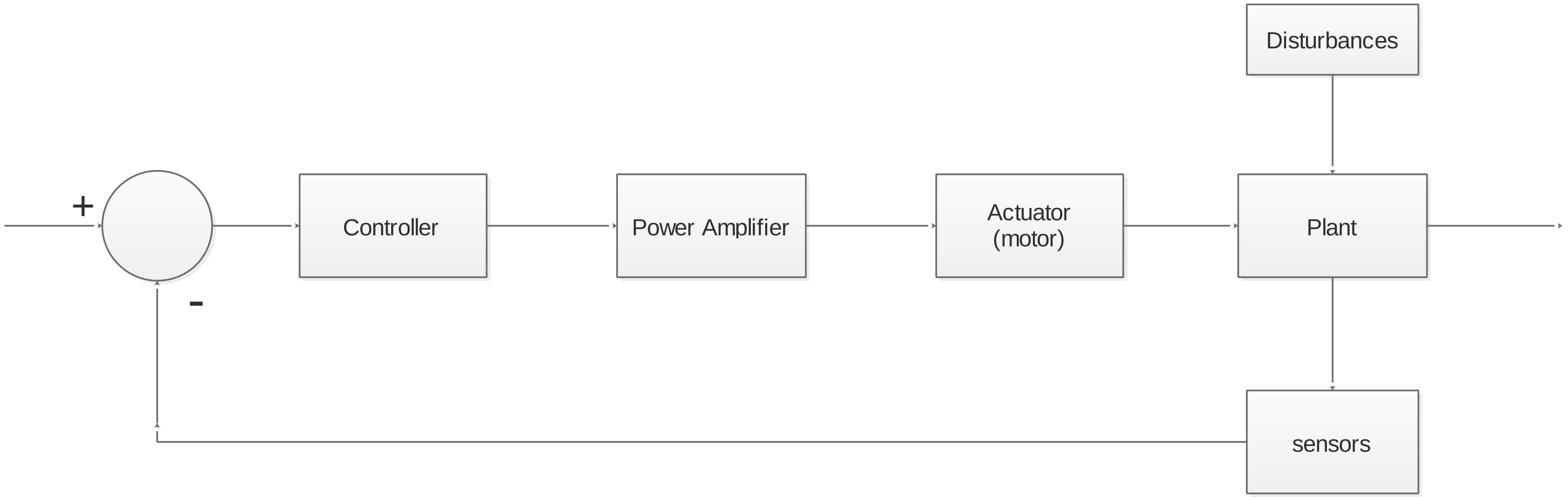
Analyzing Feedback Systems with Signal-Flow Graphs

Tuesday April 26 2016

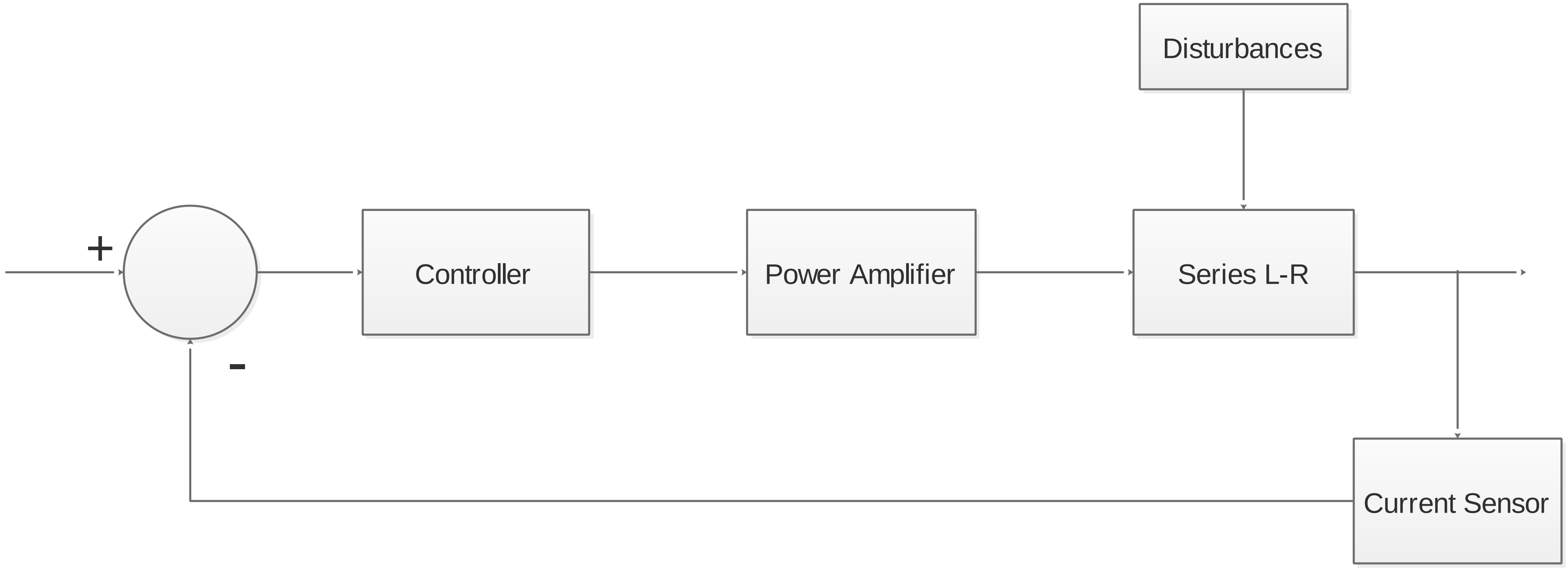
PURPOSE

- Signal flow graphs facilitate finding transfer functions of linear systems
- They aid with an intuitive understanding
- Once the flow graph is drawn Mason’s gain formula allows writing the transfer function by inspection of the graph
- They don’t aid in analyzing the effects of nonlinearities or initial conditions

BLOCK DIAGRAM



CURRENT AMPLIFIER



SIGNAL-FLOW GRAPHS

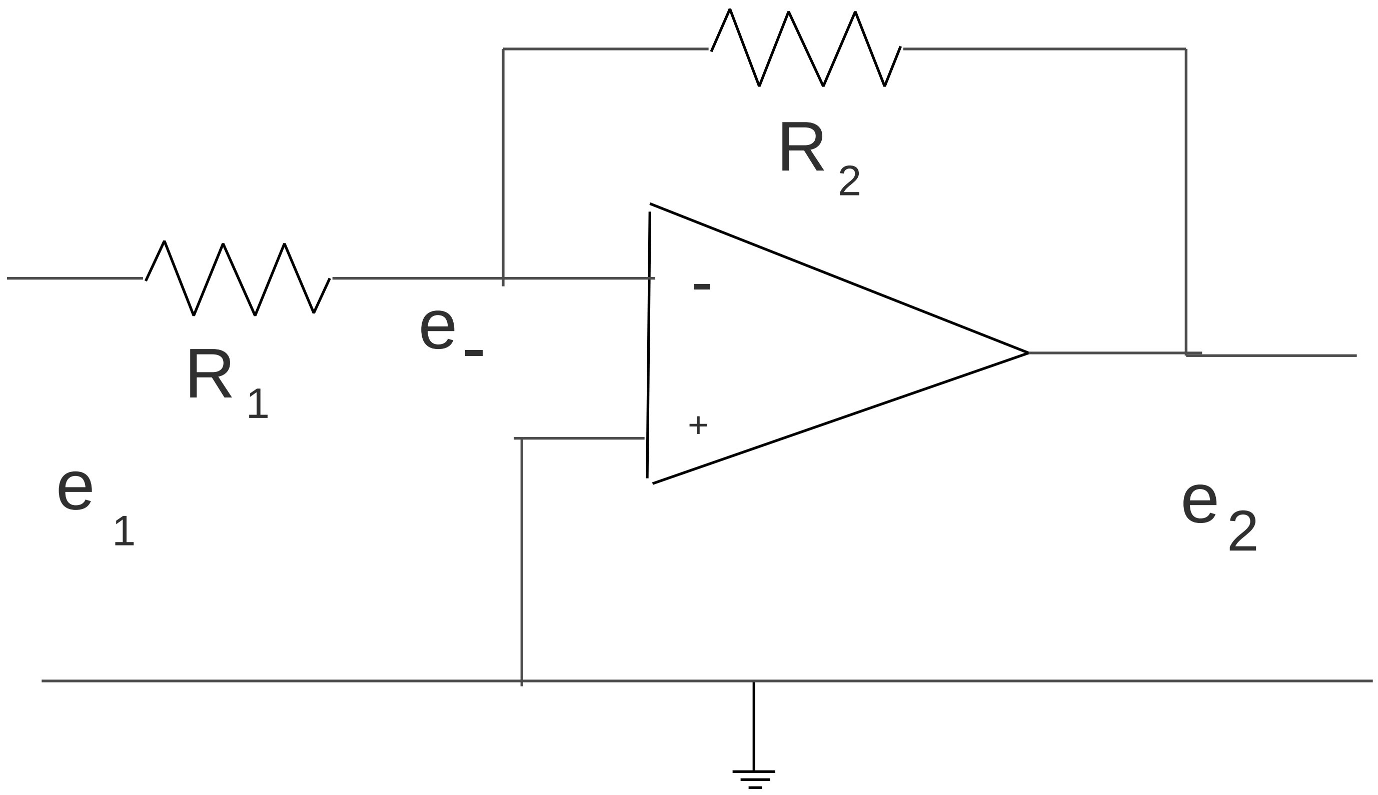
Consists of nodes and branches:

- Signals flow along branches in direction of arrows
- Node signal is sum of all signals entering
- Signals at node drive all outgoing branches
- The signal at the output of a branch is the product of the signal on the node driving it times the transmittance of the branch

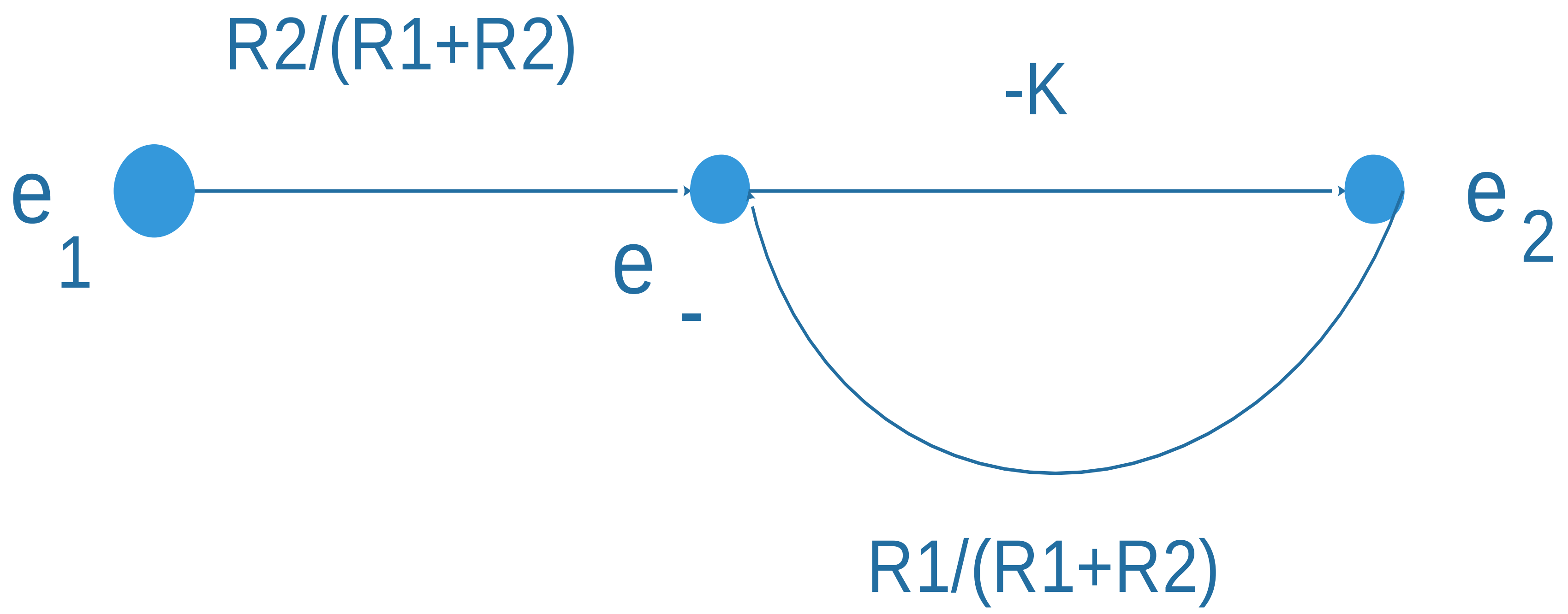
MASON'S GAIN FORMULA

- G = $\frac{\sum Gk \; \Delta k}{\Delta}$
- $\Delta = 1 - \sum L1 + \sum L2 - \cdots$
- Gk = gain of kth forward path
- Δk = Value of Δ not touching kth forward path

INVERTING OP-AMP



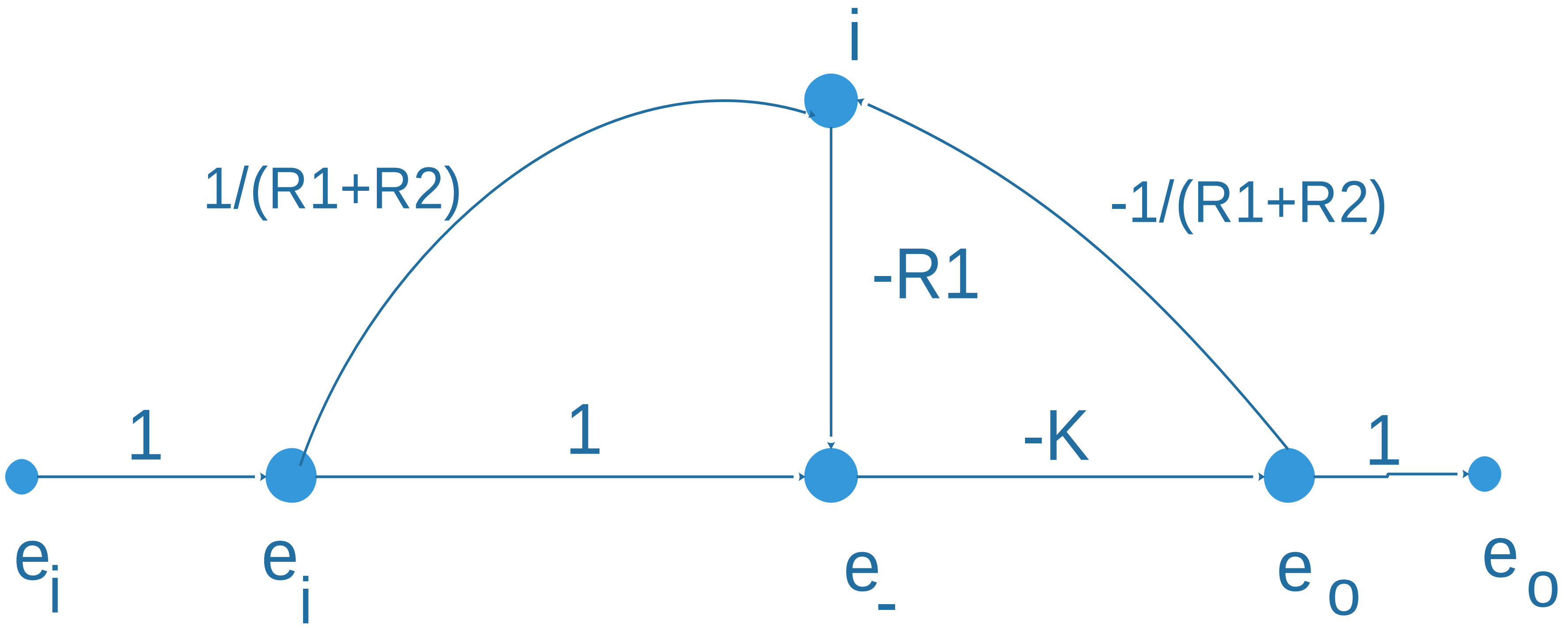
SIGNAL-FLOW GRAPH



INVERTING AMPLIFIER TRANSFER FUNCTION

- $\frac{e2}{e1} = \frac{-KR2/(R1+R2)}{1+KR1/(R1+R2)}$
- For kR1/(R1+R2) >>1
- $\frac{e2}{e1}$ = - $\frac{R2}{R1}$

INVERTING AMPLIFIER



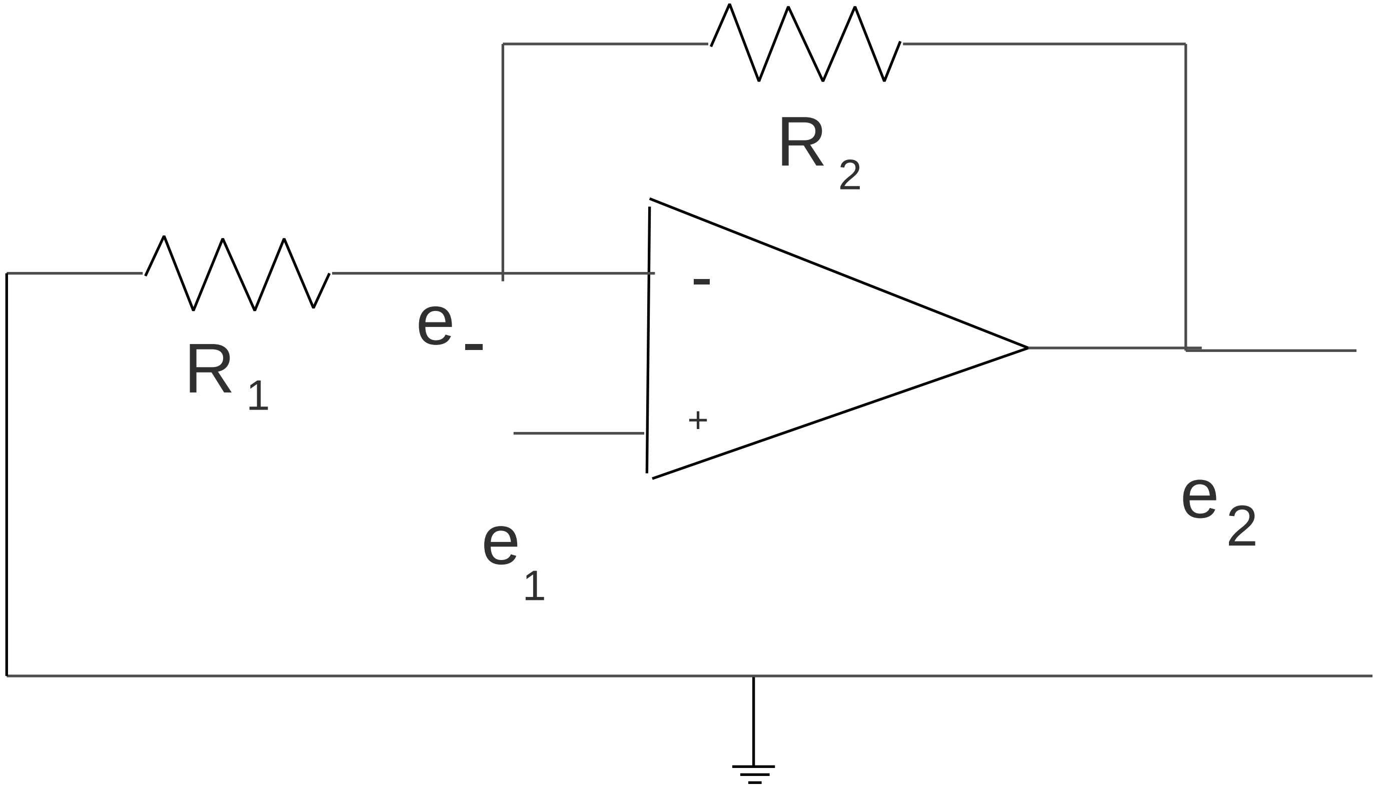
INVERTING AMPLIFIER TRANSFER FUNCTION

- $\frac{eo}{ei} = \frac{K(\frac{R1}{R1-R2}-1)}{1+\frac{R1K}{R1+R2}} = \frac{R2/(R1+R2)}{R1/(R1+R2)} = \frac{R2}{R1}$

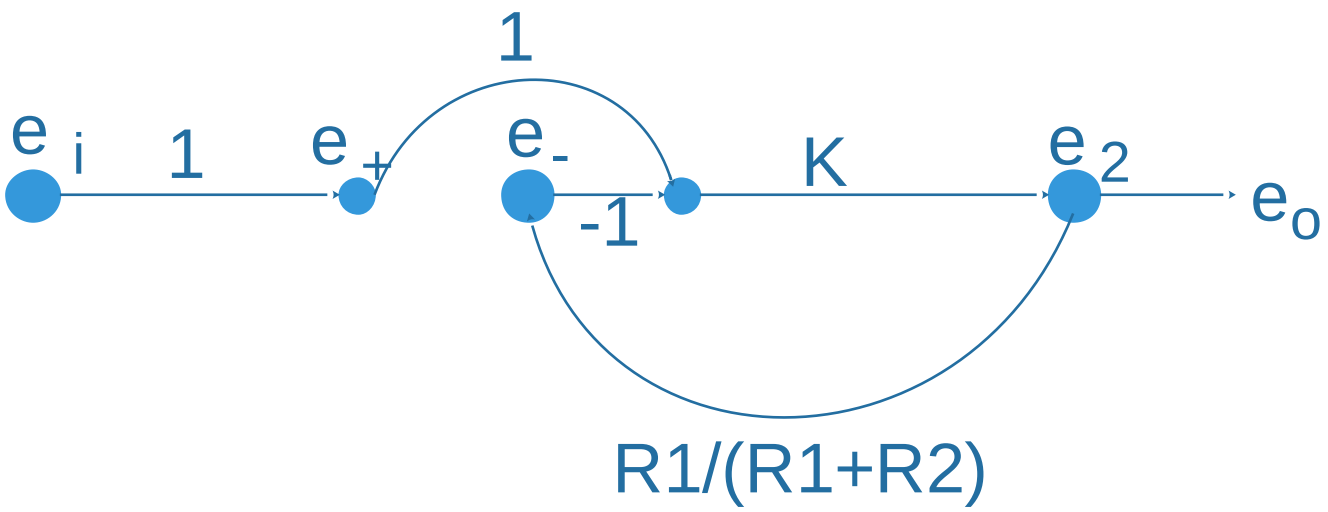
INTEGRATER

- {Int}f(t)dt < --->1/Ts
- Substitute $z=1/sc$ for R2
- $G(s) = 1/R1cs = 1/Ts$

NONINVERTING OP-AMP



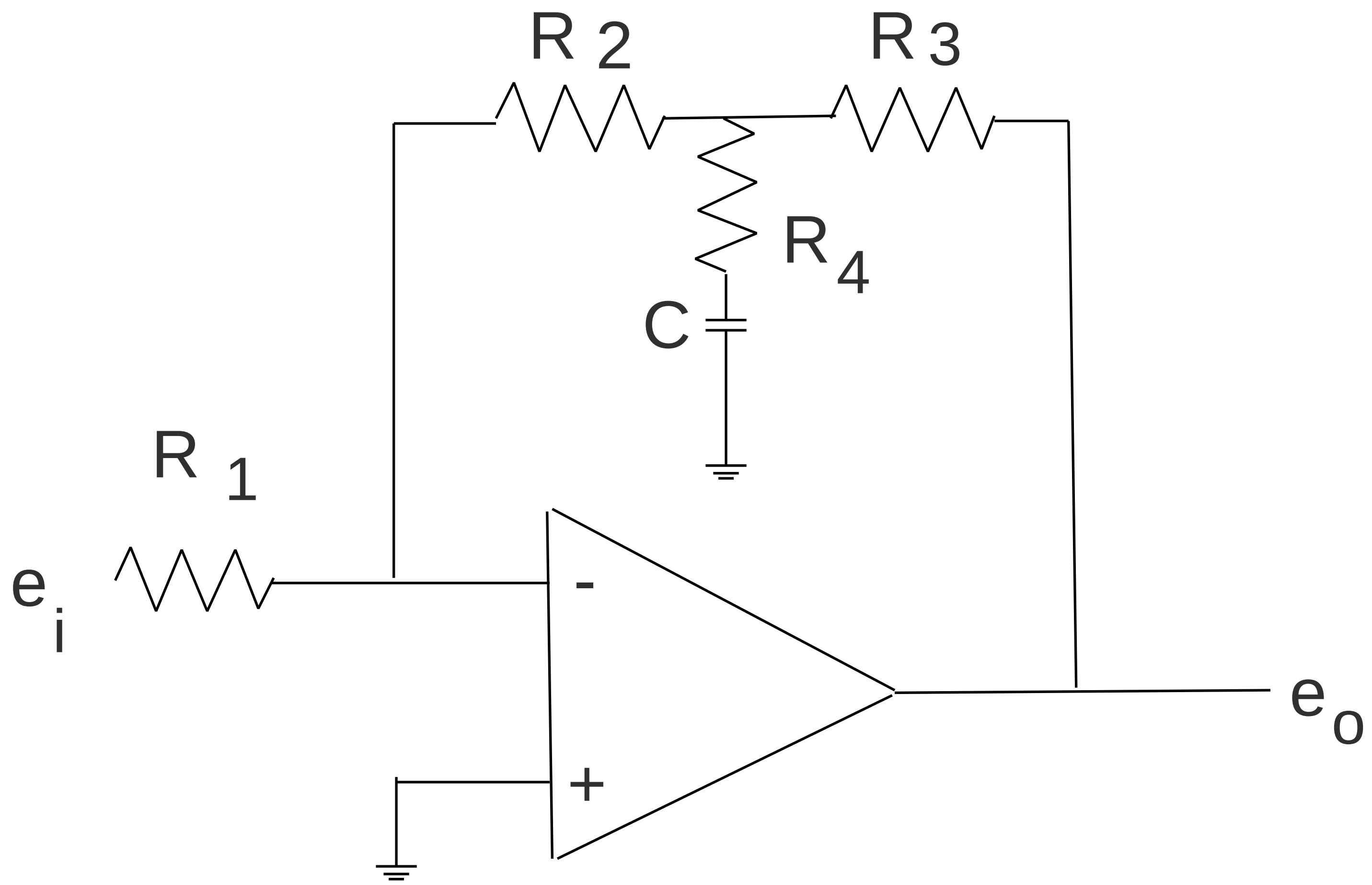
FLOW GRAPH NONINVERTING OP-AMP



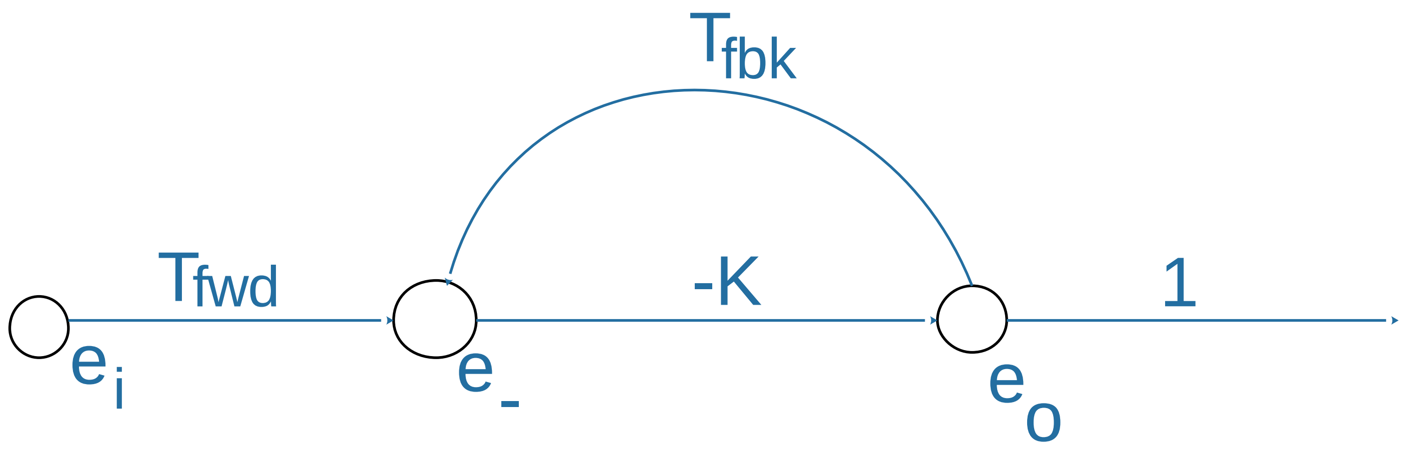
NONINVERTING AMPLIFIER TRANSFER FUNCTION

- $\frac{eo}{ei} = \frac{K}{1+KR1/(R1+R2)} = 1+\frac{R2}{R1}$

LEAD-LAG NETWORK



FLOW GRAPH NONINVERTING AMPLIFIERS



TRANSFER FUNCTION

- $\frac{eo}{ei} = \frac{Tfwd(-K)}{1-KTfbk} = \frac{-Tfwd}{Tfbk}$
- Let Z4 = R4 + 1/sc
- $\frac{eo}{ei} = \frac{R2+R3Z4/(R3+Z4)}{R1Z4/(R3+Z4)} = \frac{R2+R3}{R1}$ For Z4 ->∞
- $\frac{eo}{ei} = \frac{\frac{R2R3}{R2+R3}+Z4}{Z4}\frac{R2+R3}{R1}$
- $\frac{eo}{ei} = \frac{\frac{(R2R3)}{R2+R3}+R4+1/sc}{R4+1/sc}\frac{R2+R3}{R1} = \frac{sc\left[\frac{R2R3}{R2+R3}+R4\right]+1}{scR4+1}\frac{R2+R3}{R1}$

GENERAL FORM OF LEAD-LAG EQUATION

- A$\frac{(STld+1)}{(STlg+1)}$
- General form networks:

$$\frac{A\prod(Complex\ Conjugate\ roots)\prod(sTi+1)}{\prod(Complex\ Conjugate\ roots)\prod(sTk+1)}$$

MOST NETWORK TRANSFER FUNCTIONS

G(S) = A$\frac{\prod(STi+1)}{\prod(STK+1)}$

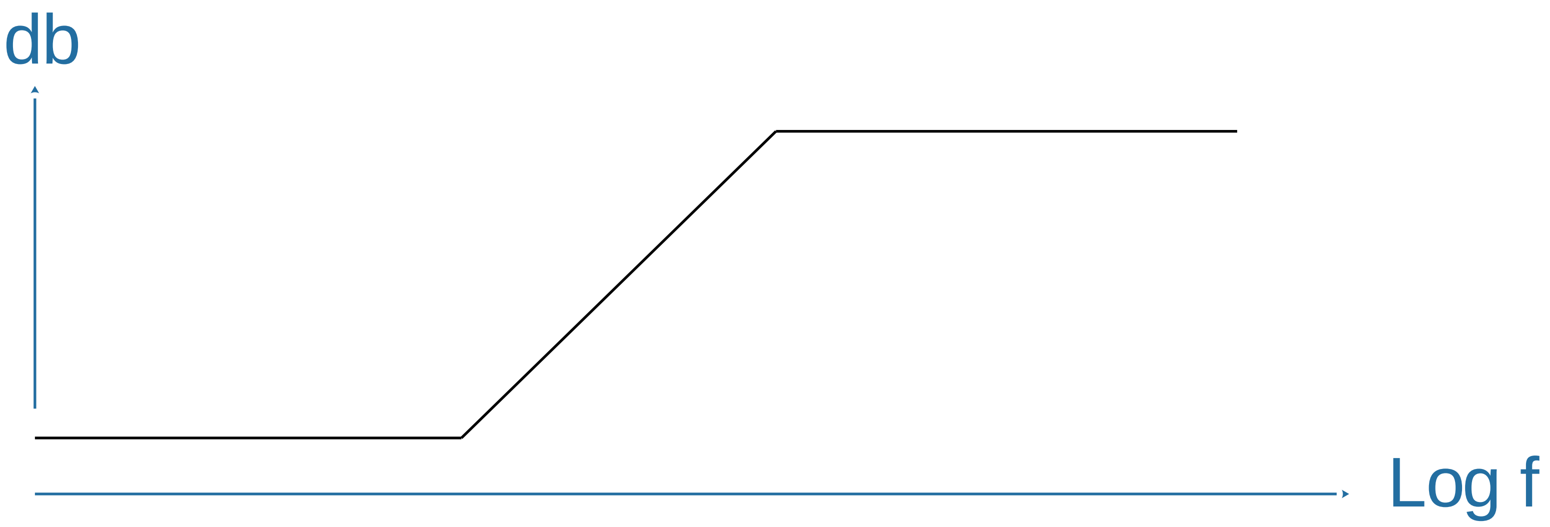
GAIN THAT IS A FUNCTION OF FREQUENCY ALSO IMPLIES PHASE THAT IS A FUNCTION OF FREQUENCY

CONTINUED

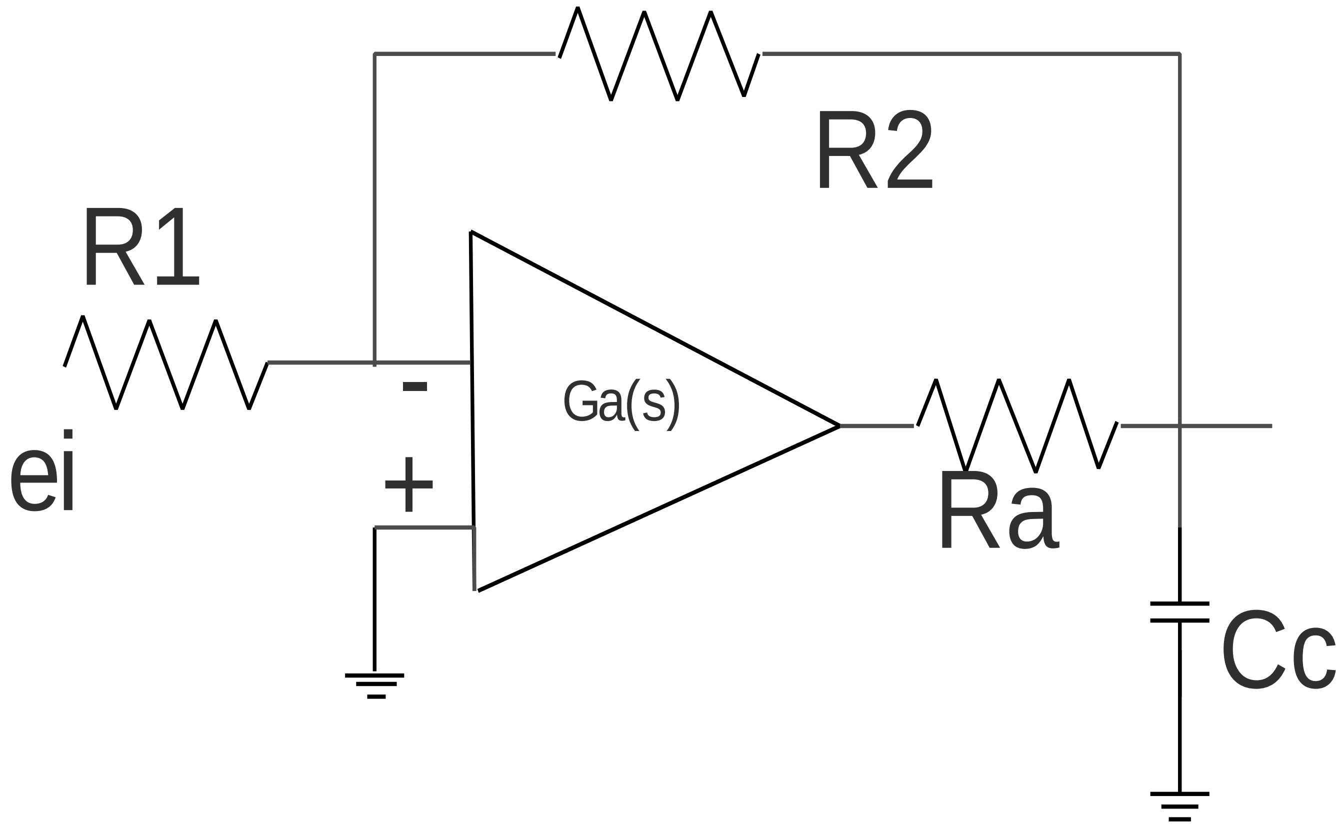
- Examine term (ST +1) as S -> jω
- Term becomes (jωT +1)
- At jωT equal 1 amplitude is $\sqrt{2}$ and phase is 45 degrees
- When this term is in numerator phase positive
- When in denominator phase is negative

GRAPH db VERSUS LOG OF FREQUENCY

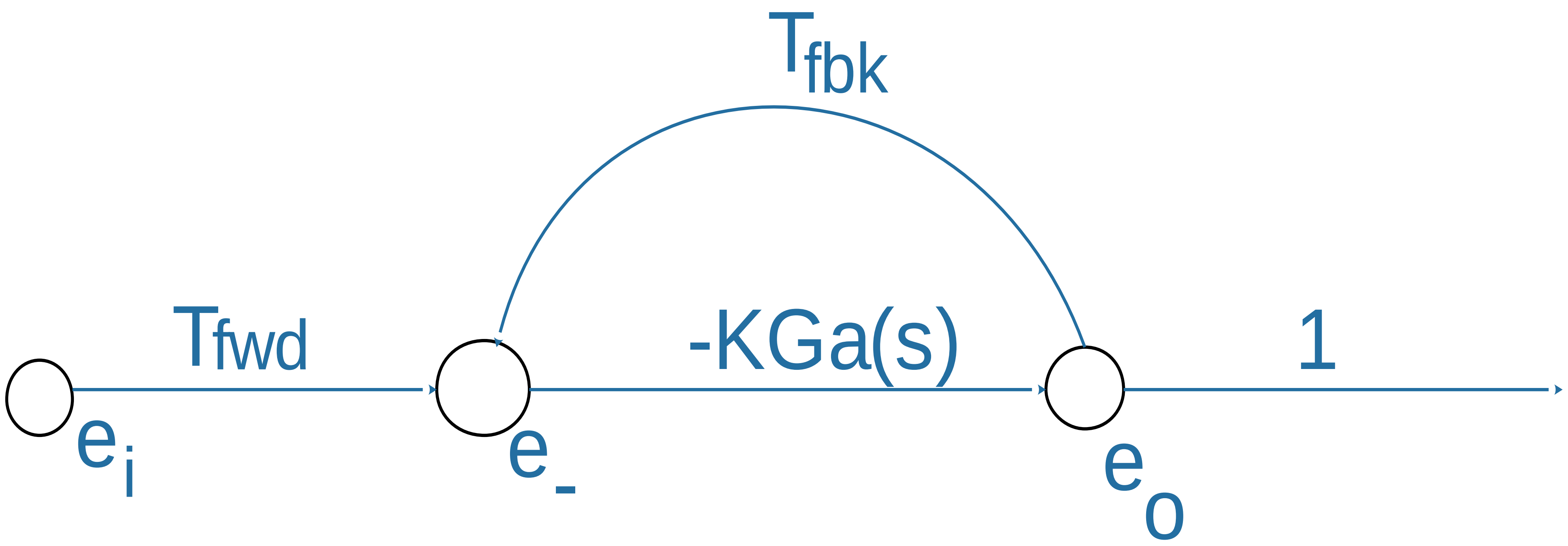
- Assyptotic gain versus frequency



OP-AMP LOADED WITH CABLE CAPACITANCE



FLOW GRAPH



OPEN LOOP GAIN

- $G(s)_{oL} = -kG_a(s) \bullet Tfbk$
- TYPICAL:
- $G_a(s) = \frac{1}{(sTd+1)\prod(sTi+1)}$ $\quad$ *the terms* $\prod(sTi+1)$ are above gain equals 1
- $Tfbk = \frac{R1/(R1+R2+Ra)}{\frac{sCc(R1+R2)Ra}{(Ra+R1+R2)}+1}$
- (R2+R1) >> Ra
- $Tfbk = \frac{R1/(R1+R2)}{(sCcRa+1)}$

STABILITY

- At the frequency where Tfbk•K•Ga(s), the open loop gain, equals 1 the phase should be significantly positive (say 45 degrees). If it is negative oscillation grows. Much below 45 degrees circuit rings on noise.

OPEN LOOP GAIN (CONT)

- G(S) = $\dfrac{\frac{R1}{R1+R2}}{(sCcRa+1)} \bullet \dfrac{K}{(sTd+1)\prod(sTi+1)}$
- The (sTi + 1) terms add negative phase shift and limit maximum amplifier bandwidth
- CcRa is time constant caused by cable capacitance
- If it occurs before gain falls to one it adds between 45 and 90 degrees additional phase shift causing the amplifier to become unstable