpose of this article it will not be addressed in any more detail).

Parameters of Passive Components

Before discussing in detail the calculations used to determine component properties from the S-parameters measured, these properties should be addressed more closely. It is advantageous to know qualitatively both the ideal and typical characteristic plots in order to judge the plausibility of the calculated values.

Impedance
Figure 1 illustrates the impedance of a component. The diagram comprises a real and imaginary element, described in cartesian form by $Z = R + jX$ or, in polar form by $Z = |Z| \times e^{-j\varphi}$.

Resistors
In case of an ideal resistor Z=R, hence the impedance is independent of frequency. In practice, however, the resistor's construction causes parasitic elements. A purely ohmic resistor does not exist in reality, it is always a complex mixture, sometimes more so than others, in which parasitic inductance(s) and capacitance(s), or both, can be present alongside the actual resistor.
Experience shows that frequency dependency can be observed in wirewound resistors down in the low megahertz range (short wave), and in film resistors from around 100 MHz.

Inductances
Inductances are usually described by the parameters inductance, L, quality factor, Q, and impedance, Z, over a given frequency range. Figures 3, 4 and 5 show the qualitative characteristics of a measured inductance.
The impedance of an ideal coil is given by $Z = j2\pi fL$, the impe-

Basic Equation for Passive Components

	Resistor	Coil	Capacitor
Impedance, ideal	Z=R	$Z = j2\pi fL$	$Z = \dfrac{1}{j2\pi fC}$
Impedance, real		$Z = \dfrac{(R + j2\pi fL) \cdot \dfrac{1}{j2\pi fC}}{R + j2\pi fL + \dfrac{1}{j2\pi fC}}$	$Z = ESR + j2\pi fL + \dfrac{1}{j2\pi fC}$
Quality of the component		Q-factor: $Q = \dfrac{Z}{Rs} = \dfrac{2\pi fL}{Rs}$	Dissipation-factor: $D = \dfrac{ESR}{j2\pi fC} = 2\pi fC \cdot ESR$
Thomson's vibration equation		$f = \dfrac{1}{2\pi\sqrt{LC}}$	
Impedance from the reflection factor		$Z = Z_0 \dfrac{1+r}{1-r}$	

dance increases linearly with frequency. In reality, however, a parallel resonant circuit is formed between the inductance and the parasitic capacitance which results from the construction of the component. The circuit is characterised by its Self Resonant Frequency, SRF. Around the SRF the change in impedance is non-linear. (Incidentally, where the inductance, L, and SRF of a coil are known, Thomson's vibration equation can be used to calculate its parasitic capacitance (see inset). This can be helpful in determining the equivalent circuit diagram). Above the SRF the impedance becomes smaller as frequency increases. This, however, is the characteristic behaviour of a capacitor, as will be shown later. Ideally, the inductance of a coil is independent of frequency. Around the SRF, the parasitic capacitance previously described introduces an apparent increase in the inductive value - rendering the coil useless.
The ratio of reactive impedance (= impedance of the ideal coil) to loss resistance is described by the quality factor, Q. The greater the Q-factor, the closer the component under consideration is to the ideal. The loss results from the ohmic resistance and additional high frequency losses. Under the assumption that the impedance increases linearly with frequency, theoretically, the Q-factor also increases linearly with frequency. In reality the Q-factor begins to drop as the parasitic capacitance emerges, since the capacitive-Q opposes the inductive-Q.

Capacitors

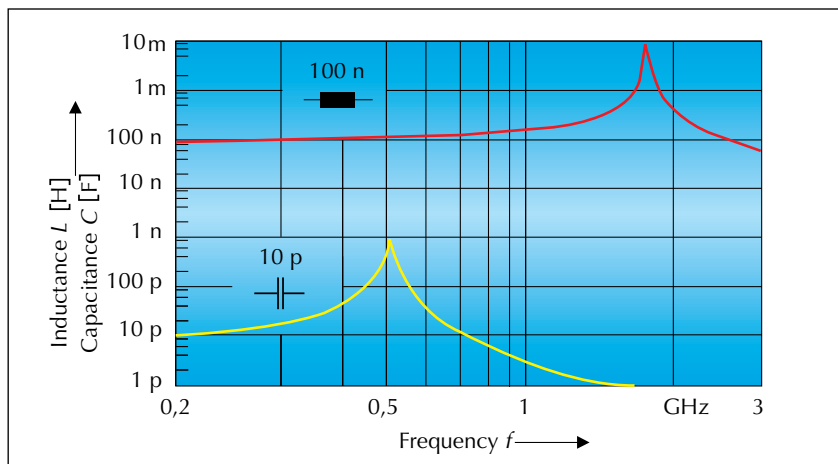Capacitors are described by their capacitance, C, their dissipation

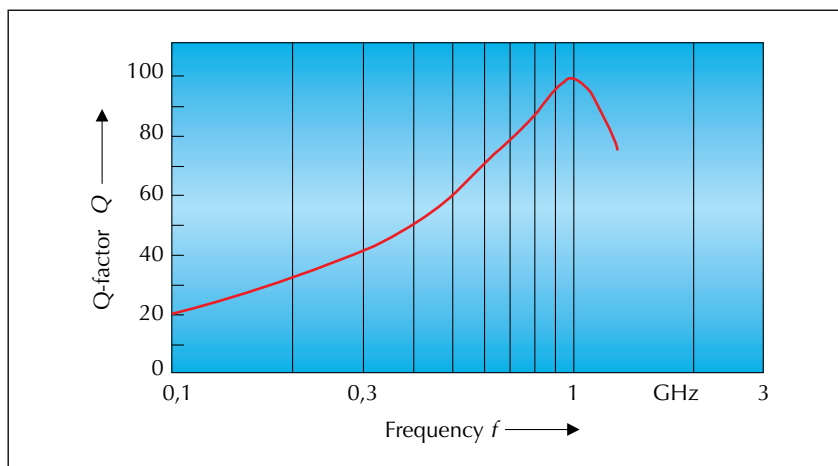Fig. 3: Typical frequency-dependent characteristics of an inductor L and capacitor C.

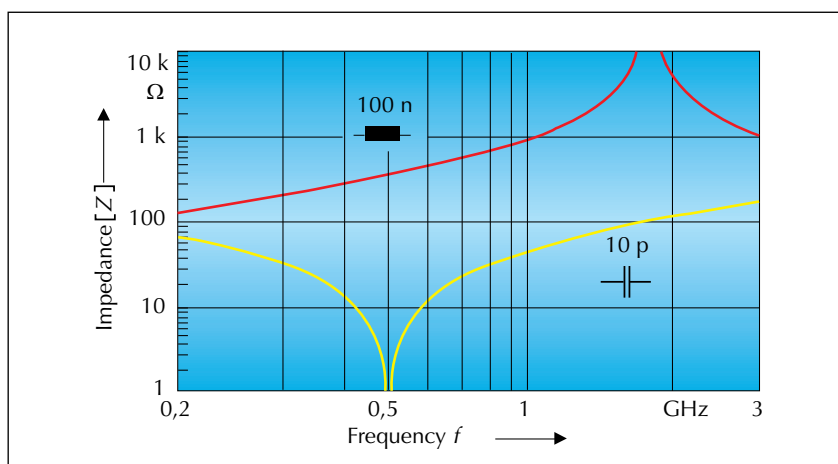Fig. 4: Typical Q-factor characteristics with respect to frequency.

Fig. 5: Typical impedance characteristics Z of an inductor and a capacitor with respect to frequency.

Given			
Resistive value R		50	Only for resistors
System impedance Z_0		50	
Frequency f		100000000	
S11_value		0,3333	
S11_phase		30	In degrees
Obtaining Z			
	S11_re	=S.11_value*COS(S.11_phase*PI()/180)	Conversion into
	S11_im	=S.11_value*SIN(S.11_phase*PI()/180)	Cartesian co-ordinates
	Z_numerator_re	=1+S.11_re	Numerator addition
	Z_numerator_im	=S.11_im	
	Z_denominator_re	=1-S.11_re	Denominator addition
	Z_denominator_im	=S.11_im*(-1)	
	Z_numerator_value	=SQRT(Z_numerator_re^2+Z_numerator_im^2)	Conversion into
	Z_numerator_phase	=ATAN(Z_numerator_im/Z_numerator_re)	Polar co-ordinates
	Z_denominator_value	=SQRT(Z_denominator_re^2+Z_denominator_im^2)	
	Z_denominator_phase	=ATAN(Z_denominator_im/Z_denominator_re)	
	Z_quotient_value	=Z_numerator_value/Z_denominator_value	Division
	Z_quotient_phase	=Z_numerator_phase-Z_denominator_phase	
Z_value		=Z0*Z_quotient_value	Multiplication by Z_0
Z_phase	=Z_quotient_phase	In radians	
		=Z_quotient_phase*180/PI()	In degrees
[Z]/R	=Z_value/R		
	Z_re	=Z_value*COS(Z_phase)	Conversion into
	Z_im	=Z_value*SIN(Z_phase)	Cartesian co-ordinates
Obtaining L, Q			
L		=Z_im/(2*PI()*f)	
Q		=Z_im/Z_re	
Obtaining C, D			
C		=1/(2*PI()*f)	
D		=Z_re/Z_im	

Table 1: Calculation of the real values Z, L, Q, C, D: This variation calculates the required factors from the reflection factor, S_{11}, using simple mathematical standard functions.

factor, D (= tan δ) and their impedance, Z, related to frequency. The qualitative characteristics are shown in figures 3, 5 and 6. The impedance of an ideal capacitor is given by $Z = 1/(j2\pi f C)$, Z sinks with increasing frequency. But the capacitor also has a resonant frequency resulting from a parasitic inductance. Above this resonant frequency the impedance increases again; the capacitor becomes inductive.

Ideally the value of a capacitor is independent of frequency. However, around the resonant frequency, the parasitic inductance discussed above results in an apparent increase in the value of the capacitor; the component is then useless. The quality of capacitors is described, not by a Q-factor, but rather by its reciprocal; a dissipation factor, tan δ. The dissipation factor represents the ratio between loss resistance(s) (Equivalent Series

Resistances, ESR) and reactive impedance (= impedance of the ideal capacitor).

From S-Parameters to Component Characteristics

It is usual to employ specialised, and expensive, software in order to compute component characteristics from S-parameters. This is not specifically necessary, providing one has a basic understanding of complex numbers and complex arithmetic, and a knowledge of a standard software package with integrated functions such as Excel or Origin. This solution provides just as reliable an evaluation of the S-parameters obtained, but more cost-effectively. Tables 1 and 2 present two possible ways of calculating the desired characteristics. The first variation applies simple mathematical standard functions to the reflection coefficient, S_{11}, in order to arrive at the result. The second variation represents a more elegant solution, if an Excel package with integrated complex arithmetic functions is available. Both variations are suitable for calculating values of R, L and C.

First Version with Mathematical Standard Functions (Table 1)

Lines 2 to 5 show the values of system impedance, Z_0, frequency f and size and phase of the reflection factor, S_{11}. Firstly Z is calculated using the reflection factor equation with r = S_{11} (see inset). S_{11} is then initially converted into cartesian co-ordinates in lines 6 and 7 (assuming the measurements are not already in
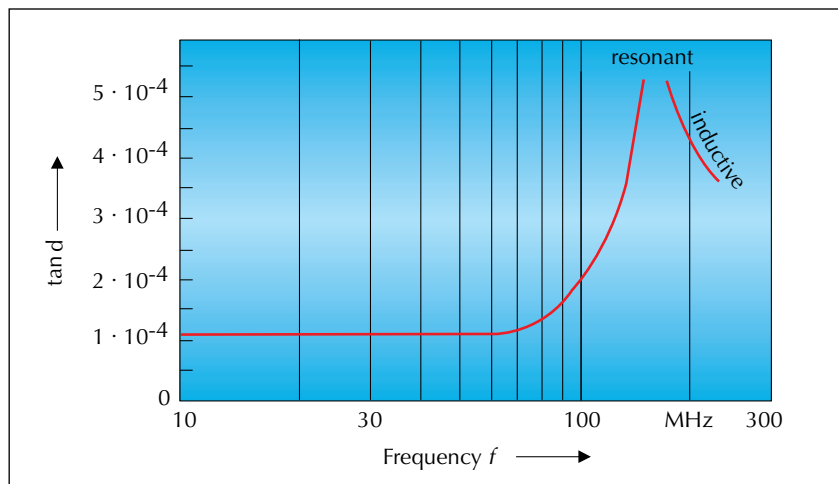

Fig. 6: Typical loss factor progression (tan δ) for a capacitor.

this form). Almost all the conversions which follow are carried out purely because addition and subtraction are simpler with cartesian co-ordinates, whereas division and multiplication are simpler with polar co-ordinates. Simple computations reduce the likelihood of error in entering formula and aid trouble shooting when unexpected results arise. Following conversion, numerator and denominator are calculated in lines 8 to 11, after which - following renewed conversion into polar co-ordinates (lines 12 to 15) - numerator and denominator are divided in lines 16 and 17. The resulting value is then multiplied by Z_0 in line 18. The phase angle is not affected by Z_0 since Z_0 is real (line 19). Should it be desired that the phase information not only in radians but also in degrees, the phase value is multiplied by 180/PI() (line 20).
The following calculations are component-specific. Should it be the aim to show the characteristics of a resistor, the value already calculated represents the result. It is preferable to scale it to the original nominal value of the resistor, making it easy to compare

several resistance values on the same diagram (line 21).
Calculation of the frequency dependency of the inductance and Q-factor of coils is achieved with the following two equations:

$$Z = j2\pi fL \qquad \rightarrow \quad L = \frac{Z}{j2\pi f} \qquad (1)$$

$$Q = \frac{Z}{R_s} = \frac{2\pi fL}{R_s} \quad \rightarrow \quad Q = \frac{Imaginary(Z)}{Real(Z)}$$
$$(2)$$

Since, according to the definition above, inductance is purely imaginary (although both the desired inductive element and also the undesirable parasitic capacitive element are represented) renewed conversion into cartesian co-ordinates separates the real and imaginary elements in lines 22 and 23. After which L results from the division of the imaginary element of Z by the angular frequency. The Q-factor is obtained by dividing the imaginary element of Z by the real element (line 25).
In this way values for the parasitic inductance of resistors, and the Q-factor of capacitors can be calculated.
Analogous to inductances, the

Given			
Resistive value R		50	Only for resistors
System impedance Z_0		50	
Frequency f		100000000	
S11_value		0,3333	
S11_phase		30	In degrees
Obtaining Z			
	S11_Re	=S.11_value*COS(S.11_phase*PI()/180)	Conversion into
	S11_Im	=S.11_value*SIN(S.11_phase*PI()/180)	Cartesian co-ordinates
	S11_complex	=COMPLEX(S.11_re;S.11_im;"i")	Formation of complex number
	Z_complex	=IMPRODUCT(Z0;IMDIV(IMSUM(1;S.11_complex); IMSUB(1;s.11_complex)))	
Z_value		=IMABS(Z_complex)	
Z_phase		=IMARGUMENT(Z_complex)	In radians
		=IMARGUMENT(Z_complex=*180/PI()	In degrees
$|Z|/R$		=Z_value/R	
	Z_Re	=IMREAL(Z_complex)	Conversion into
	Z_Im	=IMAGINARY(Z_complex)	Cartesian co-ordinates
Obtaining L, Q			
L		=Z_im/(2*PI()*f)	
Q		=Z_im/Z_re	
Obtaining C, D			
C		=1/(2*PI()*f*Z_im)	
D		=Z_re/Z_im	

Table 2: Calculation of the real values of Z, L, Q, C, D: this variation represents a more elegant solution, assuming an Excel package with integrated complex functions is available.

following is true for capacitors:

$$Z = \frac{1}{j2\pi fC} \quad \rightarrow \quad C = \frac{1}{j2\pi fZ} \qquad (3)$$

$$D = \frac{ESR}{j2\pi fC} \quad \rightarrow \quad D = \frac{Real(Z}{Imaginary(Z)} \qquad (4)$$

The conversion in lines 26 and 27 is the equivalent of the route described above for coils.

Second Version with the Aid of Excel (Table 2)

This approach does not differ from the first through the first seven lines. From line 8 complex functions which are available in Excel are applied. This simplifies the exercise as conversions are dispensed with. The line of calculation is, however, the same as that taken in the first variation. For this reason only the issues related specifically to the complex functions will be addressed here.

In line 8 the function COMPLEX is used to create a complex number from the previously separate real and imaginary elements. This complex number is necessary for the application of practically all complex functions. The function parameters are the values in cartesian format, calculated in lines 6 and 7.

Calculation of the impedance is now straight forward using the function IMPRODUCT in the multiplication of complex numbers (line 9). Function parameters are Z_0 and the result of the complex division (IMDIV) of the previously calculated numerator (IMSUM) and denominator (IMSUB). In lines 10 to 12 follows the reduction of the complex result into the required components: IMABS (line 10) provides the value of the complex number,

whereas INARGUMENT (lines 11 and 12) provides the phase of the complex number. IMREAL (line 14) and IMAGINARY (line 15) return the results of the real and imaginary components of Z, which are further processed in the equations of lines 16 to 19. These are not different to lines 24 to 27 of the first variation.

Of course, it is possible to inter-lace the functions and, hence, to reduce the number of compu-tational steps. However, this brings the associated disadvantage that clarity of calculation is lost, and the likelihood of errors in the formulae increases.

For the purpose of clarity the calculation has been arranged in a column. This representation enables the calculation for single frequencies. Arranging the calcu-lation in a row permits the calcu-lation for any number of frequen-cies arranged in a column. Additionally, it is helpful to hide columns which simply contain conversions, to apply background colours to results columns and to lock all cells in the worksheet, with the exception of f, $S_{11\text{-value}}$ and $S_{11\text{-phase}}$.

Equivalent Circuit Diagrams - and their Limitations

In HF development it is a common next step to calculate and simulate a circuit with the aid of a computer. Since the simulation can only ever be as good as the model upon which it is based, it is well worth paying close attention to the model. Models should be simple, but also precise and, in addition, be valid over a wide frequency range. The equivalent circuits for resistors, coils and capacitors depicted in figures 7
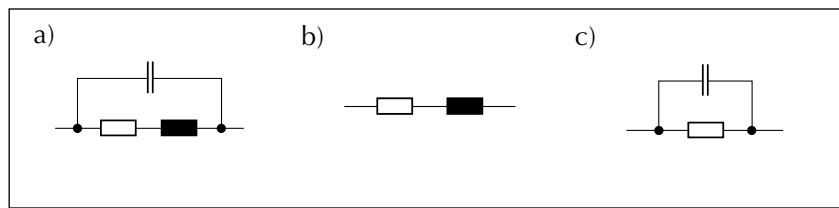

Fig. 7: (a) Equivalent circuit diagram for a resistor and inductor respectively; (b) variations for resistors < 100 Ω and inductances with value << SRF and (c) variations for resistors > 200 Ω .

and 8 are commonly used in literature and, hence, generally known. It can be seen that there is no qualitative difference between inductance and resistance. When the construction of these compo-nents is considered, this comes as no surprise: when a wire is wound around a body the result, depen-dent upon the properties of the wire, can be either a resistor or an inductor. The quantitative, and thus decisive, difference is in the fact that a resistor should have the smallest possible parasitic reactance, and a coil should have the smallest possible resistance and low parasitic capacitance. The equivalent circuits for both components can be simplified through the basic conditions detailed alongside figure 7. When the impedance equation is applied to the illustrations to calculate the progression of impe-dance against frequency, the diagrams for R, L and C are arrived at. It should be noted that, beyond around 3 GHz, the simple equivalent circuits illustrated become increasingly inaccurate; additional elements are necessary in order to provide an accurate description of the high frequency characteristics. Whereby the circuit simulation also becomes more complex. How, then, can the equivalent circuit values be determined without purchasing expensive software? The answer is very simple: the free demo version of PSpice enables the calculation

of the equivalent circuit diagram. The measured and calculated characteristics merely have to be brought into line with those calcu-lated using PSpice. This route may be slightly laborious, but has ad-vantages, including the fact that one is obliged to consider each individual element of the equiva-lent circuit in detail.

The base circuit for the calculation of the equivalent circuit diagram is shown in figures 7 and 8. The symbols IPRINT and VPRINT write the values calculated for current and voltage to a file. In this way the values can be processed further in other applications (e.g. Origin, Excel). This enables the simul-taneous depiction of the measured values and those calculated with PSpice.
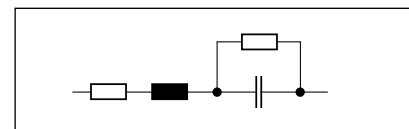

Fig 8: Equivalent circuit diagram for a capacitor.

Advantages and Disadvantages of some Methods of Depiction

A simple yet comprehensive method is required to represent the frequency response of resis-tance, or impedance. Below the advantages and disadvantages of various possible options are discussed.

Smith Diagram

The Smith diagram is one of the methods of depiction most frequently used in HF development. It allows the qualitative value and phase characteristics to be shown against frequency, as well as any resonant frequencies. The non-linearity of the representation, however, makes a quantitative estimation comparatively. The Smith diagram is particularly useful where resistors of the same value, but different designs, are to be compared: the longer the response in the frequency range considered, the worse the high frequency characteristics of the component (fig. 9).
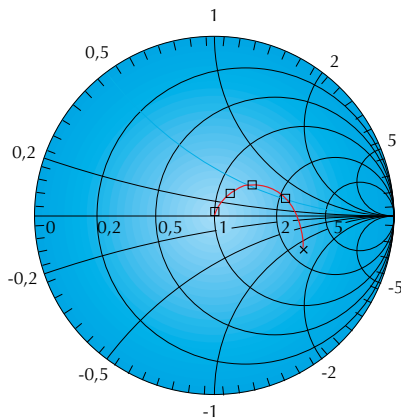

Fig. 9: Smith diagram: The longer the characteristics is in the frequency range under consideration, the worse the high frequency behaviour of the resistor.

Depiction of Magnitude

This is another common representation, often using a logarithmic scale. In the case of resistors a uniform representation can prove deceptive with regard to resonances.

Standing Wave Ratio

The standing wave ratio diagram can be used to show resonances as well as changes in the resistor's value. However, the method of calculation of the standing wave ratio has the associated disadvantage that it is not possible to differentiate between inductive and capacitive behaviour.

Depiction of Impedance

Representation of impedance against frequency provides a simple representation of the qualitative and quantitative changes in resistance, including any resonances. Scaling the calculated impedance to the (nominal) value of the resistor also enables the comparison of different resistor values on the same diagram.

The Author

Thomas Bluhm studied information technology at Ilmenau Technical University. Following graduation his first professional position was as a development engineer in industrial control. Since 1995 he has worked for the resistor manufacturer, BCcomponents BEYSCHLAG, where he has been involved in resistor applications, providing technical customer support and in components standardisation.