Digital Signal Processing For Radar Applications

Altera Corporation

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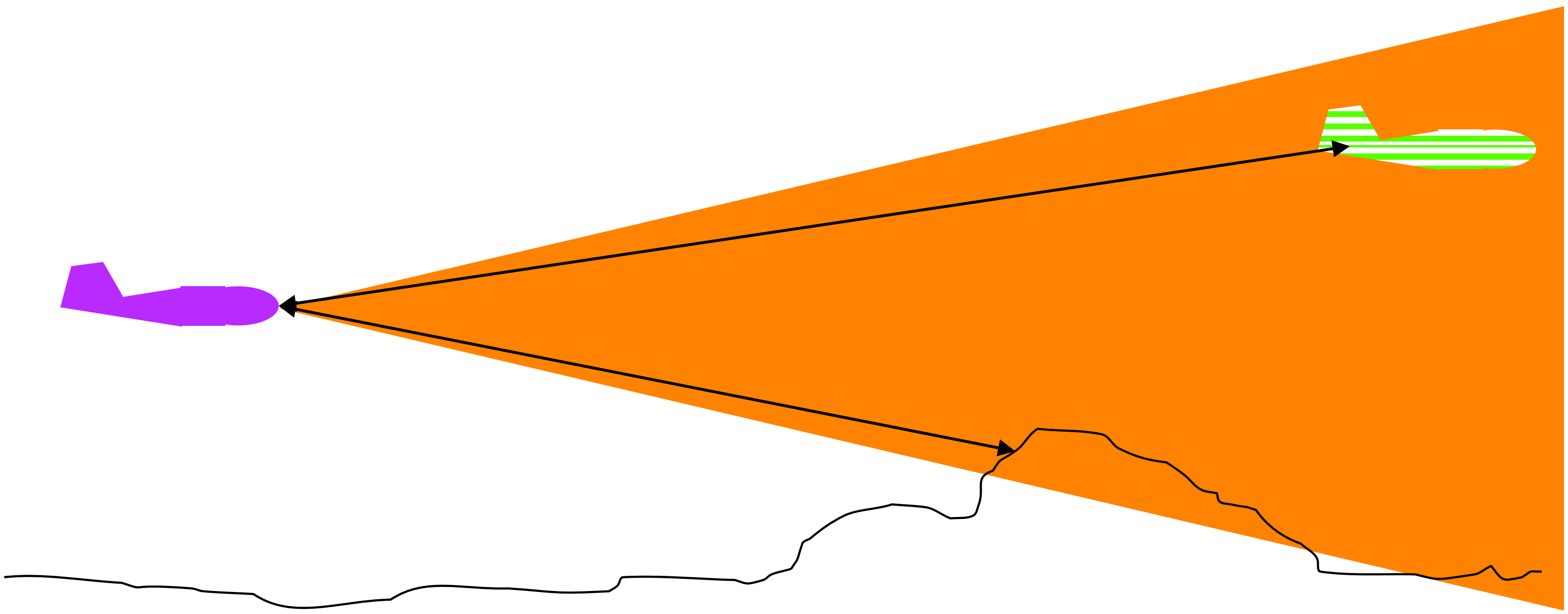
Radar: RAdio Detection And Ranging

Need a directional radio beam

Measure time between transmit pulse and receive pulse

Find Distance: Divide speed of light by interval time



Radar Band (Frequency) Terminology

Radar Band	Frequency (GHz)	Wavelength (cm)
Millimeter	40 to 100	0.75 to 0.30
Ka	26.5 to 40	1.1 to 0.75
K	18 to 26.5	1.7 to 1.1
Ku	12.5 to 18	2.4 to 1.7
X	8 to 12.5	3.75 to 2.4
C	4 to 8	7.5 to 3.75
S	2 to 4	15 to 7.5
L	1 to 2	30 to 15
UHF	0.3 to 1	100 to 30

$\lambda = v / f$ where

f = wave frequency (Hz or cycles per second)

λ = wavelength (centimeters)

v = speed of light (approximately 3×10^{10} centimeters/second)

Radar Band often dictated by antenna size requirements

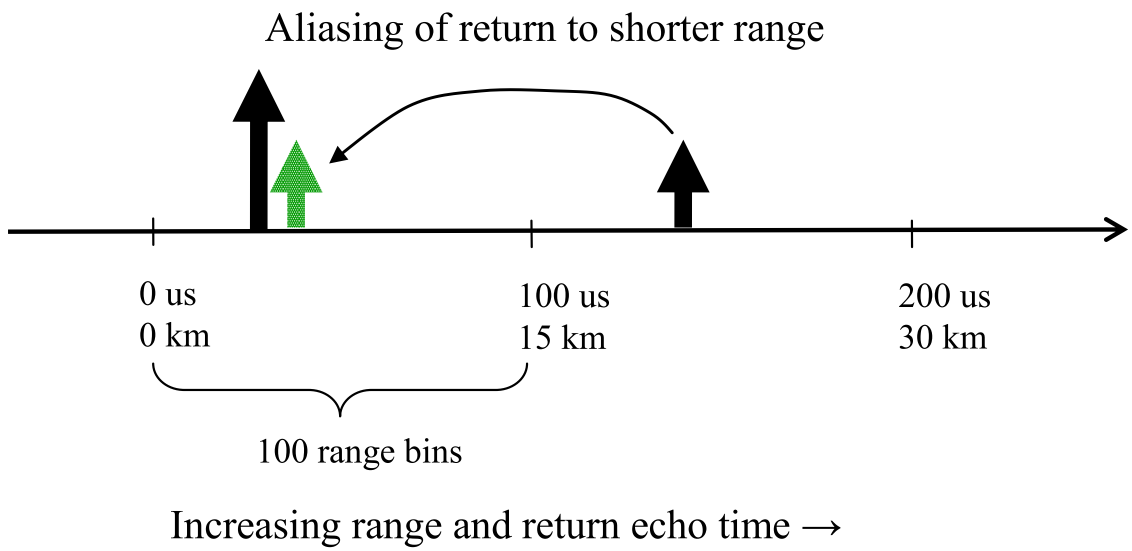
Radar Range Equation

Receiver Power $P_{receive} = P_t G_t A_r \sigma F^4 (t_{pulse} / T) / ((4\pi)^2 R^4)$
where

P_t = transmitted power
G_t = antenna transmit gain
A_r = Receive antenna aperture area
σ = radar cross section (function of target geometric cross section, reflectivity of surface, and directivity of reflections)
F = pattern propagation factor (unity in vacuum, accounts for multi-path, shadowing and other factors)
t_{pulse} = duration of receive pulse
T = duration of transmit interval
R = range between radar and target

Transmit Pulse Repetition Frequency (PRF)

- From 100s of Hz to 100s of kHz
- Can cause range "ambiguities" if too fast

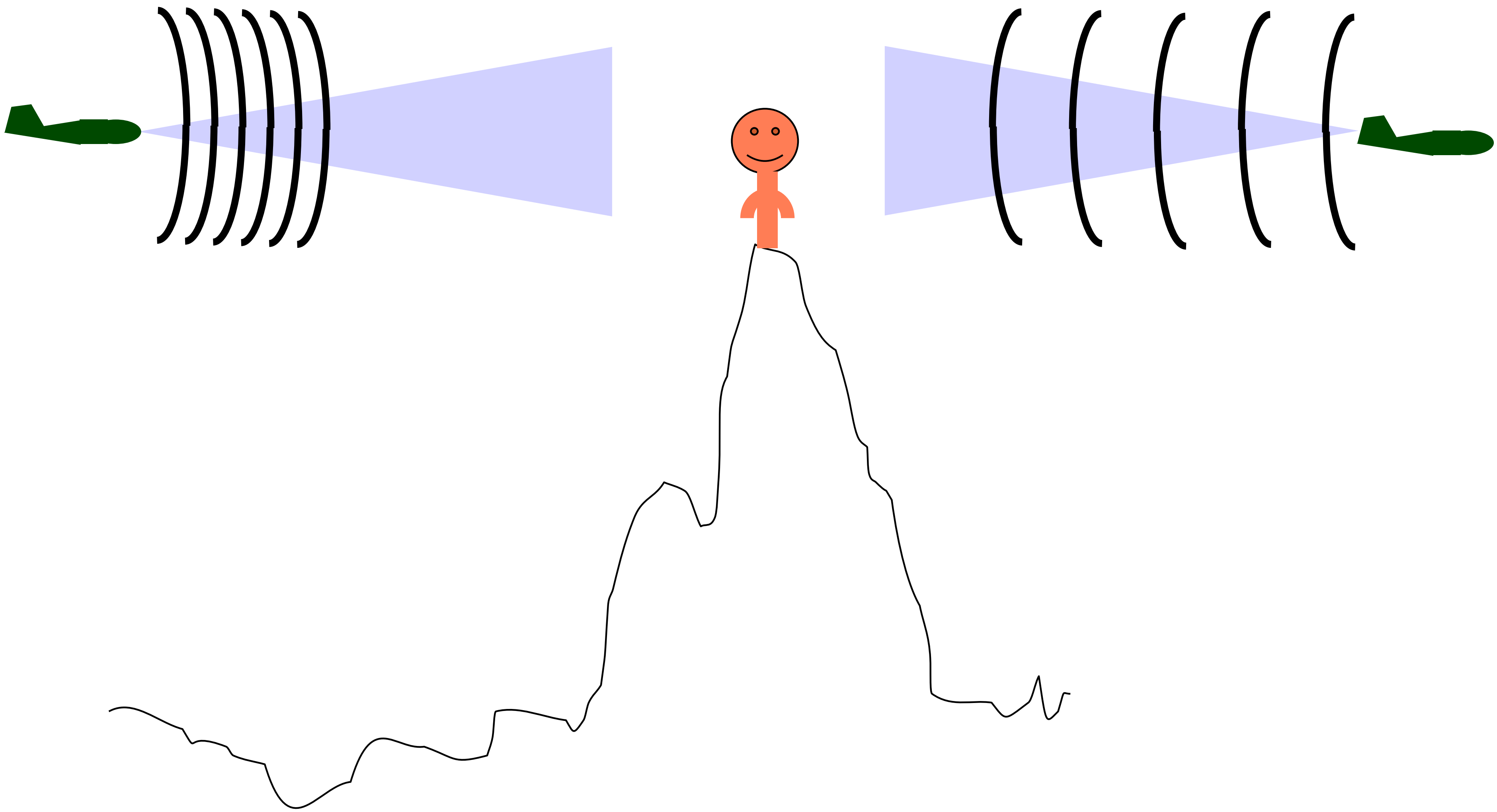


Assume PRF of 10 kHz (100 us), therefore
$R_{maximum}$ = (3x10^8 m/sec) (100x10^{-6} sec) / 2 = 15 km

Target 1 at 5 km range: t_{delay} = 2 $R_{measured}$ / v_{light} = 2 (5x10^3) / 3x10^8 = 33 us

Target 2 at 21 km range: t_{delay} = 2 $R_{measured}$ / v_{light} = 2 (21x10^3) / 3x10^8 = 140 us

Doppler concept – frequency shift through motion



Doppler effect

Frequency shift in received pulse: $f_{Doppler} = 2\, v_{relative} / \lambda$

Example: assume X band radar operating at 10 GHz (3 cm wavelength)

Airborne radar traveling at 500 mph

Target 1 traveling away from radar at 800 mph
$V_{relative}$ = 500 – 800 = -300 mph = -134 meter/s

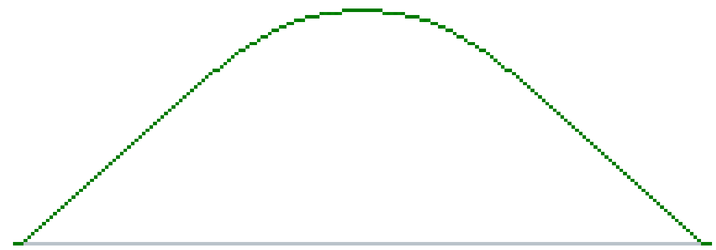
Target 2 traveling towards radar at 400 mph
$V_{relative}$ = 500 + 400 = 900 mph = 402 meter/s

First target Doppler shift = 2 (-134m/s) / (0.03m) = - 8.93 kHz

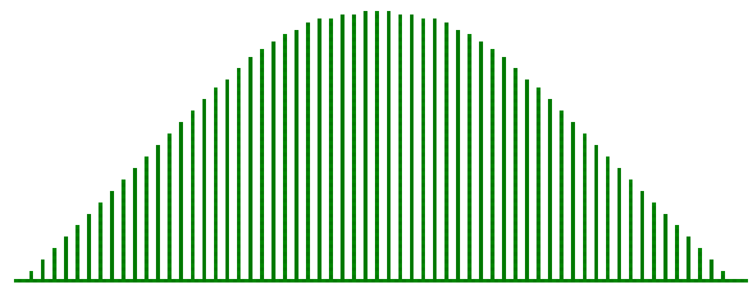
Second target Doppler shift = 2 (402m/s) / (0.03m) = 26.8 kHz

Frequency Spectrum of Pulse

Spectrum of single pulse

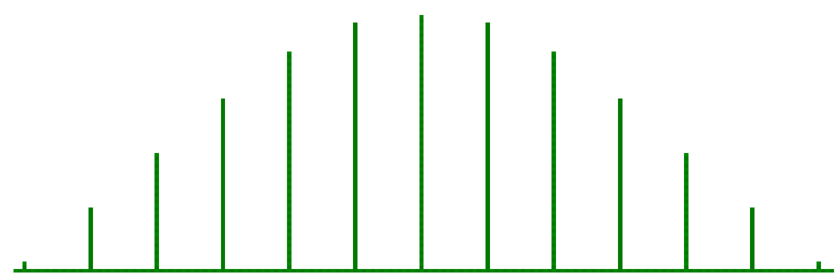


Spectrum of slowly
repeating pulse (low PRF)

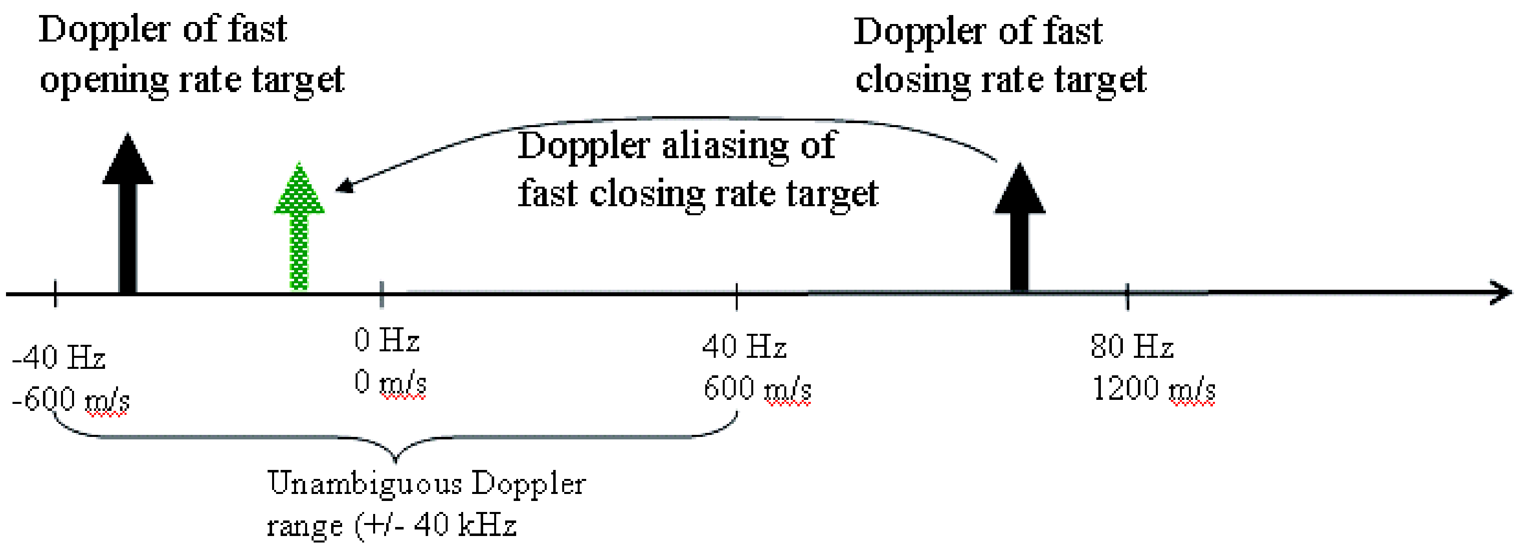


Spectrum of rapidly
repeating pulse (high PRF)

Line spacing equal to PRF



Doppler Ambiguities



Example: PRF = 80 kHz, with 10 GHz radar

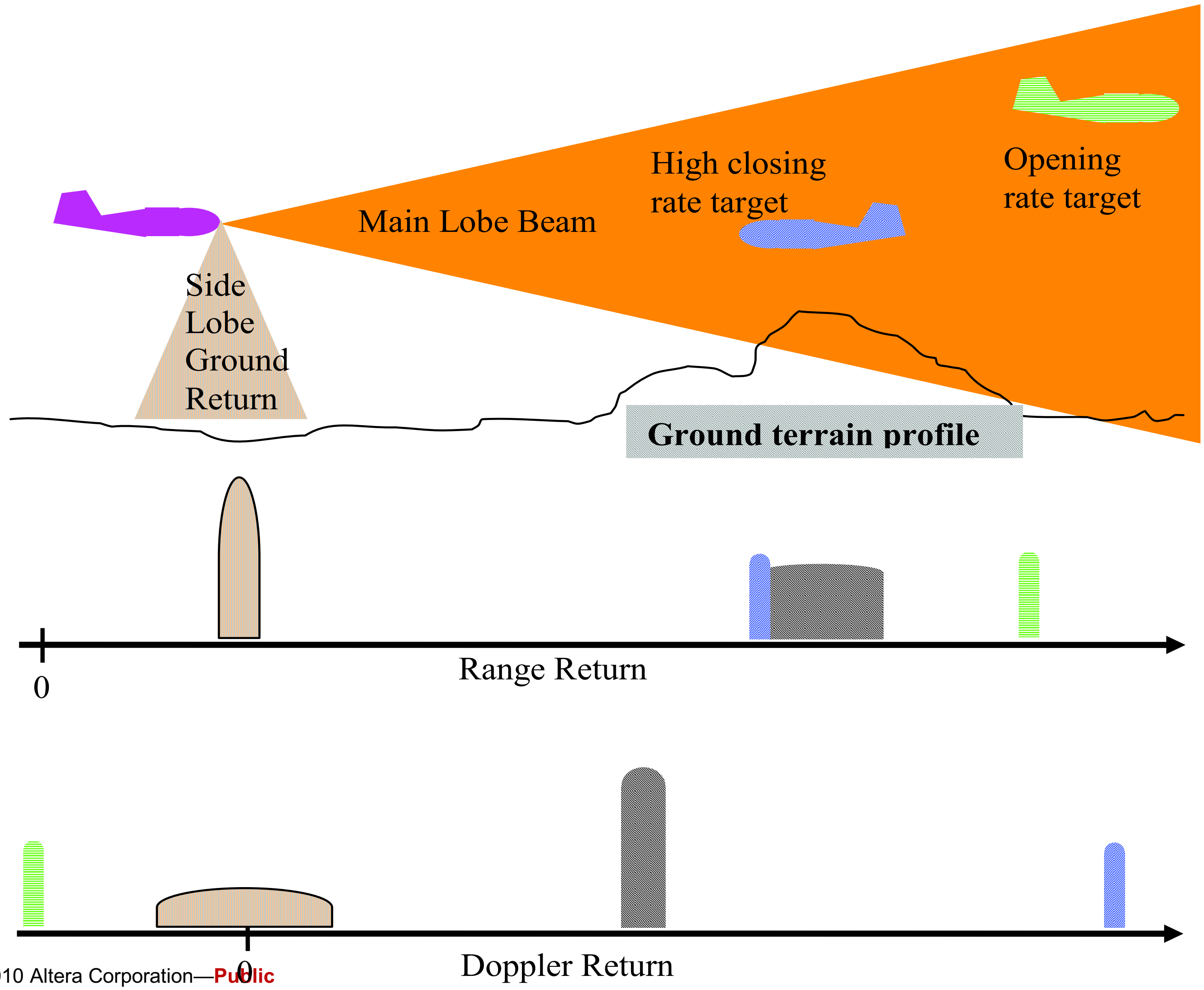
Radar–bearing aircraft maximum speed: 1200 mph = 536 m/s
Target aircraft maximum speed 1200 mph = 536 m/s
Maximum positive Doppler = 2 (1072m/s) / (0.03m) = **71.5 kHz**

Radar–bearing aircraft minimum speed: 300 mph = 134 m/s
Effective radar–bearing aircraft minimum speed with θ = 30 degree angle from target track is sin(30) = 0.5 150 mph = 67 m/s
Target aircraft maximum speed 1200 mph = 536 m/s
Maximum negative Doppler = 2 (67-536 m/s) / (0.03m) = **-31.3 kHz**

Critical Choice of PRF

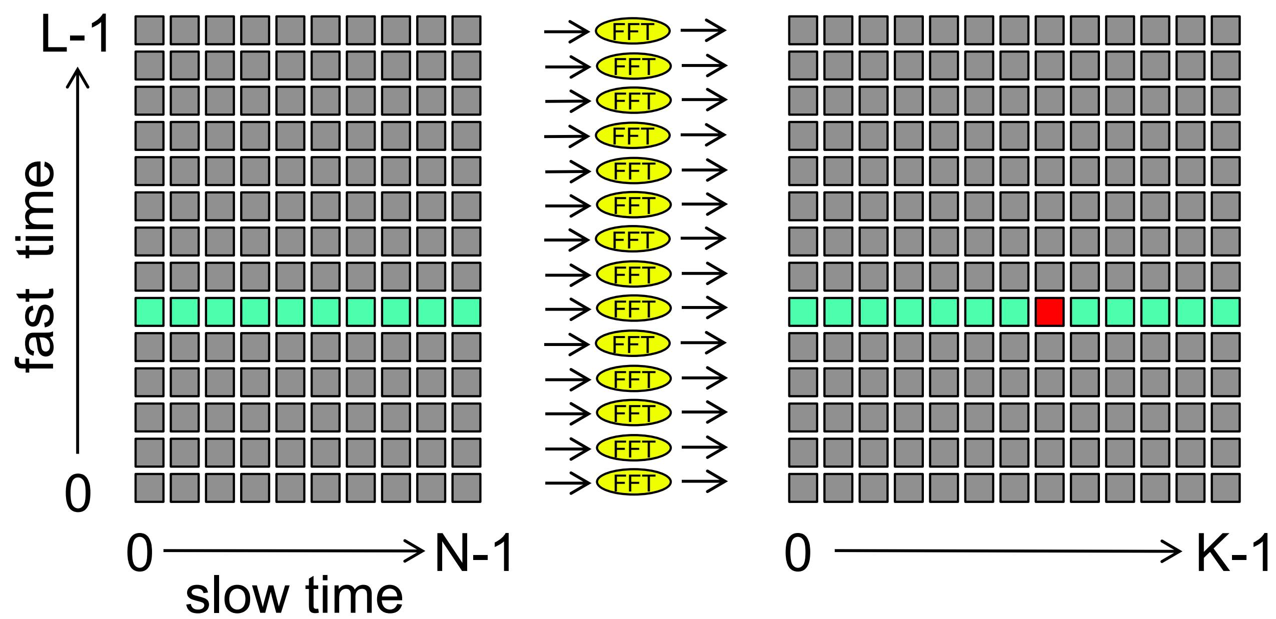
- Low PRF: generally 1-8 kHz
  - Good for maximum range detection
  - Requires long transmit pulse duration to achieve adequte transmit power – means more pulse compression
  - Excessive Doppler ambiguity
  - Difficult to reject ground clutter in main antenna lobe
- Medium PRF: generally 8-30 kHz
  - Compromise: get both range and Doppler ambiguities, but less severe
  - Good for scanning – use other PRF for precise range or for isolating fast moving targets from clutter
- High PRF: generally 30-250 kHz
  - Moving target velocity measurement and detection
  - Allows highest transmit power → greater detection ranges
  - FFT → Spectral masking → IFFT → detection processing

Using both Range and Doppler detection



Assume no range or Doppler ambiguities

Doppler Pulse Processing Basics



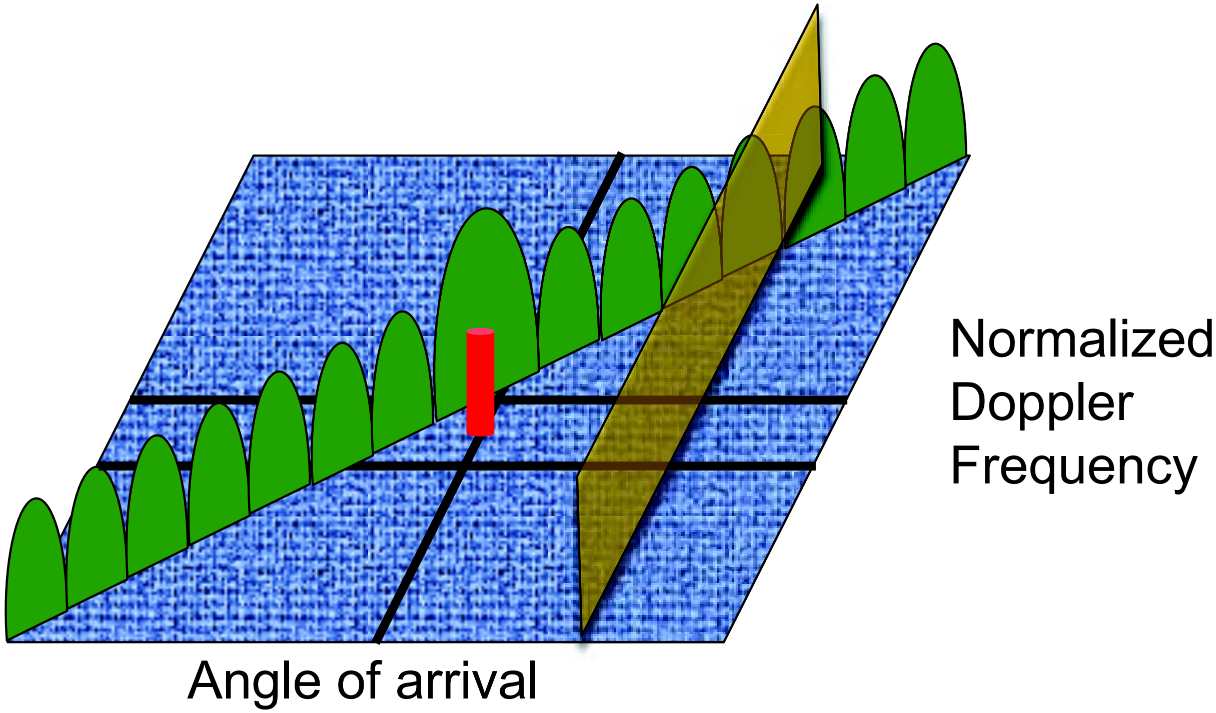
- Form L range bins in fast time
- Perform FFT across N pulse intervals for each of L ranges
- Doppler filter into K frequency bins
- Coherent processing interval (CPI) of N radar pulses
- Target discrimination in both range and Doppler frequency

STAP Algorithm

MTI Detection among clutter, jamming

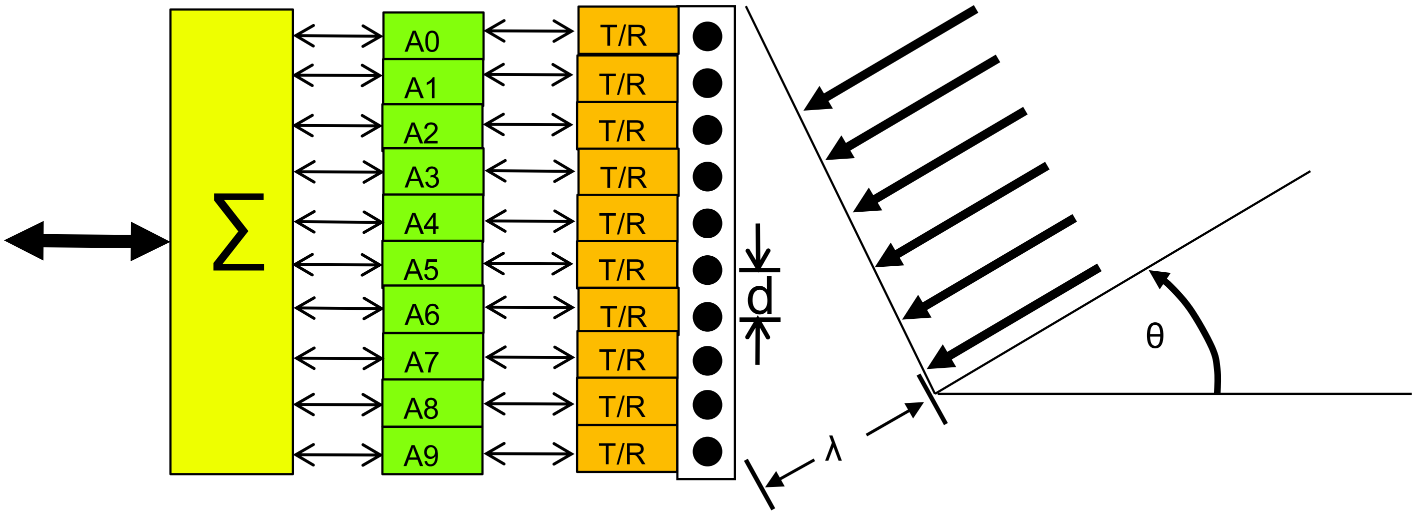
- Pulse Compression – matched filtering to optimize SINR
- MTI filtering – for gross clutter removal
- Pulse Doppler filtering – effective to resolve targets with significant motion from clutter
- STAP – temporal and spatial filtering to separate slow moving targets from clutter and null jammers
  - Very high processing requirements
  - Low latency, fast adaptation
  - Dynamic range requires floating point processing

Target Detection with Spatial Dimension



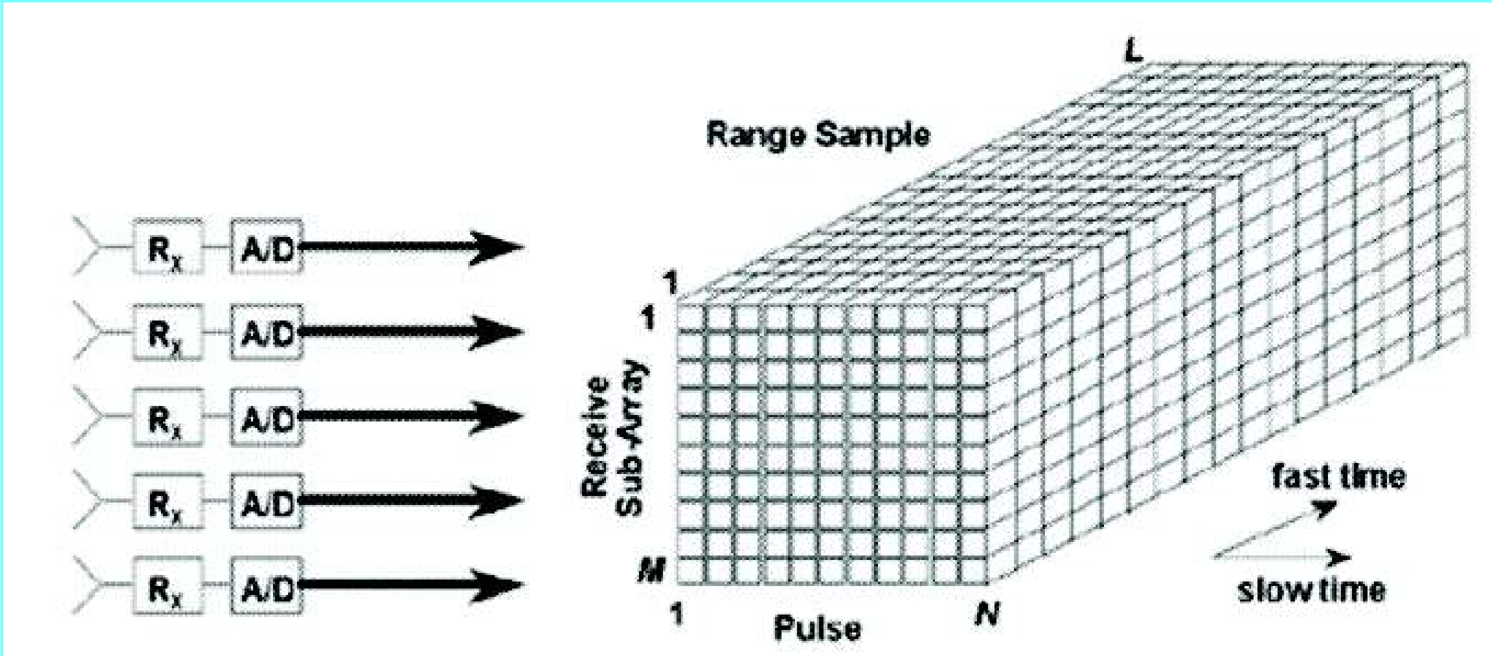
- Jammer is across all Doppler bins
- Target is same angle as main clutter
- Target is close to clutter Doppler frequency
- Spatial and Temporal filtering needed to discriminate

Beamforming using ESA



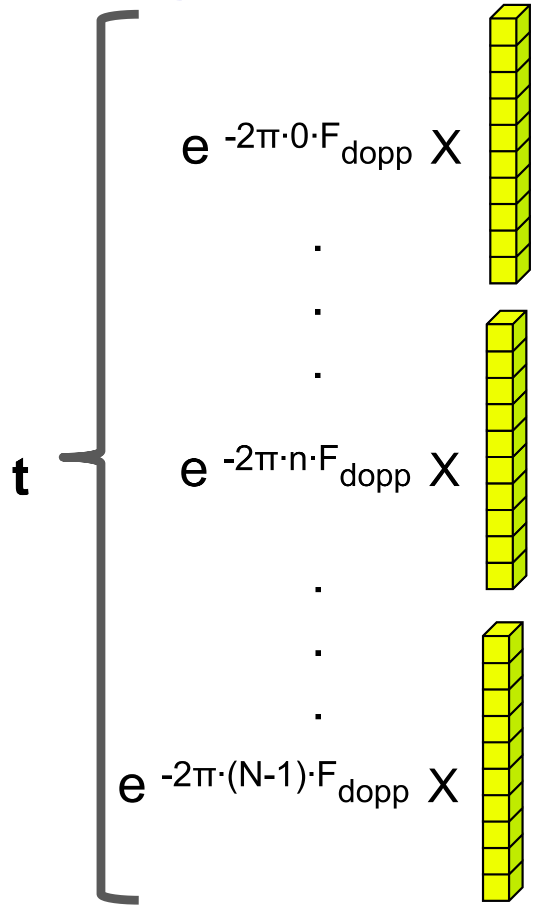
- Each T/R module sample multiplied by complex value A_m
- $A_m = e^{-2\pi d \cdot m \cdot \sin(\theta/\lambda)}$ for m = 1..M-1, for each angle θ
- Main lobe shape, side lobe height can be adjusted by multiplying angle vector with tapering window – similar to FIR filter coefficients
- All complex receive samples sum/split for single feed to/from processor
- Angular steering, determined by vector **A**

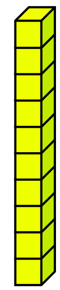
Add Spatial dimension: Radar "Datacube"



- M sub-array weighted samples (*no longer summed into single feed*)
- L sampling intervals per pulse interval (*fast time*)
- CPI of N radar pulses (*slow time*)
- Doppler pulse processing across L & N dimensions
- STAP processing across M & N dimensions

Target steering vector t = *f* (angle, Doppler)



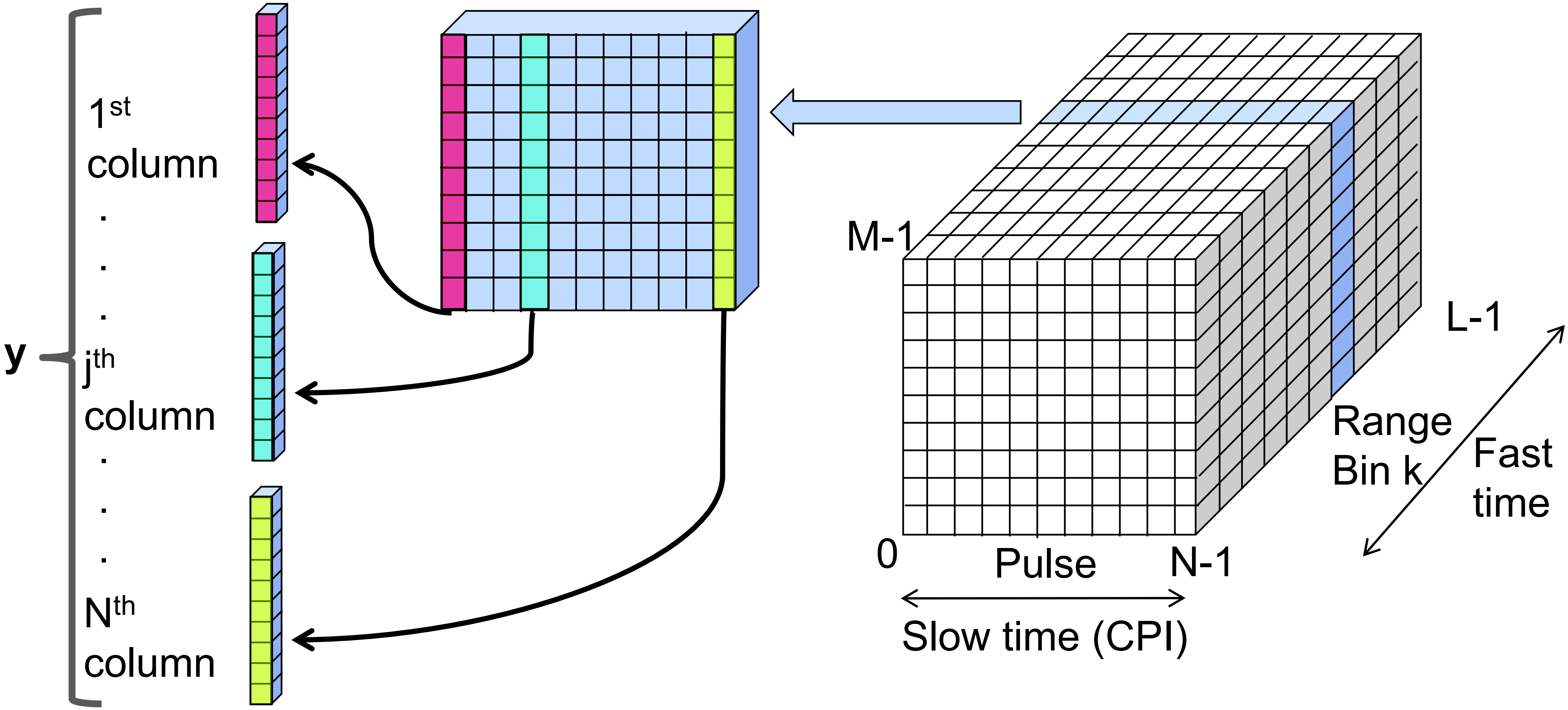
where  $\mathbf{A}_{\theta} = e^{-2\pi d\cdot m\cdot \sin(\theta/\lambda)}$

(M long vector)

t is vector of length N · M

- Temporal Steering Vector $\mathbf{F}_d$ for each Doppler frequency $F_{doppler}$
- $\mathbf{F}_d = e^{-2\pi\cdot n\cdot F_{dopp}}$ for n = 1..N-1
- $\mathbf{A}_{\theta} = e^{-2\pi d\cdot m\cdot \sin(\theta/\lambda)}$ for m = 1..M-1, for given angle of arrival θ
- $\mathbf{t} = \mathbf{F}_d \otimes \mathbf{A}_{\theta}$, Kronecker product of target Doppler and Steering vectors

Computing Interference Covariance Matrix



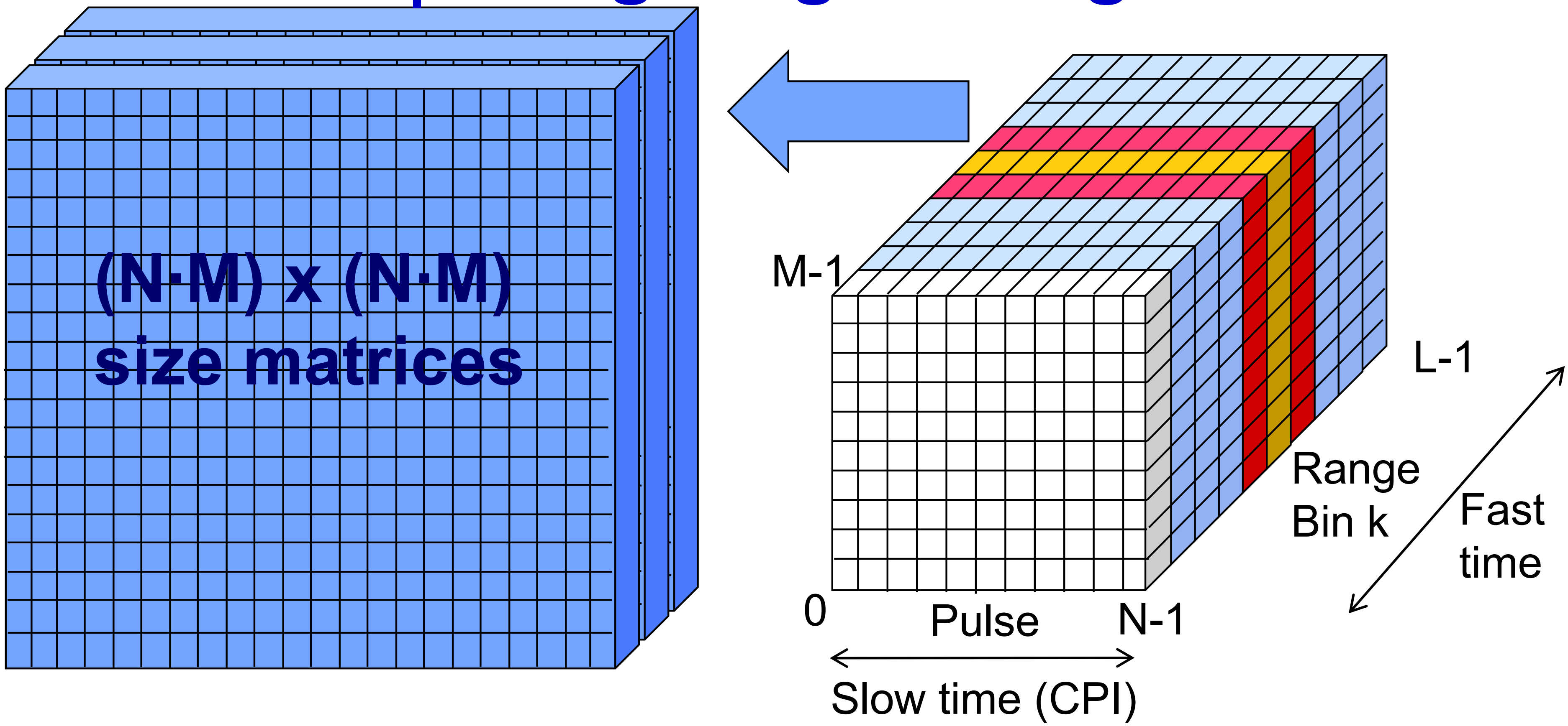
$\mathbf{S}_I = \mathbf{y}^* \cdot \mathbf{y}^T$ (vector cross product)

$\mathbf{y}$ is column vector of length N·M, so $\mathbf{S}_I$ is (N·M) x (N·M) size

$\mathbf{S}_{Interference} = \mathbf{S}_{noise} + \mathbf{S}_{jammer} + \mathbf{S}_{clutter}$ and is hermitian

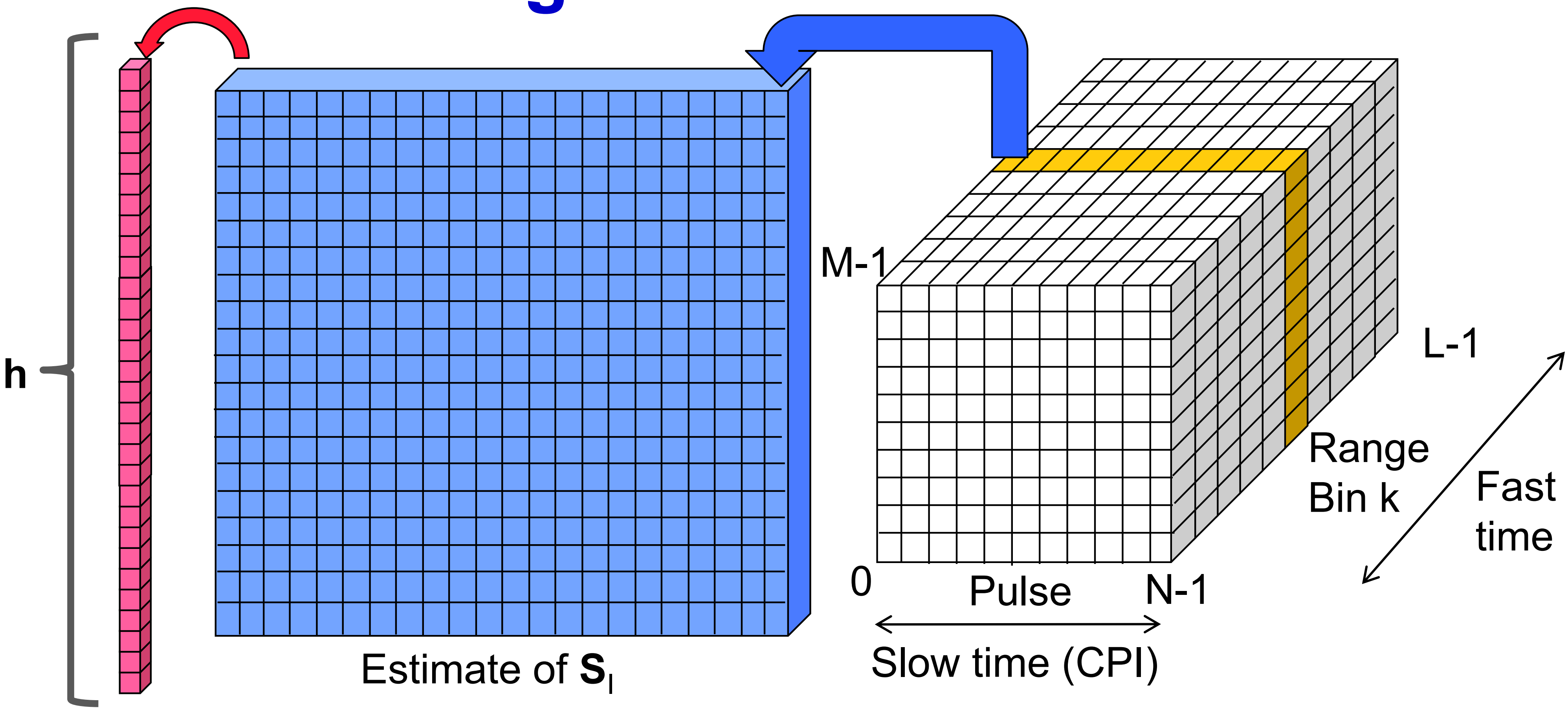
where $\mathbf{S}_{noise} = \sigma^2 \cdot \mathbf{I}$, $\mathbf{S}_{jammer}$ is block diagonal matrix, $\mathbf{S}_{clutter}$ depends upon environment

Estimate S_I using neighboring bins



- $\mathbf{S}_I$ is calculated for each neighboring range bin
- Do not want target information in covariance matrix, so estimate $\mathbf{S}_I$ in k^{th} bin by averaging, element by element, nearby $\mathbf{S}_I$ matrices
- Orange range cell k contains suspected target under test
  - Red range cells are guard bands
  - Assume nearby bins have same clutter statistics as orange cell
  - Use the nearby cells to for estimate of $\mathbf{S}_I$ to use in STAP algorithm

Calculate weight vector to maximize SIR



- Calculate optimal weighting vector $\mathbf{h} = k \cdot \mathbf{S}_I^{-1} \cdot \mathbf{t}^*$
  - k is scalar constant, $\mathbf{S}_I$ is interference covariance matrix, $\mathbf{t}$ is target steering vector
- Then find $z = \mathbf{h}^T \cdot \mathbf{y}$, where $\mathbf{y}$ is output from bin k of radar cube data
- $z(F_{doppler}, \theta)$ is then subject to target threshold detection process

Using QRD to find weighting vector **h**

- $\mathbf{h} = \kappa \cdot \mathbf{S}_l^{-1} \cdot \mathbf{t}^*$ (**h** is M·N long vector)
- $\mathbf{S}_l \cdot \mathbf{u} = \mathbf{t}^*$, where **u** includes scaling constant κ
- $\mathbf{Q} \cdot \mathbf{R} \cdot \mathbf{u} = \mathbf{t}^*$, where $\mathbf{Q} \cdot \mathbf{Q}^H = \mathbf{I}$ and $\mathbf{Q}^{-1} = \mathbf{Q}^H$
- **Q** is constructed of orthogonal normalized vectors
- **R** will become upper triangular
- $\mathbf{R} \cdot \mathbf{u} = \mathbf{Q}^H \cdot \mathbf{t}^*$
- Solve for vector **u** using back substitution
- $\mathbf{h} = \mathbf{u} / (\mathbf{t}^H \cdot \mathbf{u}^*)$ $(\kappa = \mathbf{t}^H \cdot \mathbf{u}^*)$
- $z = \mathbf{h}^T \cdot \mathbf{y}$ gives optimal output detection result

  where z is a complex scalar

STAP using covariance matrix summary

- For each range bin of interest:
  - Compute covariance matrix for each range bin
  - Estimate interference covariance matrix $\mathbf{S}_I$ by averaging the surrounding covariance matrices
  - Perform QRD upon $\mathbf{S}_I$
- Then for each $F_{doppler}$ and $\mathbf{A}_\theta$ of interest:
  - Compute $\mathbf{Q}^H \cdot \mathbf{t}^*$
  - $\mathbf{R} \cdot \mathbf{u} = \mathbf{Q}^H \cdot \mathbf{t}^* \rightarrow$ find $\mathbf{u}$ with back substitution
  - Compute $\mathbf{h} = \mathbf{u} / (\mathbf{t}^H \cdot \mathbf{u}^*)$
  - Final result $z = \mathbf{h}^T \cdot \mathbf{y}$
- This is known as power domain method

Example GFLOPs estimate (power domain)

- Process over 12 $\mathbf{A}_\theta$, 16 $F_{doppler}$ and assume prosecute 32 target steering vectors
- Use 10 range bins with a PRF = 1 kHz

▪ Compute Covariance Matrix	1.1 GFLOPS
▪ Average over 10 covariance matrices	0.4 GFLOPs
▪ QR Decomposition	37.7 GFLOPs
▪ Compute $\mathbf{Q}^H \cdot \mathbf{t}^*$ (each $\mathbf{A}_\theta$ and Doppler)	9.4 GFLOPs
▪ Solve for **u** using back substitution	4.7 GFLOPs
▪ Compute $\mathbf{h} = \mathbf{u} / (\mathbf{t}^H \cdot \mathbf{u}^*)$ and $z = \mathbf{h}^T \cdot \mathbf{y}$	0.2 GFLOPS
▪ Total	53.5 GFLOPs

- Detection for 8 possible targets over 4 possible velocities over narrow angle and Doppler, low PRF

Alternate method

“voltage domain”

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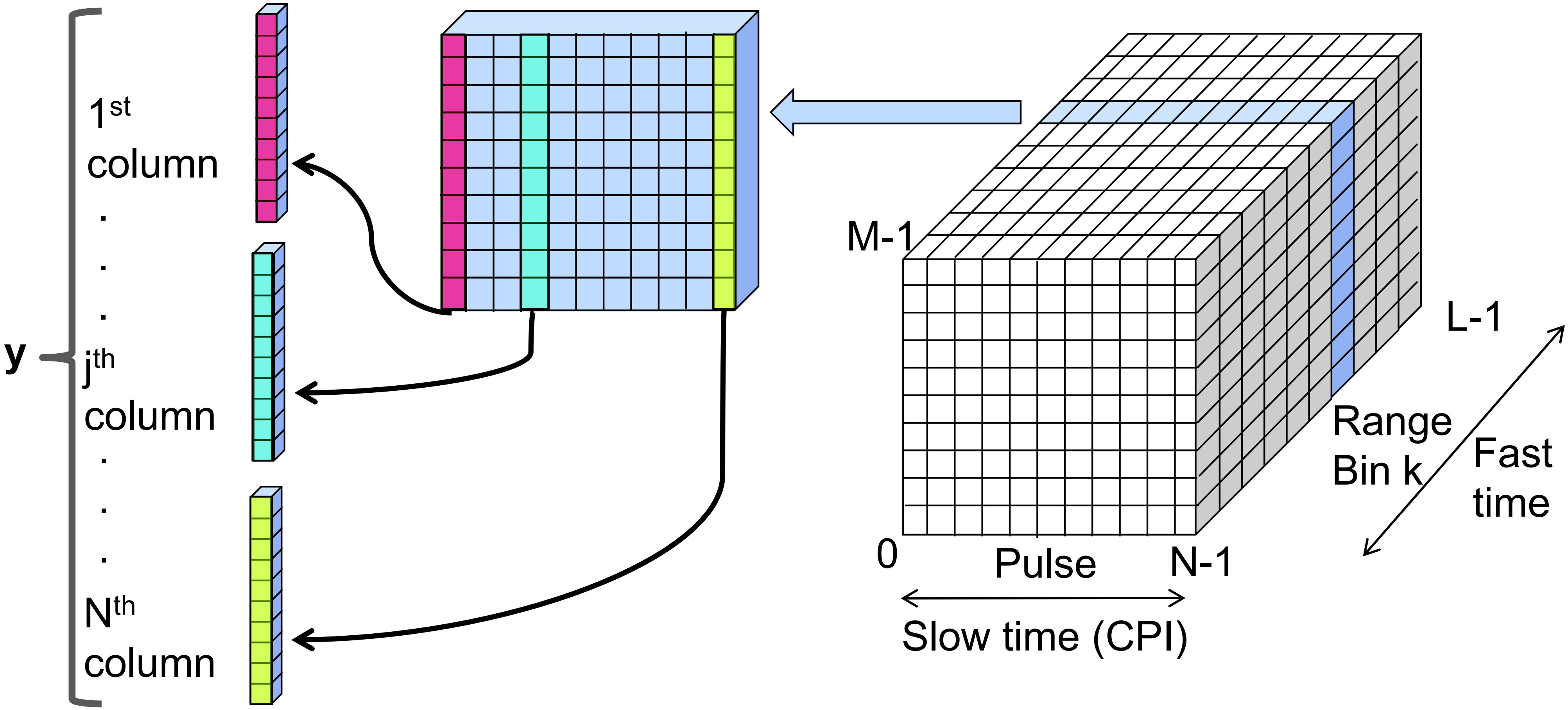
STAP using radar cube data directly



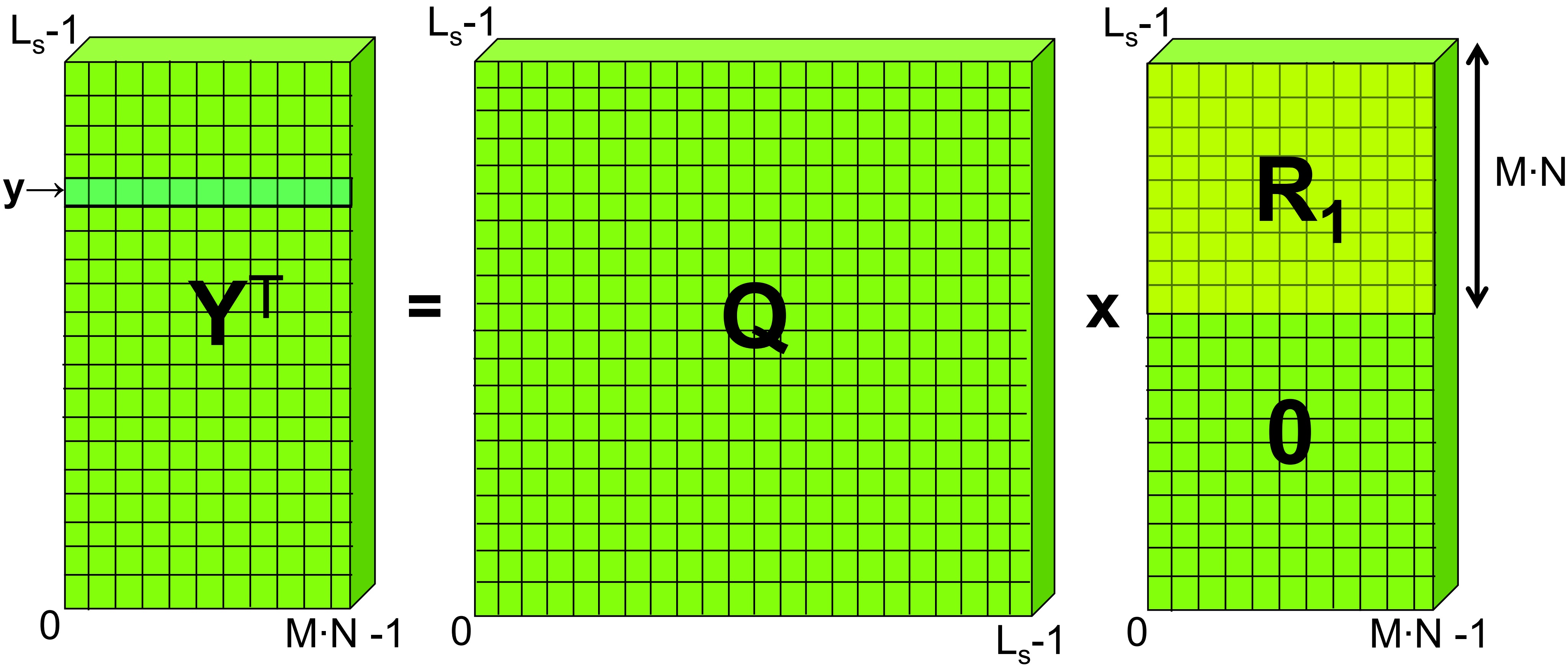
- Operate directly on **y** data vectors
  - One vector **y** per range bin
  - Use L_s range bins , where L_s > M·N range bins
  - Construct matrix **Y** = [$\mathbf{y}_0$ $\mathbf{y}_1$ $\mathbf{y}_2$ ….. $\mathbf{y}_{L_{s-1}}$], dimension [M·N x L_s]

STAP using **Y** data matrix summary

- $\mathbf{S}_l = \mathbf{Y}^* \cdot \mathbf{Y}^T = \mathbf{R}^H \cdot \mathbf{Q}^H \cdot \mathbf{Q} \cdot \mathbf{R}$ $\quad (\mathbf{Y}^T = \mathbf{Q} \cdot \mathbf{R})$

  $= \mathbf{R}^H \cdot \mathbf{R} = \mathbf{R}_1^H \cdot \mathbf{R}_1$
- Recall $\mathbf{S}_l \cdot \mathbf{u} = \mathbf{t}^*$, so substitute $\mathbf{R}_1^H \cdot \mathbf{R}_1 \cdot \mathbf{u} = \mathbf{t}^*$
- Define $\mathbf{r} \equiv \mathbf{R}_1 \cdot \mathbf{u}$
- Then for each $F_{doppler}$ and $\mathbf{A}_\theta$ of interest:
  - Solve for $\mathbf{r}$ in $\mathbf{R}_1^H \cdot \mathbf{r} = \mathbf{t}^*$ using forward substitution ($\mathbf{R}_1^H$ is lower triangular)
  - Solve for $\mathbf{u}$ in $\mathbf{R}_1 \cdot \mathbf{u} = \mathbf{r}$ using backward substitution ($\mathbf{R}_1$ is upper triangular)
  - Compute $\mathbf{h} = \mathbf{u} / (\mathbf{t}^H \cdot \mathbf{u}^*)$
  - Final result $z = \mathbf{h}^T \cdot \mathbf{y}$
- This is known as voltage domain method

Apply QRD to **Y** data matrix



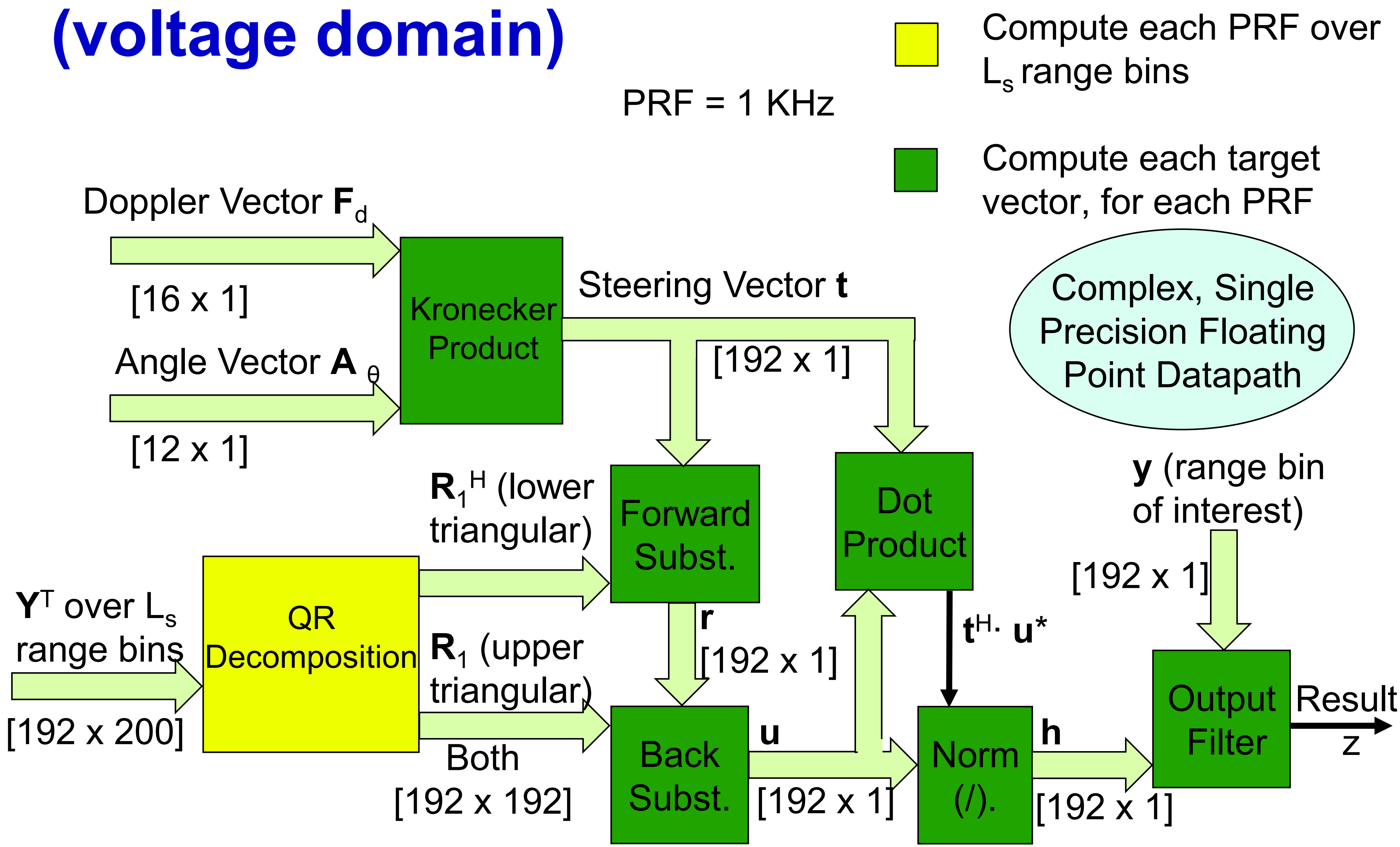
- $\mathbf{Y}^T$ = **Q** x **R**, where **R** is composed of $\mathbf{R}_1$ and **0**
  - $\mathbf{R}_1$ is upper triangular, $\mathbf{R}_1^H$ is lower triangular, [M·N x M·N]

Example GFLOPs estimate (voltage domain)

- 12 $\mathbf{A}_{\theta}$, 16 $F_{doppler}$ and 32 target steering vectors, PRF = 1 kHz, with 200 range bins
  - QR Decomposition — 40.1 GFLOPs
  - Solve for **r** using forward substitution — 4.7 GFLOPs
  - Solve for **u** using back substitution — 4.7 GFLOPs
  - Compute $\mathbf{h} = \mathbf{u} / (\mathbf{t}^{H} \cdot \mathbf{u}^*)$ and $z = \mathbf{h}^{T} \cdot \mathbf{y}$ — 0.2 GFLOPs
  - Total — 49.7 GFLOPs

- Higher rate and resolution system: 48 $\mathbf{A}_{\theta}$, 16 $F_{doppler}$ 64 target vectors, PRF = 1 kHz, 1000 range bins
  - QR Decomposition — 3.51 TFLOPs
  - Solve for **r** using forward substitution — 37.7 GFLOPs
  - Solve for **u** using back substitution — 37.7 GFLOPs
  - Compute $\mathbf{h} = \mathbf{u} / (\mathbf{t}^{H} \cdot \mathbf{u}^*)$ and $z = \mathbf{h}^{T} \cdot \mathbf{y}$ — 0.8 GFLOPs
  - Total — 3.59 TFLOPs

FPGA Processing Flow Implementation (voltage domain)



Voltage verses Power domain methods

- Voltage Domain
  - Operates directly on the data
  - Solve over-determined rectangular matrix, using QRD
  - Each steering vector requires both forward and backward substitution to solve for optimal filter

- Power Domain
  - Estimate Covariance matrix (Hermitian matrix)
  - Covariance matrix inversion → ideal for Choleski algorithm
  - Requires higher dynamic range (none issue with floating point)
  - Choleski more efficient to implement, using "Fused Datapath"
  - Each steering vector requires only backward substitution to solve for optimal filter