Chapter 16
Power And Harmonics in Nonsinusoidal Systems

16.1. Average power

Observe transmission of energy through surface S

Source $v(t)$ Load

Surface S
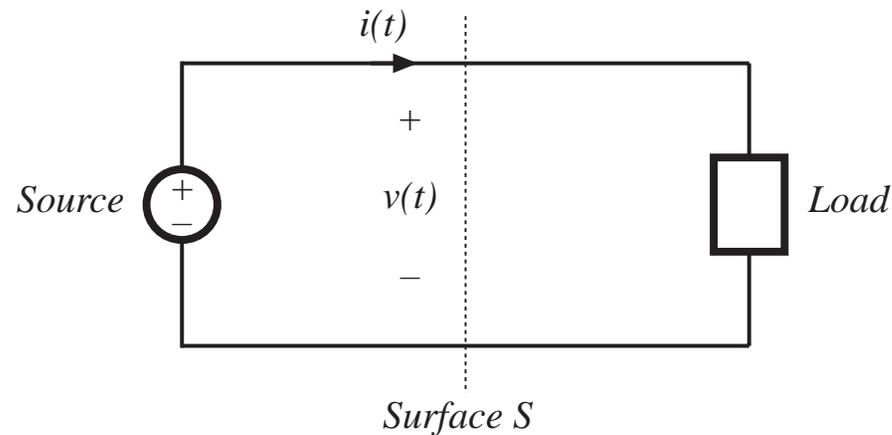
Express voltage and current as Fourier series:

$$v(t) = V_0 + \sum_{n=1}^{\infty} V_n \cos\left(n\omega t - \varphi_n\right)$$

$$i(t) = I_0 + \sum_{n=1}^{\infty} I_n \cos\left(n\omega t - \theta_n\right)$$

relate energy transmission to harmonics

Energy transmittted to load, per cycle

$$W_{cycle} = \int_0^T v(t)i(t)\,dt$$

This is related to average power as follows:

$$P_{av} = \frac{W_{cycle}}{T} = \frac{1}{T} \int_0^T v(t)i(t)\,dt$$

Investigate influence of harmonics on average power: substitute Fourier series

$$P_{av} = \frac{1}{T} \int_0^T \left(V_0 + \sum_{n=1}^{\infty} V_n \cos\left(n\omega t - \varphi_n\right) \right) \left(I_0 + \sum_{n=1}^{\infty} I_n \cos\left(n\omega t - \theta_n\right) \right) dt$$

Evaluation of integral

Orthogonality of harmonics: Integrals of cross-product terms are zero

$$\int_0^T \left(V_n \cos \left(n\omega t - \varphi_n \right) \right) \left(I_m \cos \left(m\omega t - \theta_m \right) \right) dt = \begin{cases} 0 & \text{if } n \neq m \\ \dfrac{V_n I_n}{2} \cos \left(\varphi_n - \theta_n \right) & \text{if } n = m \end{cases}$$

Expression for average power becomes

$$P_{av} = V_0 I_0 + \sum_{n=1}^{\infty} \frac{V_n I_n}{2} \cos \left(\varphi_n - \theta_n \right)$$

So net energy is transmitted to the load only when the Fourier series of *v(t)* and *i(t)* contain terms at the same frequency. For example, if the voltage and current both contain third harmonic, then they lead to the average power
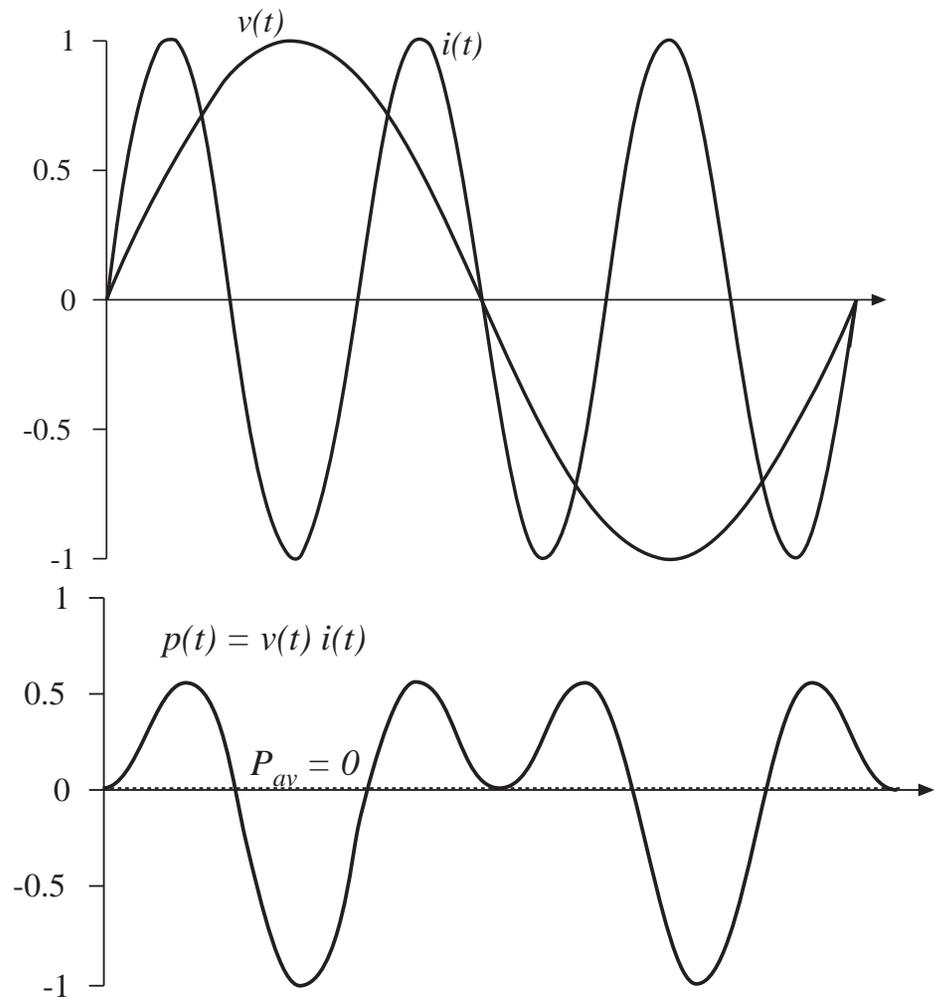
$$\frac{V_3 I_3}{2} \cos \left(\varphi_3 - \theta_3 \right)$$

Example 1

Voltage: fundamental only

Current: third harmonic only

Power: zero average
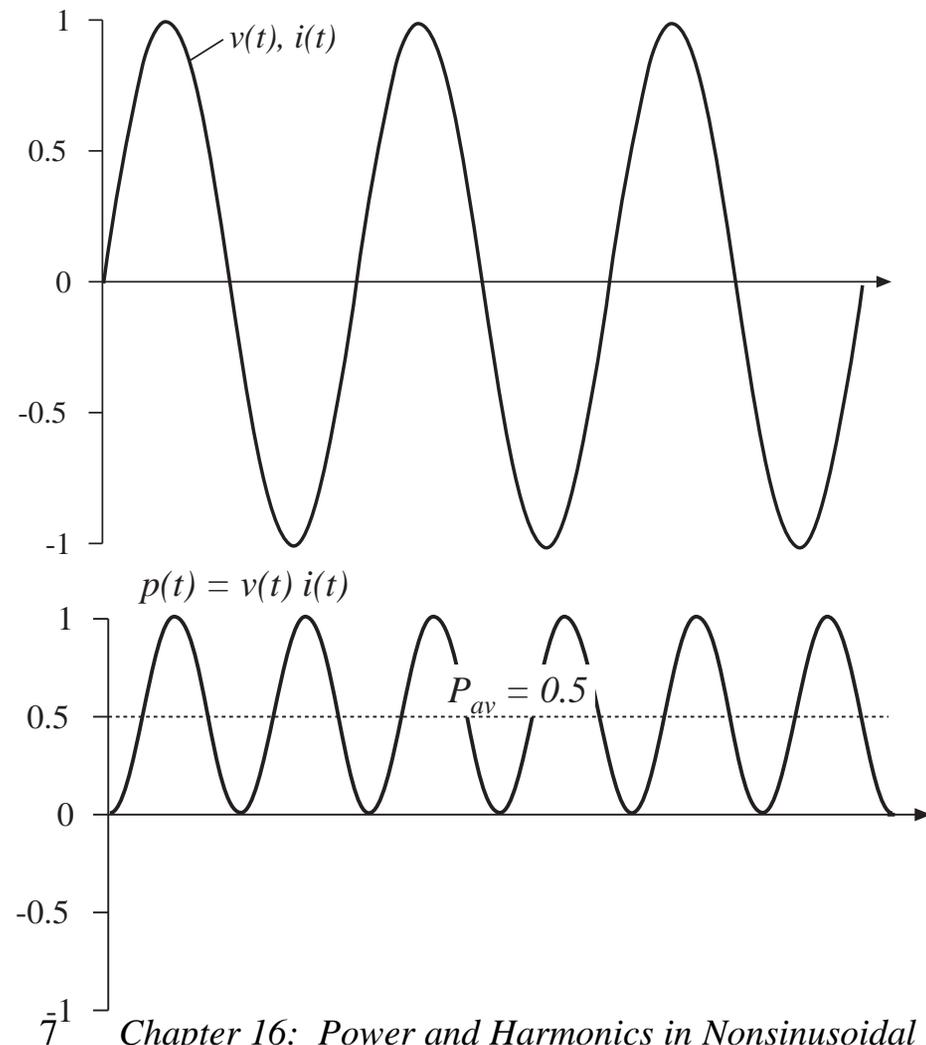
Example 2

Voltage: third harmonic only

Current: third harmonic only, in phase with voltage

Power: nonzero average

$v(t), i(t)$

$p(t) = v(t)\, i(t)$

$P_{av} = 0.5$

Example 3

Fourier series:

$$v(t) = 1.2 \cos (\omega t) + 0.33 \cos (3\omega t) + 0.2 \cos (5\omega t)$$

$$i(t) = 0.6 \cos (\omega t + 30°) + 0.1 \cos (5\omega t + 45°) + 0.1 \cos (7\omega t + 60°)$$
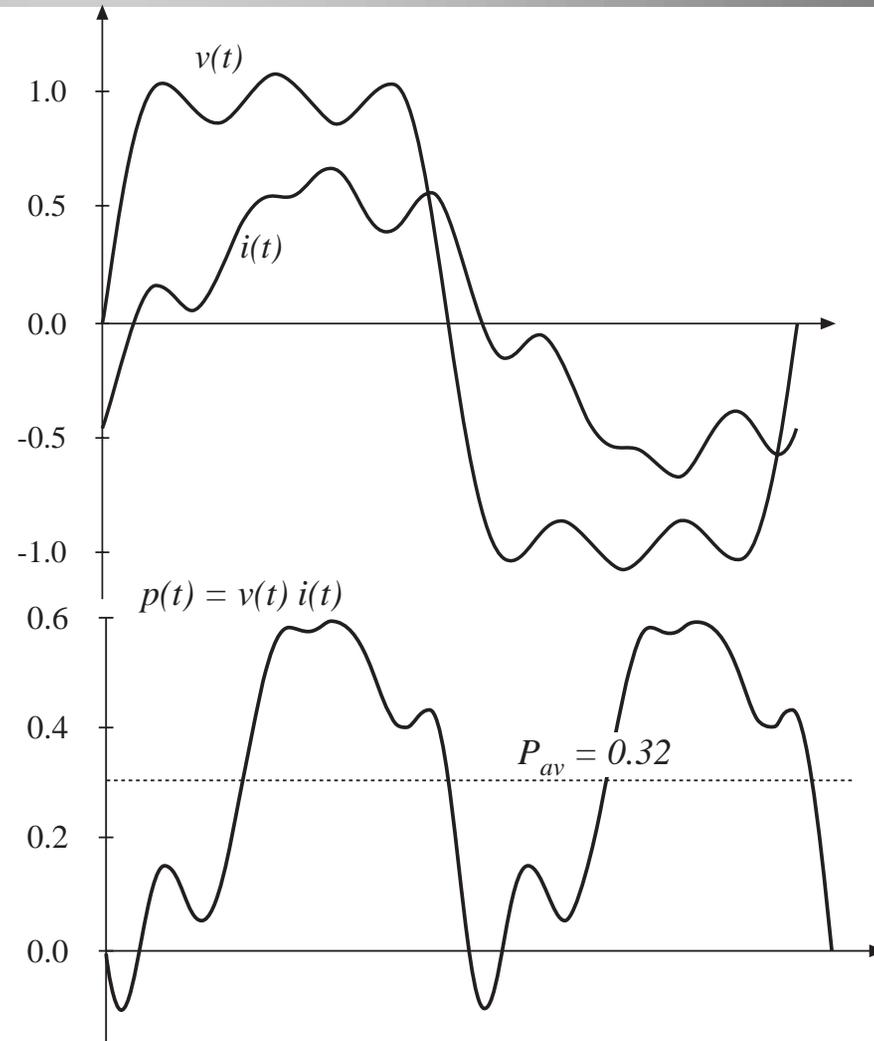
Average power calculation:

$$P_{av} = \frac{(1.2)(0.6)}{2} \cos (30°) + \frac{(0.2)(0.1)}{2} \cos (45°) = 0.32$$

Example 3

Voltage: 1st, 3rd, 5th

Current: 1st, 5th, 7th

v(t)

1.0

0.5

i(t)

0.0

-0.5

-1.0

p(t) = v(t) i(t)

0.6

Power: net energy is transmitted at fundamental and fifth harmonic frequencies

0.4

$P_{av} = 0.32$

0.2

0.0

16.2. Root-mean-square (RMS) value of a waveform, in terms of Fourier series

$$(\text{rms value}) = \sqrt{\frac{1}{T} \int_0^T v^2(t)\,dt}$$

Insert Fourier series. Again, cross-multiplication terms have zero average. Result is

$$(\text{rms value}) = \sqrt{V_0^2 + \sum_{n=1}^{\infty} \frac{V_n^2}{2}}$$

- Similar expression for current
- Harmonics always increase rms value
- Harmonics do not necessarily increase average power
- Increased rms values mean increased losses

16.3. Power factor

For efficient transmission of energy from a source to a load, it is desired to maximize average power, while minimizing rms current and voltage (and hence minimizing losses).

Power factor is a figure of merit that measures how efficiently energy is transmitted. It is defined as

$$\text{power factor} = \frac{(\text{average power})}{(\text{rms voltage}) \, (\text{rms current})}$$

Power factor always lies between zero and one.

16.3.1. Linear resistive load, nonsinusoidal voltage

Then current harmonics are in phase with, and proportional to, voltage harmonics. All harmonics result in transmission of energy to load, and unity power factor occurs.

$$I_n = \frac{V_n}{R} \qquad\qquad \theta_n = \varphi_n \qquad \text{so} \quad \cos(\theta_n - \varphi_n) = 1$$

$$(\text{rms voltage}) = \sqrt{V_0^2 + \sum_{n=1}^{\infty} \frac{V_n^2}{2}}$$

$$(\text{rms current}) = \sqrt{I_0^2 + \sum_{n=1}^{\infty} \frac{I_n^2}{2}} = \sqrt{\frac{V_0^2}{R^2} + \sum_{n=1}^{\infty} \frac{V_n^2}{2R^2}}$$

$$= \frac{1}{R}(\text{rms voltage})$$

$$P_{av} = V_0 I_0 + \sum_{n=1}^{\infty} \frac{V_n I_n}{2} \cos(\varphi_n - \theta_n)$$

16.3.2. Nonlinear dynamical load, sinusoidal voltage

With a sinusoidal voltage, current harmonics do not lead to average power. However, current harmonics do increase the rms current, and hence they decrease the power factor.

$$P_{av} = \frac{V_1 I_1}{2} \cos(\varphi_1 - \theta_1)$$

$$(\text{rms current}) = \sqrt{I_0^2 + \sum_{n=1}^{\infty} \frac{I_n^2}{2}}$$

$$(\text{power factor}) = \left(\frac{\frac{I_1}{\sqrt{2}}}{\sqrt{I_0^2 + \sum_{n=1}^{\infty} \frac{I_n^2}{2}}} \right) \left(\cos(\varphi_1 - \theta_1) \right)$$

$$= (\text{distortion factor})\,(\text{displacement factor})$$

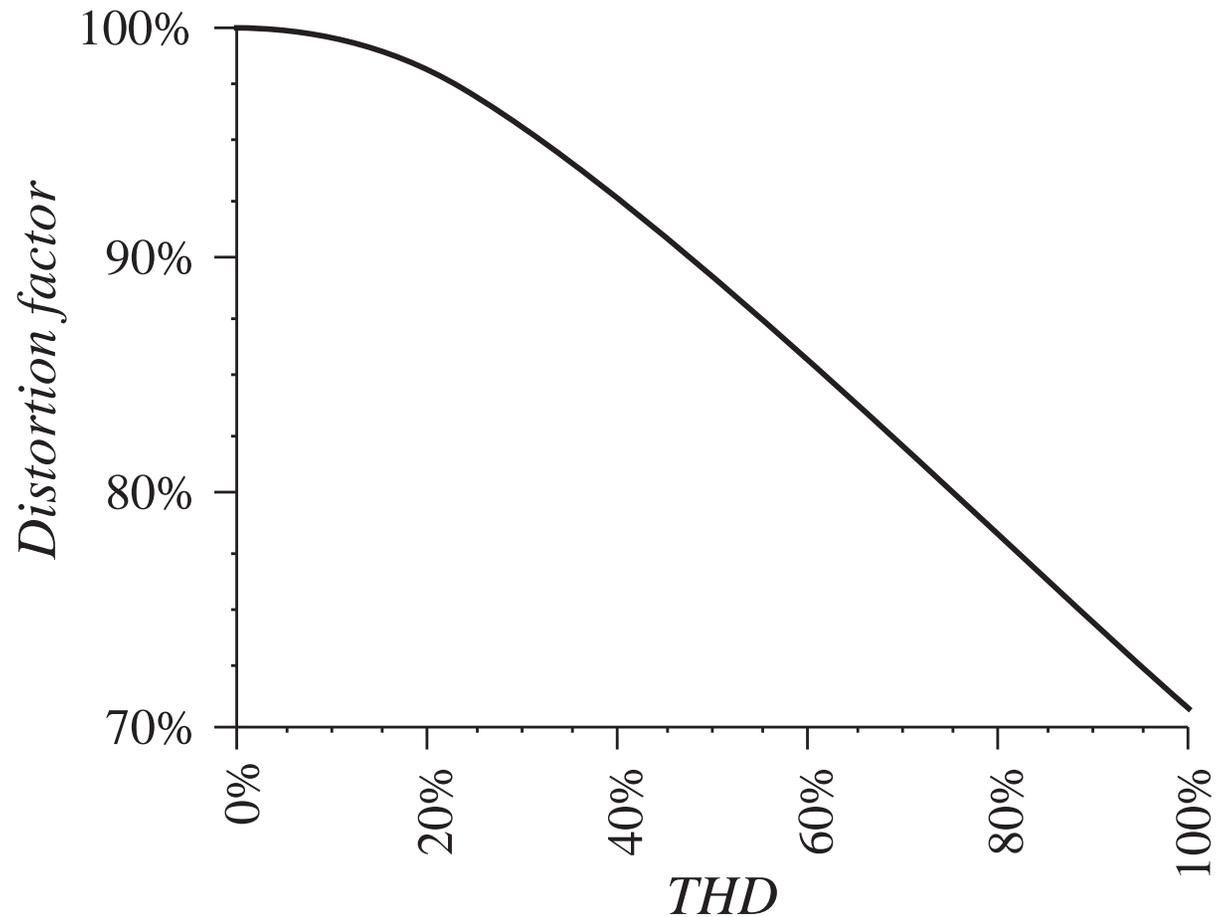# Distortion factor

Defined only for sinusoidal voltage.

$$(\text{distortion factor}) = \left(\frac{\dfrac{I_1}{\sqrt{2}}}{\sqrt{I_0^2 + \sum_{n=1}^{\infty} \dfrac{I_n^2}{2}}} \right) = \frac{(\text{rms fundamental current})}{(\text{rms current})}$$

Related to Total Harmonic Distortion (THD):

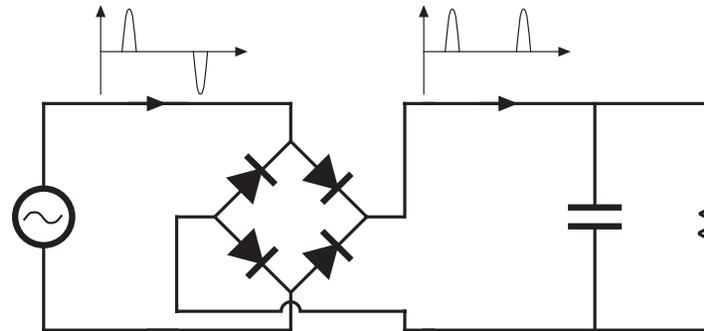$$(\text{THD}) = \frac{\sqrt{\sum_{n=2}^{\infty} I_n^2}}{I_1}$$

$$(\text{distortion factor}) = \frac{1}{\sqrt{1 + (\text{THD})^2}}$$
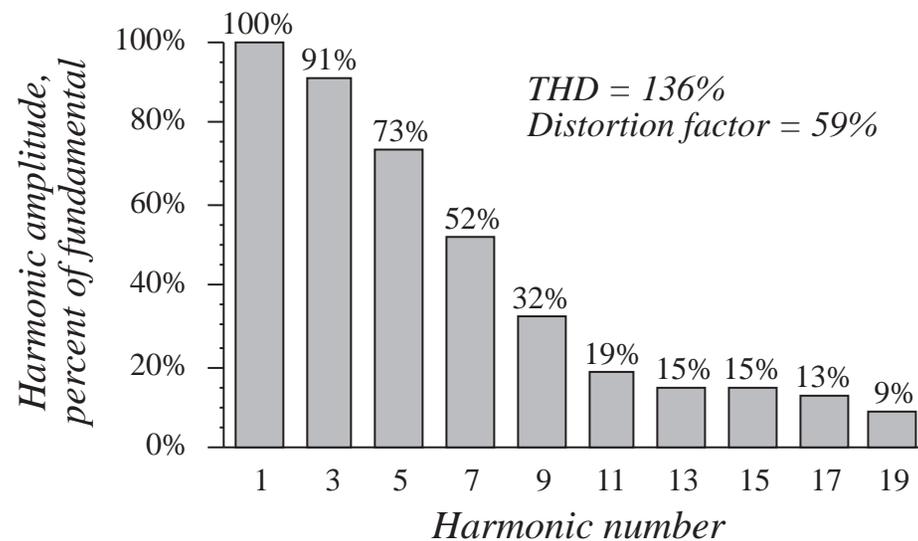
Distortion factor vs. THD

Peak detection rectifier example

Conventional single-phase peak detection rectifier

Typical ac line current spectrum

THD = 136%
Distortion factor = 59%

Maximum power obtainable from 120V 15A wall outlet

with peak detection rectifier

(ac voltage) (derated breaker current) (power factor) (rectifier efficiency)

= (120 V)   (80% of 15 A)   (0.55)   (0.98)

= 776 W

at unity power factor

(ac voltage) (derated breaker current) (power factor) (rectifier efficiency)

= (120 V)   (80% of 15 A)   (0.99)   (0.93)

= 1325 W

16.4. Power phasors in sinusoidal systems

Apparent power is the product of the rms voltage and rms current

> It is easily measured —simply the product of voltmeter and ammeter readings
>
> Unit of apparent power is the volt-ampere, or VA
>
> Many elements, such as transformers, are rated according to the VA that they can supply

So power factor is the ratio of average power to apparent power

With sinusoidal waveforms (no harmonics), we can also define the

> *real power* P $\qquad$ *reactive power* Q $\qquad$ *complex power* S

If the voltage and current are represented by phasors V and I, then

$$ S = VI^* = P + jQ $$

with I^* = complex conjugate of I, j = square root of -1. The magnitude of S is the apparent power (VA). The real part of S is the average power P (watts). The imaginary part of S is the reactive power Q (reactive volt-amperes, or VARs).

Example: power phasor diagram

The phase angle between the voltage and current, or $(\varphi_1 - \theta_1)$, coincides with the angle of S. The power factor is

$$\text{power factor} = \frac{P}{\|S\|} = \cos\left(\varphi_1 - \theta_1\right)$$

In this purely sinusoidal case, the distortion factor is unity, and the power factor coincides with the displacement factor.
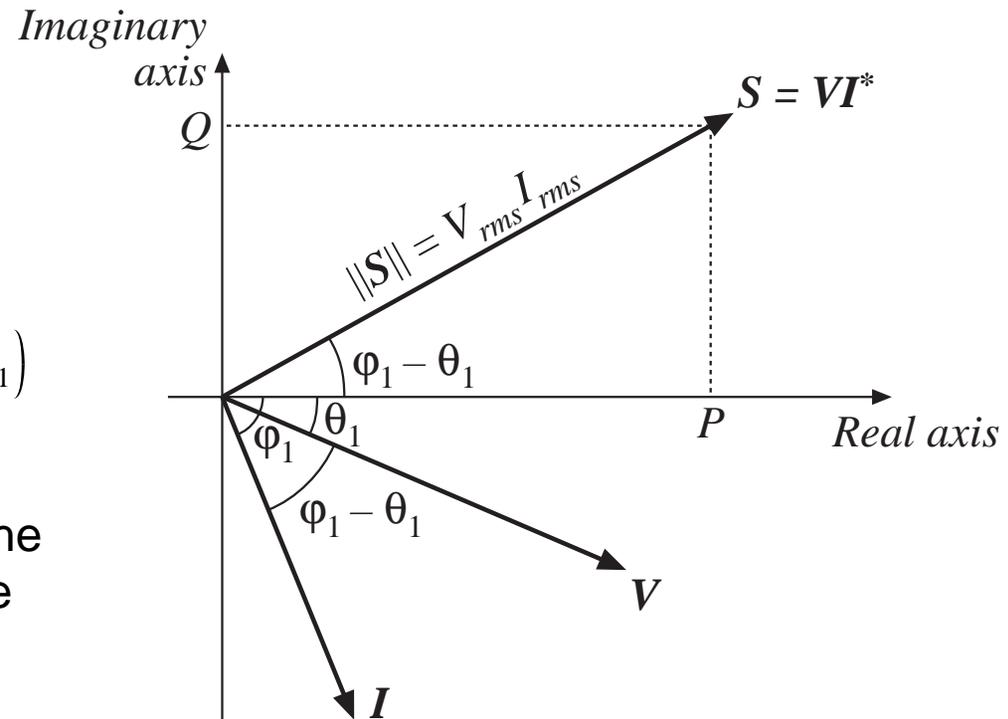
Reactive power Q

The reactive power Q does not lead to net transmission of energy between the source and load. When $Q \neq 0$, the rms current and apparent power are greater than the minimum amount necessary to transmit the average power P.

Inductor: current lags voltage by 90˚, hence displacement factor is zero.

The alternate storing and releasing of energy in an inductor leads to current flow and nonzero apparent power, but $P = 0$.

Just as resistors consume real (average) power P, inductors can be viewed as consumers of reactive power Q.

Capacitor: current leads voltage by 90˚, hence displacement factor is zero.

Capacitors supply reactive power Q.

They are often placed in the utility power distribution system near inductive loads. If Q supplied by capacitor is equal to Q consumed by inductor, then the net current (flowing from the source into the capacitor-inductive-load combination) is in phase with the voltage, leading to unity power factor and minimum rms current magnitude.

Lagging fundamental current of phase-controlled rectifiers

It will be seen in the next chapter that phase-controlled rectifiers produce a nonsinusoidal current waveform whose fundamental component lags the voltage.

This lagging current does not arise from energy storage, but it does nonetheless lead to a reduced displacement factor, and to rms current and apparent power that are greater than the minimum amount necessary to transmit the average power.

At the fundamental frequency, phase-controlled rectifiers can be viewed as consumers of reactive power Q, similar to inductive loads.

# 16.5. Harmonic currents in three phase systems

The presence of harmonic currents can also lead to some special problems in three-phase systems:

- In a four-wire three-phase system, harmonic currents can lead to large currents in the neutral conductors, which may easily exceed the conductor rms current rating

- Power factor correction capacitors may experience significantly increased rms currents, causing them to fail

In this section, these problems are examined, and the properties of harmonic current flow in three-phase systems are derived:

- Harmonic neutral currents in 3ø four-wire networks
- Harmonic neutral voltages in 3ø three-wire wye-connected loads

16.5.1. Harmonic currents in three-phase four-wire networks
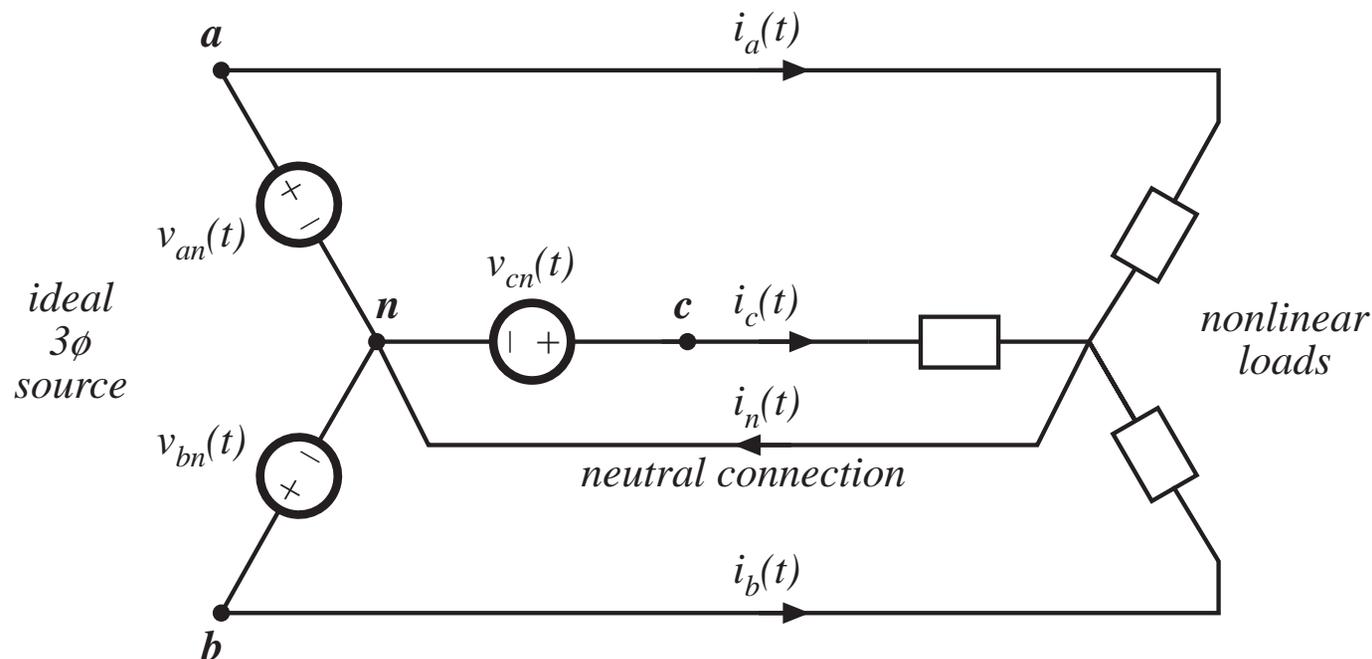
Fourier series of line currents and voltages:

$$i_a(t) = I_{a0} + \sum_{k=1}^{\infty} I_{ak} \cos\left(k\omega t - \theta_{ak}\right)$$

$$i_b(t) = I_{b0} + \sum_{k=1}^{\infty} I_{bk} \cos\left(k(\omega t - 120°) - \theta_{bk}\right)$$

$$i_c(t) = I_{c0} + \sum_{k=1}^{\infty} I_{ck} \cos\left(k(\omega t + 120°) - \theta_{ck}\right)$$

$$v_{an}(t) = V_m \cos\left(\omega t\right)$$

$$v_{bn}(t) = V_m \cos\left(\omega t - 120°\right)$$

$$v_{cn}(t) = V_m \cos\left(\omega t + 120°\right)$$

Neutral current

$$i_n(t) = I_{a0} + I_{b0} + I_{c0} +$$

$$\sum_{k=1}^{\infty} \left[I_{ak} \cos(k\omega t - \theta_{ak}) + I_{bk} \cos(k(\omega t - 120°) - \theta_{bk}) + I_{ck} \cos(k(\omega t + 120°) - \theta_{ck}) \right]$$

If the load is unbalanced, then there is nothing more to say. The neutral connection may contain currents having spectrum similar to the line currents.

In the balanced case, $I_{ak} = I_{bk} = I_{ck} = I_k$ and $\theta_{ak} = \theta_{bk} = \theta_{ck} = \theta_k$, for all k; i.e., the harmonics of the three phases all have equal amplitudes and phase shifts. The neutral current is then

$$i_n(t) = 3I_0 + \sum_{k=3,6,9,\cdots}^{\infty} 3I_k \cos(k\omega t - \theta_k)$$

Neutral currents

$$i_n(t) = 3I_0 + \sum_{k=3,6,9,\cdots}^{\infty} 3I_k \cos(k\omega t - \theta_k)$$

- Fundamental and most harmonics cancel out

- Triplen (triple-n, or 0, 3, 6, 9, ...) harmonics do not cancel out, but add. Dc components also add.

- Rms neutral current is

$$i_{n,\,rms} = 3 \sqrt{I_0^2 + \sum_{k=3,6,9,\cdots}^{\infty} \frac{I_k^2}{2}}$$

Example

A balanced nonlinear load produces line currents containing fundamental and 20% third harmonic: $i_{an}(t) = I_1 \cos(\omega t - \theta_1) + 0.2\ I_1 \cos(3\omega t - \theta_3)$. Find the rms neutral current, and compare its amplitude to the rms line current amplitude.

Solution

$$i_{n,\,rms} = 3 \sqrt{\frac{(0.2 I_1)^2}{2}} = \frac{0.6\ I_1}{\sqrt{2}}$$

$$i_{1,\,rms} = \sqrt{\frac{I_1^2 + (0.2 I_1)^2}{2}} = \frac{I_1}{\sqrt{2}} \sqrt{1 + 0.04} \approx \frac{I_1}{\sqrt{2}}$$

- The neutral current magnitude is 60% of the line current magnitude!

- The triplen harmonics in the three phases add, such that 20% third harmonic leads to 60% third harmonic neutral current.

- Yet the presence of the third harmonic has very little effect on the rms value of the line current. Significant unexpected neutral current flows.

16.5.2. Harmonic currents in three-phase three-wire networks

Wye-connected nonlinear load, no neutral connection:
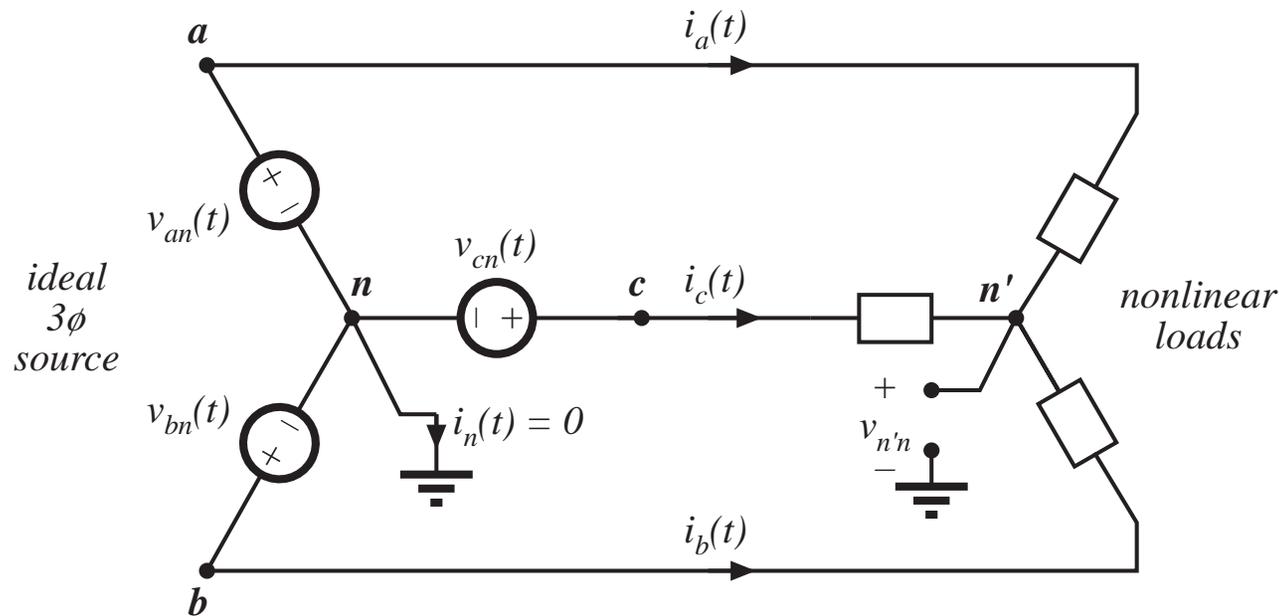
No neutral connection

If the load is
balanced, then it
is still true that

$$i_n(t) = 3I_0 + \sum_{k=3,6,9,\cdots}^{\infty} 3I_k \cos(k\omega t - \theta_k)$$

But $i_n(t) = 0$, since there is no neutral connection.

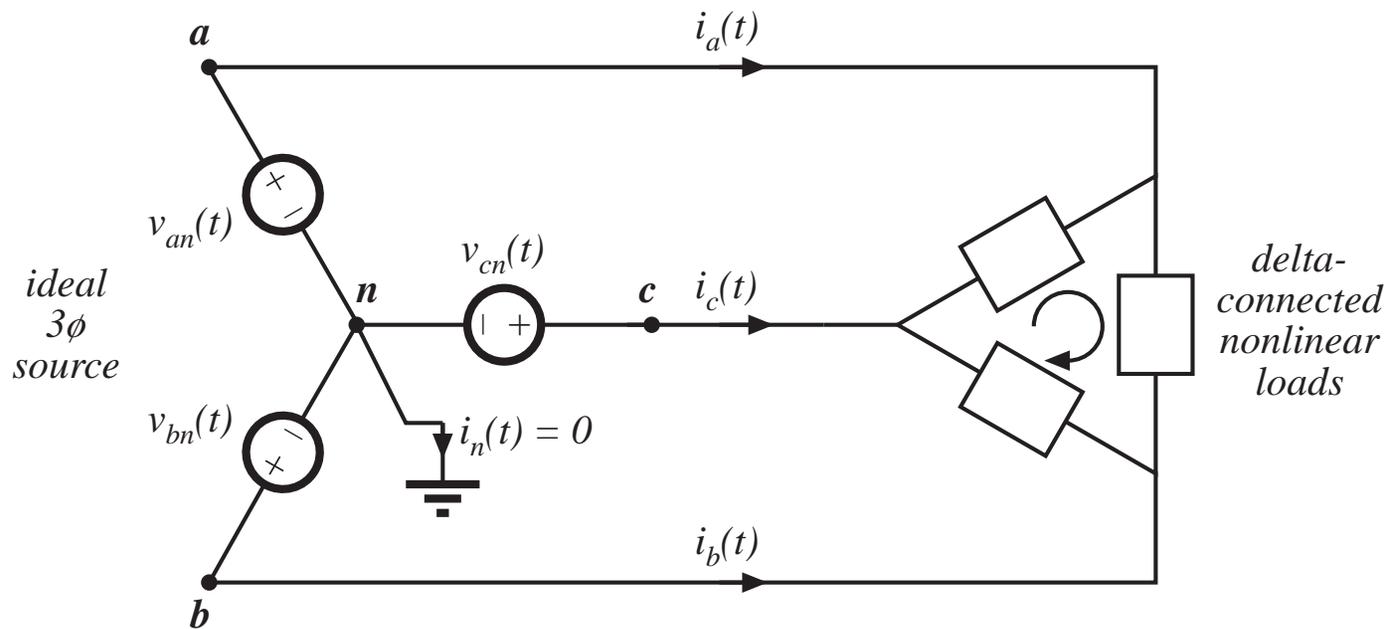So the ac line currents cannot contain dc or triplen harmonics.

What happens:

A voltage is induced at the load neutral point, that causes the line
current dc and triplen harmonics to become zero.

The load neutral point voltage contains dc and triplen harmonics.

With an unbalanced load, the line currents can still contain dc and
triplen harmonics.

Delta-connected load

- There is again no neutral connection, so the ac line currents contain no dc or triplen harmonics

- The load currents may contain dc and triplen harmonics: with a balanced nonlinear load, these circulate around the delta.

Harmonic currents in power factor correction capacitors

PFC capacitors are usually not intended to conduct significant harmonic currents.

Heating in capacitors is a function of capacitor equivalent series resistance (esr) and rms current. The maximum allowable rms current then leads to the capacitor rating:

$$\text{rated rms voltage } V_{rms} = \frac{I_{rms}}{2\pi f C}$$

$$\text{rated reactive power} = \frac{I_{rms}^2}{2\pi f C}$$
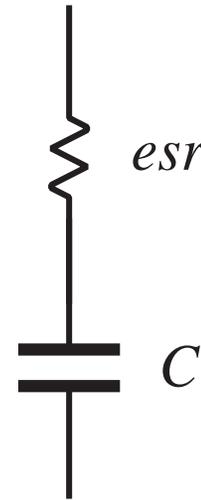
16.6. AC line current harmonic standards

US MIL-STD-461B

International Electrotechnical Commission Standard 1000

IEEE/ANSI Standard 519

US MIL-STD-461B

- An early attempt to regulate ac line current harmonics generated by nonlinear loads

- For loads of 1kW or greater, no current harmonic magnitude may be greater than 3% of the fundamental magnitude.

- For the nth harmonic with $n > 33$, the harmonic magnitude may not exceed $(1/n)$ times the fundamental magnitude.

- Harmonic limits are now employed by all of the US armed forces. The specific limits are often tailored to the specific application.

- The shipboard application is a good example of the problems faced in a relatively small stand-alone power system having a large fraction of electronic loads.

International Electrotechnical Commission Standard 1000

- First draft of their IEC-555 standard:1982. It has since undergone a number of revisions. Recent reincarnation: IEC-1000-3-2

- Enforcement of IEC-1000 is the prerogative of each individual country, and hence it has been sometimes difficult to predict whether and where this standard will actually be given the force of law.

- Nonetheless, IEC-1000 is now enforced in Europe, making it a de facto standard for commercial equipment intended to be sold worldwide.

- IEC-1000 covers a number of different types of low power equipment, with differing harmonic limits. Harmonics for equipment having an input current of up to 16A, connected to 50 or 60 Hz, 220V to 240V single phase circuits (two or three wire), as well as 380V to 415V three phase (three or four wire) circuits, are limited.

Low-power harmonic limits

- In a city environment such as a large building, a large fraction of the total power system load can be nonlinear

- Example: a major portion of the electrical load in a building is comprised of fluorescent lights, which present a very nonlinear characteristic to the utility system.

- A modern office may also contain a large number of personal computers, printers, copiers, etc., each of which may employ peak detection rectifiers.

- Although each individual load is a negligible fraction of the total local load, these loads can collectively become significant.

IEC-1000: Class A and B

Class A: Balanced three-phase equipment, and any equipment which does not fit into the other categories. This class includes low harmonic rectifiers for computer and other office equipment. These limits are given in Table 16.1, and are absolute ampere limits.

Class B: Portable tools, and similar devices. The limits are equal to the Table 16.1 limits, multiplied by 1.5.

Classes C, D, and E: For other types of equipment, including lighting (Class C) and equipment having a "special waveshape" (Class D).

Class A limits

Table 16.1. IEC-1000 Harmonic current limits, Class A

Odd harmonics		Even harmonics	
Harmonic number	Maximum current	Harmonic number	Maximum curre
3	2.30A	2	1.08A
5	1.14A	4	0.43A
7	0.77A	6	0.30A
9	0.40A	$8 \leq n \leq 40$	$0.23A \cdot (8/n)$
11	0.33A		
13	0.21A		
$15 \leq n \leq 39$	$0.15A \cdot (15/n)$		

16.6.2. IEEE/ANSI Standard 519

- In 1993, the IEEE published a revised draft standard limiting the amplitudes of current harmonics, *IEEE Guide for Harmonic Control and Reactive Compensation of Static Power Converters*.

- Harmonic limits are based on the ratio of the fundamental component of the load current IL to the short circuit current at the point of common (PCC) coupling at the utility I_{sc}.

- Stricter limits are imposed on large loads than on small loads. The limits are similar in magnitude to IEC-1000, and cover high voltage loads (of much higher power) not addressed by IEC-1000. Enforcement of this standard is presently up to the local utility company.

- The odd harmonic limits are listed in Table 16.2. The limits for even harmonics are 25% of the odd harmonic limits. Dc current components and half-wave rectifiers are not allowed.

IEEE-519 current limits, low voltage systems

Table 16.2. IEEE-519 Maximum odd harmonic current limits for general distribution systems, 120V through 69kV

I_{sc}/I_L	$n < 11$	$11 \leq n < 17$	$17 \leq n < 23$	$23 \leq n < 35$	$35 \leq n$	THD
<20	4.0%	2.0%	1.5%	0.6%	0.3%	5.0%
20–50	7.0%	3.5%	2.5%	1.0%	0.5%	8.0%
50–100	10.0%	4.5%	4.0%	1.5%	0.7%	12.0%
100–1000	12.0%	5.5%	5.0%	2.0%	1.0%	15.0%
>1000	15.0%	7.0%	6.0%	2.5%	1.4%	20.0%

IEEE-519 voltage limits

Table 16.3. IEEE-519 voltage distortion limits

Bus voltage at PCC	Individual harmonics	THD
69kV and below	3.0%	5.0%
69.001kV–161kV	1.5%	2.5%
above 161kV	1.0%	1.5%

It is the responsibility of the utility to meet these limits.