Working with transistor S-parameters

Transistors are generally considered two-port devices with a common emitter or source. However, they can also be applied to other configurations.

By Duangrat Eungdamrong and Devendra Misra

Manufacturers generally supply the data sheets for transistors containing S-parameters with respect to the emitter (or the source). In other words, the transistor is considered a two-port device with a common emitter (or a common source). However, design engineers may want to use other transistor configurations for certain circuits. This tutorial summarizes the necessary transformations for such applications.

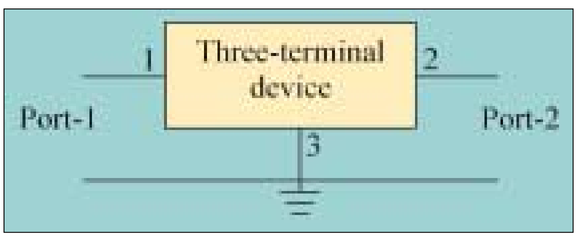

Figure 1. A three-terminal device as a two-port network.

Consider the two-port circuit illustrated in Figure 1. Terminal Three of the device is connected to the ground to form a two-port network. Assume that the scattering matrix of this network is given as follows:

$$[S] = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \qquad (1)$$

This two-port network will be unconditionally stable if both of the following conditions are satisfied: $|D| < 1$, (2) and $k > 1$ (3), where:

$$\Delta = S_{11}S_{22} - S_{21}S_{12} \qquad (4)$$

and,

$$k = \frac{1 - |S_{11}|^2 - |S_{22}|^2 + |\Delta|^2}{2|S_{12}S_{21}|} \qquad (5)$$

If these conditions are not satisfied, then the device is potentially unstable. Such unstable regions on the Smith chart can be identified after plotting the input and output stability circles[1]. Centers and radii of these two circles can be found as follows:

$$C_1 = \frac{(S_{11} - \Delta S_{22}^*)^*}{|S_{11}|^2 - |\Delta|^2} \quad (6), \quad r_1 = \left|\frac{(S_{12}S_{21})}{|\Delta|^2 - |S_{11}|^2}\right| \quad (7)$$

$$C_2 = \frac{(S_{22} - \Delta S_{11}^*)^*}{|S_{22}|^2 - |\Delta|^2} \quad (8), \quad r_2 = \left|\frac{(S_{12}S_{21})}{|\Delta|^2 - |S_{22}|^2}\right| \quad (9)$$

Furthermore, the device will be unstable if even one of the conditions is not satisfied. However, these conditions do not provide means to compare the level of instability of two transistors. An alternative parameter for measuring a transistor for certain circuits, such as the oscillators, is evaluated as follows:

$$\mu = \frac{1 - |S_{22}|^2}{|S_{11} - \Delta S_{22}^*| + |S_{21}S_{12}|} \qquad (10)$$

For the oscillator design, the magnitude of the μ-factor is kept to a value of less than unity. For magnitudes greater than unity, the transistor will be unconditionally stable .

Transformation of the two-port network

A circuit designer may sometimes need to consider the transistor as a three-port device by disconnecting Terminal Three from the ground. It may be dictated by certain applications to either use a different transistor configuration (common base, common gate, etc.) or to add external feedback that enhances the desired characteristics of the device. This can be achieved after transforming the two-port device into a three-port device. Scattering parameters of this three-port transistor description can be found as follows[2, 3]:

$$[S'] \begin{bmatrix} \left(S_{11} + \frac{\Delta_{11}\Delta_{12}}{4-\xi}\right) & \left(S_{12} + \frac{\Delta_{11}\Delta_{21}}{4-\xi}\right) & \frac{2\Delta_{11}}{4-\xi} \\ \left(S_{21} + \frac{\Delta_{22}\Delta_{12}}{4-\xi}\right) & \left(S_{22} + \frac{\Delta_{22}\Delta_{21}}{4-\xi}\right) & \frac{2\Delta_{22}}{4-\xi} \\ \frac{2\Delta_{12}}{4-\xi} & \frac{2\Delta_{21}}{4-\xi} & \frac{\xi}{4-\xi} \end{bmatrix} \qquad (11)$$

where:

$$\Delta_{12} = 1 - S_{11} - S_{21} \qquad (12)$$

$$\Delta_{21} = 1 - S_{12} - S_{22} \qquad (13)$$

$$\xi = S_{11} + S_{22} + S_{12} + S_{21} =$$

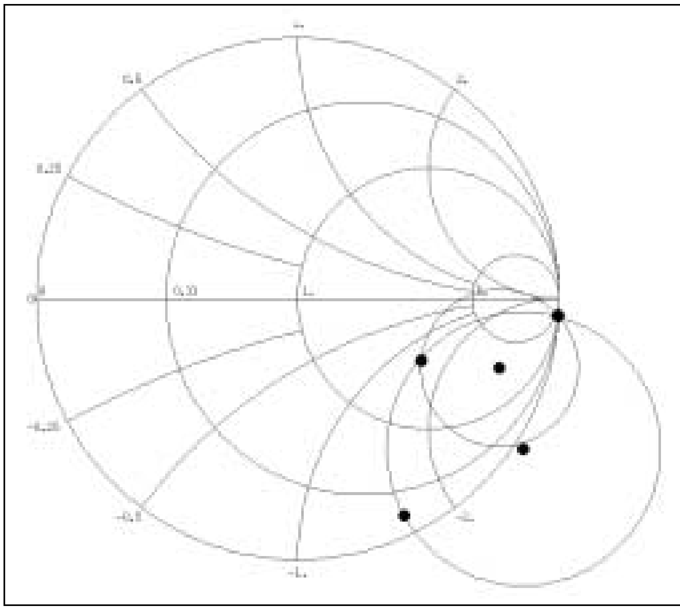

Figure 2. Impedance circle plots for the common source configuration.

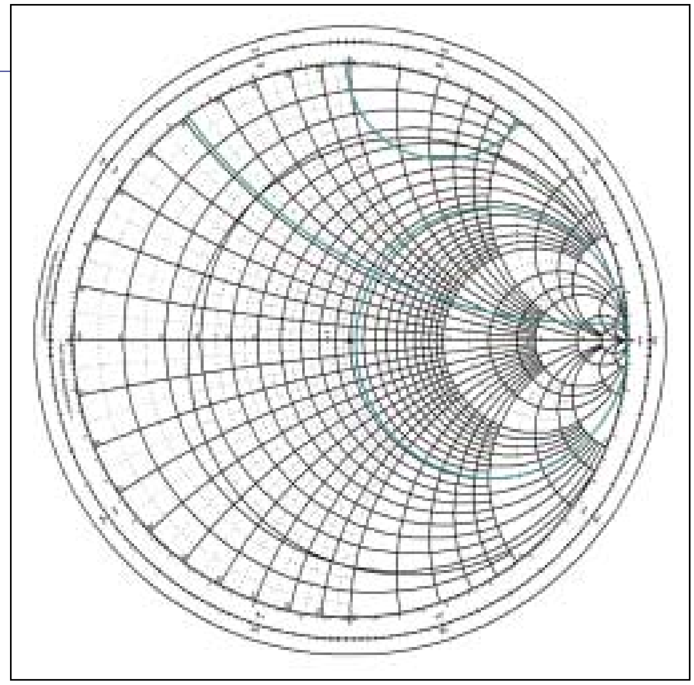

Figure 3. Common source configuration stability circles with and without feedback at the source.

$$2 - \Delta_{12} - \Delta_{21} = 2 - \Delta_{11} - \Delta_{22} \quad (14)$$

$$\Delta_{11} = 1 - S_{11} - S_{12} \quad (15)$$

$$\Delta_{22} = 1 - S_{21} - S_{22} \quad (16)$$

If the gate (base) is connected to ground and the source (emitter) represents new port one (while the drain (collector) remains as port two), then its S-parameters are found to be:

$$[S^n] = \begin{bmatrix} S'_{33} - \dfrac{S'_{31}S'_{13}}{1+S'_{11}} & S'_{32} - \dfrac{S'_{31}S'_{12}}{1+S'_{11}} \\ S'_{23} - \dfrac{S'_{21}S'_{13}}{1+S'_{11}} & S'_{22} - \dfrac{S'_{21}S'_{12}}{1+S'_{11}} \end{bmatrix} \quad (17)$$

On the other hand, if a normalized impedance $\overline{Z}$ is connected between Terminal Three and the ground to provide a feedback, then the scattering parameters of the resulting two-port network are found as follows:

$$[S_n] = \begin{bmatrix} S'_{11} - \dfrac{S'_{13}S'_{31}}{S'_{33}+\Gamma^{-1}} & S'_{12} - \dfrac{S'_{13}S'_{32}}{S'_{33}+\Gamma^{-1}} \\ S'_{21} - \dfrac{S'_{23}S'_{31}}{S'_{33}+\Gamma^{-1}} & S'_{22} - \dfrac{S'_{23}S'_{32}}{S'_{33}+\Gamma^{-1}} \end{bmatrix} \quad (18)$$

where:

$$\Gamma = \frac{\overline{Z}-1}{\overline{Z}+1} \quad (19)$$

As (18) indicates, S-parameters of the new two-port network can be modified to a certain extent by adjusting the normalized impedance. Note that its reflection coefficient, Γ, will satisfy the following two equations:

$$\Gamma = \frac{S_{11n} - S'_{11}}{S_{11n}S'_{33} - \Lambda_1} \quad (20)$$

and,

$$\Gamma = \frac{S_{22n} - S'_{22}}{S_{22n}S'_{33} - \Lambda_2} \quad (21)$$

where:

$$\Lambda_1 = S'_{11}S'_{33} - S'_{31}S'_{13} \quad (22)$$

and,

$$\Lambda_2 = S'_{22}S'_{33} - S'_{32}S'_{23} \quad (23)$$

In the case of a reactive termination at port three, $|\Gamma| = 1$. It maps a circle in the S_{11n}-plane, the center C_1 and radius R_1, which are given as follows:

$$C_1 = \frac{S'_{11} - \Lambda_1 S'^{*}_{33}}{1-|S'_{33}|^2}\,3 \quad (24)$$

and,

$$R_1 = \frac{|S'_{31}S'_{13}|}{\left|1-|S'_{33}|^2\right|} \quad (25)$$

Similarly, center C_2 and radius R_2 of the circle for S_{22n} are found as follows:

$$C_2 = \frac{S'_{22} - \Lambda_2 S'^{*}_{33}}{1-|S'_{33}|^2} \quad (26)$$ and,

$$R_2 = \frac{|S'_{32}S'_{23}|}{\left|1-|S'_{33}|^2\right|} \quad (27)$$

These circles may be drawn on the Smith chart, and a suitable feedback circuit can be identified to meet the desired properties of the resulting two-port network.

A case study

Consider the common-source S-parameters of SHF-0198 HFET, as shown in Equation 28. These are supplied by the manufacturer at V_{ds} = 9 VDC, I_{ds} = 150 mA, and f = 500 MHz.

$$[S] = \begin{bmatrix} 0.928\angle -64^\circ & 0.023\angle 70^\circ \\ 10.84\angle 150^\circ & 0.529\angle -27^\circ \end{bmatrix} \quad (28)$$

Using equations (2) – (5), find that $|\Delta|$ = 0.3776 and k = 0.0031. This shows that the transistor is potentially unstable. Its stability circles are found from equations (6) – (10) as follows:

Input stability circle C1 = 1.0595Ð71.51°, R1 = 0.3469, outside is stable.

Output stability circle C2 = 2.0759Ð64.79°, R2 = 1.8161, outside is stable.

And, m = 0.7125

For the common-source configura-

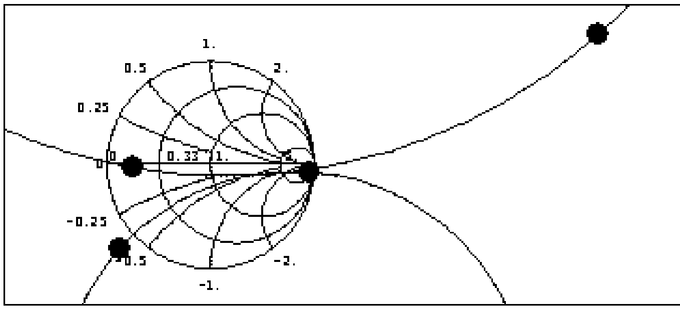

Figure 4. Common-gate configuration stability circles with $\bar{Z}$ = *j12*.

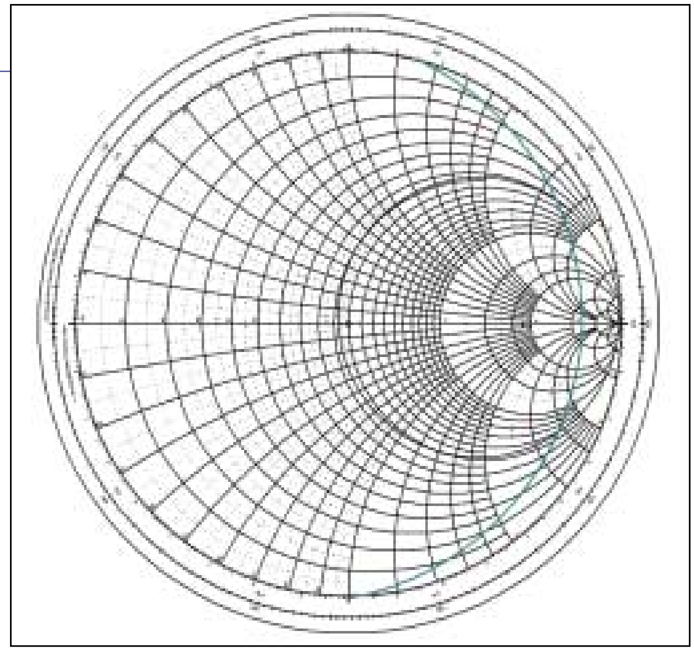

Figure 5. Stability circles for both cases (with and without feedback element at the gate terminal).

tion, the feedback reactance can be identified via equations (24) – (27) as follows:

S_{11n} circle $C_1 = 1.0369\angle{-33.73°}$, $R_1 = 0.5237$.
S_{22n} circle $C_2 = 0.8155\angle{-18.99°}$, $R_2 = 0.3008$.

These circles are illustrated in Figure 2. If the $\bar{Z}= j25$ is connected between the source and the ground, scattering parameters of the resulting two-port are found to be:

$$[S_n]=\begin{bmatrix} 0.9962\angle -4° & 0.705\angle 90.57° \\ 0.1462\angle 86.68° & 1.0002\angle -3.88° \end{bmatrix} \quad (29)$$

Again, using equations (2) – (5), find that $|\Delta| = 1.0067$ and $k = 0.9975$. Therefore, the transistor with this feedback element involved is potentially unstable. For this configuration, the stability circles are found as follows:

Input stability circle, $C_1 = 0.5107\angle 0.94°$, $R_1 = 0.4917$, inside is stable.
Output stability circle, $C_2 = 0.2153\angle 15.17°$, $R_2 = 0.7943$, inside is stable.
And, $\mu_n = -0.015$

Because μ_n is now much smaller than it was without feedback, this arrangement is preferable if one is designing an oscillator with a common-source configuration. As shown in Figure 3, it becomes obvious when the stability circles are plotted on a Smith chart for the transistor with and without feedback at its source lead.

If the configuration is switched to common-gate, then its S-parameters are found from equations (11) – (17) as follows:

$$[S^n]=\begin{bmatrix} 0.75\angle -178.23° & 0.0614\angle 2.79° \\ 1.7266\angle -4.26° & 0.9448\angle -4.14° \end{bmatrix} \quad (30)$$

And again, using equations (2) – (5), find that $|\Delta| = 0.8146$ and $k = 0.983$ in this configuration. Therefore, the transistor is still potentially unstable. Its stability circles are found from equations (6) – (10) as follows:

Input stability circle, $C_1 = 0.1949\angle 173.68°$, $R_1 = 1.0488$, inside is stable.
Output stability circle: $C_2 = 1.4574\angle 4.35°$, $r_2 = 0.4628$, outside is stable.
And, $\mu_n = 0.8534$

The feedback reactance can be identified via equations (24) – (27) as follows:

S_{11n} circle, $C_1 = 5.2851 \angle 88.27°$, $R_1 = 5.3832$. S_{22n} circle: $C_2 = 2.4646\angle{-70.97°}$, $R_2 = 2.266$.

These circles are illustrated in Figure 4. With $\bar{Z} = j12$ connected between the gate and the ground, scattering parameters of the resulting two-port are found to be:

$$[S_n]=\begin{bmatrix} 3.9426\angle 18.77° & 2.0431\angle 28.94° \\ 2.9946\angle -163.91° & 1.1881\angle -137.34° \end{bmatrix} \quad (31)$$

Going back to the standard equations (2) – (5), we find that $|\Delta| = 2.0323$ and $k = -1.0066$. Therefore, the transistor with this feedback element is potentially unstable. Its stability circles are found as follows:

Input stability circle, $C_1 = 0.4775\angle 2.88°$, $R_1 = 0.5155$, outside is stable.
Output stability circle: $C_2 = 3.2851\angle 9.3°$, R2 = 2.2805, inside is stable. $\mu_n = -0.0379$

Stability circles for both of these cases (with and without feedback element at its gate terminal) are shown in Figure 5. As it indicates, the feedback provides a lot of flexibility to the oscillator designer in selecting the source and the drain side circuits.

Conclusion

Some fundamental techniques can be used for transforming scattering parameters of transistors for different configurations. Feedback networks can be designed to obtain the desired device behavior. It also shows that three-port transistor devices are flexible and able to function in various not-so-common configurations.

RF

References:

[1] D. K. Misra, "Radio - Frequency and Microwave Communication Circuits," NY:Wiley, 2001, Chapter - 10.
[2] A. P. S. Khanna, "Oscillators, Microwave Solid State Circuit Design," I Bahl and P. Bhartia (Editors), Chapter - 9, NY:Wiley, 1988.
[3] D. K. Misra, "Radio - Frequency and Microwave Communication Circuits," NY:Wiley, 2001, Chapter - 11.

About the authors

Duangrat Eungdamrong received her B.S. and M.S. degrees in electrical engineering from the University of Wisconsin at Madison in 1994 and 1996. She is currently working toward a Ph.D. She joined the electrical and computer engineering department at the university as a teaching assistant in 1997. Research interests include design and simulation of oscillators and study of RF nonlinear circuits.

Devendra Misra received his B.S. and M.S. degrees in electronics engineering from B.H.U. (India), and a Ph. D. in electrical engineering from Michigan State University. Currently, he is a professor and the chair for the Electrical Engineering, College of Engineering and Applied Science, University of Wisconsin, Milwaukee. They can be contacted at: *duangrat@uwm.edu*, or *misra@uwm.edu*