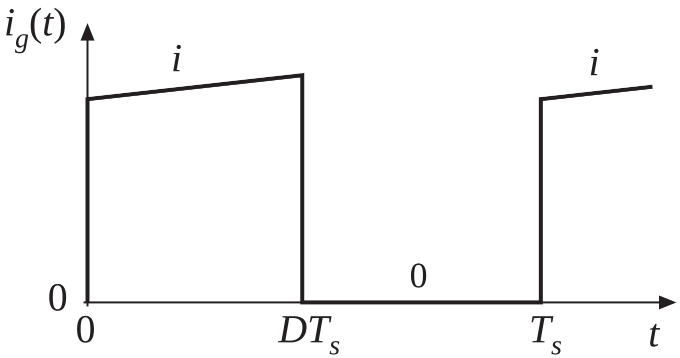
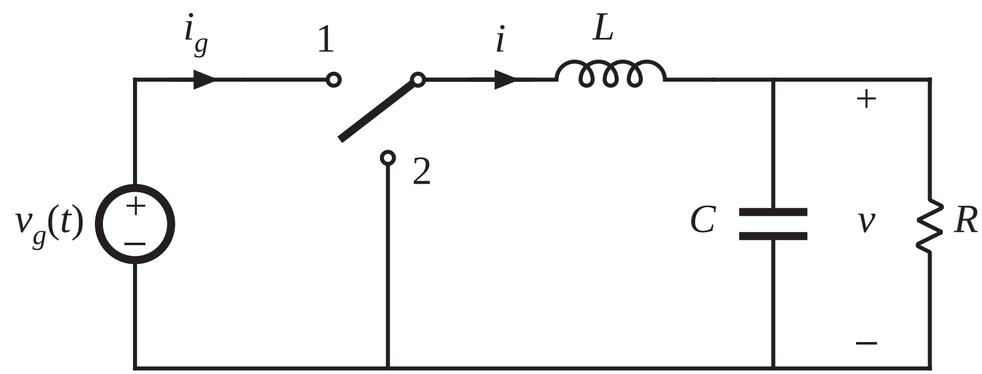
Chapter 10
Input Filter Design

10.1.1 Conducted Electromagnetic Interference (EMI)

Input current $i_g(t)$ is *pulsating*:



Buck converter example

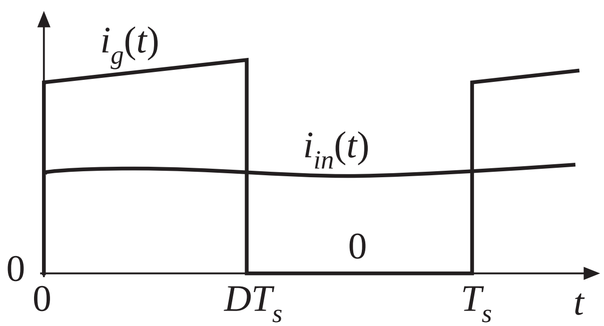


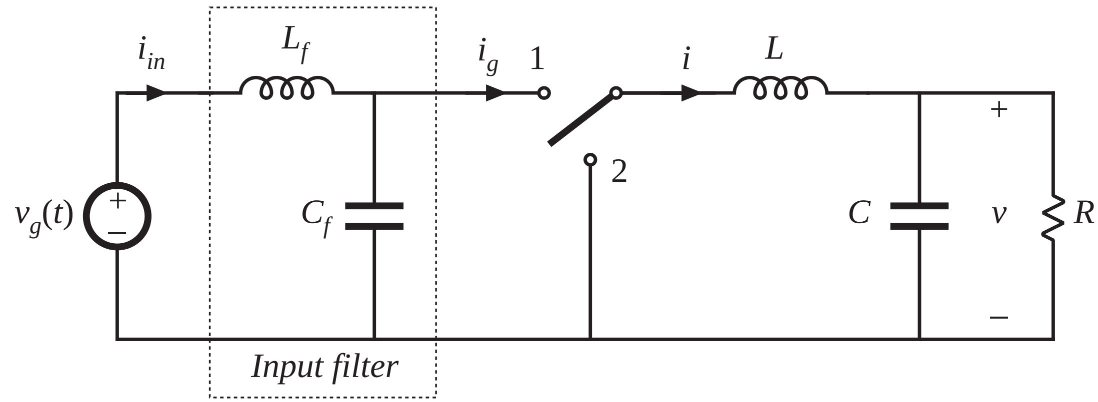
Approximate Fourier series of $i_g(t)$:

$$i_g(t) = DI + \sum_{k=1}^{\infty} \frac{2I}{k\pi} \sin\left(k\pi D\right) \cos\left(k\omega t\right)$$

High frequency current harmonics of substantial amplitude are injected back into $v_g(t)$ source. These harmonics can interfere with operation of nearby equipment. Regulations limit their amplitude, typically to values of 10 µA to 100 µA.

Addition of Low-Pass Filter





Magnitudes and phases of input current harmonics are modified by input filter transfer function $H(s)$:

$$i_{in}(t) = H(0)DI + \sum_{k=1}^{\infty} \left\| H(kj\omega) \right\| \frac{2I}{k\pi} \sin\left(k\pi D\right) \cos\left(k\omega t + \angle H(kj\omega)\right)$$

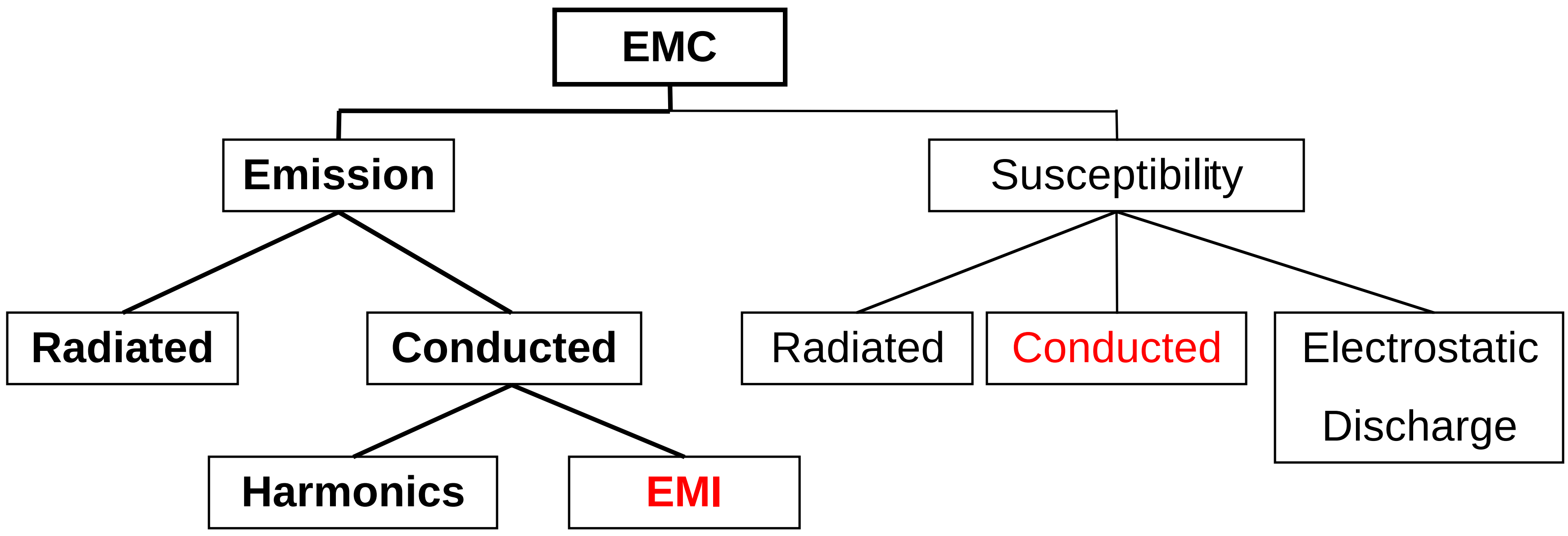
The input filter may be required to attenuate the current harmonics by factors of 80 dB or more.

Electromagnetic Compatibility

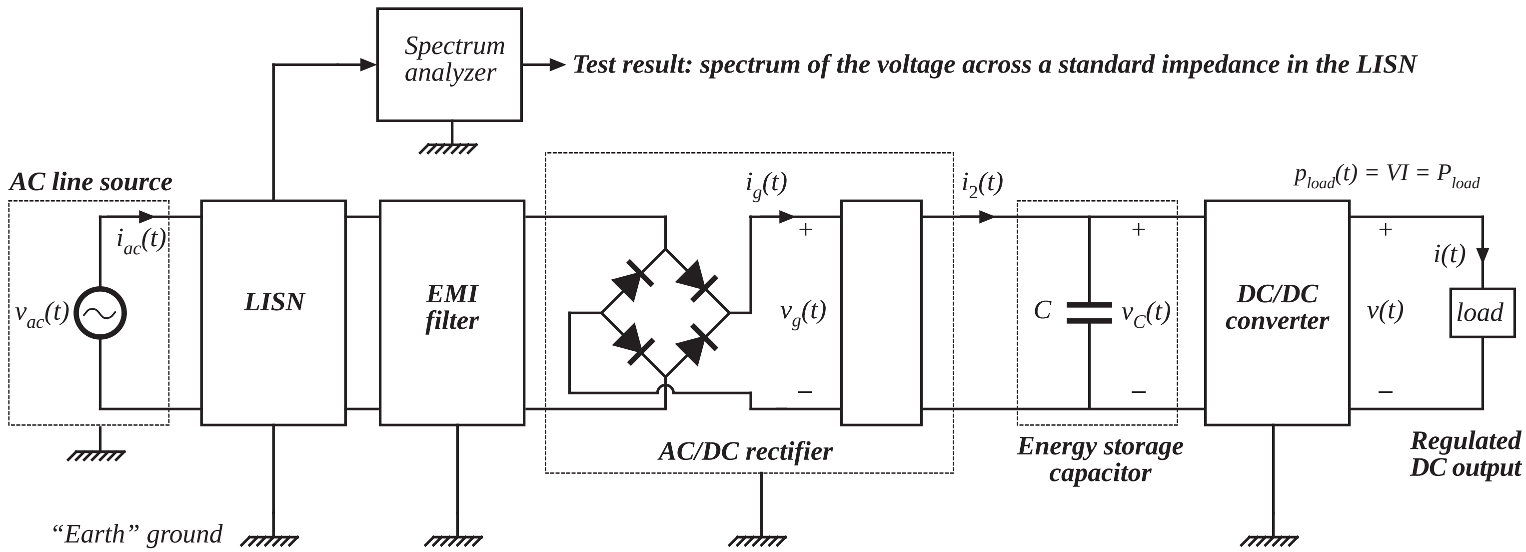
Ability of the device (e.g. power supply) to:

function satisfactorily in its electromagnetic environment (susceptibility or immunity aspect)

without introducing intolerable electromagnetic disturbances (**emission** aspect)



Conducted EMI

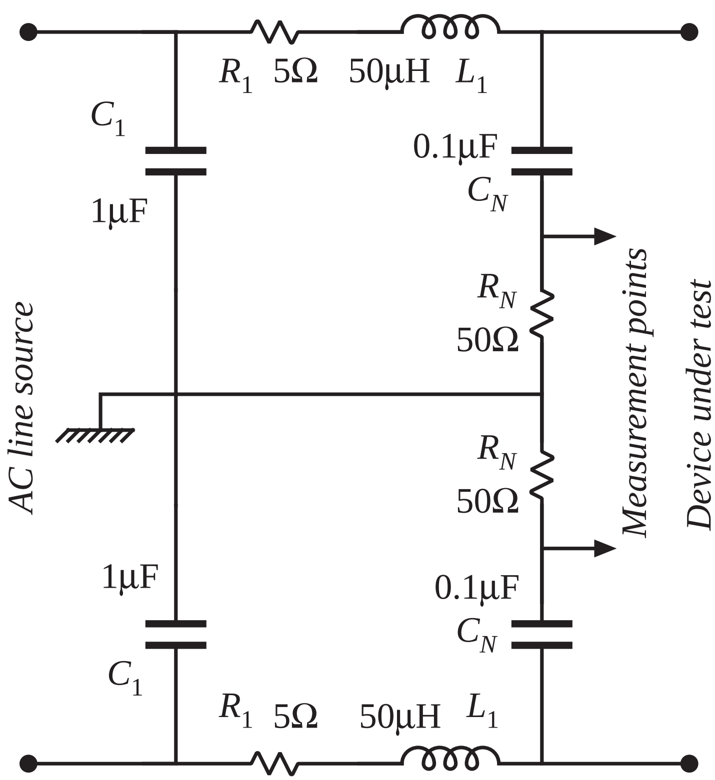


Sample of EMC regulations that include limits on radio-frequency emissions:

European Community Directive on EMC: Euro-Norm EN 55022 or 55081, earlier known as CISPR 22

National standards: VDE (German), FCC (US)

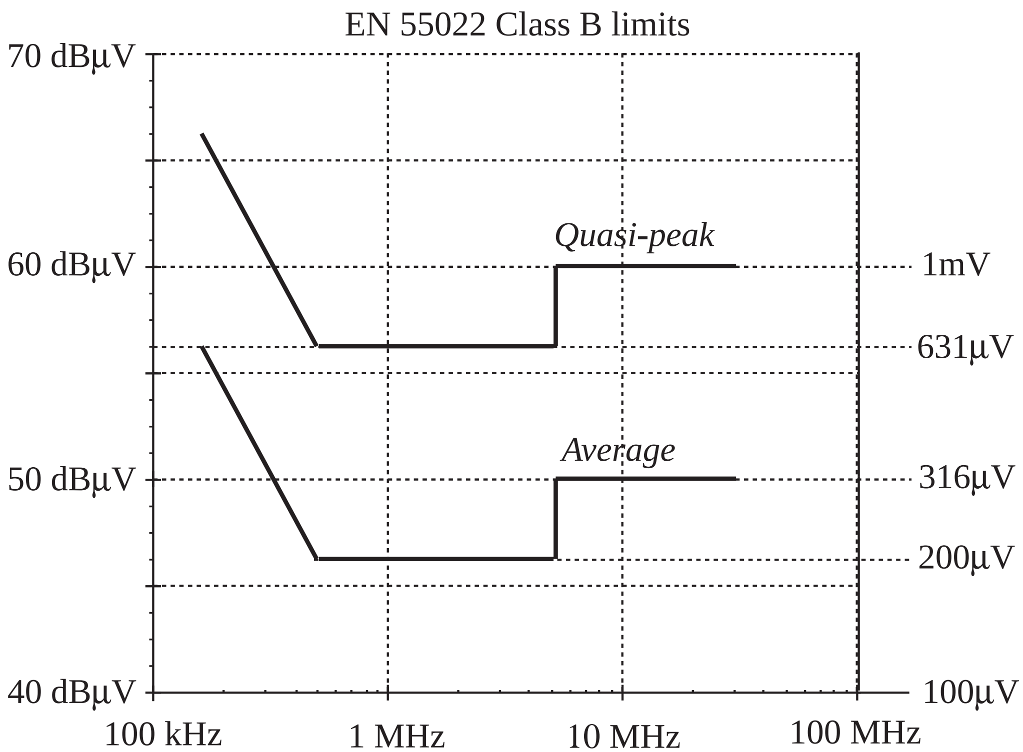
LISN



LISN example

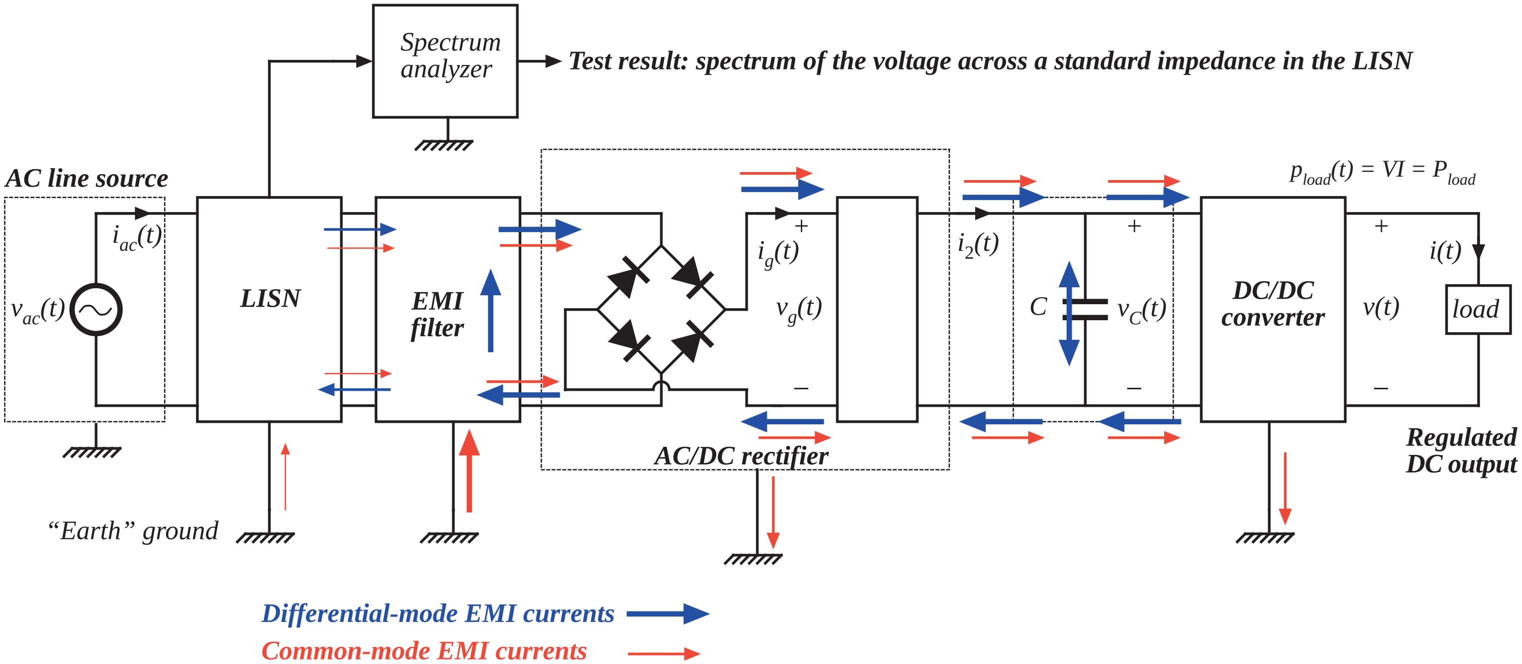
- **LISN:** “Line Impedance Stabilization Network,” or “artificial mains network”
- **Purpose:** to standardize impedance of the power source used to supply the device under test
- Spectrum of conducted emissions is measured across the standard impedance (50Ω above 150kHz)

An Example of EMI Limits



- Frequency range: 150kHz-30MHz
- Class B: residential environment
- Quasi-peak/Average: two different setups of the measurement device (such as narrow-band voltmeter or spectrum analyzer)
- Measurement bandwidth: 9kHz

Differential and Common-Mode EMI



- **Differential mode EMI**: input current waveform of the PFC. Differential-mode noise depends on the PFC realization and circuit parameters.
- **Common-mode EMI**: currents through parasitic capacitances between high *dv/dt* points and earth ground (such as from transistor drain to transistor heat sink). Common-mode noise depends on: *dv/dt*, circuit and mechanical layout.

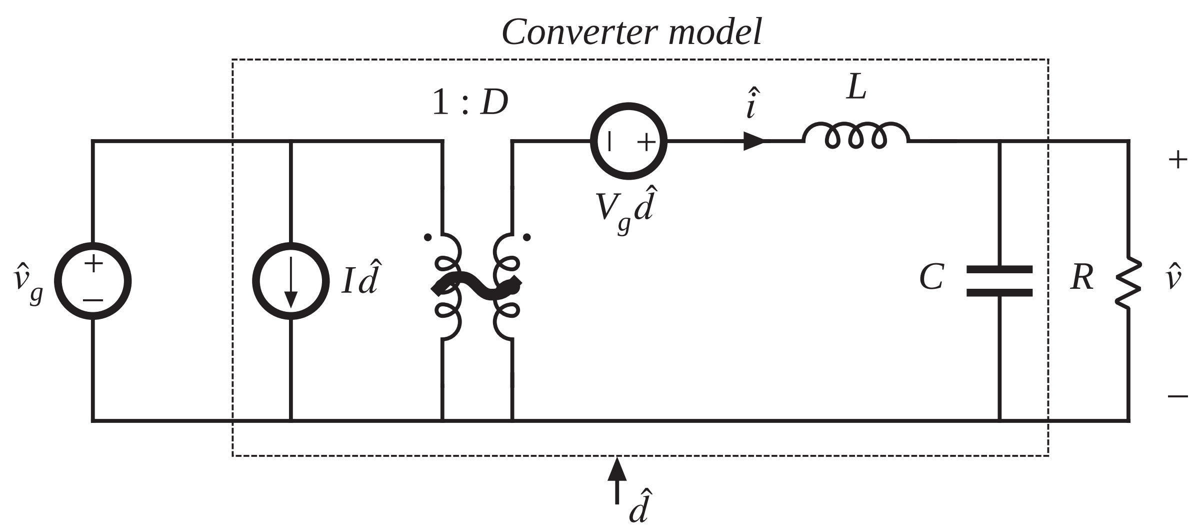
10.1.2 The Input Filter Design Problem

A typical design approach:

1. Engineer designs switching regulator that meets specifications (stability, transient response, output impedance, etc.). In performing this design, a basic converter model is employed, such as the one below:

(no input filter)

(buck converter example)

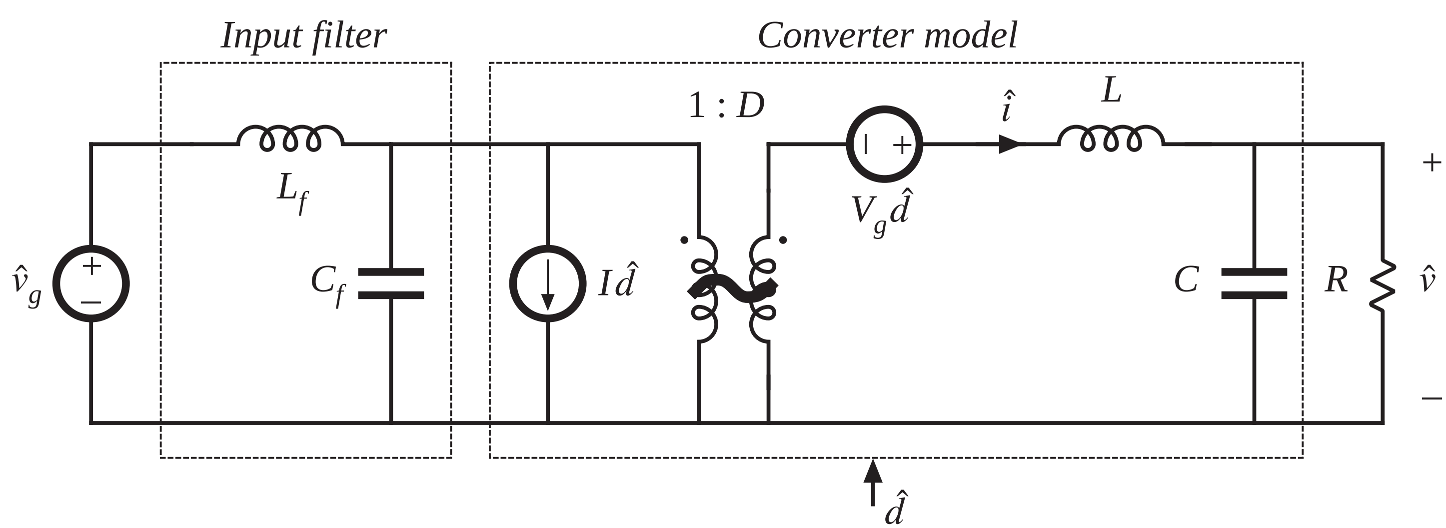


Input Filter Design Problem, p. 2

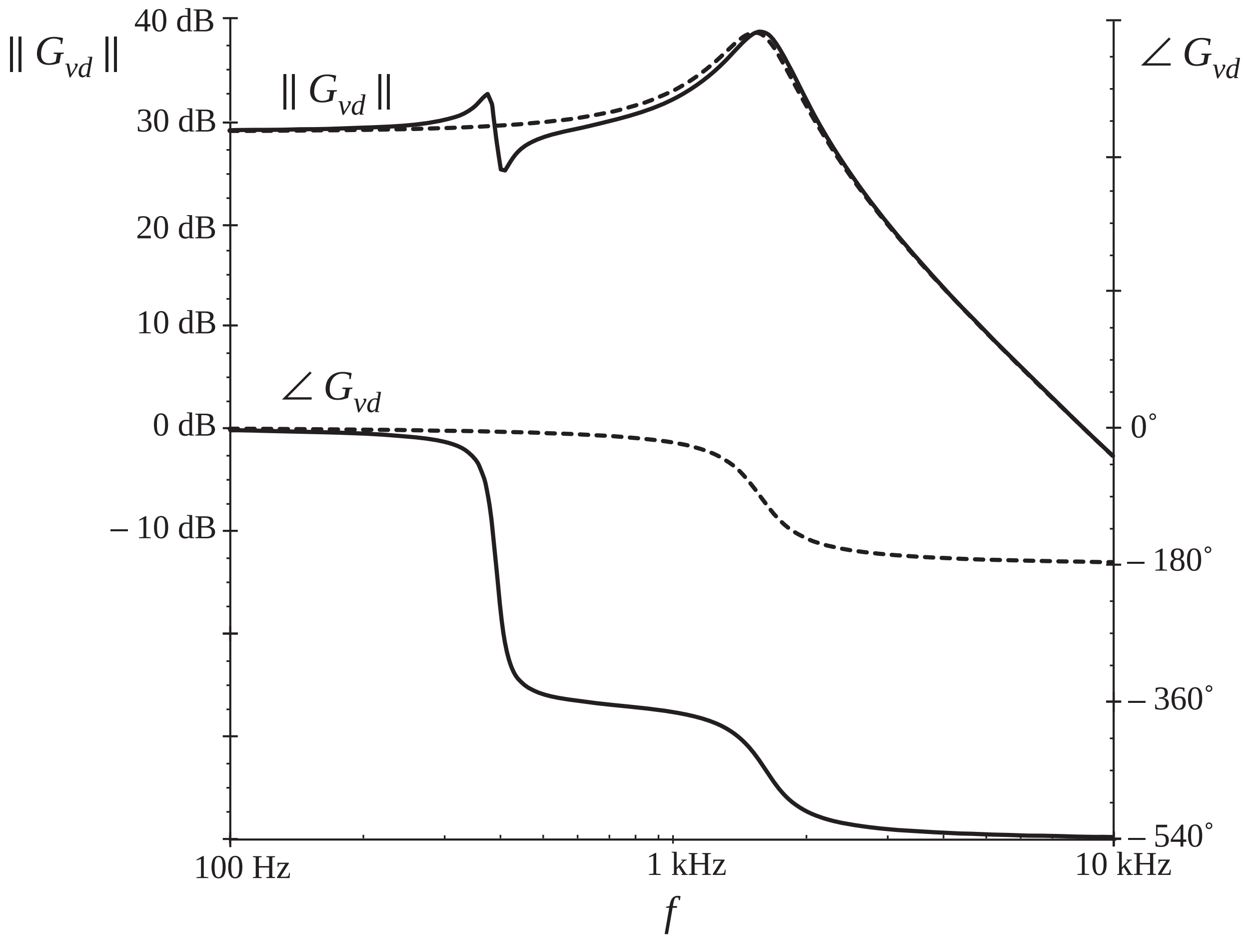
2. Later, the problem of conducted EMI is addressed. An input filter is added, that attenuates harmonics sufficiently to meet regulations.
3. A new problem arises: the controller no longer meets dynamic response specifications. The controller may even become unstable.

Reason: input filter changes converter transfer functions

Converter ac model is modified by input filter



Input Filter Design Problem, p. 3

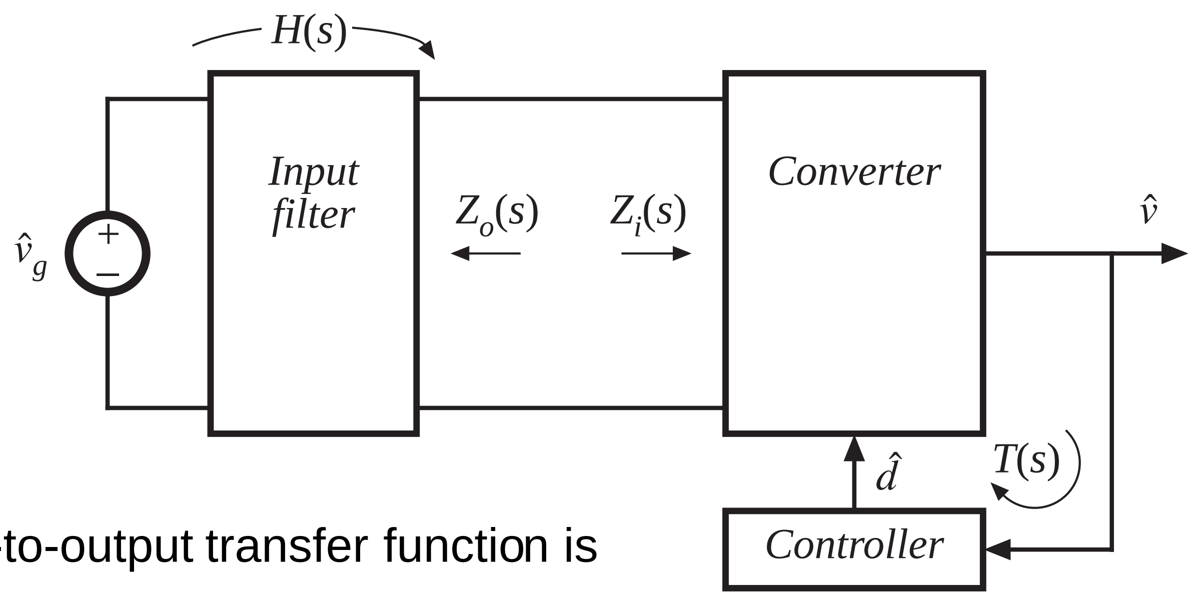


Effect of L-C input filter on control-to-output transfer function $G_{vd}(s)$, buck converter example.

Dashed lines: original magnitude and phase

Solid lines: with addition of input filter

10.2 Effect of an Input Filter on Converter Transfer Functions



Control-to-output transfer function is

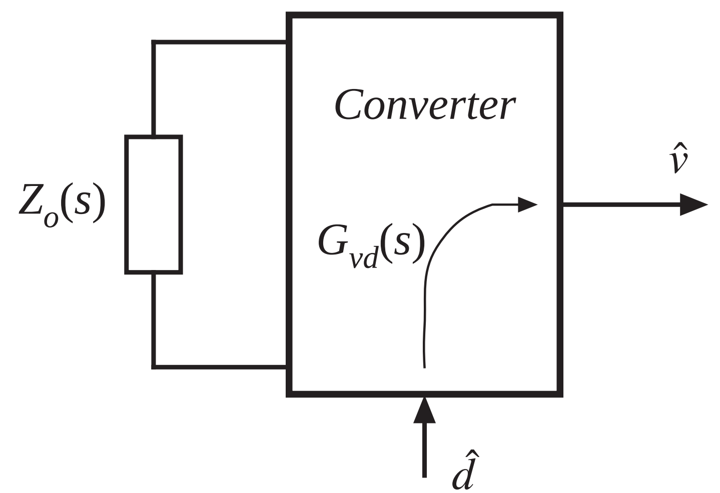
$$G_{vd}(s) = \left. \frac{\hat{v}(s)}{\hat{d}(s)} \right|_{\hat{v}_g(s) = 0}$$

Determination of $G_{vd}(s)$

$$G_{vd}(s) = \left. \frac{\hat{v}(s)}{\hat{d}(s)} \right|_{\hat{v}_g(s) = 0}$$

$\hat{v}_g(s)$ source → short circuit

$Z_o(s)$ = output impedance of input filter



We will use Middlebrook's Extra Element Theorem to show that the input filter modifies $G_{vd}(s)$ as follows:

$$G_{vd}(s) = \left(\left. G_{vd}(s) \right|_{Z_o(s) = 0} \right) \frac{\left(1 + \dfrac{Z_o(s)}{Z_N(s)} \right)}{\left(1 + \dfrac{Z_o(s)}{Z_D(s)} \right)}$$

How an input filter changes $G_{vd}(s)$
Summary of result

$$G_{vd}(s) = \left(G_{vd}(s) \Big|_{Z_o(s)=0} \right) \frac{\left(1 + \frac{Z_o(s)}{Z_N(s)}\right)}{\left(1 + \frac{Z_o(s)}{Z_D(s)}\right)}$$

$G_{vd}(s) \Big|_{Z_o(s)=0}$ is the original transfer function, before addition of input filter

$Z_D(s) = Z_i(s) \Big|_{\hat{d}(s)=0}$ is the converter input impedance, with $\hat{d}$ set to zero

$Z_N(s) = Z_i(s) \Big|_{\hat{v}(s) \xrightarrow[\text{null}]{} 0}$ is the converter input impedance, with the output $\hat{v}$ nulled to zero

(see Appendix C for proof using EET)

Design criteria for basic converters

Table 10.1 Input filter design criteria for basic converters

Converter	$Z_N(s)$	$Z_D(s)$	$Z_e(s)$
Buck	$-\frac{R}{D^2}$	$\frac{R}{D^2}\frac{\left(1 + s\frac{L}{R} + s^2LC\right)}{\left(1 + sRC\right)}$	$\frac{sL}{D^2}$
Boost	$-D'^2R\left(1 - \frac{sL}{D'^2R}\right)$	$D'^2R\frac{\left(1 + s\frac{L}{D'^2R} + s^2\frac{LC}{D'^2}\right)}{\left(1 + sRC\right)}$	sL
Buck–boost	$-\frac{D'^2R}{D^2}\left(1 - \frac{sDL}{D'^2R}\right)$	$\frac{D'^2R}{D^2}\frac{\left(1 + s\frac{L}{D'^2R} + s^2\frac{LC}{D'^2}\right)}{\left(1 + sRC\right)}$	$\frac{sL}{D^2}$

10.2.2 Impedance Inequalities

$$G_{vd}(s) = \left(G_{vd}(s) \Big|_{Z_o(s)=0} \right) \frac{\left(1 + \frac{Z_o(s)}{Z_N(s)}\right)}{\left(1 + \frac{Z_o(s)}{Z_D(s)}\right)}$$

The *correction factor* $\frac{\left(1 + \frac{Z_o(s)}{Z_N(s)}\right)}{\left(1 + \frac{Z_o(s)}{Z_D(s)}\right)}$ shows how the input filter modifies the transfer function $G_{vd}(s)$.

The correction factor has a magnitude of approximately unity provided that the following inequalities are satisfied:

$$\| Z_o \| \ll \| Z_N \|, \quad \text{and}$$
$$\| Z_o \| \ll \| Z_D \|$$

These provide design criteria, which are relatively easy to apply.

Effect of input filter on converter output impedance

A similar analysis leads to the following inequalities, which guarantee that the converter output impedance is not substantially affected by the input filter:

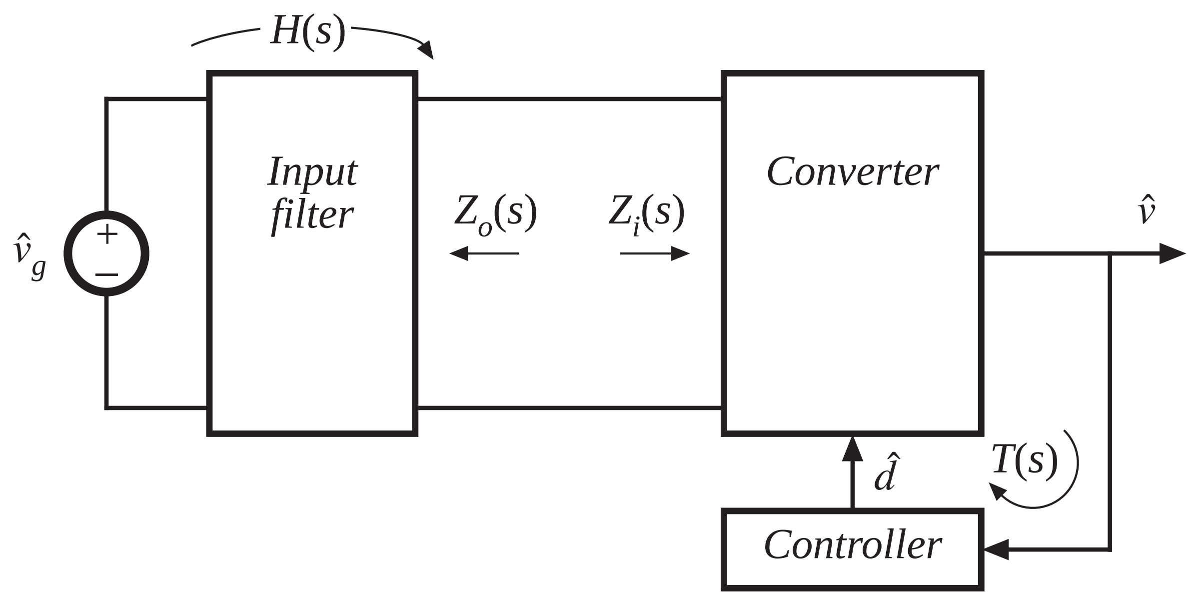
$$\| Z_o \| \ll \| Z_e \|, \quad \text{and}$$
$$\| Z_o \| \ll \| Z_D \|$$

The quantity Z_e is given by:

$$Z_e = Z_i \Big|_{\hat{v} = 0}$$

(converter input impedance when the output is shorted)

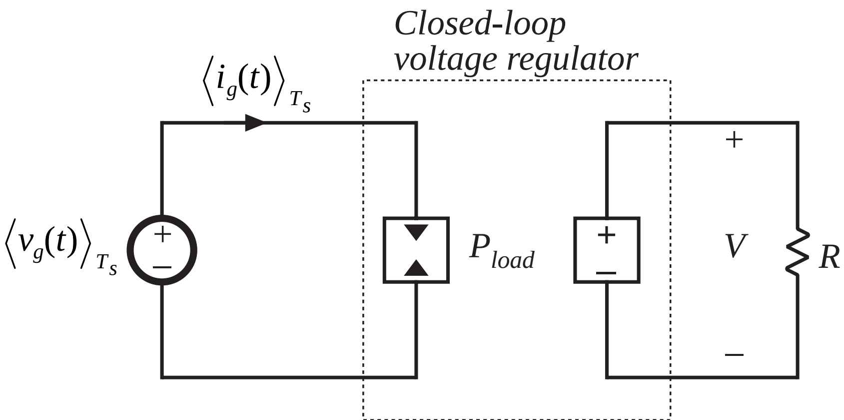
10.2.1 Discussion

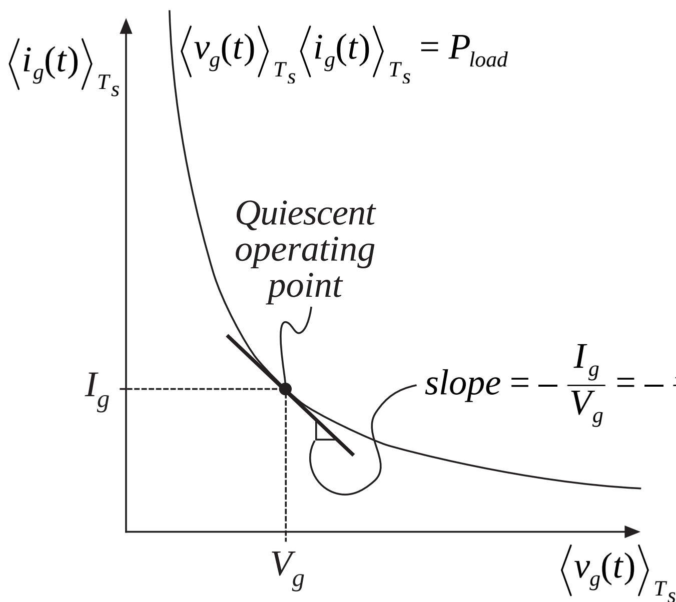


$$Z_N(s) = Z_i(s)\Big|_{\hat{v}(s) \xrightarrow[\text{null}]{} 0}$$ is the converter input impedance, with the output $\hat{v}$ nulled to zero

Note that this is the same as the function performed by an ideal controller, which varies the duty cycle as necessary to maintain zero error of the output voltage. So Z_N coincides with the input impedance when an ideal feedback loop perfectly regulates the output voltage.

When the output voltage is perfectly regulated





- For a given load characteristic, the output power P_{load} is independent of the converter input voltage
- If losses are negligible, then the input port i-v characteristic is a power sink characteristic, equal to P_{load}:

$$\langle v_g(t) \rangle_{T_s} \langle i_g(t) \rangle_{T_s} = P_{load}$$

- Incremental input resistance is negative, and is equal to:

$$-\frac{R}{M^2}$$

(same as dc asymptote of Z_N)

Negative resistance oscillator

It can be shown that the closed-loop converter input impedance is given by:

$$\frac{1}{Z_i(s)} = \frac{1}{Z_N(s)} \frac{T(s)}{1 + T(s)} + \frac{1}{Z_D(s)} \frac{1}{1 + T(s)}$$

where $T(s)$ is the converter loop gain.

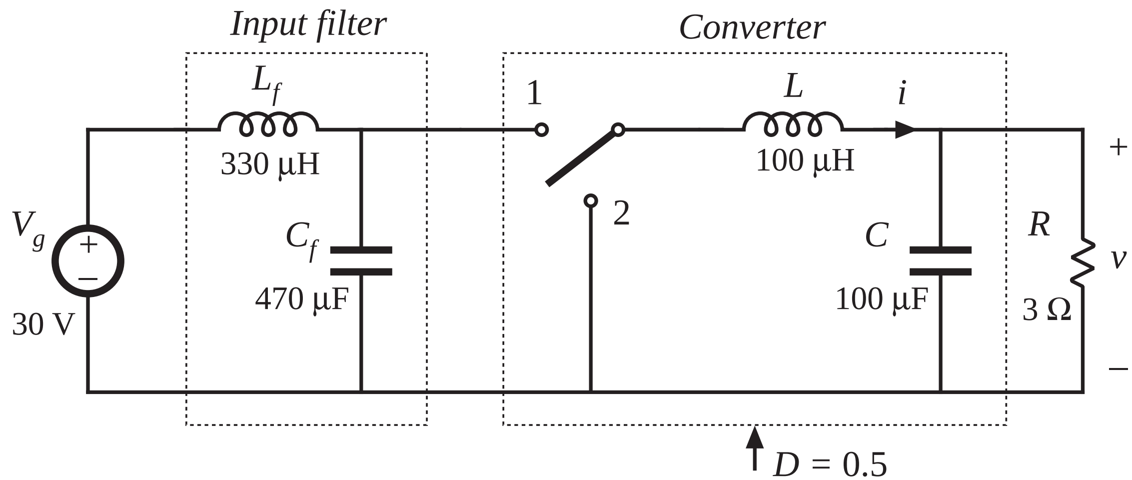
At frequencies below the loop crossover frequency, the input impedance is approximately equal to Z_N, which is a negative resistance.

When an undamped or lightly damped input filter is connected to the regulator input port, the input filter can interact with Z_N to form a *negative resistance oscillator*.

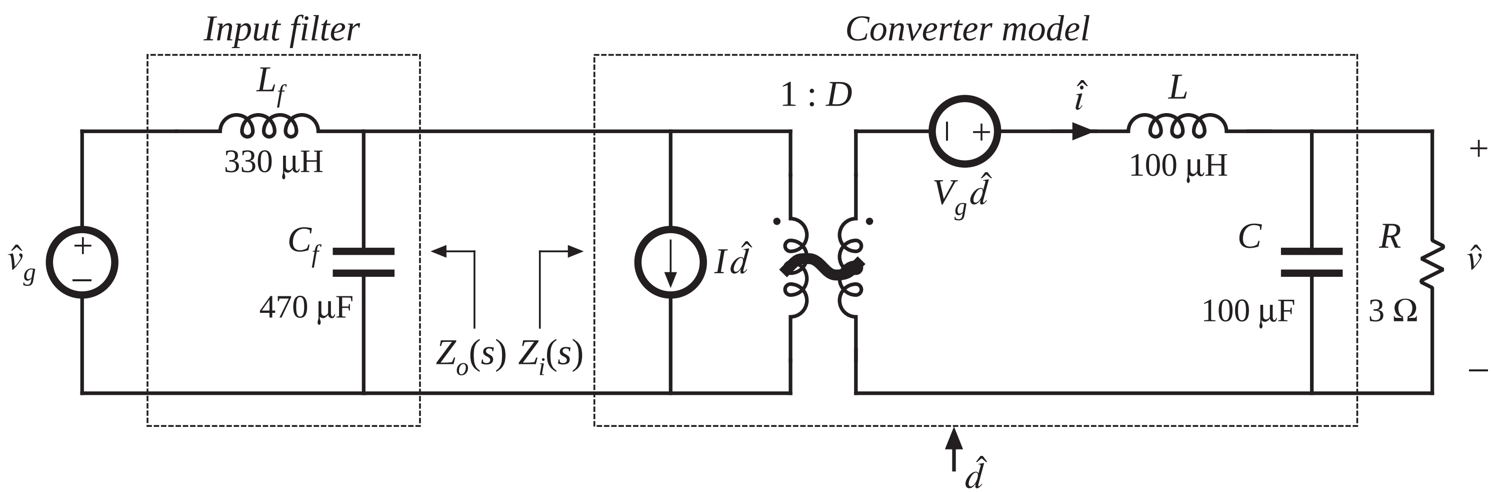
10.3 Buck Converter Example

10.3.1 Effect of undamped input filter

Buck converter with input filter

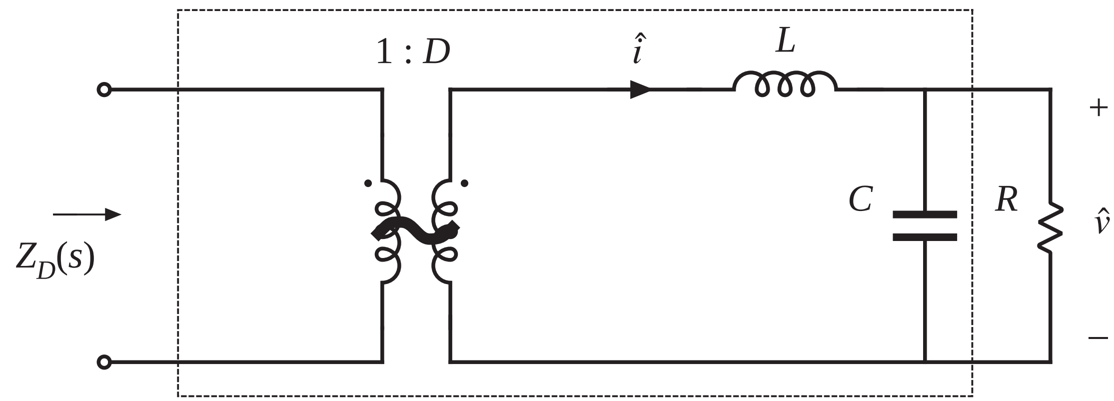


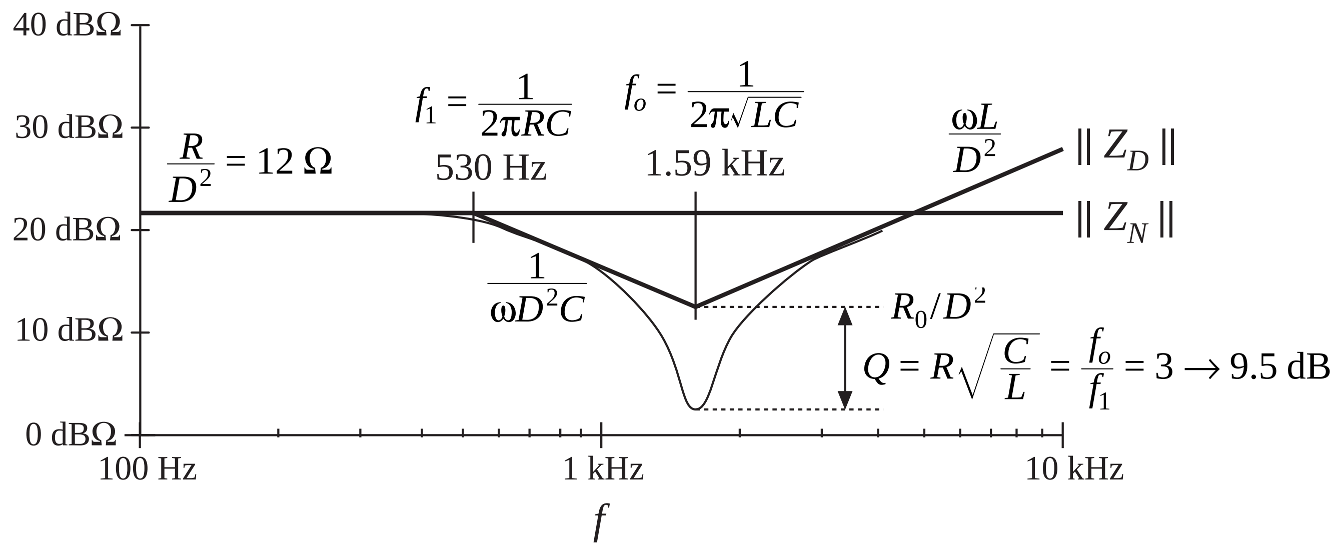
Small-signal model



Determination of Z_D

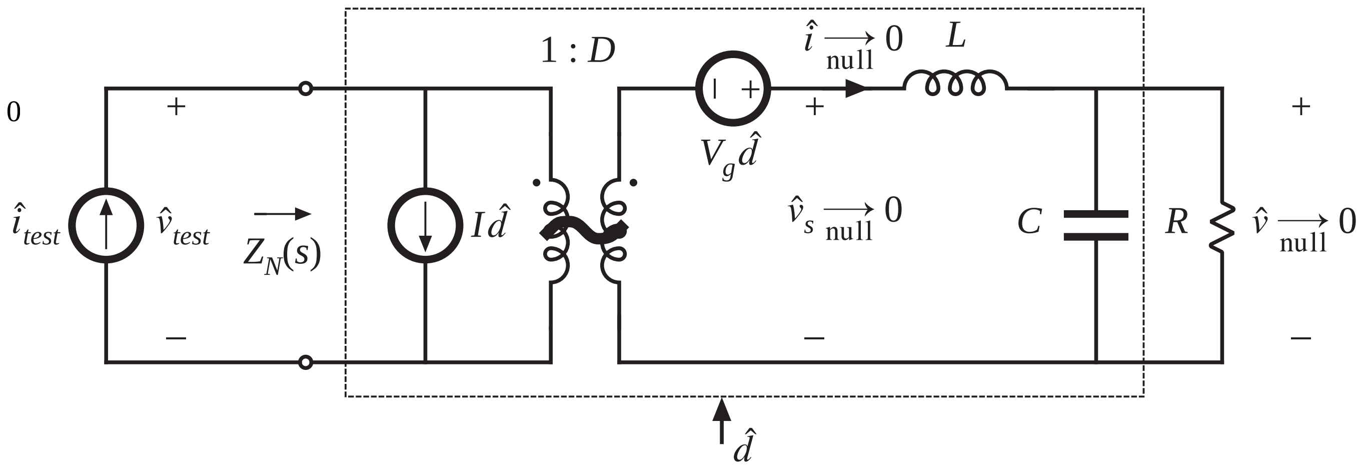
$$Z_D(s) = \frac{1}{D^2}\left(sL + R \,||\, \frac{1}{sC}\right)$$





Determination of Z_N

$$Z_N(s) = \left. \frac{\hat{v}_{test}(s)}{\hat{i}_{test}(s)} \right|_{\hat{v} \xrightarrow[\text{null}]{} 0}$$



Solution:

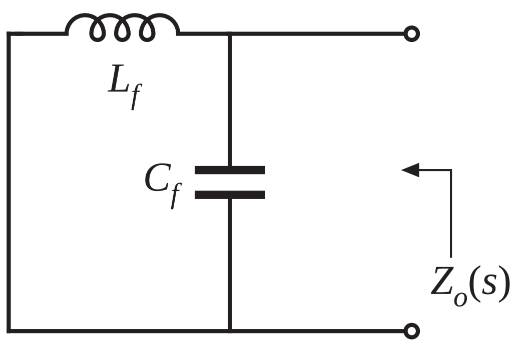
$$\hat{i}_{test}(s) = I\hat{d}(s)$$

$$\hat{v}_{test}(s) = -\frac{V_g \hat{d}(s)}{D}$$

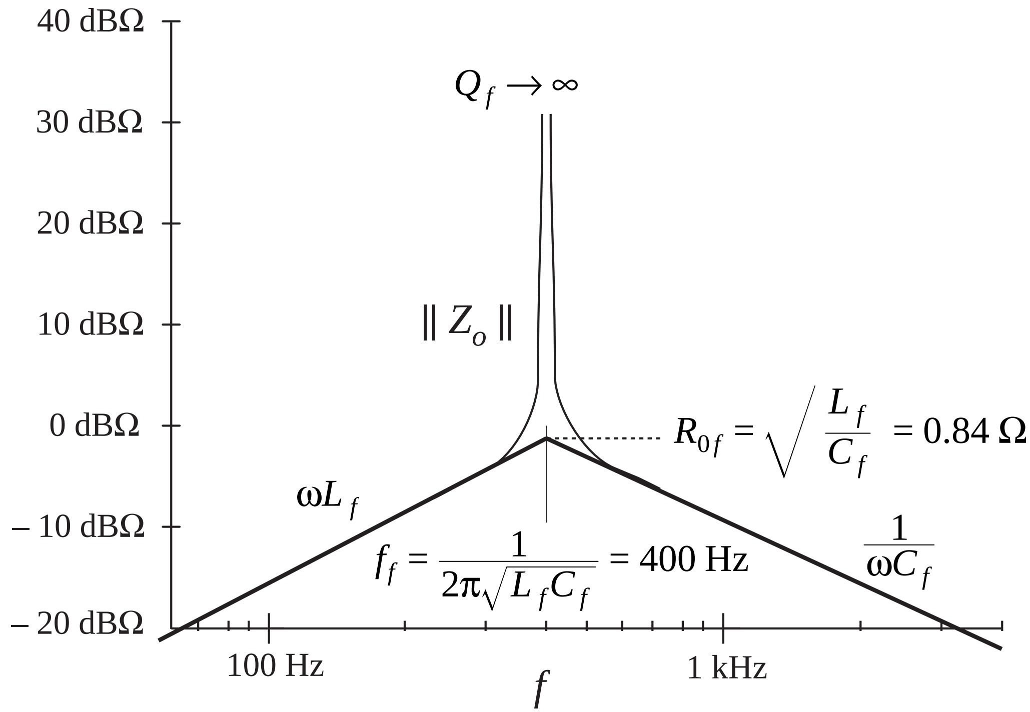
Hence,

$$Z_N(s) = \frac{\left(-\frac{V_g \hat{d}(s)}{D}\right)}{\left(I\hat{d}(s)\right)} = -\frac{R}{D^2}$$

Z_o of undamped input filter



$$Z_o(s) = sL_f \,||\, \frac{1}{sC_f}$$

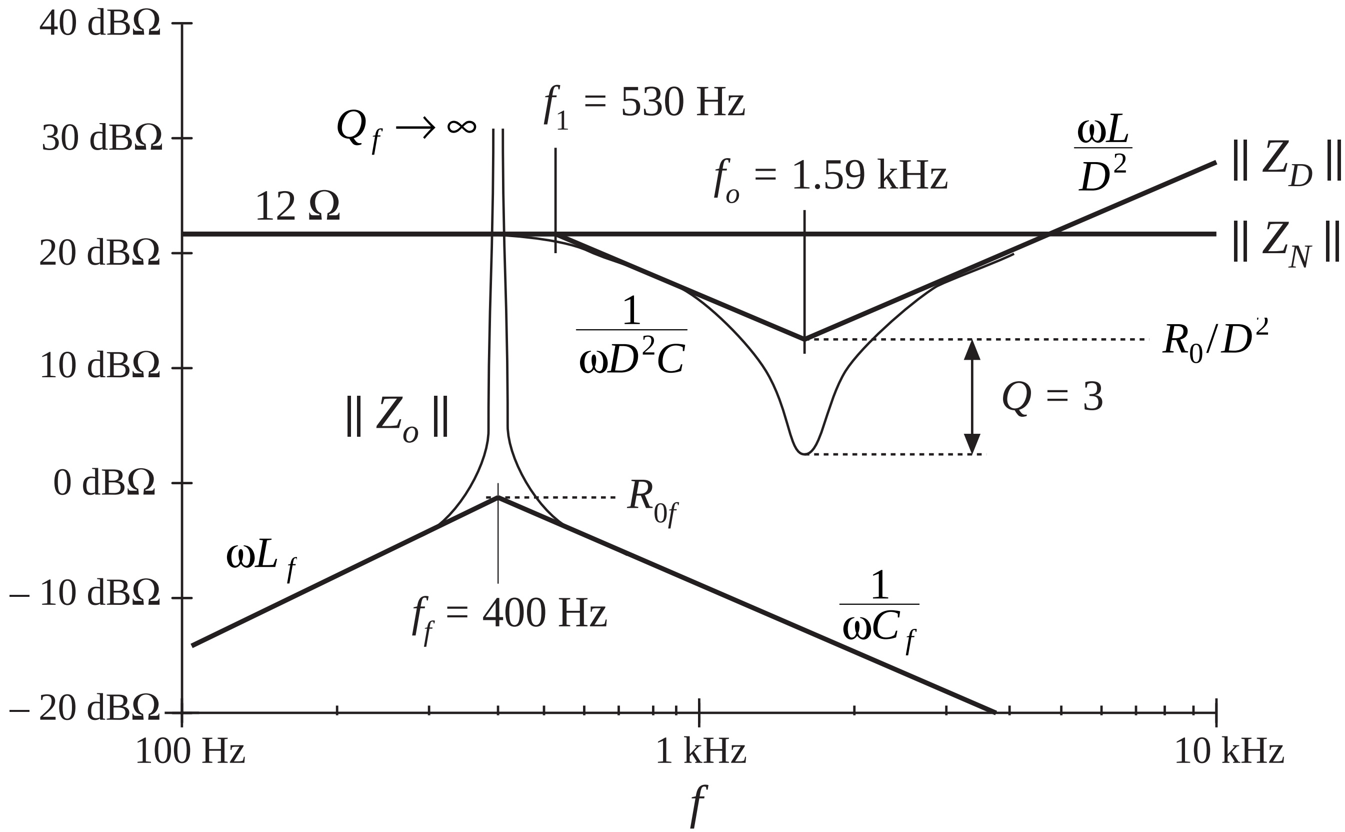


No resistance, hence poles are undamped (infinite Q-factor).

In practice, losses limit Q-factor; nonetheless, Q_f may be very large.

Design criteria

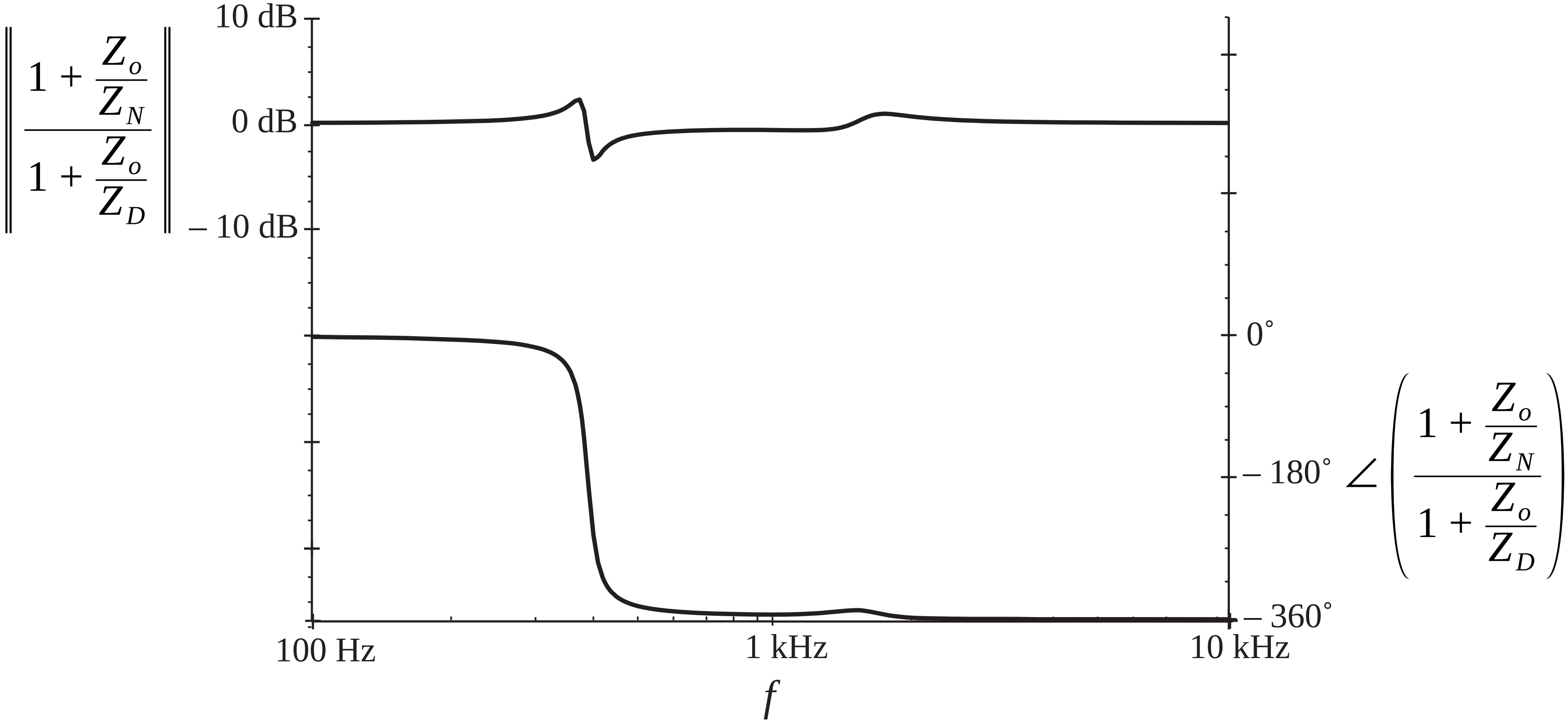
$$\| Z_o \| \ll \| Z_N \|, \quad \text{and}$$
$$\| Z_o \| \ll \| Z_D \|$$



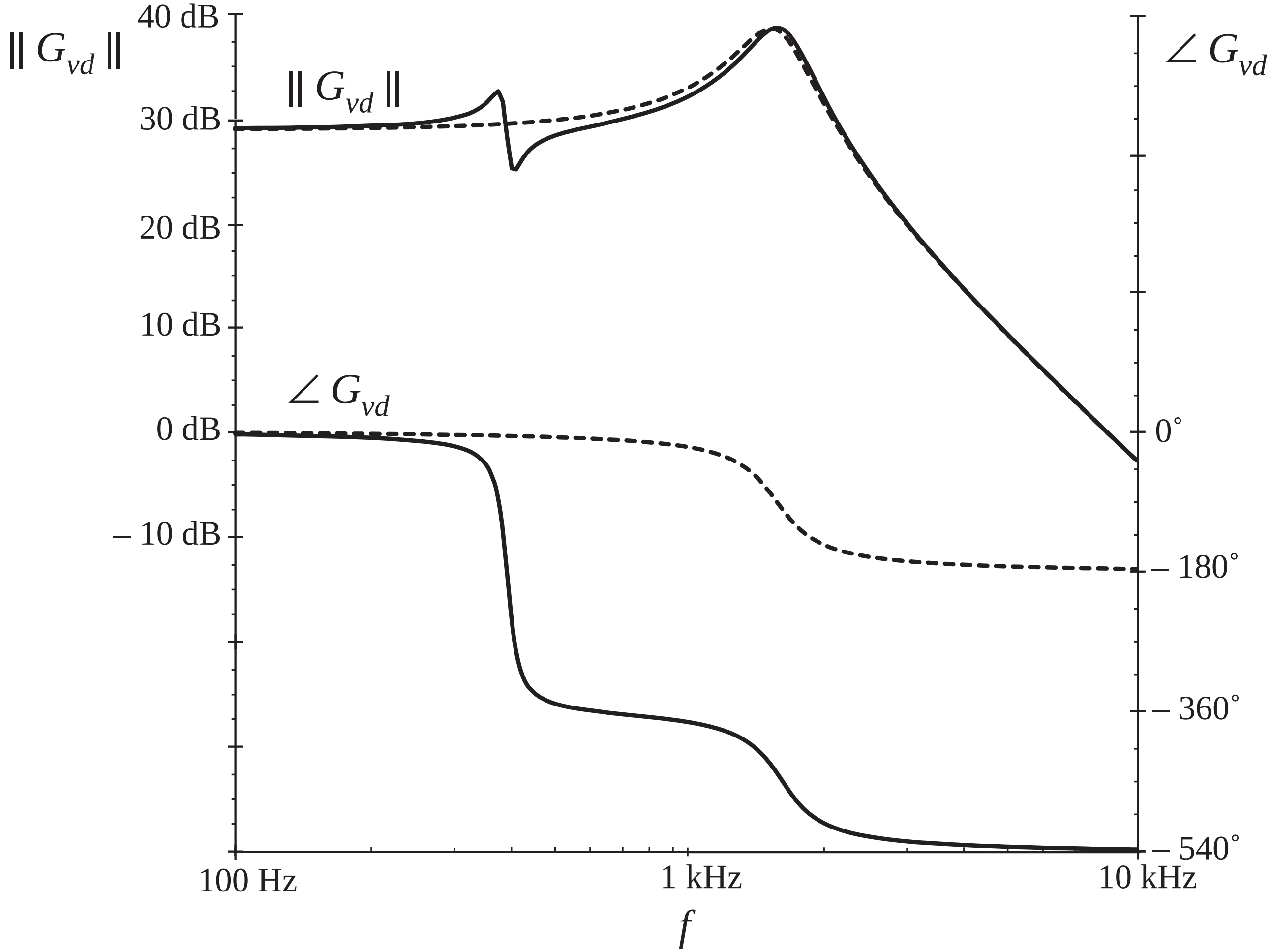
Can meet inequalities everywhere except at resonant frequency f_f.

Need to damp input filter!

Resulting correction factor



Resulting transfer function

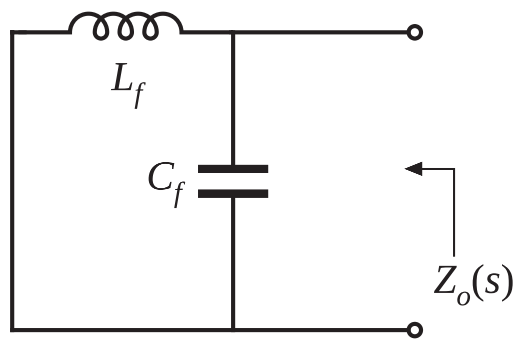


Dashed lines: no input filter

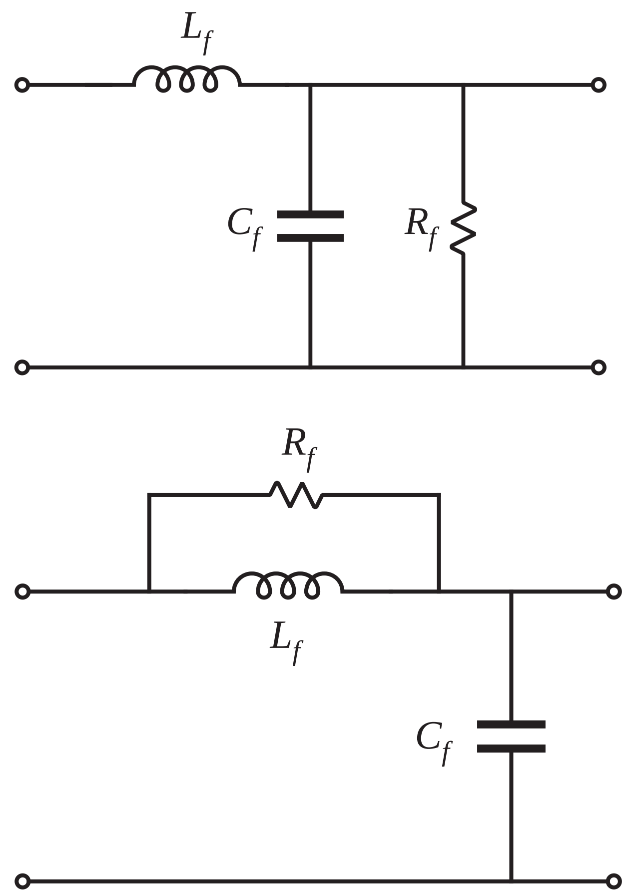
Solid lines: including effect of input filter

10.3.2 Damping the input filter

Undamped filter:



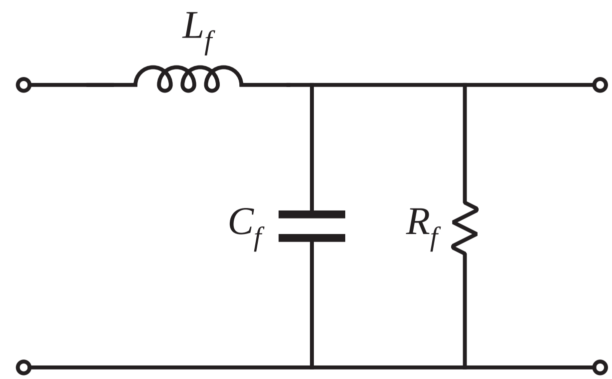
Two possible approaches:



Addition of R_f across C_f

To meet the requirement $R_f \ll \| Z_N \|$:

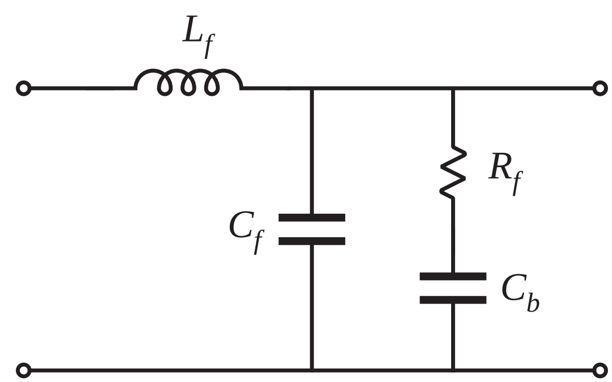
$$R_f \ll \frac{R}{D^2}$$



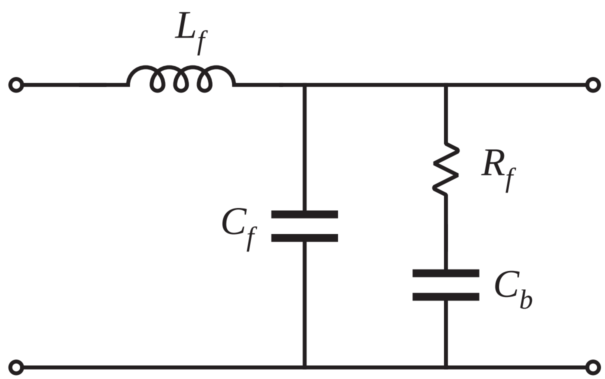
The power loss in R_f is V_g^2/ R_f, which is larger than the load power!

A solution: add dc blocking capacitor C_b.

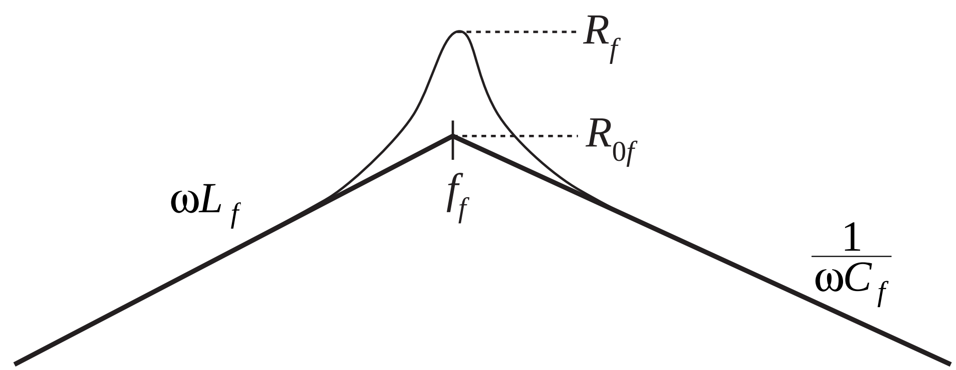
Choose C_b so that its impedance is sufficiently smaller than R_f at the filter resonant frequency.



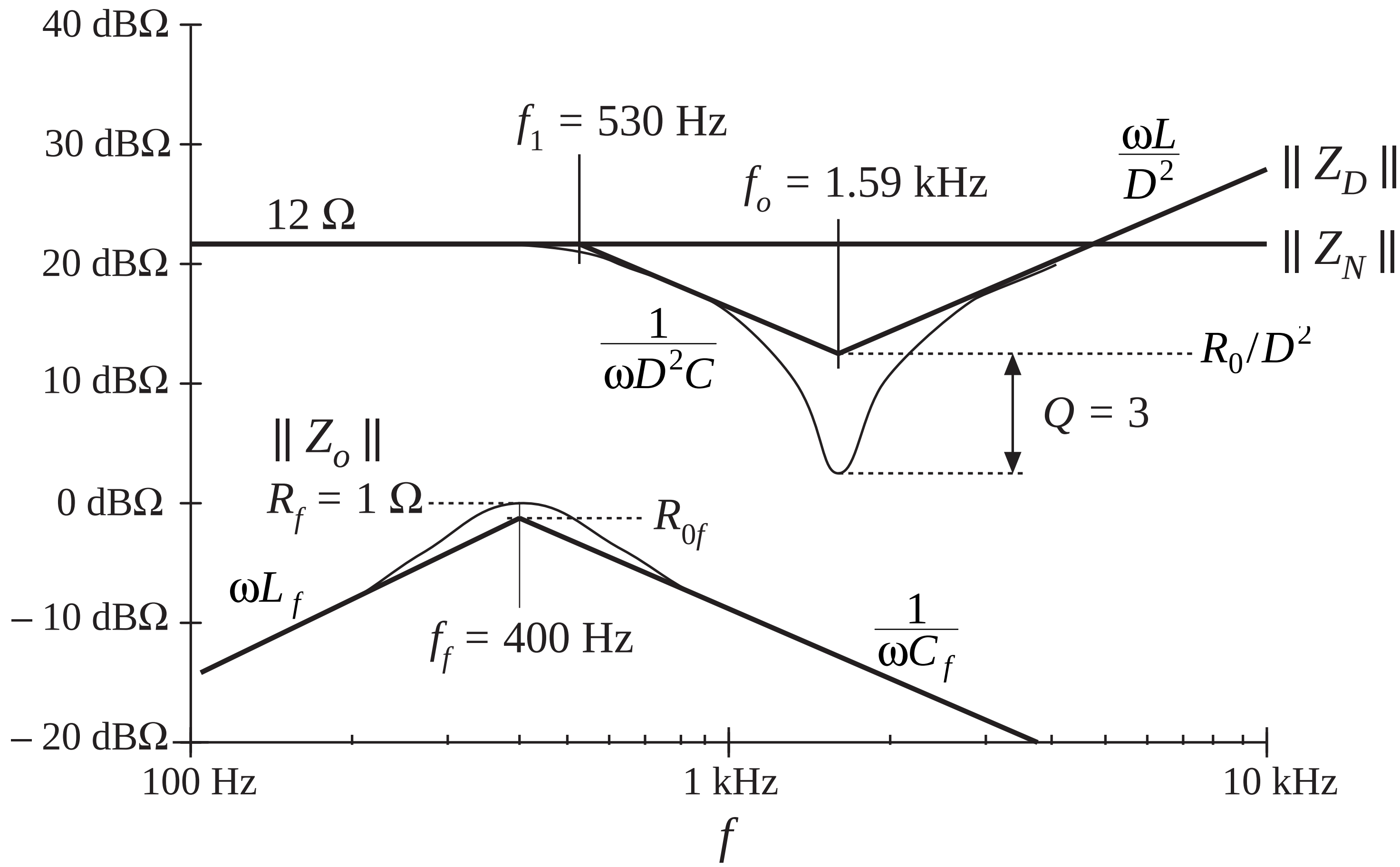
Damped input filter



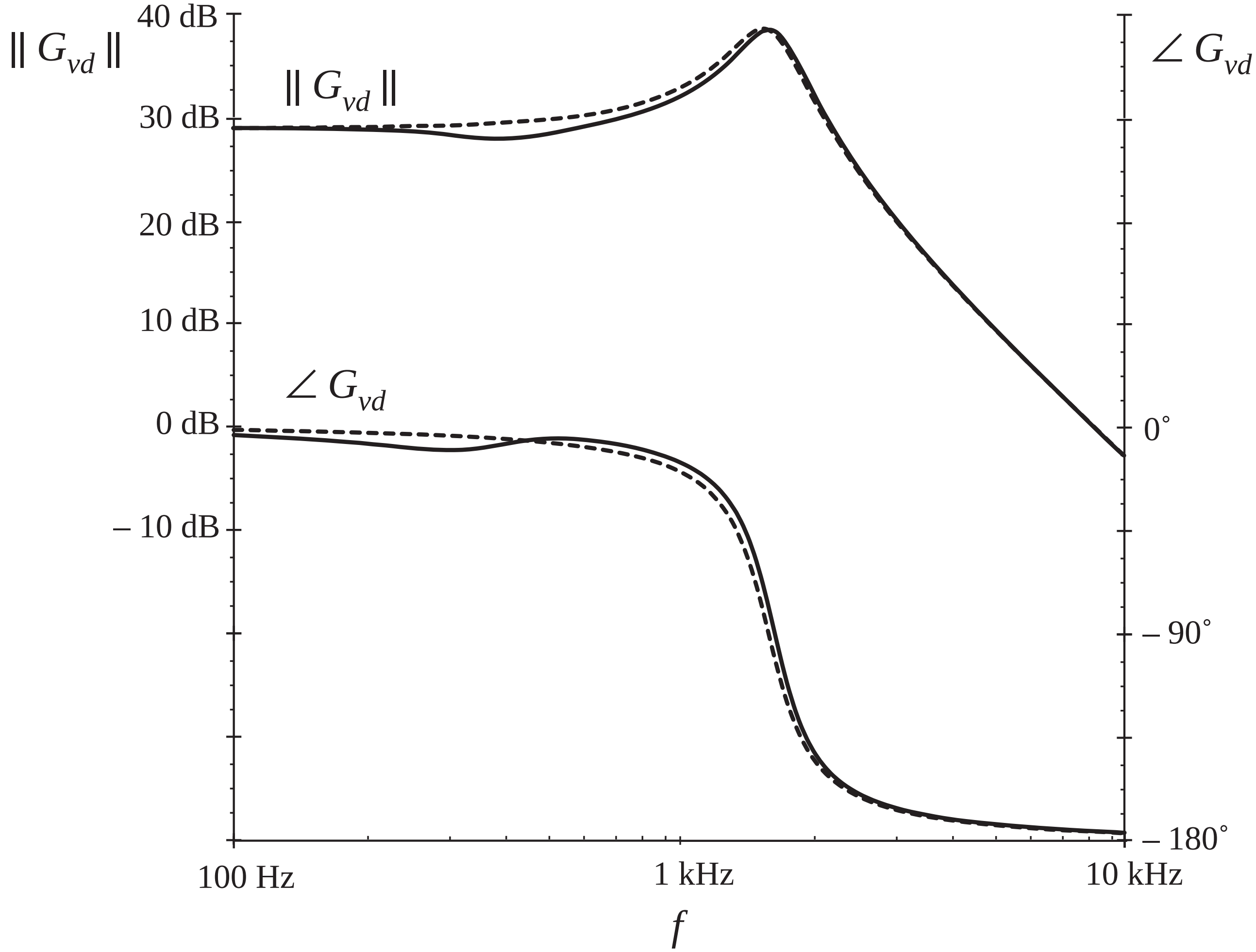
$\| Z_o \|$, with large C_b



Design criteria, with damped input filter



Resulting transfer function

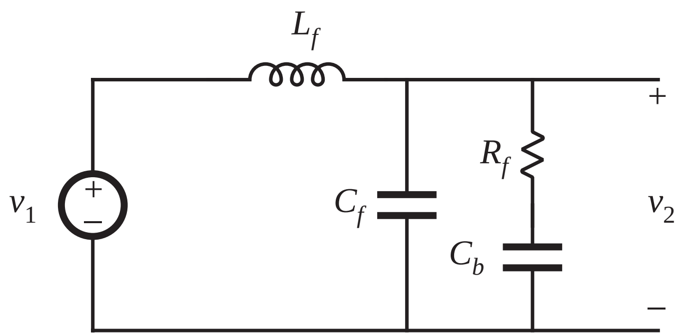


Dashed lines: no input filter

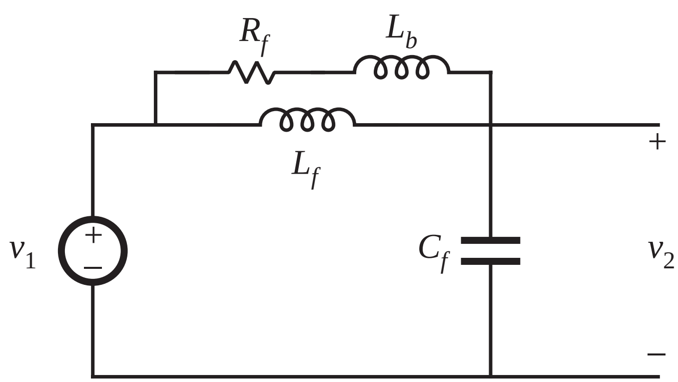
Solid lines: including effect of input filter

10.4 Design of a Damped Input Filter

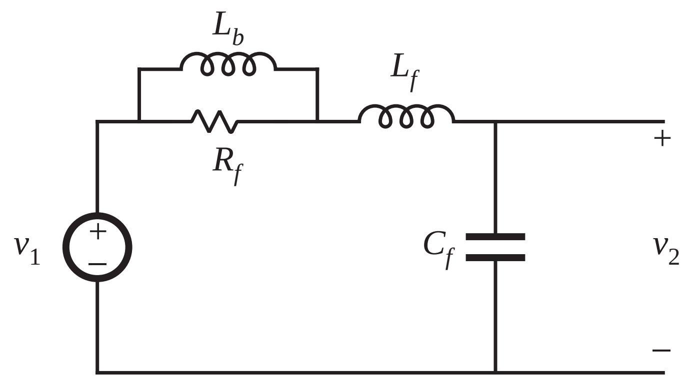
R_f–C_b Parallel Damping



R_f–L_b Parallel Damping

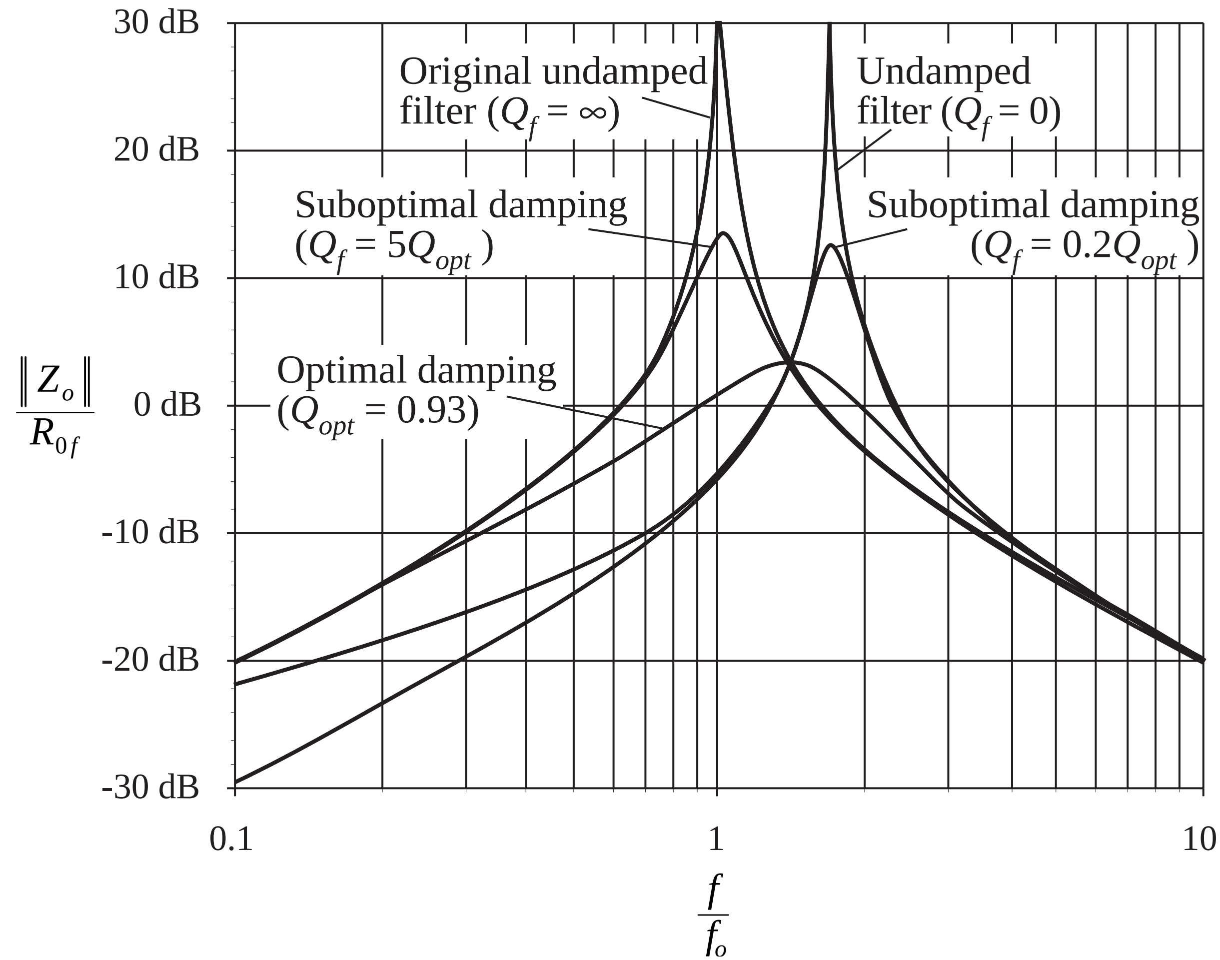


R_f–L_b Series Damping



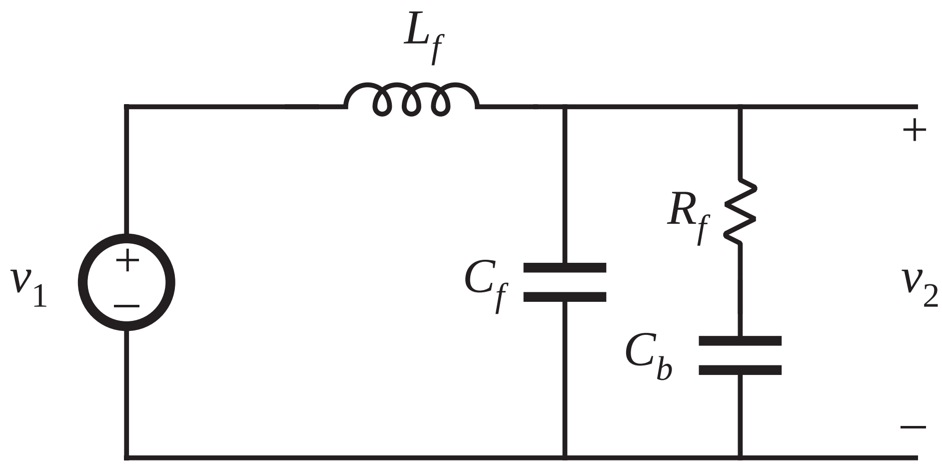
- Size of C_b or L_b can become very large
- Need to optimize design

Dependence of $\| Z_o \|$ on R_f
R_f–L_b Parallel Damping



For this example,
$n = L_b/L = 0.516$

10.4.1 R_f–C_b Parallel Damping



- Filter is damped by R_f
- C_b blocks dc current from flowing through R_f
- C_b can be large in value, and is an element to be optimized

Optimal design equations

R_f–C_b Parallel Damping

Define $n = \dfrac{C_b}{C_f}$

The value of the peak output impedance for the optimum design is

$$\| Z_o \|_{mm} = R_{0f} \frac{\sqrt{2(2+n)}}{n}$$

where R_{0f} = characteristic impedance of original undamped input filter

Given a desired value of the peak output impedance, can solve above equation for n. The required value of damping resistance R_f can then be found from:

$$Q_{opt} = \frac{R_f}{R_{0f}} = \sqrt{\frac{(2+n)(4+3n)}{2n^2(4+n)}}$$

The peak occurs at the frequency

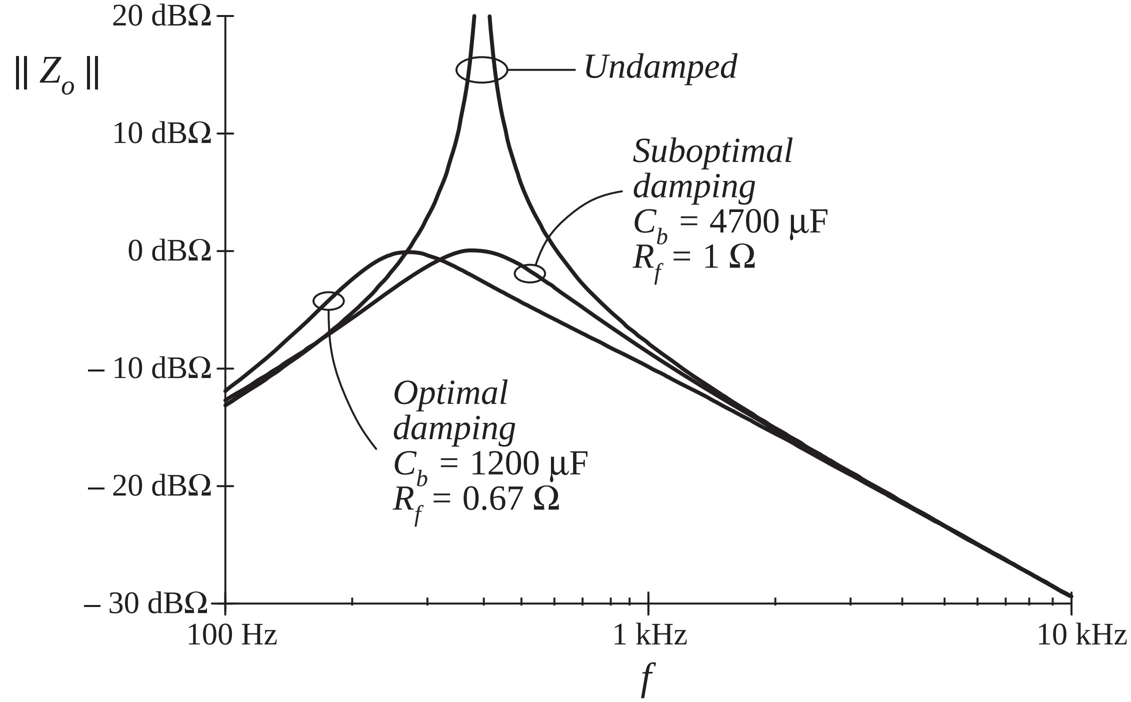
$$f_m = f_f \sqrt{\frac{2}{2+n}}$$

Example
Buck converter of Section 10.3.2

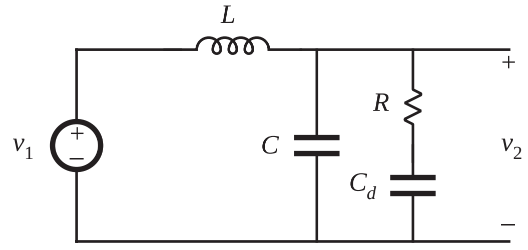
$$n = \frac{R_{0f}^2}{\| Z_o \|_{\mathrm{mm}}^2} \left(1 + \sqrt{1 + 4 \frac{\| Z_o \|_{\mathrm{mm}}^2}{R_{0f}^2}} \right) = 2.5 \qquad R_f = R_{0f} \sqrt{\frac{(2+n)(4+3n)}{2n^2(4+n)}} = 0.67\ \Omega$$

Comparison of designs

Optimal damping achieves same peak output impedance, with much smaller C_b.



Summary
Optimal R-C_d damping



Basic results

$$Q_{opt} = \frac{R}{R_0} = \sqrt{\frac{(2+n)(4+3n)}{2n^2(4+n)}}$$

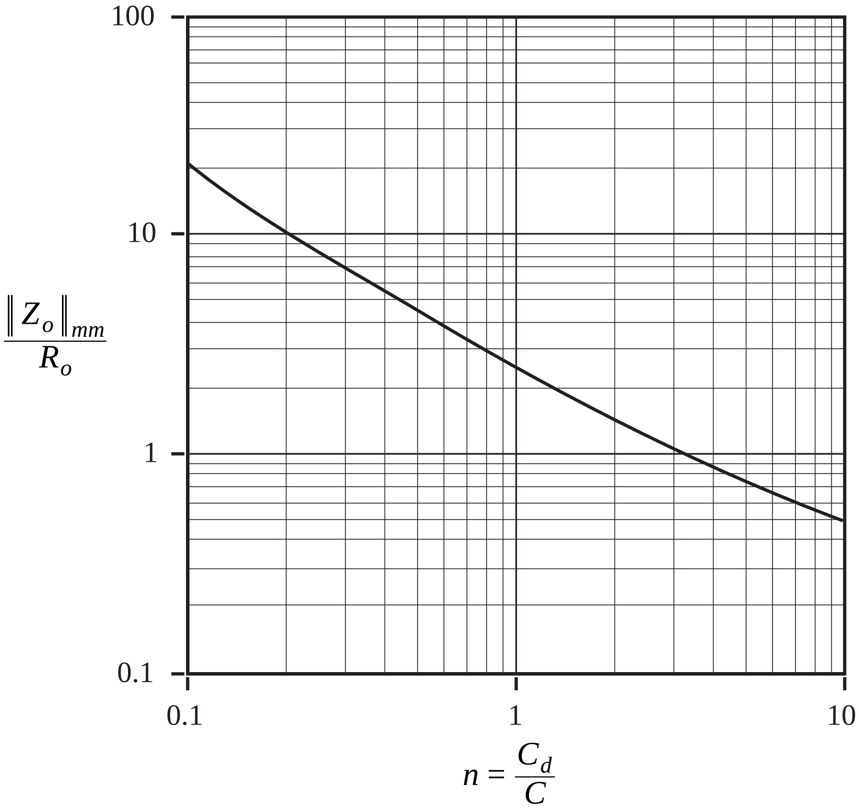
$$\frac{\| Z \|_{mm}}{R_0} = \frac{\sqrt{2(2+n)}}{n}$$

with

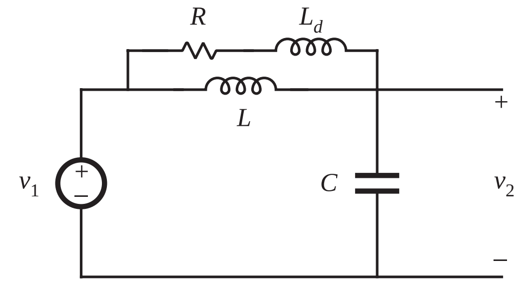
$$n = \frac{C_d}{C}$$

$$R_0 = \sqrt{\frac{L}{C}}$$

- Does not degrade HF attenuation
- No limit on $\| Z \|_{mm}$
- C_d is typically larger than C



Optimal R-L_d damping



Basic results

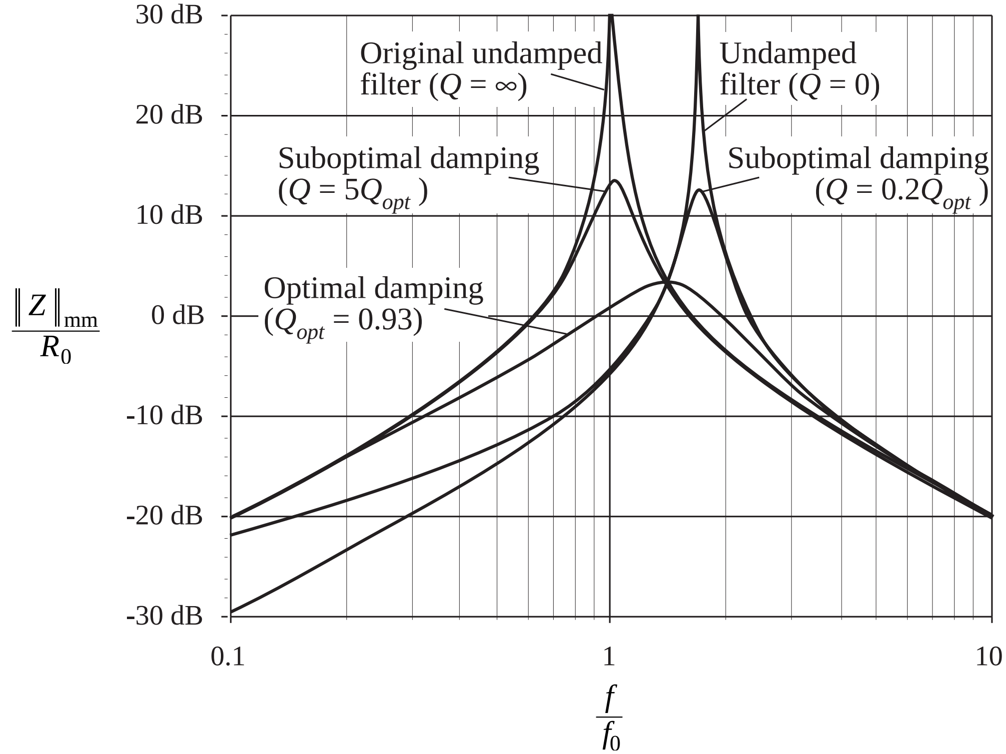
$$Q_{opt} = \sqrt{\frac{n\left(3+4n\right)\left(1+2n\right)}{2\left(1+4n\right)}}$$

$$\frac{\| Z \|_{mm}}{R_0} = \sqrt{2n\left(1+2n\right)}$$

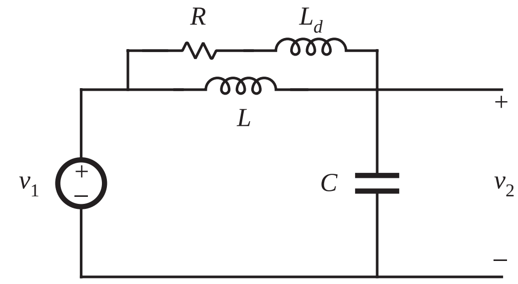
with

$$Q_{opt} = \frac{\text{optimum value of } R}{R_0}$$

$$n = \frac{L_d}{L} \qquad R_0 = \sqrt{\frac{L}{C}}$$



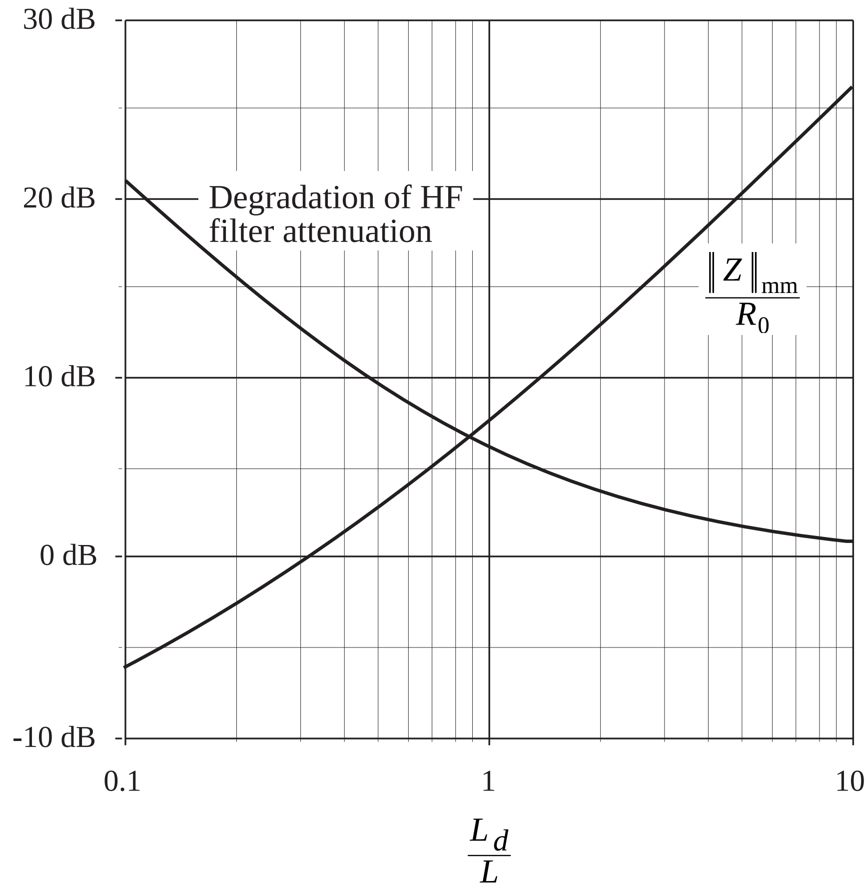
Discussion: Optimal R-L_d damping



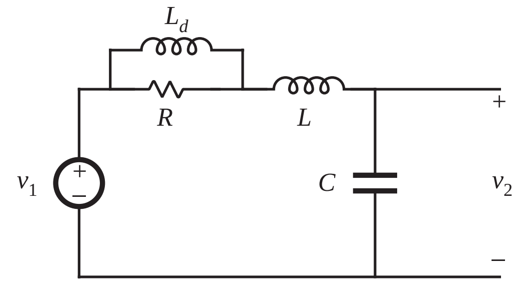
- L_d is physically very small
- A simple low-cost approach to damping the input filter
- Disadvantage: L_d degrades high-frequency attenuation of filter, by the factor

$$\frac{L}{L \| L_d} = 1 + \frac{1}{n}$$

- Basic tradeoff: peak output impedance vs. high-frequency attenuation
- Example: the choice $n = 1$ ($L_d = L$) degrades the HF attenuation by 6 dB, an leads to peak output impedance of $\| Z \|_{mm} = \sqrt{6}\ R_0$



Optimal R-L_d series damping



Basic results

$$Q_{opt} = \frac{R_0}{R} = \left(\frac{1+n}{n}\right)\sqrt{\frac{2(1+n)(4+n)}{(2+n)(4+3n)}}$$

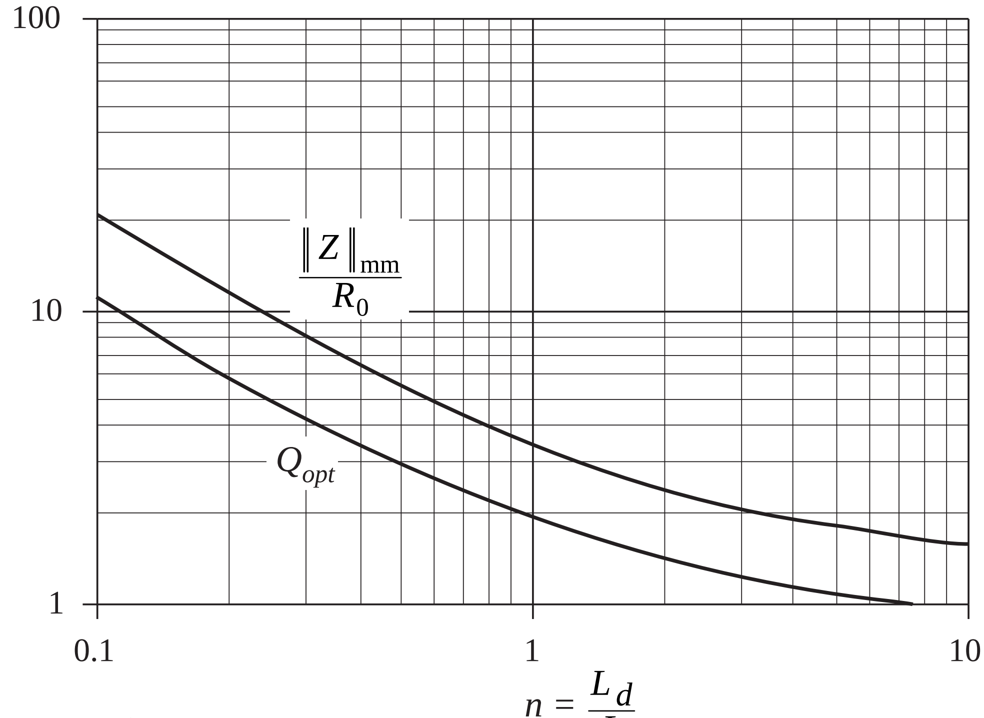
$$\frac{\| Z \|_{mm}}{R_0} = \frac{\sqrt{2(1+n)(2+n)}}{n}$$

with

$$n = \frac{L_d}{L}$$

$$R_0 = \sqrt{\frac{L}{C}}$$

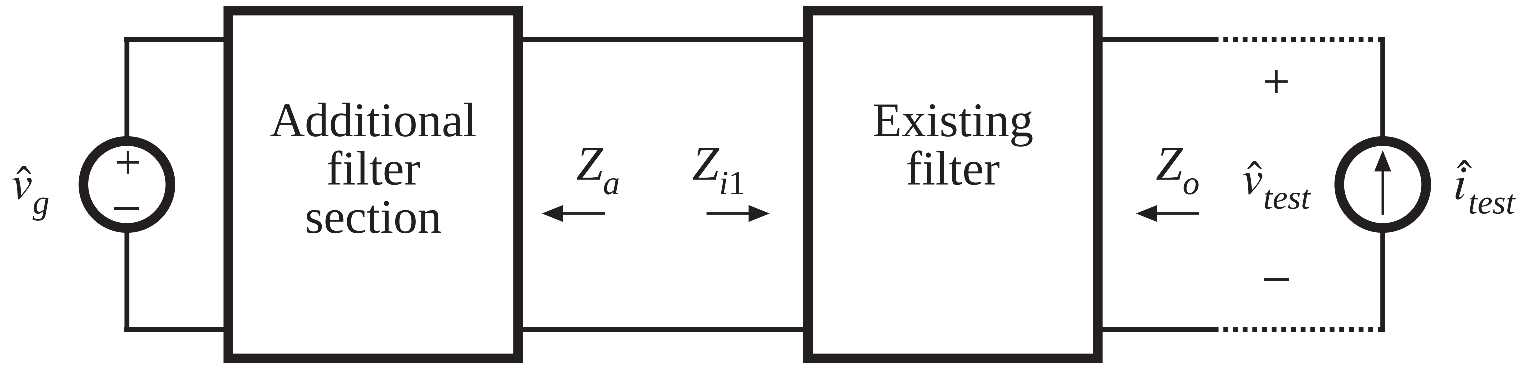
- Does not degrade HF attenuation
- L_d must conduct entire dc current
- Peak output impedance cannot be reduced below $\sqrt{2}\, R_0$



10.4.4 Cascading Filter Sections

- Cascade connection of multiple *L-C* filter sections can achieve a given high-frequency attenuation with much smaller volume and weight
- Need to damp each section of the filter
- One approach: add new filter section to an existing filter, using new design criteria
- Stagger-tuning of filter sections

Addition of filter section

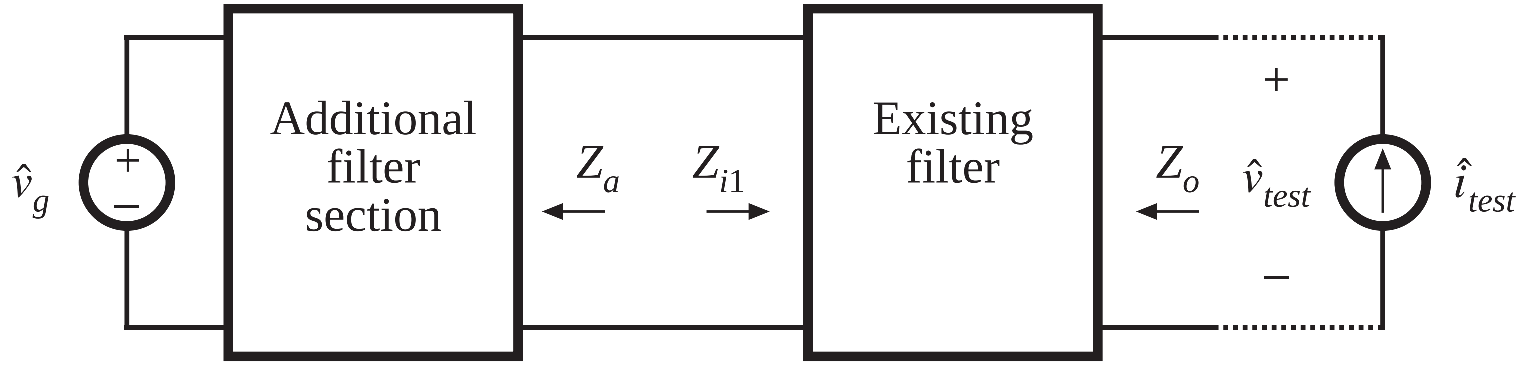


How the additional filter section changes the output impedance of the existing filter:

$$\text{modified } Z_o(s) = \left[Z_o(s) \right]_{Z_a(s)=0} \frac{\left(1 + \frac{Z_a(s)}{Z_{N1}(s)}\right)}{\left(1 + \frac{Z_a(s)}{Z_{D1}(s)}\right)}$$

$$Z_{N1}(s) = Z_{i1}(s)\Big|_{\hat{v}_{test}(s) \underset{\text{null}}{\longrightarrow} 0} \qquad Z_{D1}(s) = Z_{i1}(s)\Big|_{\hat{i}_{test}(s)=0}$$

Design criteria



$$Z_{N1}(s) = Z_{i1}(s)\Big|_{\hat{v}_{test}(s) \xrightarrow[\text{null}]{} 0}$$ (with filter output port short-circuited)

$$Z_{D1}(s) = Z_{i1}(s)\Big|_{\hat{i}_{test}(s) = 0}$$ (with filter output port open-circuited)

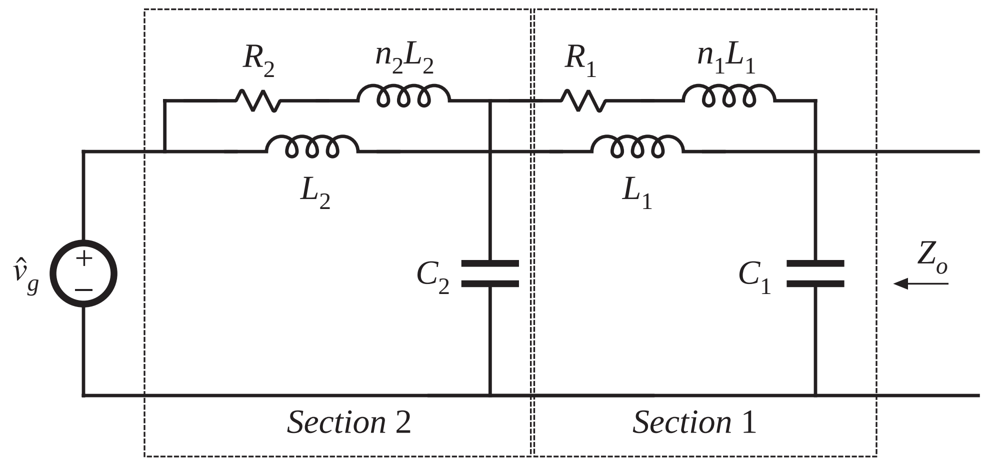
The presence of the additional filter section does not substantially alter the output impedance Z_o of the existing filter provided that

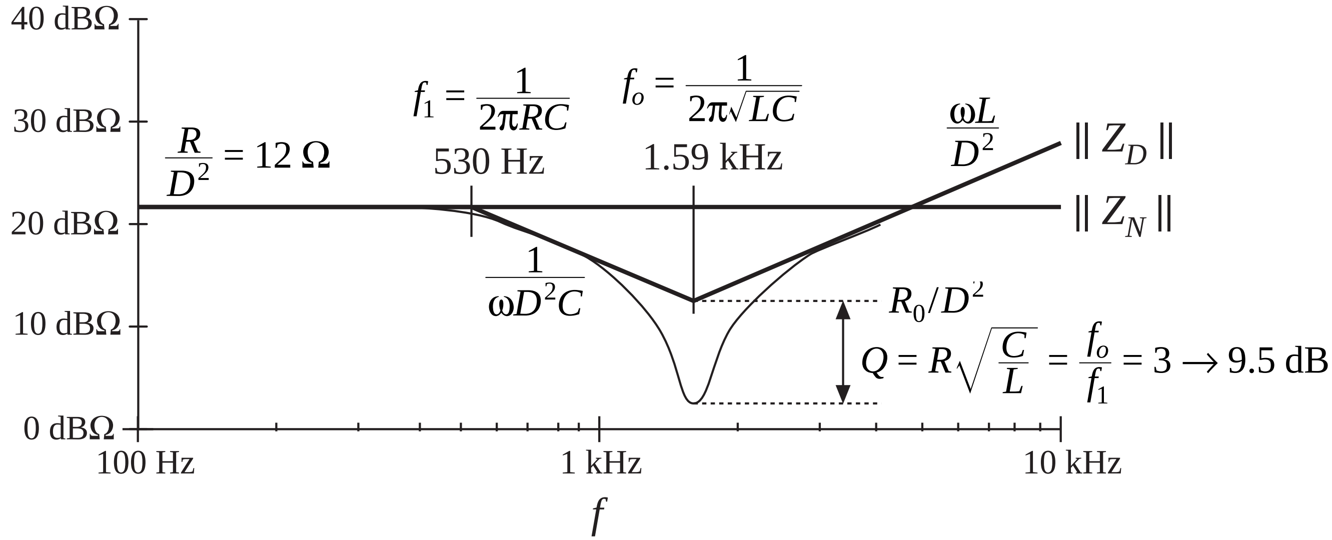
$$\| Z_a \| \ll \| Z_{N1} \| \quad \text{and}$$
$$\| Z_a \| \ll \| Z_{D1} \|$$

10.4.5 Example
Two-Stage Input Filter

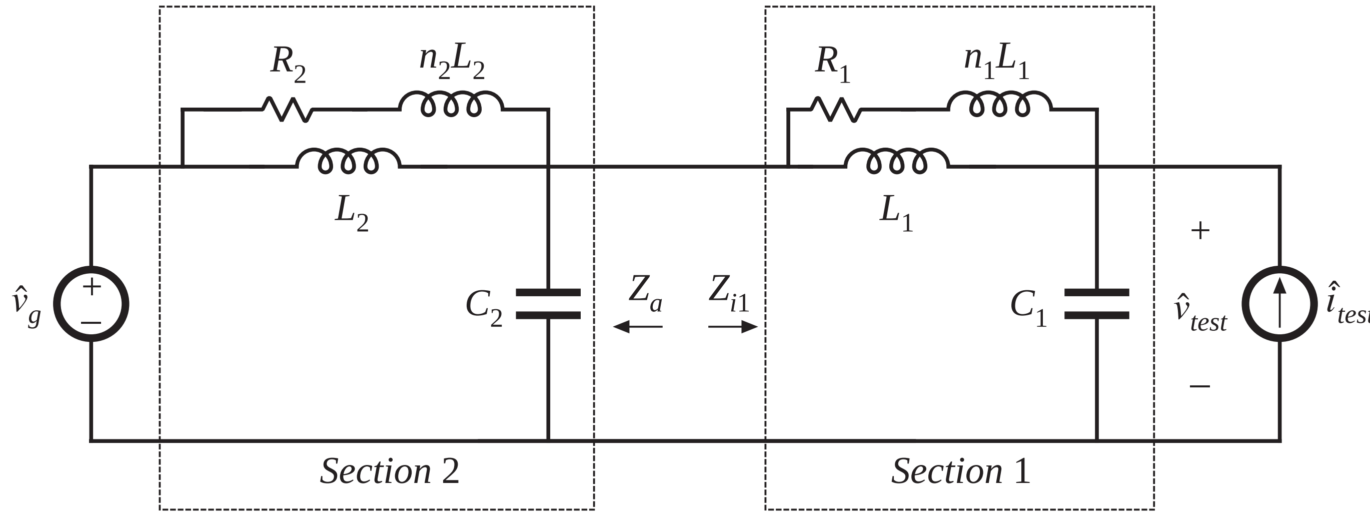
Requirements: For the same buck converter example, achieve the following:

- 80 dB of attenuation at 250 kHz
- Section 1 to satisfy Z_o impedance inequalities as before:





Section 2 impedance inequalities

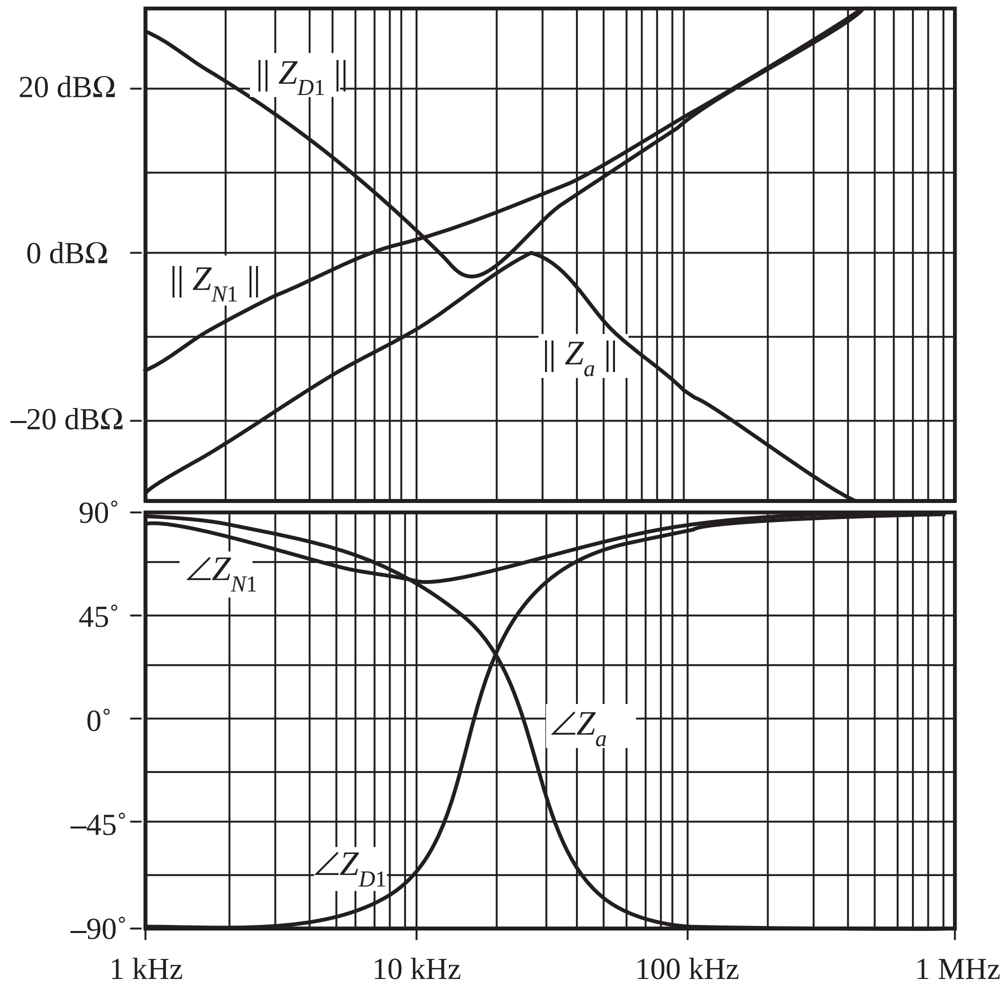


To avoid disrupting the output impedance Z_o of section 1, section 2 should satisfy the following inequalities:

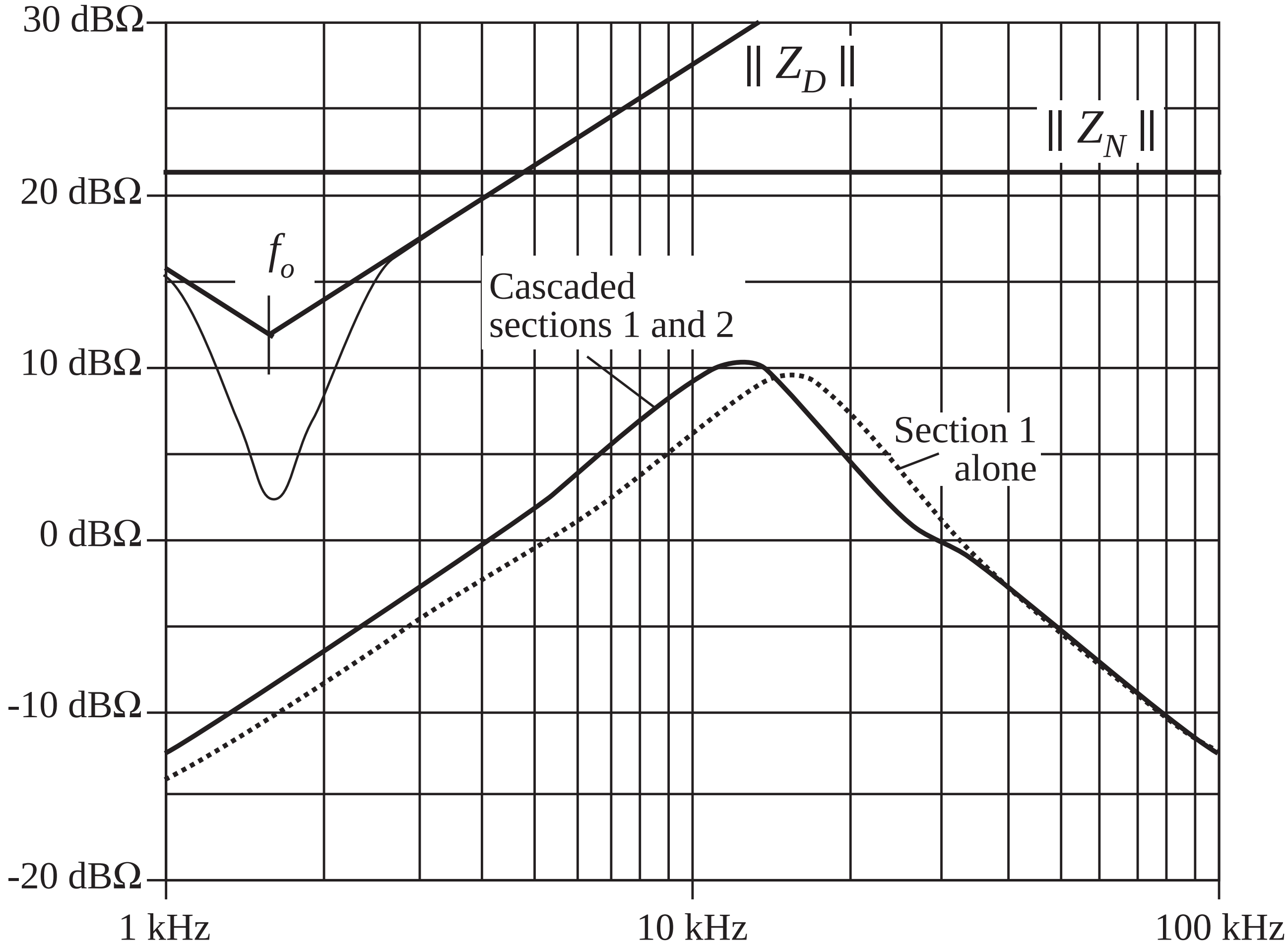
$$Z_a \ll Z_{N1} = Z_{i1}\Big|_{output\ shorted} = \left(R_1 + sn_1L_1\right) \| sL_1$$

$$Z_a \ll Z_{D1} = Z_{i1}\Big|_{output\ open\text{-}circuited} = \frac{1}{sC_1} + \left(R_1 + sn_1L_1\right) \| sL_1$$

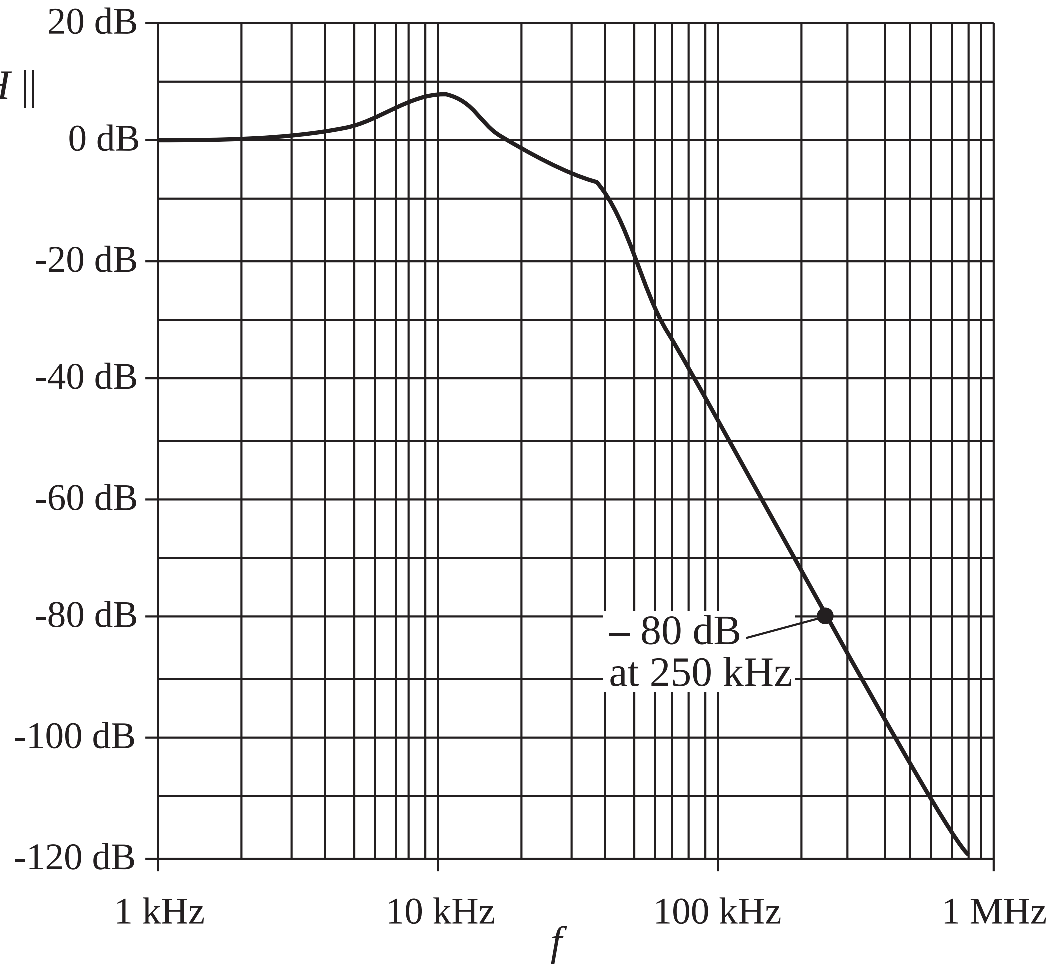
Plots of Z_{N1} and Z_{D1}



Section 1 output impedance inequalities



Resulting filter transfer function



Comparison of single-section and two-section designs