Digital Signal Processing For Radar Applications

Altera Corporation
Radar: **RAdio Detection And Ranging**

Need a directional radio beam

Measure time between transmit pulse and receive pulse

Find Distance: Divide speed of light by interval time
# Radar Band (Frequency) Terminology

<table>
<thead>
<tr>
<th>Radar Band</th>
<th>Frequency (GHz)</th>
<th>Wavelength (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Millimeter</td>
<td>40 to 100</td>
<td>0.75 to 0.30</td>
</tr>
<tr>
<td>Ka</td>
<td>26.5 to 40</td>
<td>1.1 to 0.75</td>
</tr>
<tr>
<td>K</td>
<td>18 to 26.5</td>
<td>1.7 to 1.1</td>
</tr>
<tr>
<td>Ku</td>
<td>12.5 to 18</td>
<td>2.4 to 1.7</td>
</tr>
<tr>
<td>X</td>
<td>8 to 12.5</td>
<td>3.75 to 2.4</td>
</tr>
<tr>
<td>C</td>
<td>4 to 8</td>
<td>7.5 to 3.75</td>
</tr>
<tr>
<td>S</td>
<td>2 to 4</td>
<td>15 to 7.5</td>
</tr>
<tr>
<td>L</td>
<td>1 to 2</td>
<td>30 to 15</td>
</tr>
<tr>
<td>UHF</td>
<td>0.3 to 1</td>
<td>100 to 30</td>
</tr>
</tbody>
</table>

\[ \lambda = \frac{v}{f} \quad \text{where} \]

- \( f \) = wave frequency (Hz or cycles per second)
- \( \lambda \) = wavelength (centimeters)
- \( v \) = speed of light (approximately \( 3 \times 10^{10} \) centimeters/second)

Radar Band often dictated by antenna size requirements
Radar Range Equation

Receiver Power \( P_{\text{receive}} \) = \( P_t \ G_t \ A_r \ \sigma \ F^4 \ (t_{\text{pulse}} / T) / ((4\pi)^2 \ R^4) \)

where

- \( P_t \) = transmitted power
- \( G_t \) = antenna transmit gain
- \( A_r \) = Receive antenna aperture area
- \( \sigma \) = radar cross section (function of target geometric cross section, reflectivity of surface, and directivity of reflections)
- \( F \) = pattern propagation factor (unity in vacuum, accounts for multi-path, shadowing and other factors)
- \( t_{\text{pulse}} \) = duration of receive pulse
- \( T \) = duration of transmit interval
- \( R \) = range between radar and target
Transmit Pulse Repetition Frequency (PRF)

- From 100s of Hz to 100s of kHz
- Can cause range “ambiguities” if too fast

Assume PRF of 10 kHz (100 us), therefore

\[ R_{\text{maximum}} = \frac{(3 \times 10^8 \text{ m/sec})(100 \times 10^{-6} \text{ sec})}{2} = 15 \text{ km} \]

Target 1 at 5 km range: \[ t_{\text{delay}} = \frac{2 R_{\text{measured}}}{v_{\text{light}}} = \frac{2(5 \times 10^3)}{3 \times 10^8} = 33 \text{ us} \]

Target 2 at 21 km range: \[ t_{\text{delay}} = \frac{2 R_{\text{measured}}}{v_{\text{light}}} = \frac{2(21 \times 10^3)}{3 \times 10^8} = 140 \text{ us} \]
Doppler concept – frequency shift through motion
Doppler effect

Frequency shift in received pulse: \[ f_{\text{Doppler}} = 2 \frac{v_{\text{relative}}}{\lambda} \]

Example: assume X band radar operating at 10 GHz (3 cm wavelength)

Airborne radar traveling at 500 mph

Target 1 traveling away from radar at 800 mph
\[ V_{\text{relative}} = 500 - 800 = -300 \text{ mph} = -134 \text{ meter/s} \]

Target 2 traveling towards radar at 400 mph
\[ V_{\text{relative}} = 500 + 400 = 900 \text{ mph} = 402 \text{ meter/s} \]

First target Doppler shift = \[ 2 \times \frac{-134\text{m/s}}{0.03\text{m}} = -8.93 \text{ kHz} \]

Second target Doppler shift = \[ 2 \times \frac{402\text{m/s}}{0.03\text{m}} = 26.8 \text{ kHz} \]
Frequency Spectrum of Pulse

Spectrum of single pulse

Spectrum of slowly repeating pulse (low PRF)

Spectrum of rapidly repeating pulse (high PRF)

Line spacing equal to PRF
Doppler Ambiguities

Radar–bearing aircraft maximum speed:
Target aircraft maximum speed
Maximum positive Doppler = \(2 \times \frac{1072 \text{ m/s}}{0.03 \text{ m}} = 71.5 \text{ kHz}\)

Radar–bearing aircraft minimum speed:
Effective radar–bearing aircraft minimum speed with \(\theta = 30\) degree angle from target track is \(\sin(30) = 0.5\)
Target aircraft maximum speed
Maximum negative Doppler = \(2 \times \frac{67 - 536 \text{ m/s}}{0.03 \text{ m}} = -31.3 \text{ kHz}\)
Critical Choice of PRF

- **Low PRF: generally 1-8 kHz**
  - Good for maximum range detection
  - Requires long transmit pulse duration to achieve adequate transmit power – means more pulse compression
  - Excessive Doppler ambiguity
  - Difficult to reject ground clutter in main antenna lobe

- **Medium PRF: generally 8-30 kHz**
  - Compromise: get both range and Doppler ambiguities, but less severe
  - Good for scanning – use other PRF for precise range or for isolating fast moving targets from clutter

- **High PRF: generally 30-250 kHz**
  - Moving target velocity measurement and detection
  - Allows highest transmit power → greater detection ranges
  - FFT → Spectral masking → IFFT → detection processing
Using both Range and Doppler detection

Assume no range or Doppler ambiguities
Doppler Pulse Processing Basics

- Form L range bins in fast time
- Perform FFT across N pulse intervals for each of L ranges
- Doppler filter into K frequency bins
- Coherent processing interval (CPI) of N radar pulses
- Target discrimination in both range and Doppler frequency
STAP Algorithm
MTI Detection among clutter, jamming

- Pulse Compression – matched filtering to optimize SINR
- MTI filtering – for gross clutter removal
- Pulse Doppler filtering – effective to resolve targets with significant motion from clutter
- **STAP** – temporal and spatial filtering to separate slow moving targets from clutter and null jammers
  - Very high processing requirements
  - Low latency, fast adaptation
  - Dynamic range requires floating point processing
Target Detection with Spatial Dimension

- Jammer is across all Doppler bins
- Target is same angle as main clutter
- Target is close to clutter Doppler frequency
- Spatial and Temporal filtering needed to discriminate
Beamforming using ESA

- Each T/R module sample multiplied by complex value $A_m$
  
  $A_m = e^{-2\pi d \cdot m \cdot \sin(\theta/\lambda)}$ for $m = 1..M-1$, for each angle $\theta$

- Main lobe shape, side lobe height can be adjusted by multiplying angle vector with tapering window – similar to FIR filter coefficients

- All complex receive samples sum/split for single feed to/from processor

- Angular steering, determined by vector $A$
Add Spatial dimension: Radar “Datacube”

- M sub-array weighted samples (*no longer summed into single feed*)
- L sampling intervals per pulse interval (*fast time*)
- CPI of N radar pulses (*slow time*)
- Doppler pulse processing across L & N dimensions
- STAP processing across M & N dimensions
Target steering vector $\mathbf{t} = \varphi(\text{angle, Doppler})$

- Temporal Steering Vector $\mathbf{F}_d$ for each Doppler frequency $F_{d\text{opp}}$
  
  \[ \mathbf{F}_d = e^{-2\pi n F_{d\text{opp}}} \text{ for } n = 1..N-1 \]

- $\mathbf{A}_\theta = e^{-2 \pi d \cdot m \cdot \sin(\theta / \lambda)}$ for $m = 1..M-1$, for given angle of arrival $\theta$

- $\mathbf{t} = \mathbf{F}_d \otimes \mathbf{A}_\theta$, Kronecker product of target Doppler and Steering vectors

$t$ is vector of length $N \cdot M$

where

\[ \mathbf{A}_\theta = e^{-2\pi d \cdot m \cdot \sin(\theta / \lambda)} \]

(M long vector)
Computing Interference Covariance Matrix

\[ S_{\text{I}} = y^* \cdot y^T \]  

(vector cross product)

\( y \) is column vector of length \( N \cdot M \), so \( S_{\text{I}} \) is \( (N \cdot M) \times (N \cdot M) \) size

\[ S_{\text{Interference}} = S_{\text{noise}} + S_{\text{jammer}} + S_{\text{clutter}} \] and is hermitian

where \( S_{\text{noise}} = \sigma^2 \cdot I \), \( S_{\text{jammer}} \) is block diagonal matrix, \( S_{\text{clutter}} \) depends upon environment
Estimate $S_i$ using neighboring bins

- $S_i$ is calculated for each neighboring range bin
- Do not want target information in covariance matrix, so estimate $S_i$ in $k^{th}$ bin by averaging, element by element, nearby $S_i$ matrices
- Orange range cell $k$ contains suspected target under test
  - Red range cells are guard bands
  - Assume nearby bins have same clutter statistics as orange cell
  - Use the nearby cells to for estimate of $S_i$ to use in STAP algorithm
Calculate weight vector to maximize SIR

- Calculate optimal weighting vector \( h = k \cdot S_i^{-1} \cdot t^* \)
  - \( k \) is scalar constant, \( S_i \) is interference covariance matrix, \( t \) is target steering vector
- Then find \( z = h^T \cdot y \), where \( y \) is output from bin k of radar cube data
- \( z(F_{\text{doppler}}, \theta) \) is then subject to target threshold detection process
Using QRD to find weighting vector $h$

- $h = \kappa \cdot S_{l}^{-1} \cdot t^*$ (where $h$ is $M \cdot N$ long vector)
- $S_{l} \cdot u = t^*$, where $u$ includes scaling constant $\kappa$
- $Q \cdot R \cdot u = t^*$, where $Q \cdot Q^H = I$ and $Q^{-1} = Q^H$
- $Q$ is constructed of orthogonal normalized vectors
- $R$ will become upper triangular
- $R \cdot u = Q^H \cdot t^*$

Solve for vector $u$ using back substitution

- $h = u / (t^H \cdot u^*)$ (where $\kappa = t^H \cdot u^*$)
- $z = h^T \cdot y$ gives optimal output detection result
  where $z$ is a complex scalar
STAP using covariance matrix summary

- For each range bin of interest:
  - Compute covariance matrix for each range bin
  - Estimate interference covariance matrix $S_i$ by averaging the surrounding covariance matrices
  - Perform QRD upon $S_i$

- Then for each $F_{doppler}$ and $A_\theta$ of interest:
  - Compute $Q^H \cdot t^*$
  - $R \cdot u = Q^H \cdot t^* \rightarrow$ find $u$ with back substitution
  - Compute $h = u / (t^H \cdot u^*)$
  - Final result $z = h^T \cdot y$

- This is known as power domain method
Example GFLOPs estimate (power domain)

- Process over $12 \mathbf{A}_\theta, 16 \mathbf{F}_{\text{doppler}}$ and assume prosecute 32 target steering vectors
- Use 10 range bins with a PRF = 1 kHz
  - Compute Covariance Matrix 1.1 GFLOPS
  - Average over 10 covariance matrices 0.4 GFLOPs
  - QR Decomposition 37.7 GFLOPs
  - Compute $\mathbf{Q}^H \cdot \mathbf{t}^*$ (each $\mathbf{A}_\theta$ and Doppler) 9.4 GFLOPs
  - Solve for $\mathbf{u}$ using back substitution 4.7 GFLOPs
  - Compute $\mathbf{h} = \mathbf{u} / (\mathbf{t}^H \cdot \mathbf{u}^*)$ and $z = \mathbf{h}^T \cdot \mathbf{y}$ 0.2 GFLOPs
  - Total 53.5 GFLOPs

- Detection for 8 possible targets over 4 possible velocities over narrow angle and Doppler, low PRF
Alternate method

“voltage domain”
STAP using radar cube data directly

- Operate directly on $y$ data vectors
  - One vector $y$ per range bin
  - Use $L_s$ range bins, where $L_s > M \cdot N$ range bins
  - Construct matrix $Y = [y_0 \ y_1 \ y_2 \ \ldots \ y_{L_s-1}]$, dimension $[M \cdot N \times L_s]$
STAP using $Y$ data matrix summary

- $S_i = Y^* \cdot Y^T = R^H \cdot Q^H \cdot Q \cdot R \quad (Y^T = Q \cdot R)$
  
  \[ = R^H \cdot R = R_1^H \cdot R_1 \]

- Recall $S_i \cdot u = t^*$, so substitute $R_1^H \cdot R_1 \cdot u = t^*$

- Define $r \equiv R_1 \cdot u$

- Then for each $F_{doppler}$ and $A_\theta$ of interest:
  - Solve for $r$ in $R_1^H \cdot r = t^*$ using forward substitution
    ($R_1^H$ is lower triangular)
  - Solve for $u$ in $R_1 \cdot u = r$ using backward substitution
    ($R_1$ is upper triangular)
  - Compute $h = u / (t^H \cdot u^*)$
  - Final result $z = h^T \cdot y$

- This is known as voltage domain method
Apply QRD to $Y$ data matrix

- $Y^T = Q \times R$, where $R$ is composed of $R_1$ and $0$
  - $R_1$ is upper triangular, $R_1^H$ is lower triangular, $[M \times N]$
Example GFLOPs estimate (voltage domain)

- $12 \mathbf{A}_{\theta}, 16 \mathbf{F}_{doppler}$ and 32 target steering vectors, PRF = 1 kHz, with 200 range bins
  - QR Decomposition 40.1 GFLOPs
  - Solve for $\mathbf{r}$ using forward substitution 4.7 GFLOPs
  - Solve for $\mathbf{u}$ using back substitution 4.7 GFLOPs
  - Compute $\mathbf{h} = \mathbf{u} / (\mathbf{t}^H \cdot \mathbf{u}^*)$ and $\mathbf{z} = \mathbf{h}^T \cdot \mathbf{y}$ 0.2 GFLOPs
  - Total 49.7 GFLOPs

- Higher rate and resolution system: $48 \mathbf{A}_{\theta}, 16 \mathbf{F}_{doppler}$
  - 64 target vectors, PRF = 1 kHz, 1000 range bins
  - QR Decomposition 3.51 TFLOPs
  - Solve for $\mathbf{r}$ using forward substitution 37.7 GFLOPs
  - Solve for $\mathbf{u}$ using back substitution 37.7 GFLOPs
  - Compute $\mathbf{h} = \mathbf{u} / (\mathbf{t}^H \cdot \mathbf{u}^*)$ and $\mathbf{z} = \mathbf{h}^T \cdot \mathbf{y}$ 0.8 GFLOPs
  - Total 3.59 TFLOPs
FPGA Processing Flow Implementation
(voltage domain)

PRF = 1 KHz

Compute each PRF over
L_s range bins

Compute each target
vector, for each PRF

Complex, Single
Precision Floating
Point Datapath

Doppler Vector $F_d$

[16 x 1]

Angle Vector $A_\theta$

[12 x 1]

Kronecker Product

Steering Vector $t$

[192 x 1]

QR Decomposition

$\mathbf{Y}^T$ over $L_s$
range bins

[192 x 200]

$\mathbf{R}_1^H$ (lower
triangular)

Forward Subst.

$\mathbf{R}_1$ (upper
triangular)

Both

[192 x 192]

Back Subst.

$\mathbf{H} \cdot \mathbf{u}^*$

[192 x 1]

$\mathbf{t}^H \cdot \mathbf{u}^*$

[192 x 1]

Dot Product

$\mathbf{u}$

[192 x 1]

Norm ($/.$)

$\mathbf{h}$

[192 x 1]

Output Filter

$\mathbf{y}$ (range bin of interest)

[192 x 1]

Result $z$
Voltage verses Power domain methods

- **Voltage Domain**
  - Operates directly on the data
  - Solve over-determined rectangular matrix, using QRD
  - Each steering vector requires both forward and backward substitution to solve for optimal filter

- **Power Domain**
  - Estimate Covariance matrix (Hermitian matrix)
  - Covariance matrix inversion → ideal for Choleski algorithm
  - Requires higher dynamic range (none issue with floating point)
  - Choleski more efficient to implement, using “Fused Datapath”
  - Each steering vector requires only backward substitution to solve for optimal filter