Some Interesting Filter Design Configurations and Transformations Normally Not Found in The General Literature

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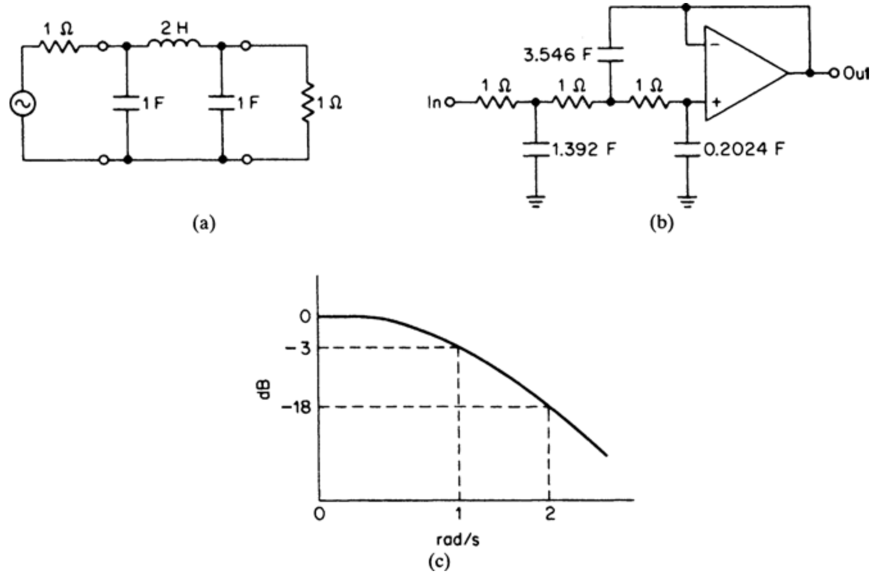
TOPICS

- Design of D-Element Active Low-Pass Filters and a Bi-directional Impedance Converter for Resistive Loads
- Active Adjustable Amplitude and Delay Equalizer Structures
- High-Q Notch filters
- Q-Multiplier Active Band-Pass filters
- A Family of Zero Phase-Shift Low-Pass Filters
- Some Useful Passive Filter Transformations to Improve Realizability
- Miscellaneous Circuits and "Tricks"

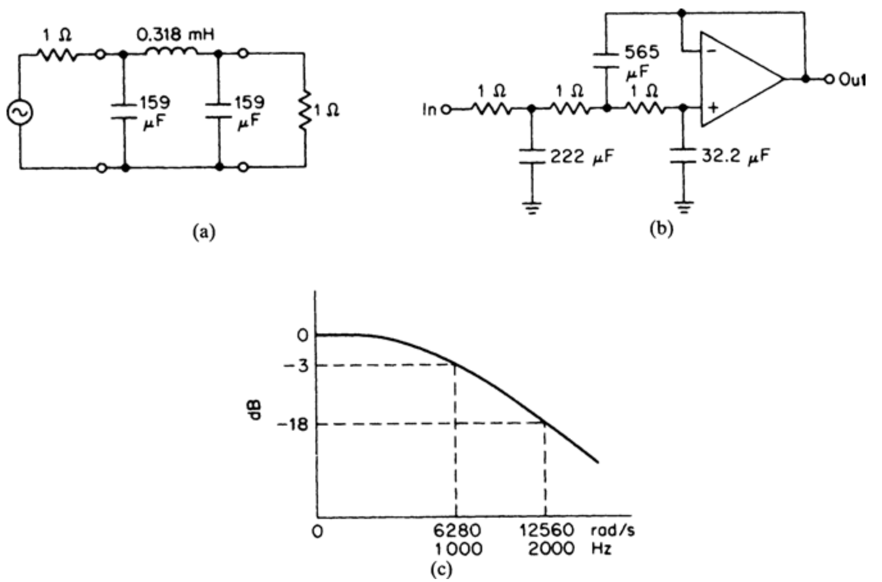
Frequency and Impedance Scaling from Normalized Circuit

Frequency Scaling

$$\mathrm{FSF} = \frac{\text{desired reference frequency}}{\text{existing reference frequency}}$$



Normalized $n = 3$ **Butterworth low-pass filter normalized to 1 rad/sec : (*a*) *LC* filter; (*b*) active filter; (*c*) frequency response**



Denormalized low-pass filter scaled to 1000Hz: (*a*) *LC* filter; (*b*) active filter; (*c*) frequency response.

Impedance scaling can be mathematically expressed as

$$R' = ZxR$$

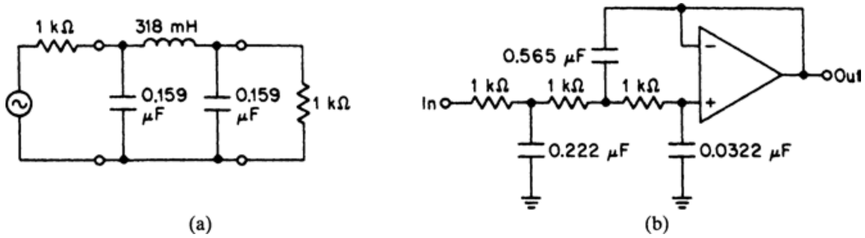
$$L' = ZxL$$

$$C' = \frac{C}{Z}$$

Frequency and impedance scaling are normally combined into one step rather than performed sequentially. The denormalized values are then given by

$$L' = ZxL/FSF$$

$$C' = \frac{C}{Z \times FSF}$$



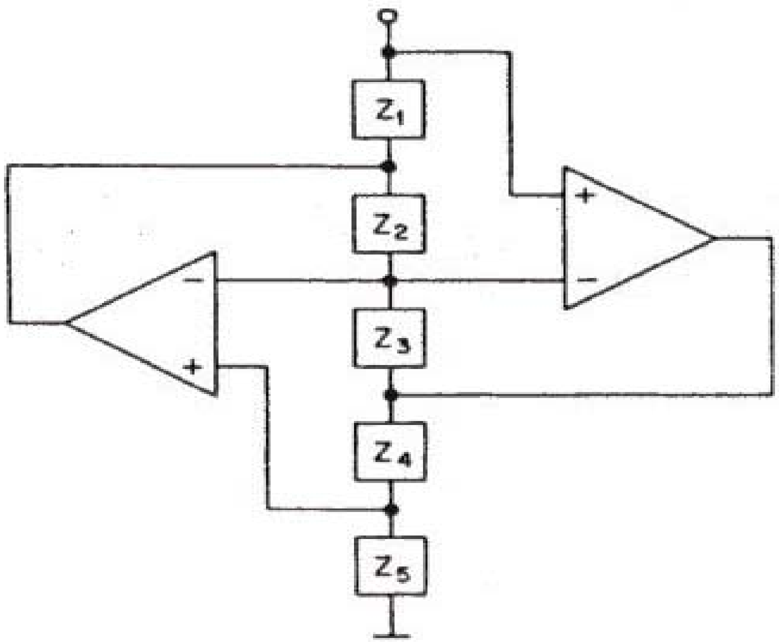
Impedance-scaled filters using Z=1K : (*a*) *LC* filter; (*b*) active filter.

Design of D-Element Active Low-Pass Filters and a Bi-Directional Impedance Converter for Resistive Loads

Generalized Impedance Converters (GIC)

$$Z_{11} = \frac{Z_1 Z_3 Z_5}{Z_2 Z_4}$$

By substituting RC combinations for Z_1 through Z_5 a variety of impedances can be realized.



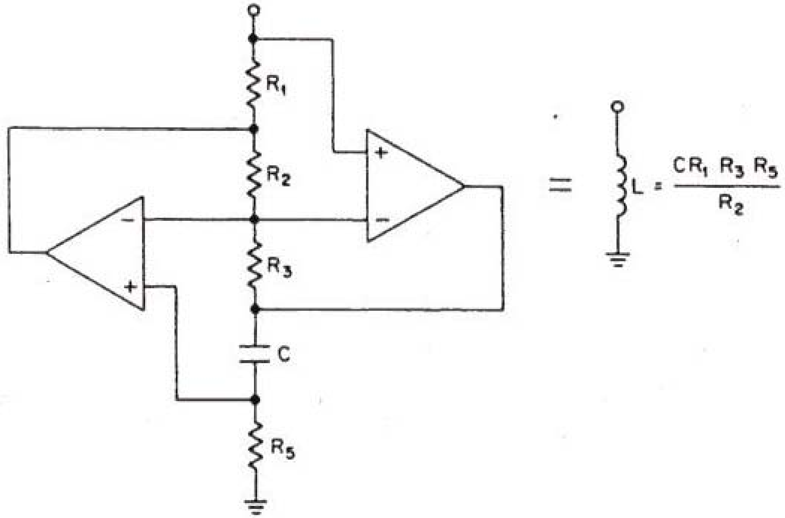
GIC Inductor Simulation

If Z_4 consists of a capacitor having an impedance 1/sC where s=jω and all other elements are resistors, the driving point impedance becomes:

$$Z_{11} = \frac{sCR_1R_3R_5}{R_2}$$

The impedance is proportional to frequency and is therefore identical to an inductor having an inductance of:

$$L = \frac{CR_1R_3R_5}{R_2}$$



Note: If R_1 and R_2 and part of a digital potentiometer the value of L can be digitally programmable.

D Element

If both Z_1 and Z_3 are capacitors C and Z_2,Z_4 and Z_5 are resistors, the resulting driving point impedance becomes:

$$Z_{11} = \frac{R_5}{s^2 C^2 R_2 R_4}$$

An impedance proportional to $1/s^2$ is called a D Element.

$$Z_{11} = \frac{1}{s^2 D} \qquad \text{where:} \qquad D = \frac{C^2 R_2 R_4}{R_5}$$

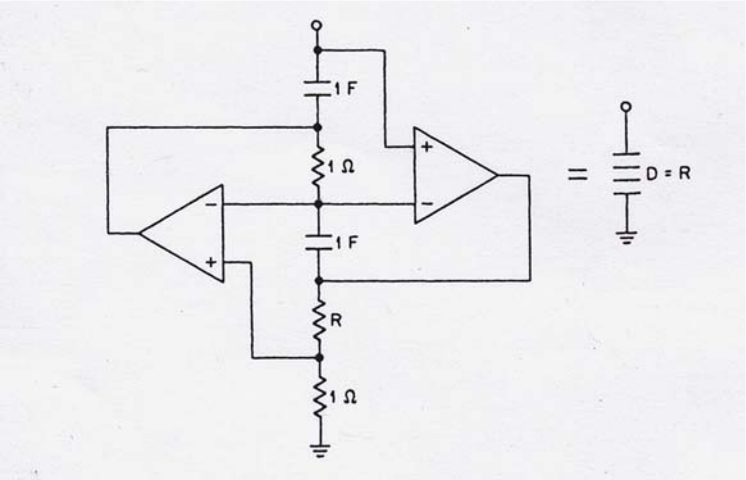
If we let $C=1F, R_2=R_5=1\ \Omega$ and $R_4=R$ we get $D=R$ so:

$$Z_{11} = \frac{1}{s^2 R}$$

If we let $s=j\omega$ the result is a Frequency Dependant Negative Resistor FDNR

$$Z_{11} = \frac{1}{-\omega^2 R}$$

D Element Circuit

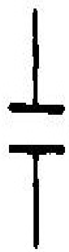
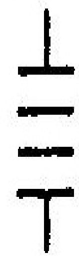
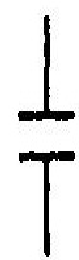


Rule

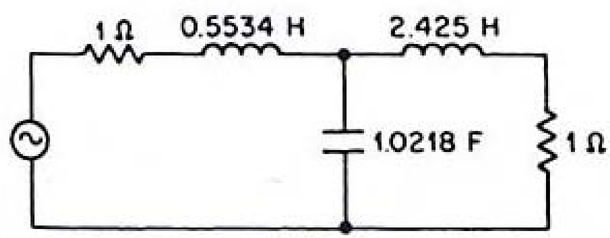
A transfer function of a network remains unchanged if all impedances are multiplied (or divided) by the same factor. This factor can be a fixed number or a variable, as long as every impedance element that appears in the transfer function is multiplied (or divided) by the same factor.

The 1/S transformation involves multiplying all impedances in a network by 1/S.

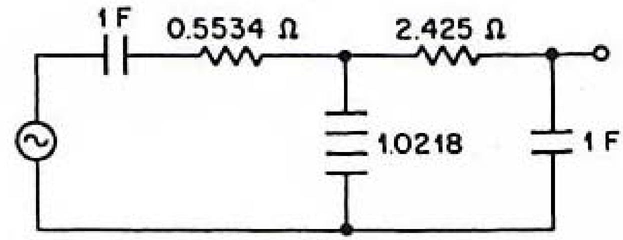
The 1/S Transformation

Element	Impedance	Transformed Element	Transformed Impedance
 L	sL	 L	L
 C	$\frac{1}{sC}$	 C	$\frac{1}{s^2C}$
 R	R	 $\frac{1}{R}$	$\frac{R}{s}$

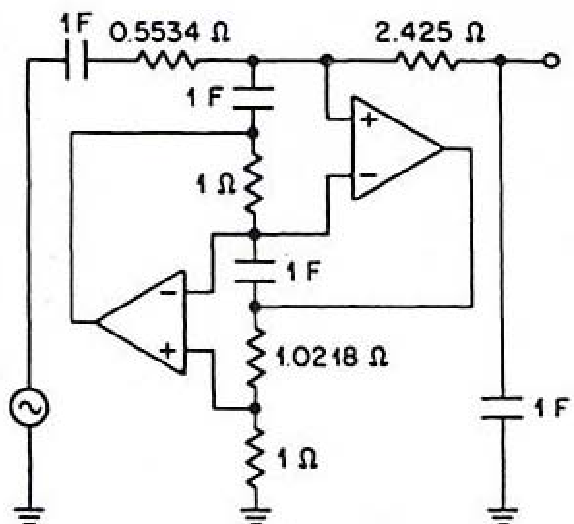
Design of Active Low-Pass filter with 3dB point at 400Hz using D Elements



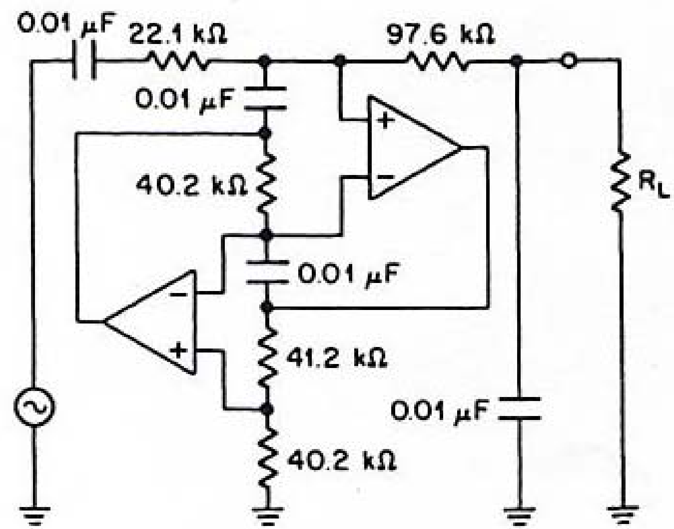
(a) Normalized Low-Pass filter



(b) 1/S Transformation



(c) Realization of D Element



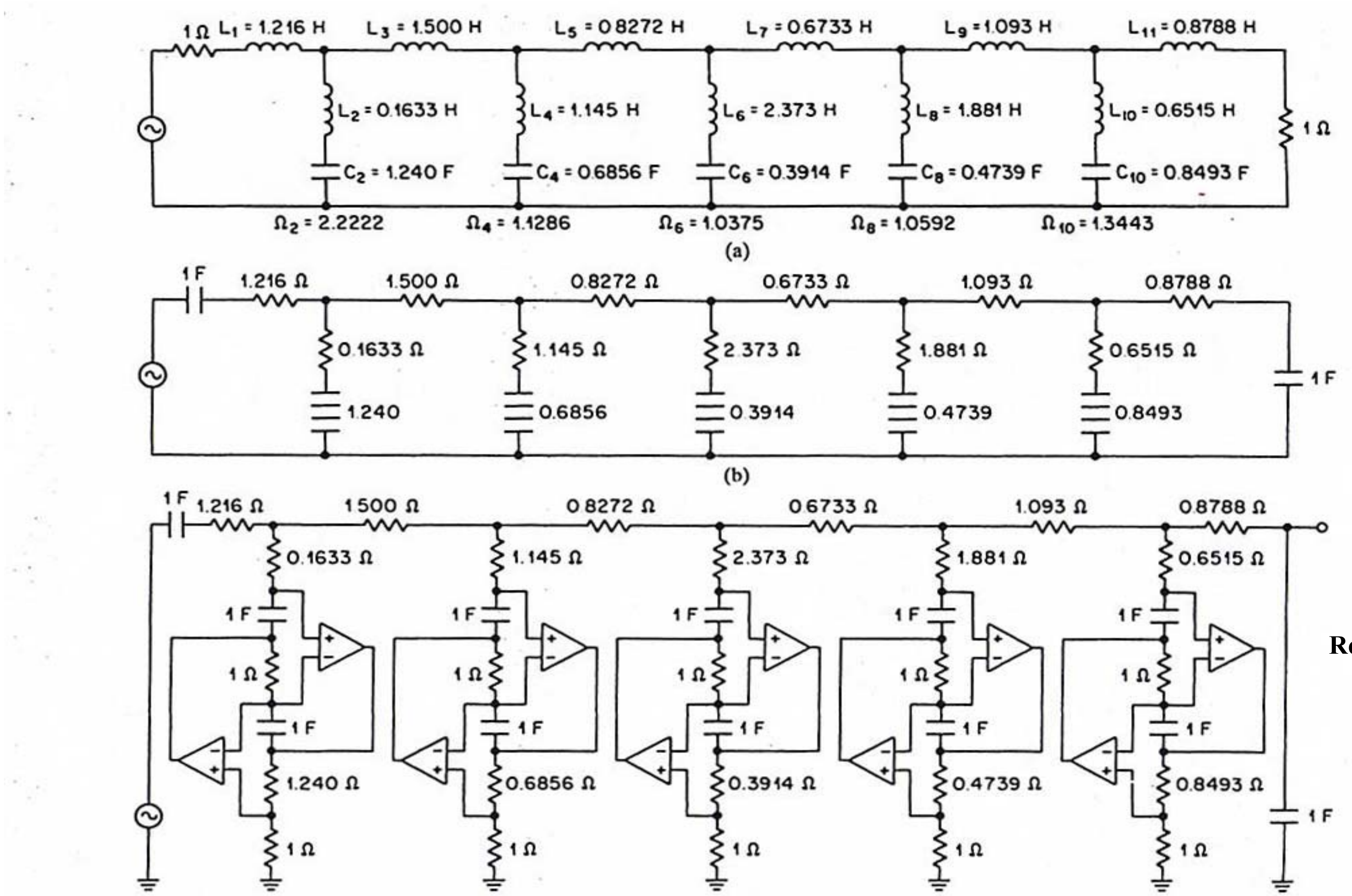
(d) Frequency and Impedance Scaled Final Circuit

Filter is Linear Phase ±0.5° Type

Elliptic Function Low-Pass filter using GICs

Requirements: 0.5dB Maximum at 260Hz
60dB Minimum at 270Hz
Steepness factor=1.0385

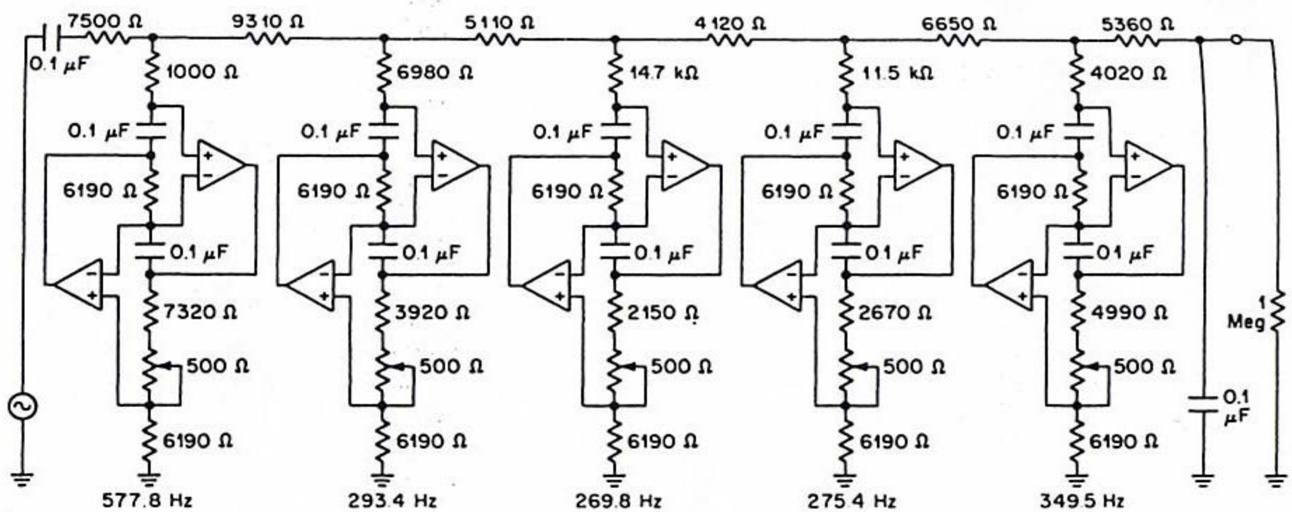
Normalized Elliptic Function Filter
C11 20 θ=75°
N=11 R_{db}=0.18dB Ωs=1.0353   60.8dB



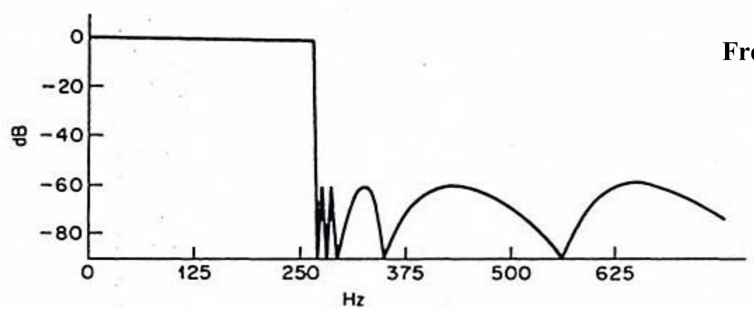
Normalized Elliptic Function Filter

1/S Transformation

Realization of D Elements



(d)

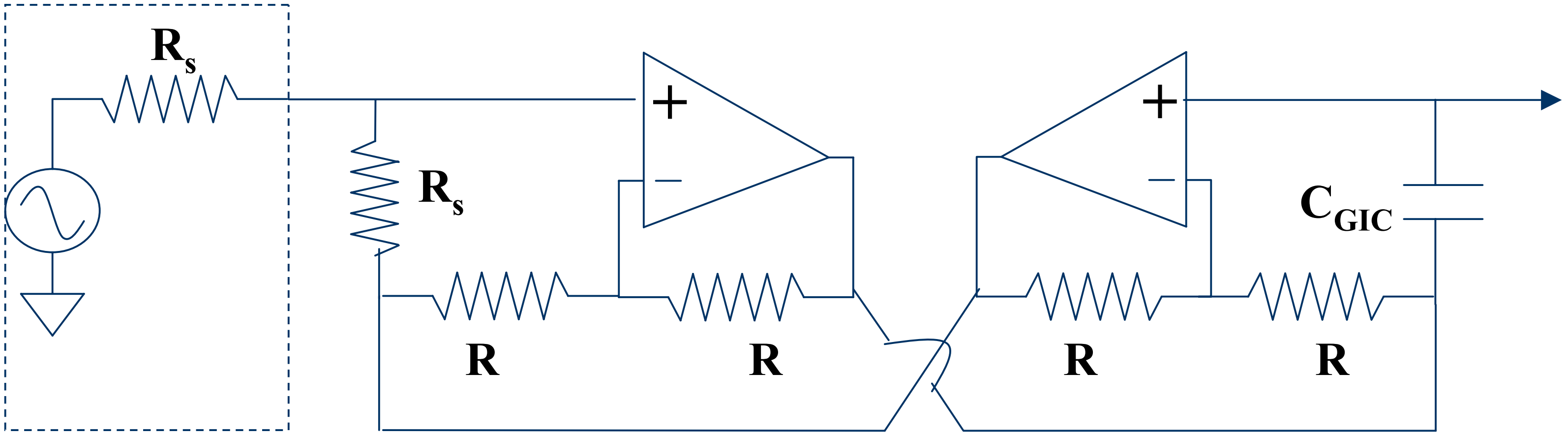


(e)

Frequency and Impedance Scaled Final Circuit

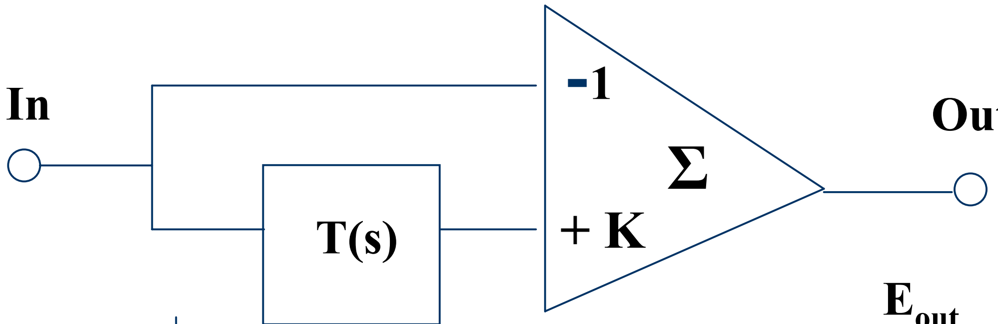
Note: 1 meg termination resistor is needed to provide DC return path.

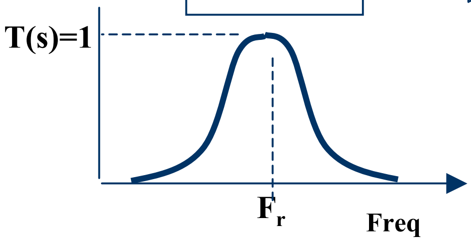
Bi-Directional Impedance Converter for Matching D Element Filters Requiring Capacitive Loads to Resistive Terminations



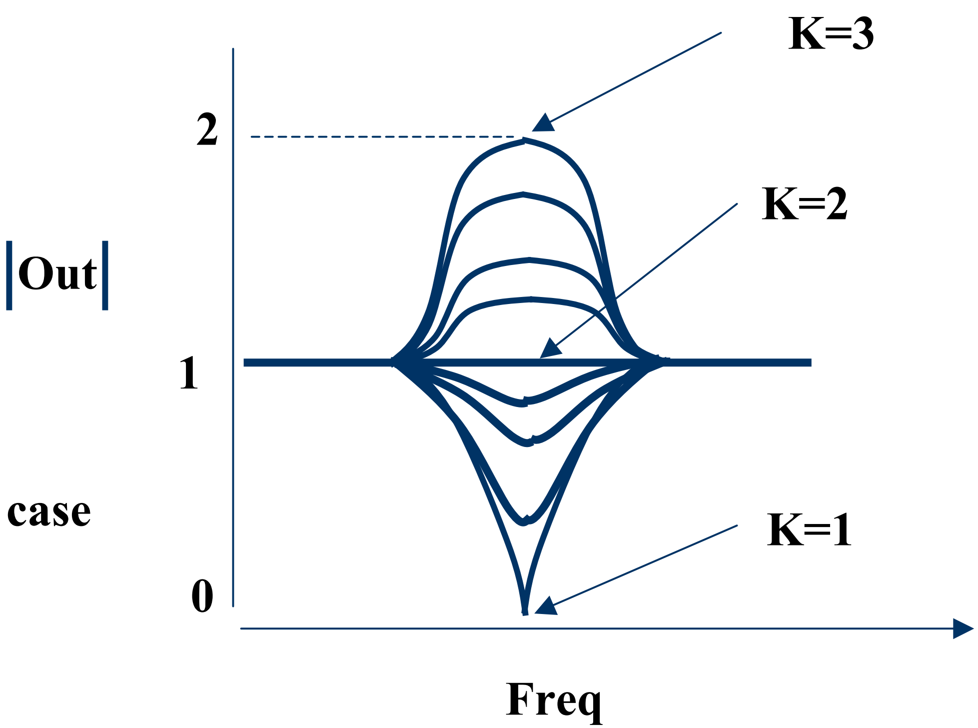
Value of R Arbitrary
R_s is source and load resistive terminations
C_{GIC} is D Element Circuit Capacitive Terminations

Active Amplitude and Delay Equalizer Structures





$$\frac{E_{out}}{E_{In}} = T(s)K-1$$



K=2 corresponds to the all-pass case

T(s) is a second-order bandpass network having a gain of +1 at center frequency F_r

$$T(s) = \frac{\frac{\omega_r}{Q} s}{s^2 + \frac{\omega_r}{Q} s + \omega_r^2}$$

$$\omega_r = \sqrt{\alpha_2 + \beta_2}$$

$$\omega_r = 2\pi F_r$$

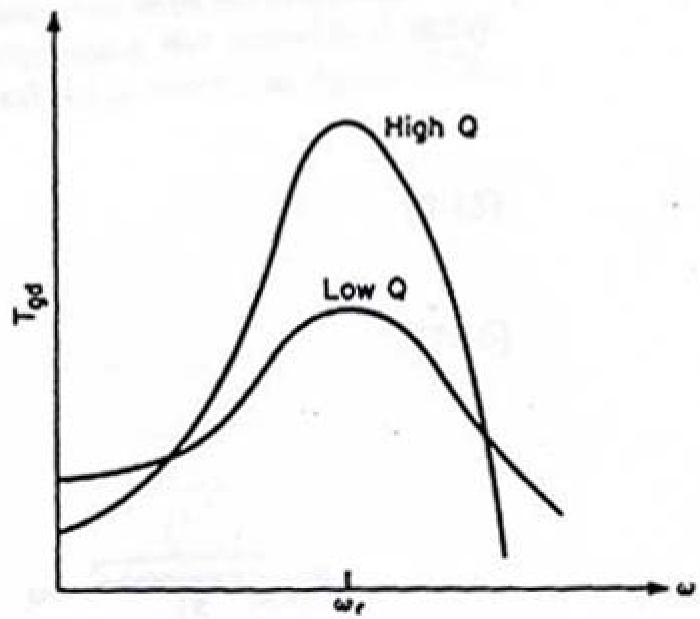
$$Q = \frac{\omega_r}{2\alpha}$$

As a function of Q the circuit has a group delay as follows:

The peak delay is equal to

$$T_{gd,max} = \frac{4Q}{\omega_r}$$

Where K=2 and Q >2

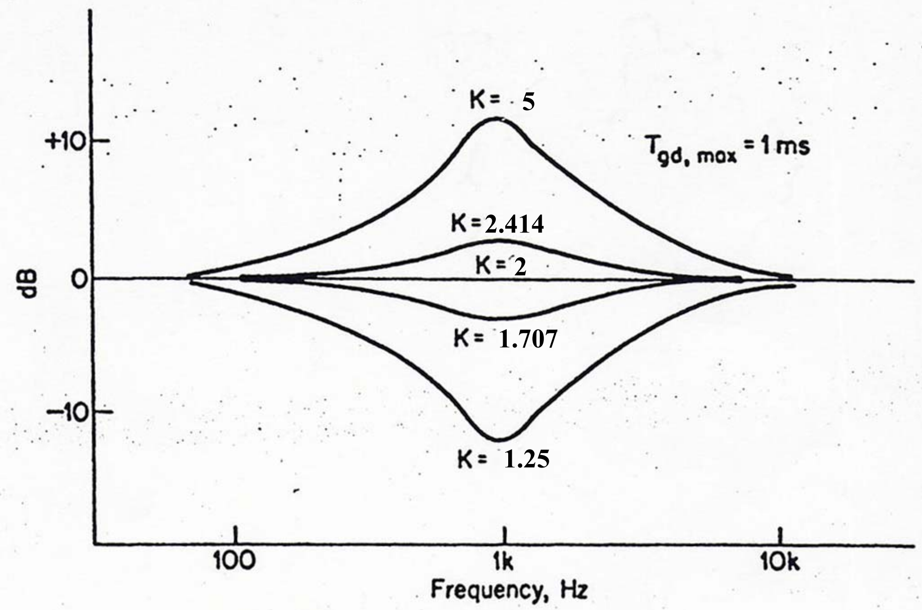


- **This architecture can be used as an amplitude equalizer by varying K.**

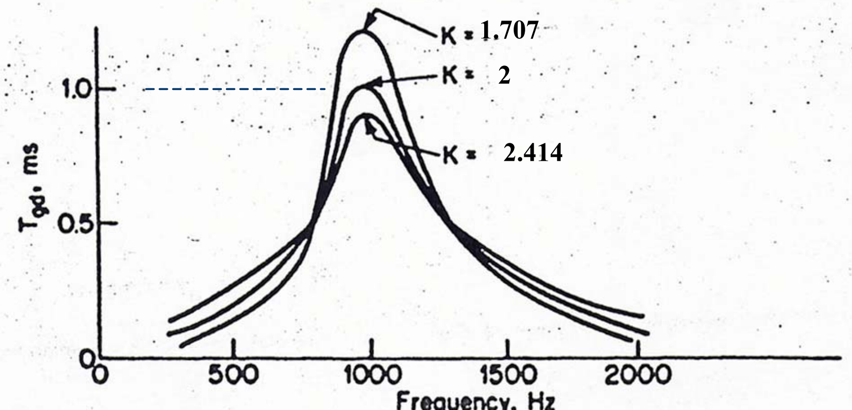
- **If K=2 the circuit can be used as a delay equalizer by varying Q.**

- **By varying both K and Q this architecture can be used for both amplitude and delay equalization.**

There will be an interaction between amplitude and delay as K is varied to change the amount of amplitude equalization.



(a)



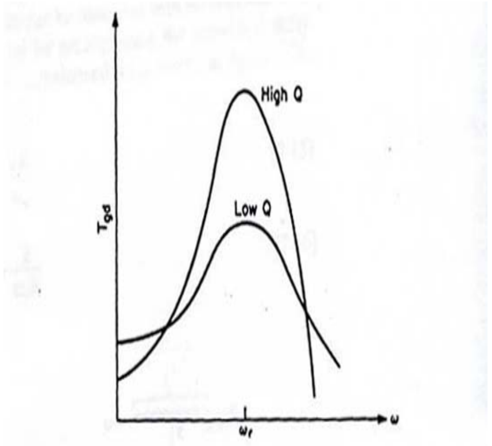
(b)

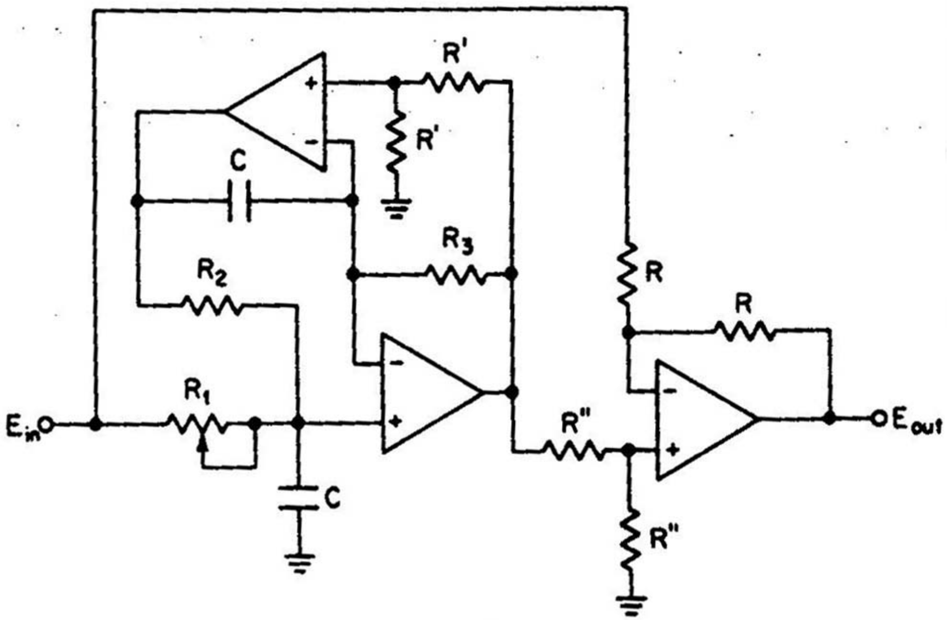
Adjustable Delay Equalizer

$T_{gd,max}=4R_1C$

$$R2=R3=\frac{1}{\omega_r C}$$

C,R,R′ and R″
can be any convenient values





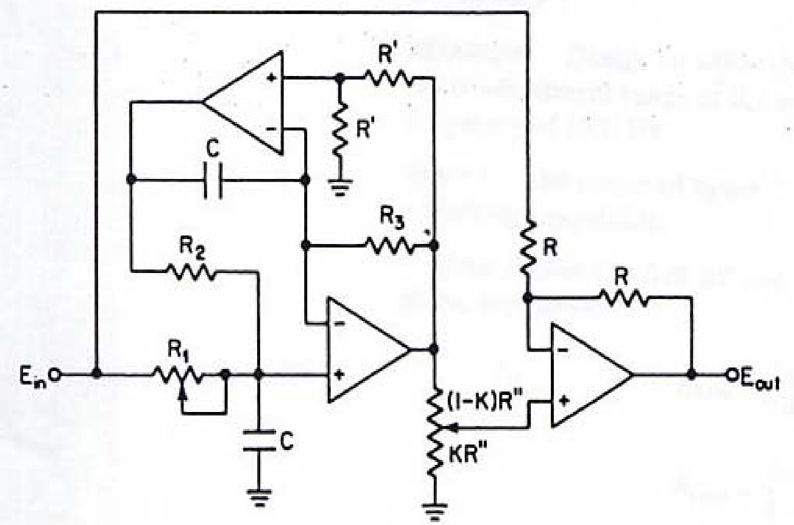
(a)

Adjustable Delay and Amplitude Equalizer

By introducing a potentiometer into the circuit amplitude equalization can also be achieved
The amplitude equalization at ω_r is given by:

$$A_{db} = 20 \, Log \, (4K-1)$$

Where K from 0.25 to 1 covers an amplitude equalization range of $-\infty$ to +9.54 dB

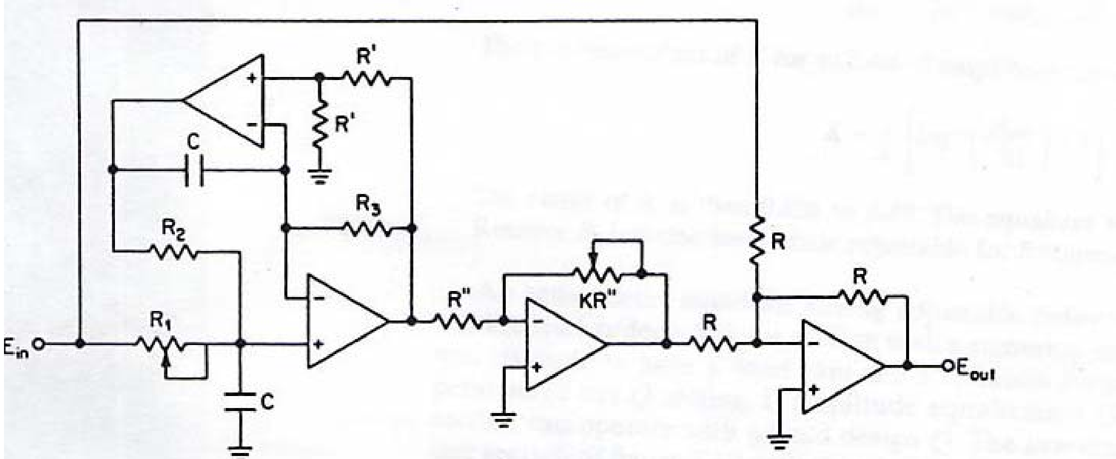


(b)

To extend the amplitude equalization range beyond +9.54 dB the following circuit can be used. The amplitude equalization at ω_r given by:

A_{db}= 20 Log (2K-1)

where a K variation from 0.5 to ∞ results in an infinite range of equalization capability. In reality ± 12dB has been found to be more than adequate so K will vary from 0.626 to 2.49.



(c)

Simplified Adjustable Amplitude Equalizer

The following circuit combines a fixed Q bandpass section with a summing amplifier to provide a low complexity adjustable amplitude equalizer. The design equations are given by:

$$R_2 = \frac{2Q}{\omega_r C}$$

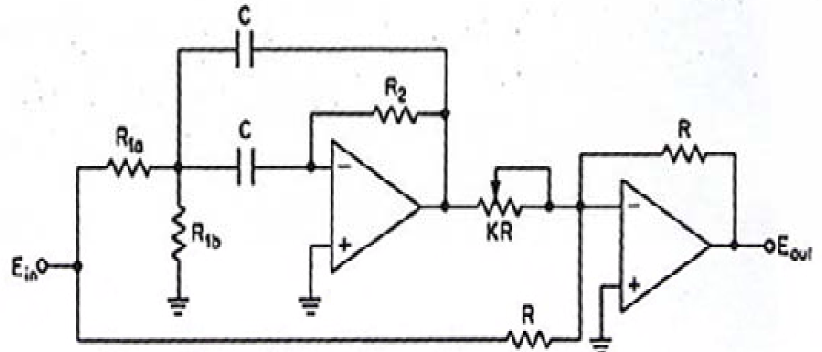
Q≥0.707

$$R_{1a} = \frac{R_2}{2}$$

$$R_{1b} = \frac{R_{1a}}{2Q^2 - 1}$$

$$A_{dB} = 20 \log \left(\frac{1}{K} - 1 \right)$$

Where K will range from 0 to 1 for an infinite range of amplitude equalization.



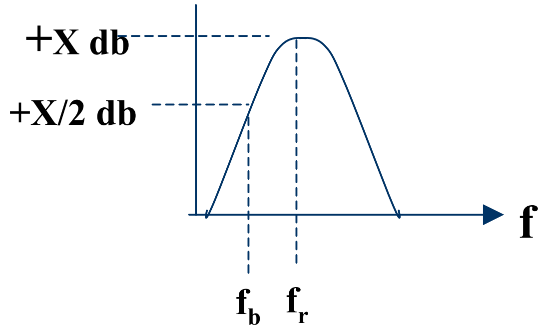
(a)

To compute the desired Q first define f_b corresponding to one-half the boost(or null) desired in dB.

Then:

$$Q = \frac{f_b b^2 \sqrt{K_r}}{f_r(b^2 - 1)}$$

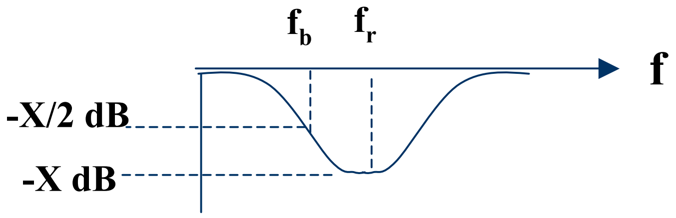
Where $K_r = \mathrm{Log}^{-1}\left(\frac{A_{db}}{20}\right) = 10^{Adb/20}$



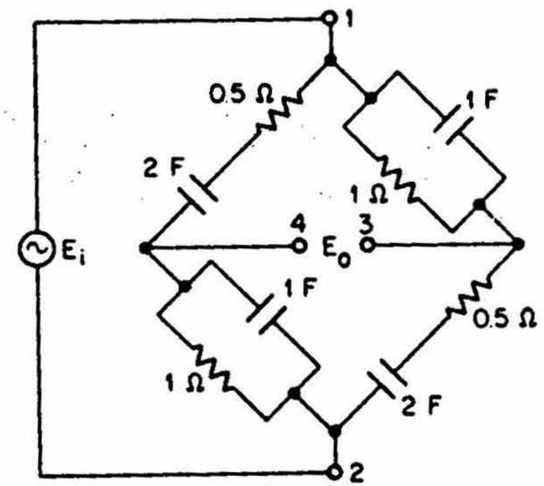
and $b = \frac{f_b}{f_r}$

or $b = \frac{f_r}{f_b}$

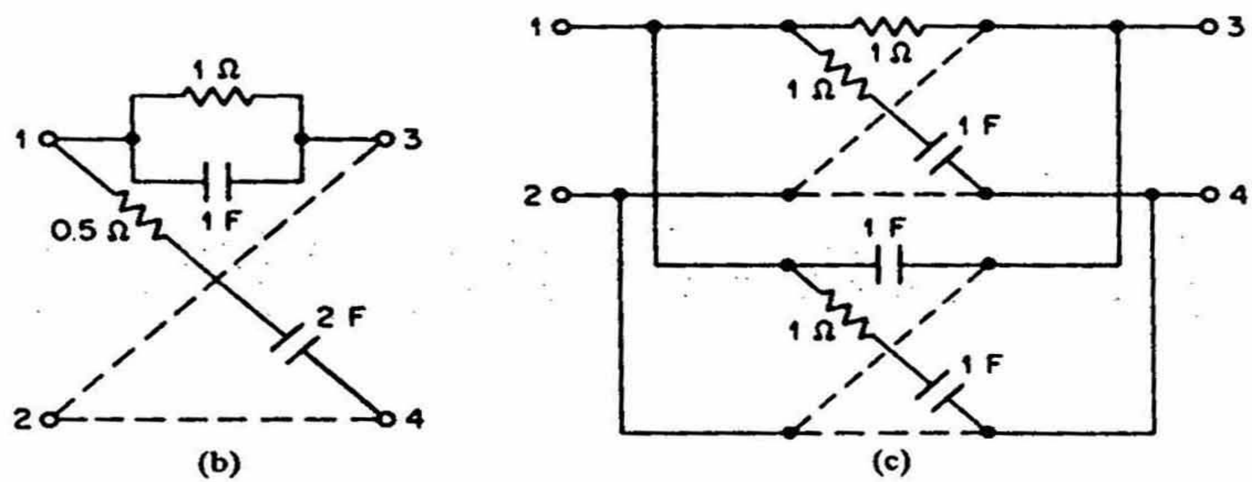
Whichever b is >1



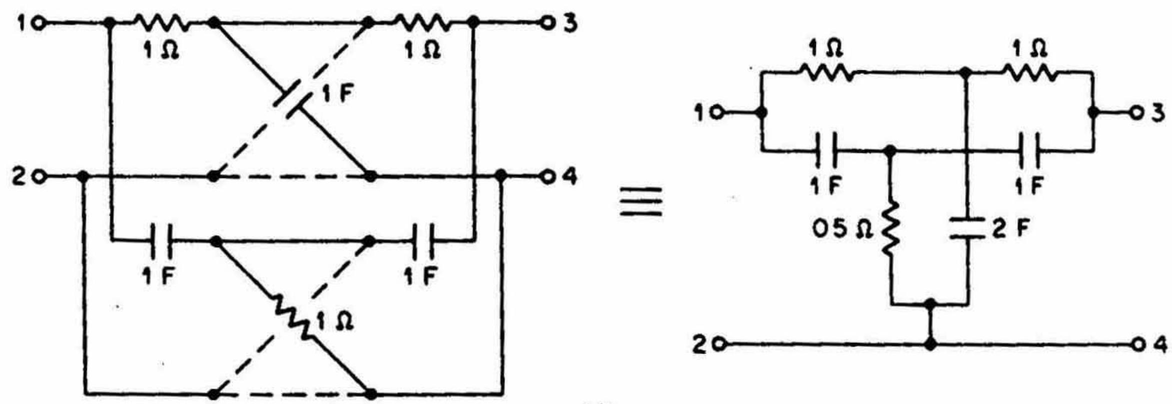
High-Q Notch Filters



This circuit is in the form of a bridge where a signal is applied across terminals' 1 and 2 the output is measured across terminals' 3 and 4. At ω=1 all branches have equal impedances of 0.707 ∠-45°so a null occurs across the output.

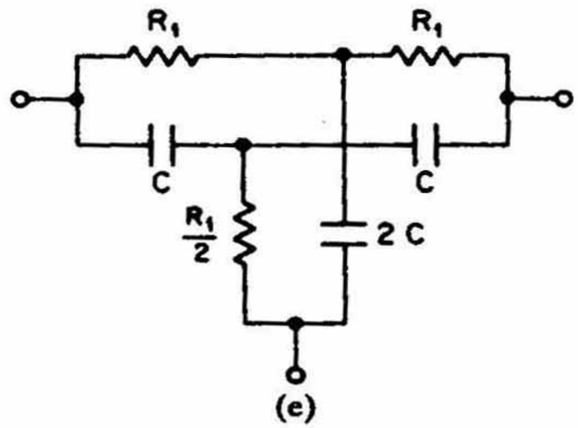


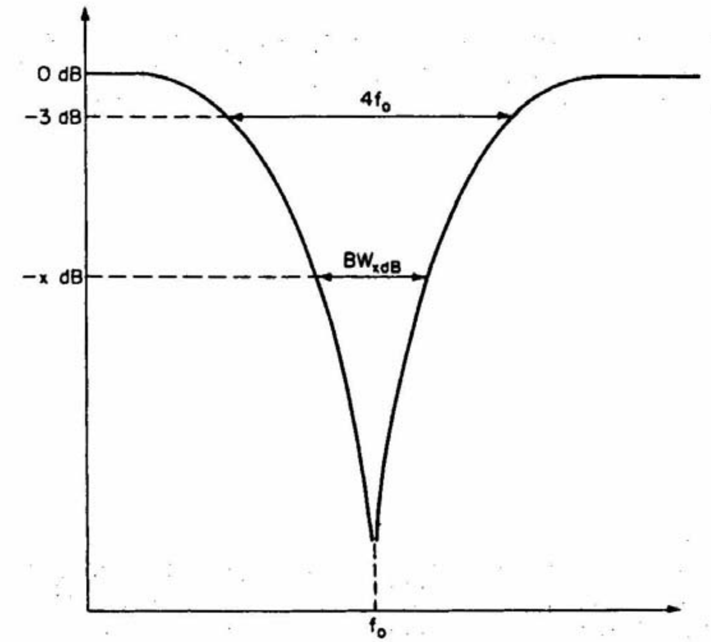
The circuit is redrawn in figure B in the form of a lattice. Circuit C is the Identical circuit shown as two lattices in parallel.



(d)

There is a theorem which states that any branch in series with both the Z_A and Z_B branches of a lattice can be extracted and placed outside the lattice. The branch is replaced by a short. This is shown in figure D above. The resulting circuit is known as a Twin-T. This circuit has a null at 1 radian for the normalized values shown.

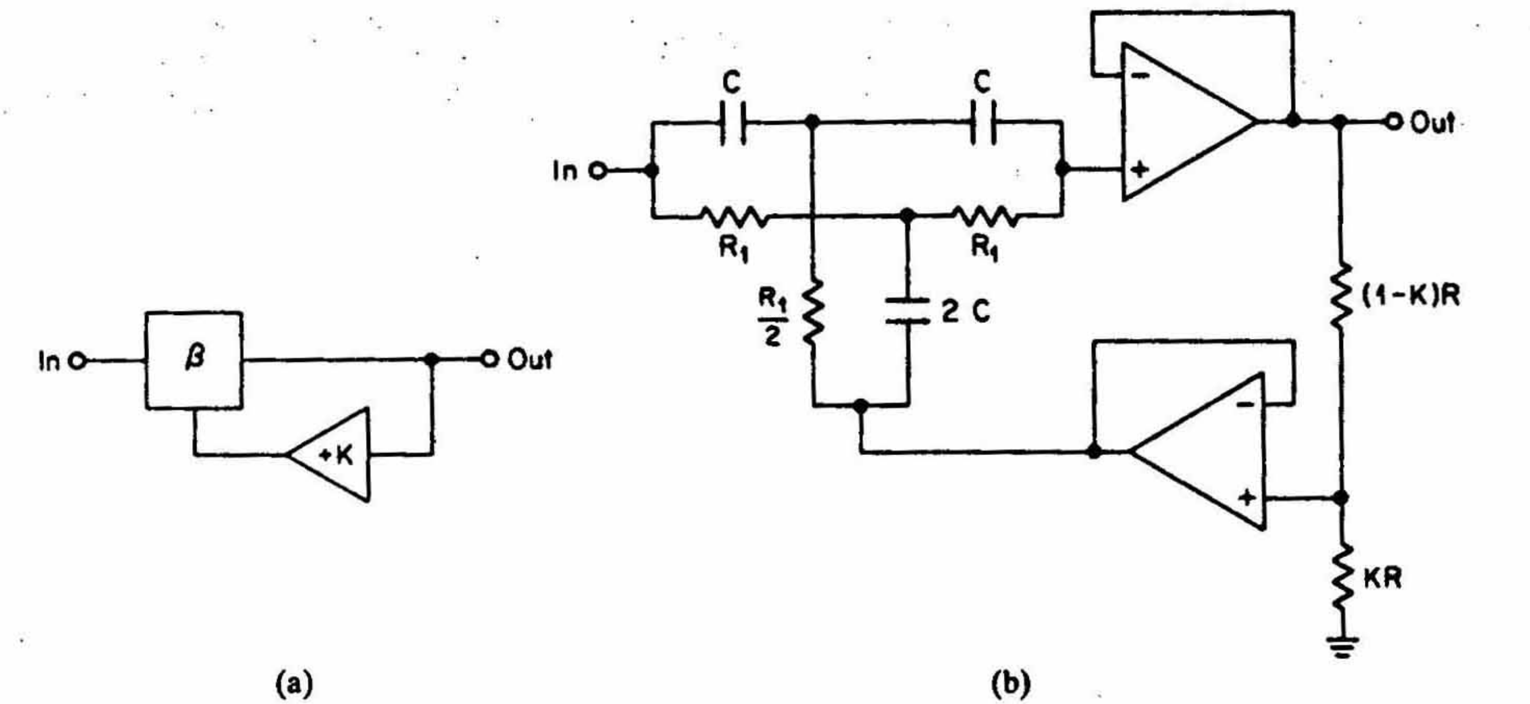




To calculate values for this circuit pick a convenient value for C. Then

$$R_1 = \frac{1}{2\pi f_o C}$$

The Twin-T has a Q (f_o/BW_{3dB}) of only ¼ which is far from selective.

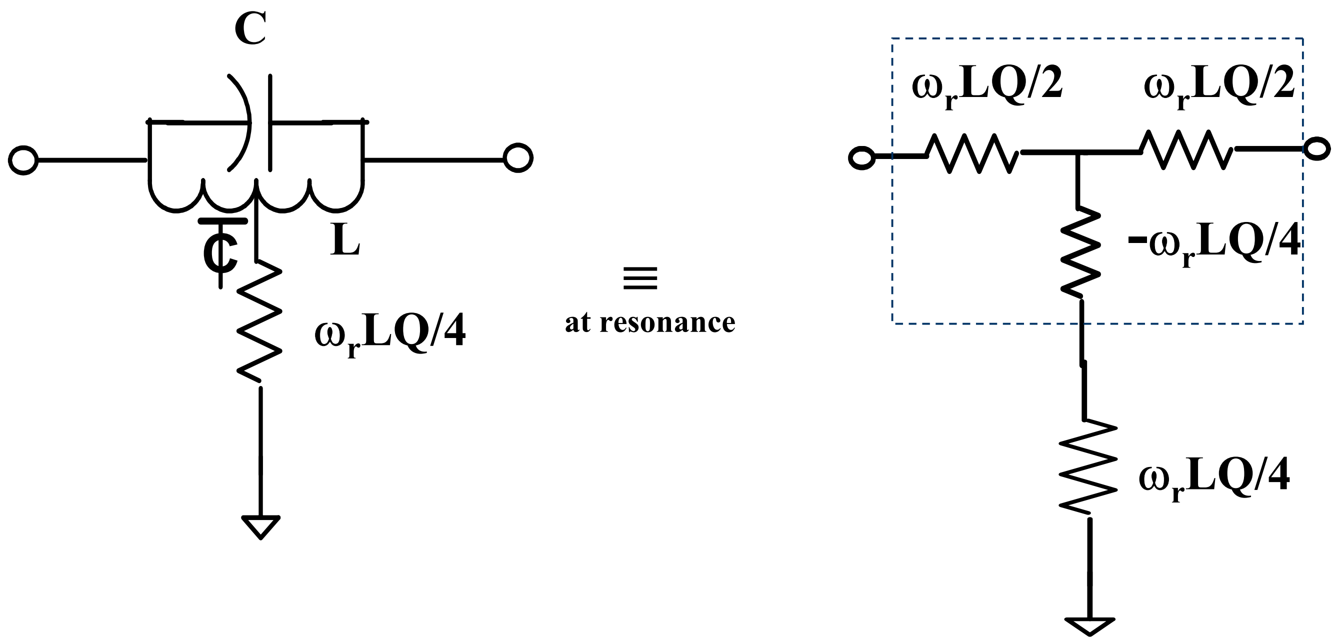


Circuit A above illustrates bootstrapping a network β with a factor K.
If β is a twin-T the resulting Q becomes:

$$Q = \frac{1}{4(1-K)}$$

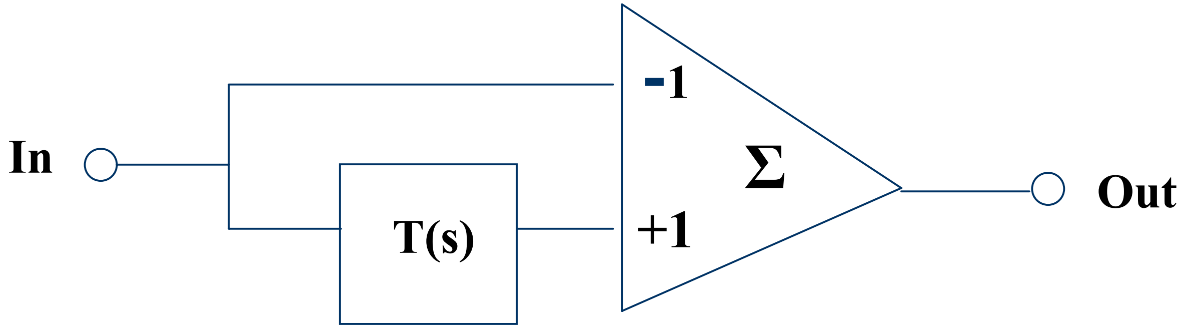
If we select a positive K <1, and sufficiently close to 1, the circuit Q can be dramatically increased. The resulting circuit is shown in figure B.

Bridged-T Null Network



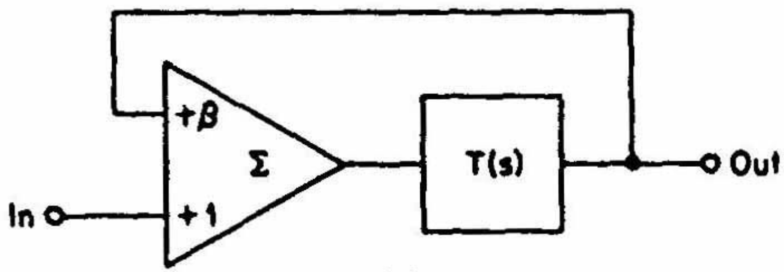
Impedance of a center-tapped parallel resonant circuit <u>at resonance</u> is $\omega_r LQ$ total and $\omega_r LQ/4$ from end to center tap (due to N^2 relationship). Hence a phantom negative resistor of $-\omega_r LQ/4$ appears in the equivalent circuit which can be cancelled by a positive resistor of $\omega_r LQ/4$ resulting in a very deep null at resonance (60dB or more).

Adjustable Q and Frequency Null Network



T(s) can be any bandpass circuit having properties of unity gain at f_r, adjustable Q and adjustable f_r.

Q Multiplier Active Bandpass Filters



(a)

If T(s) in circuit A corresponds to a bandpass transfer function of:

$$T(s) = \frac{\frac{\omega_r}{Q} s}{s^2 + \frac{\omega_r}{Q} s + \omega_r^2}$$

The overall circuit transfer function becomes:

$$\frac{\text{Out}}{\text{In}} = \frac{\frac{\omega_r}{Q} s}{s^2 + \frac{\frac{\omega_r}{Q}}{\frac{Q}{\frac{Q}{1-\beta}}} s + \omega_r^2}$$

The middle term of the denominator has been modified so the circuit Q is given by Q/(1-β) where 0<β <1. The Q can then be increased by the factor 1/(1-β) . Note that the circuit gain is increased by the same factor.

A simple implementation of this circuit is shown in figure B.

The design equations are:

First calculate β from $\beta = 1 - \dfrac{Q_r}{Q_{eff}}$

where Q_{eff} is the overall circuit Q and Q_r is the design Q of the bandpass section.

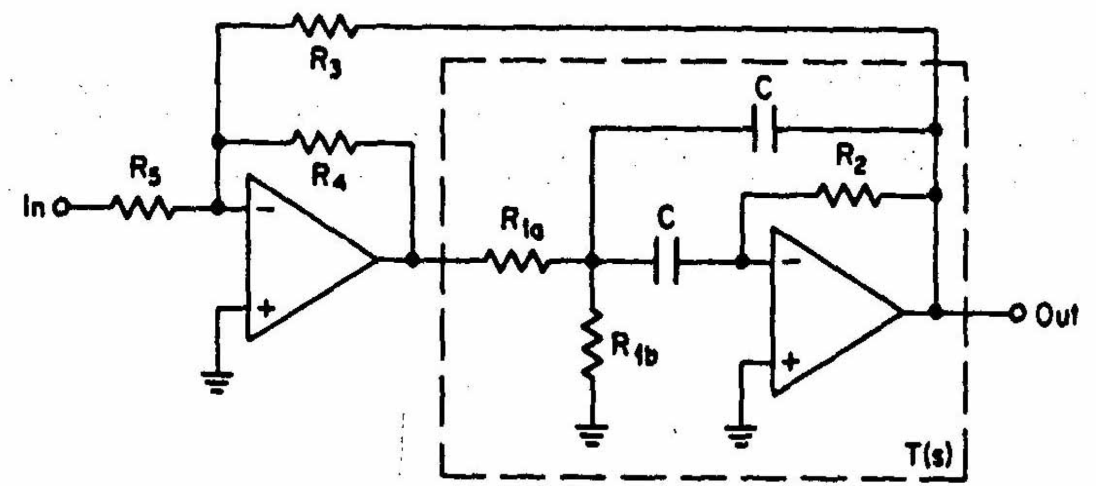
The component values can be computed from:

$$R_3 = \frac{R}{\beta} \qquad R_2 = \frac{Q_r}{\pi f_r C} \qquad R_{1b} = \frac{R_{1a}}{2Q_r^2 - 1}$$

$$R_4 = R \qquad R_{1a} = \frac{R_2}{2}$$

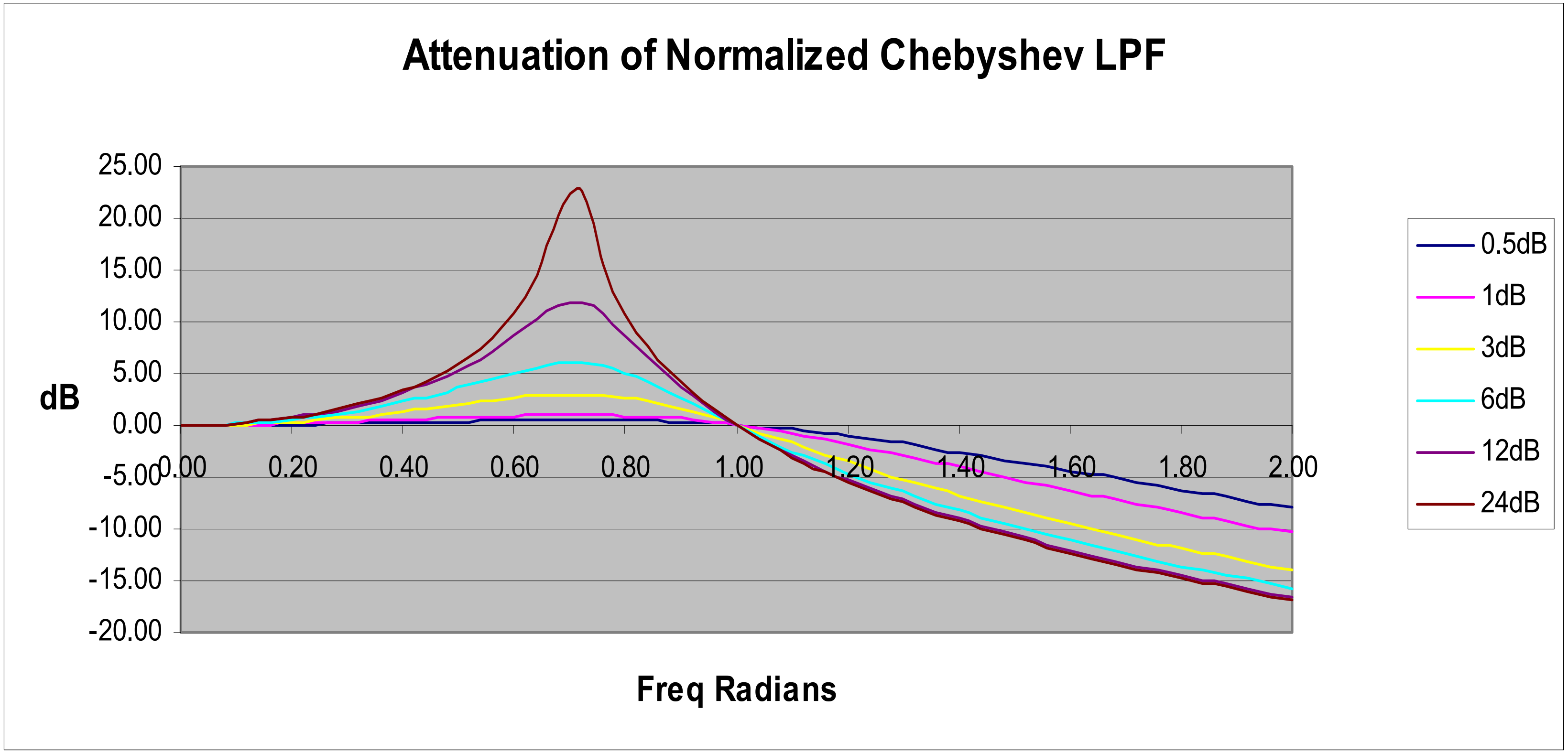
$$R_5 = \frac{R}{(1-\beta)A_r}$$

Where R and C can be conveniently chosen.

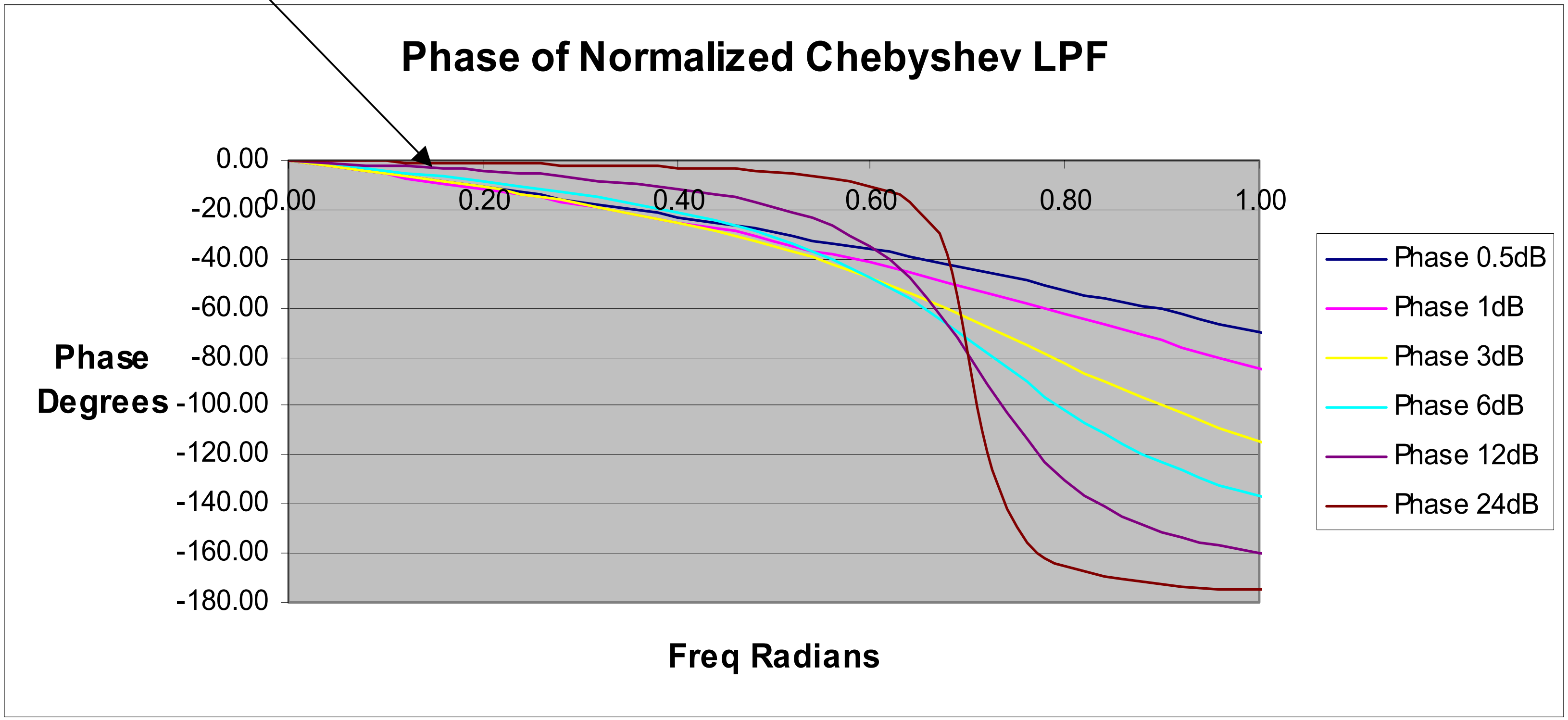


(b)

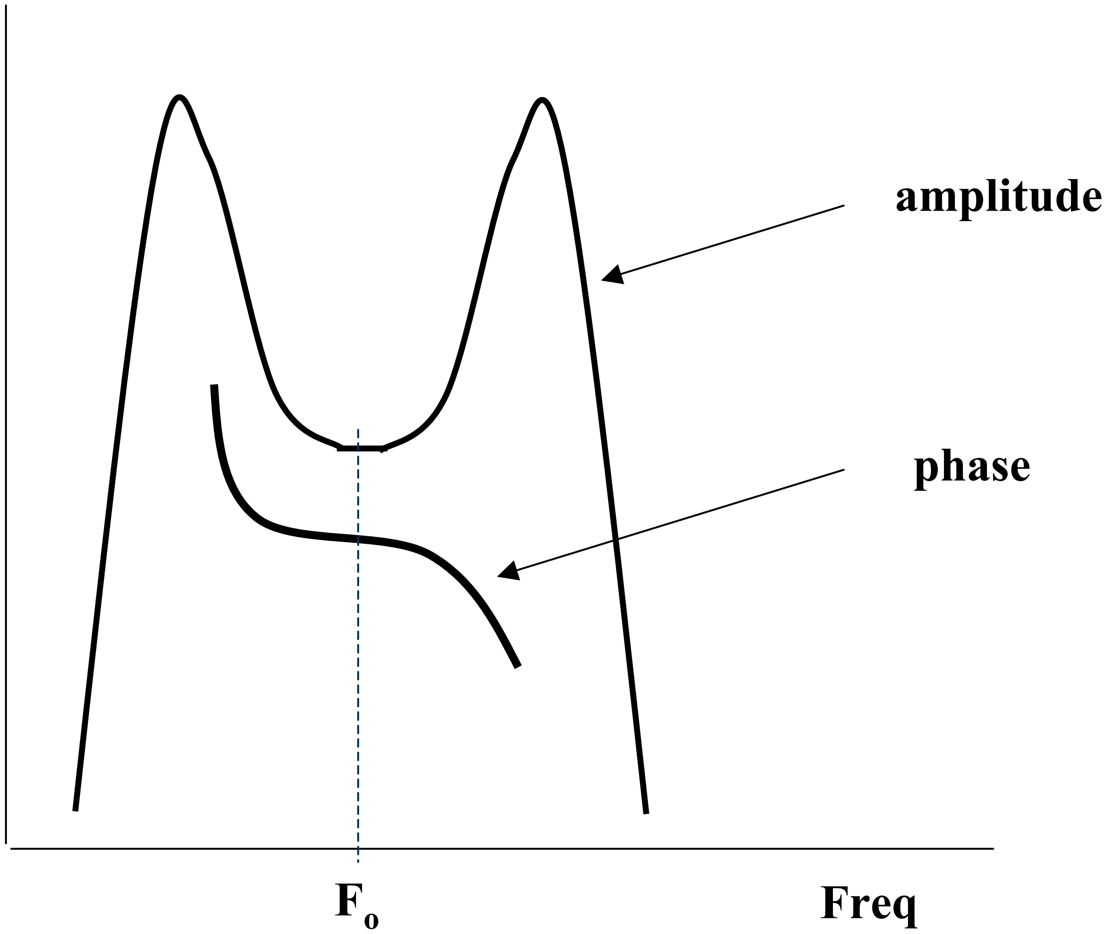
A Family of Zero Phase Shift Low-Pass Filters



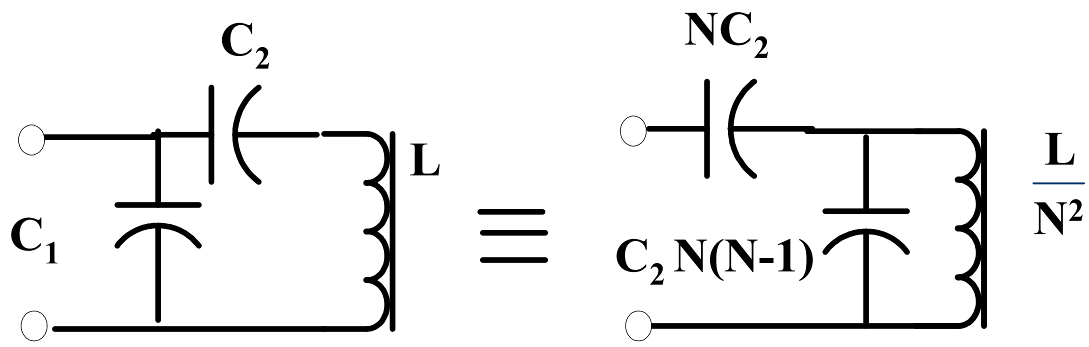
note near-zero phase slope for high ripples



Transformation to Band-Pass Filter

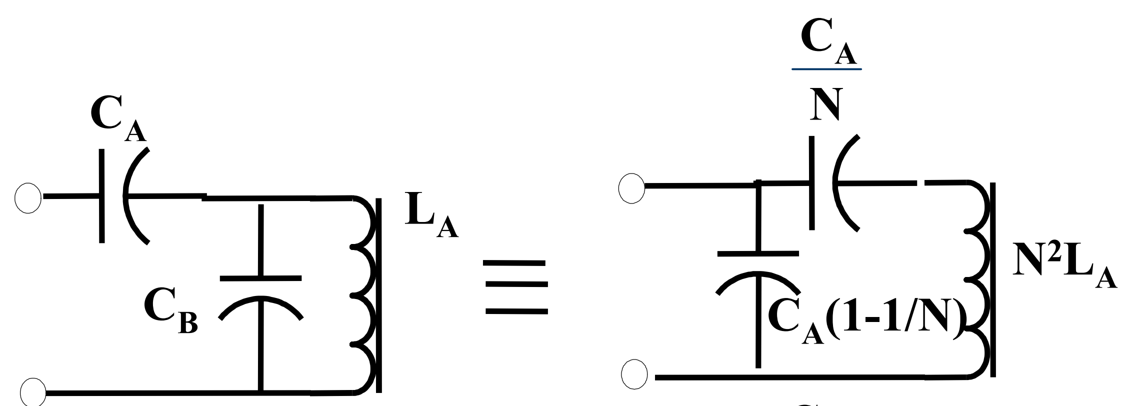


Some Useful Passive Filter Transformations to Improve Realizability



Advantages :
Reduces value of L
Allows for parasitic capacity across inductor

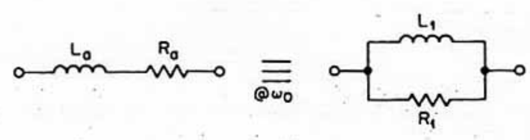
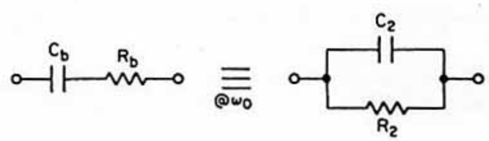
$$N = 1 + \frac{C_1}{C_2}$$



$$N = 1 + \frac{C_B}{C_A}$$

Advantages:
Increases value of L

Narrow Band Approximations

Circuit	Design Equations
	$L_1 = L_a + \frac{R_a^2}{\omega_0^2 L_a}$ $R_1 = R_a + \frac{\omega_0^2 L_a^2}{R_a}$ $L_a = \frac{L_1 R_1^2}{R_1^2 + \omega_0^2 L_1^2}$ $R_a = \frac{\omega_0^2 L_1^2 R_1}{R_1^2 + \omega_0^2 L_1^2}$
	$C_2 = \frac{C_b}{1 + \omega_0^2 C_b^2 R_b^2}$ $R_2 = R_b + \frac{1}{\omega_0^2 C_b^2 R_b}$ $C_b = C_2 + \frac{1}{\omega_0^2 R_2^2 C_2}$ $R_b = \frac{R_2}{1 + \omega_0^2 C_2^2 R_2^2}$

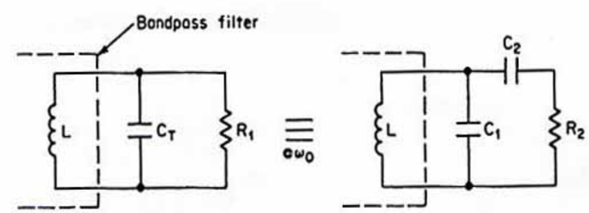
This transformation can be used to reduce the value of a terminating resistor and yet maintain the narrow-band response.

$$C_2 = \frac{1}{\omega_0 \sqrt{R_1 R_2 - R_2^2}}$$

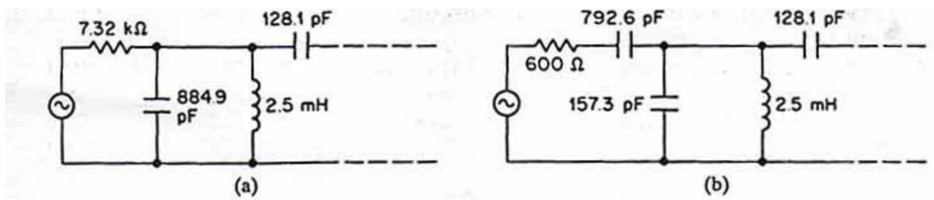
and

$$C_1 = C_T - \frac{1}{\omega_0}\sqrt{\frac{R_1 - R_2}{R_1^2 R_2}}$$

where the restrictions $R_2 < R_1$ and $(R_1 - R_2)/(R_1^2 R_2) < \omega_0^2 C_T^2$ apply.



The following example illustrates how this approximation can reduce the source impedance of a filter.



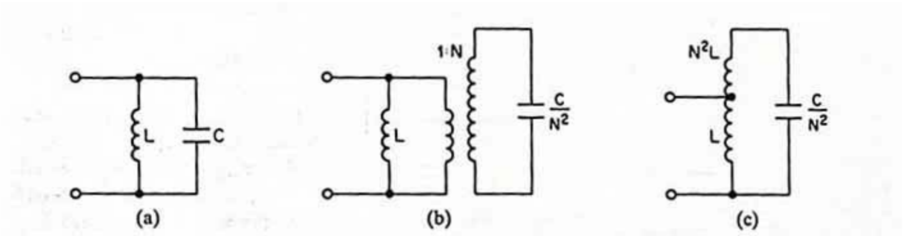
$$C_2 = \frac{1}{\omega_0 \sqrt{R_1 R_2 - R_2^2}} = \frac{1}{2\pi \times 10^5 \sqrt{7.32 \times 6 \times 10^5 - 600^2}}$$

$$= 792.6 \text{ pF}$$

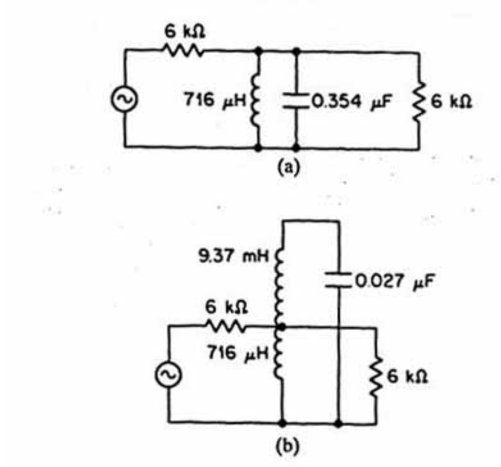
$$C_1 = C_T - \frac{1}{\omega_0} \sqrt{\frac{R_1 - R_2}{R_1^2 R_2}} = 884.9 \times 10^{-12}$$

$$- \frac{1}{2\pi \times 10^5} \sqrt{\frac{7.32 \times 10^3 - 600}{7320^2 \times 600}} = 157.3 \text{ pF}$$

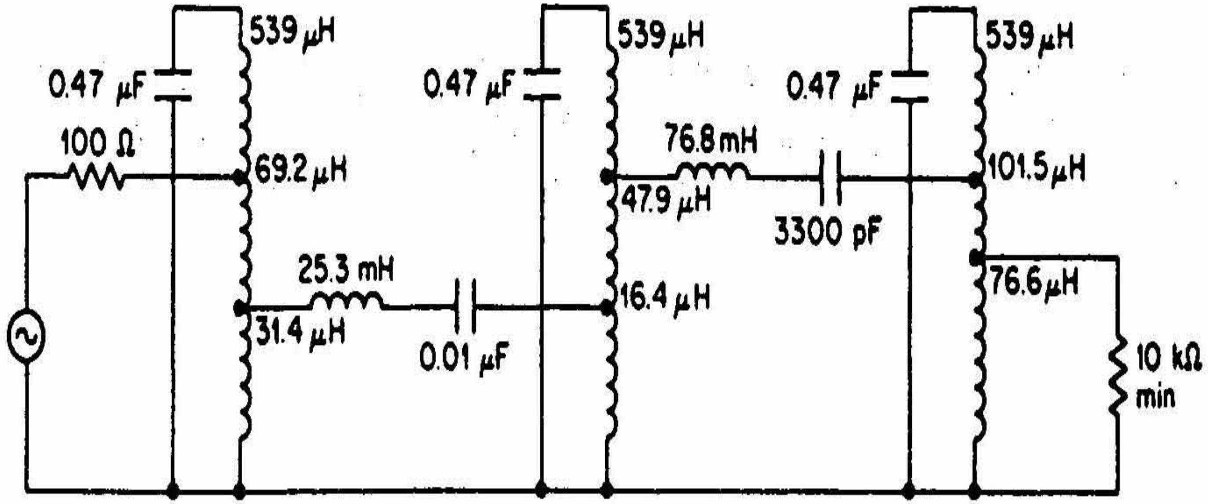
Using the Tapped Inductor



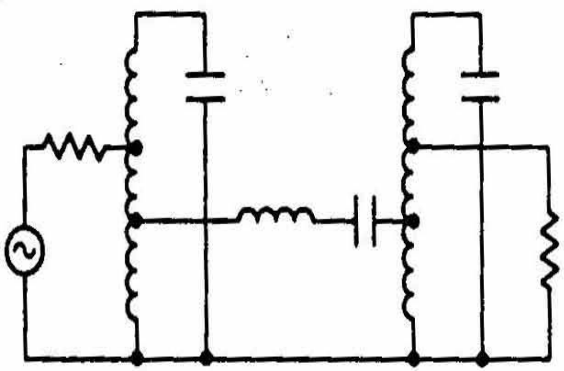
An inductor can be used as an auto-transformer by adding a tap

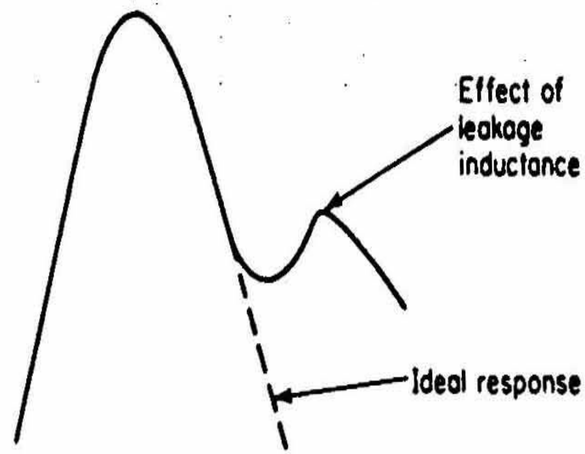


Resonant circuit capacitor values can be reduced

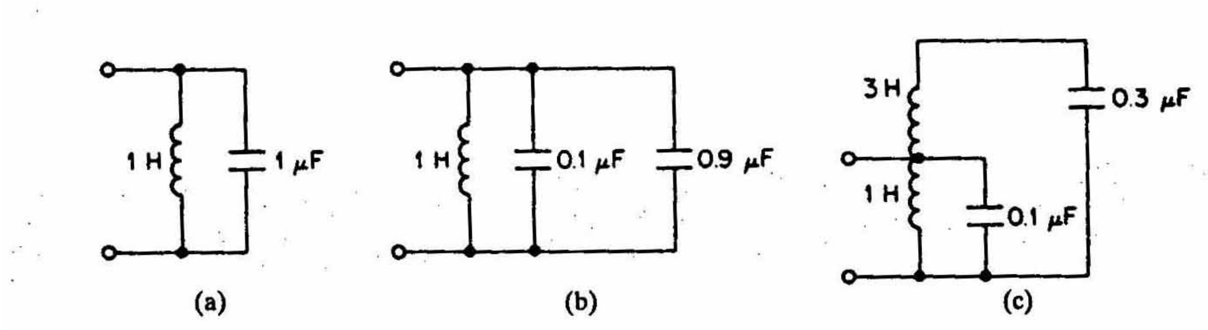


Intermediate branches can be scaled in impedance



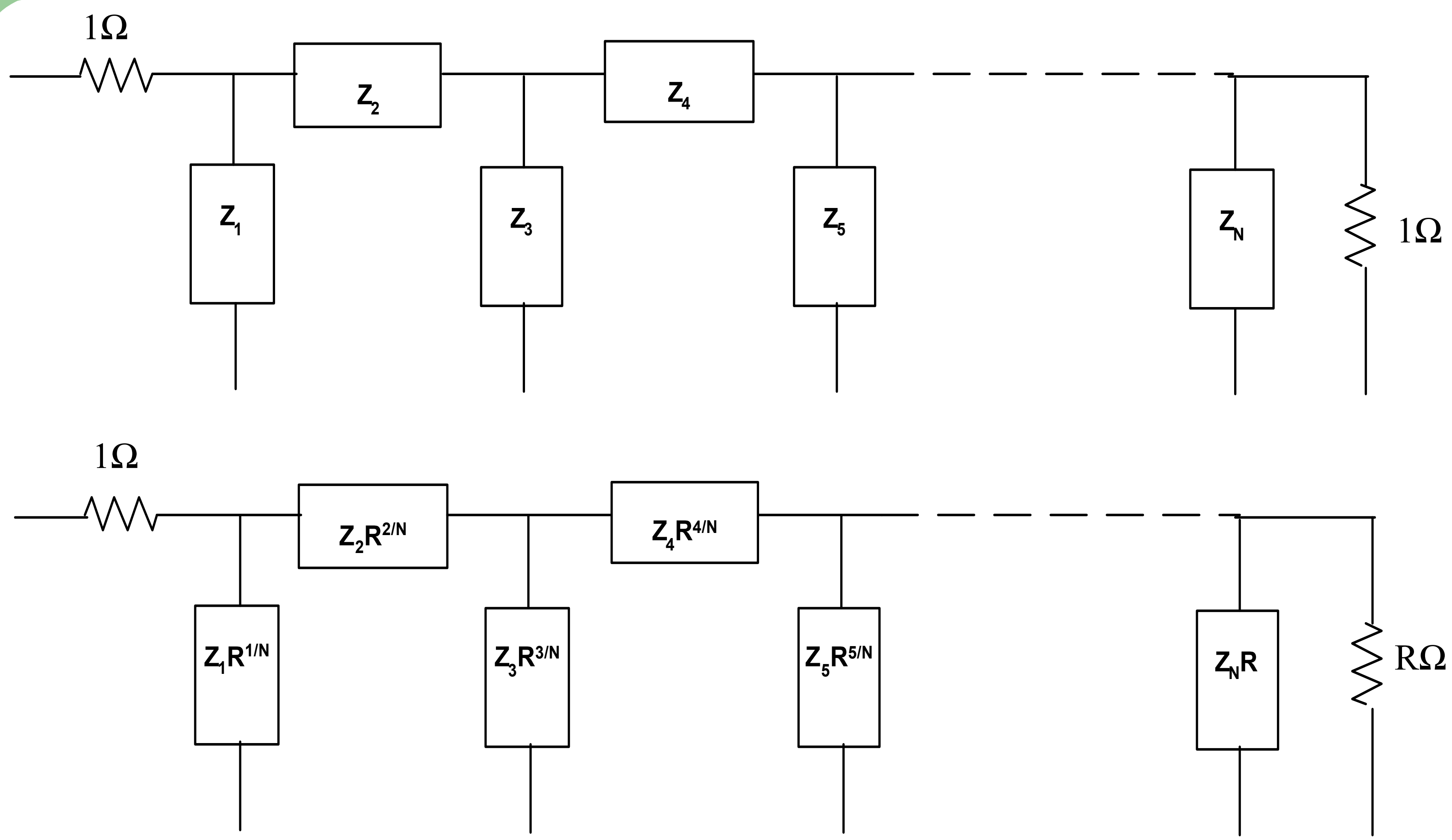


Leakage inductance can wreak havoc



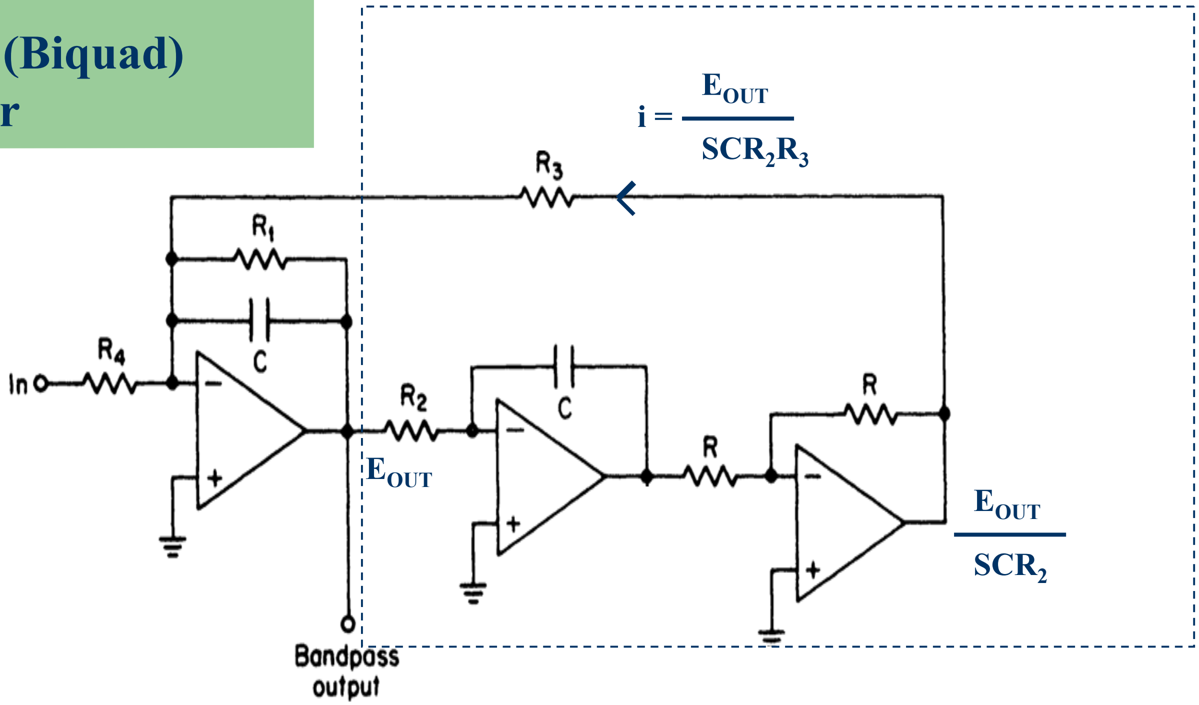
Effect of leakage inductance can be minimized by splitting capacitors which adds additional poles

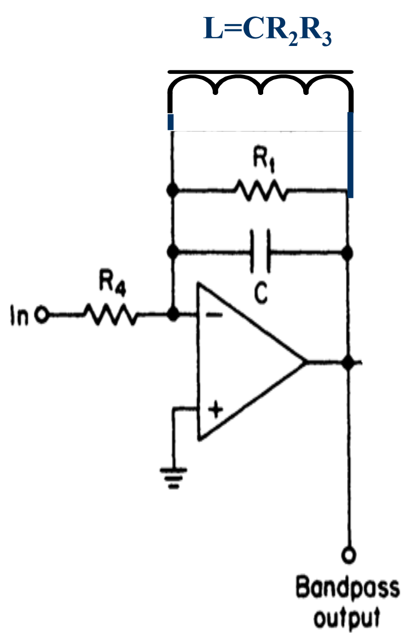
Exponentially Tapered Network



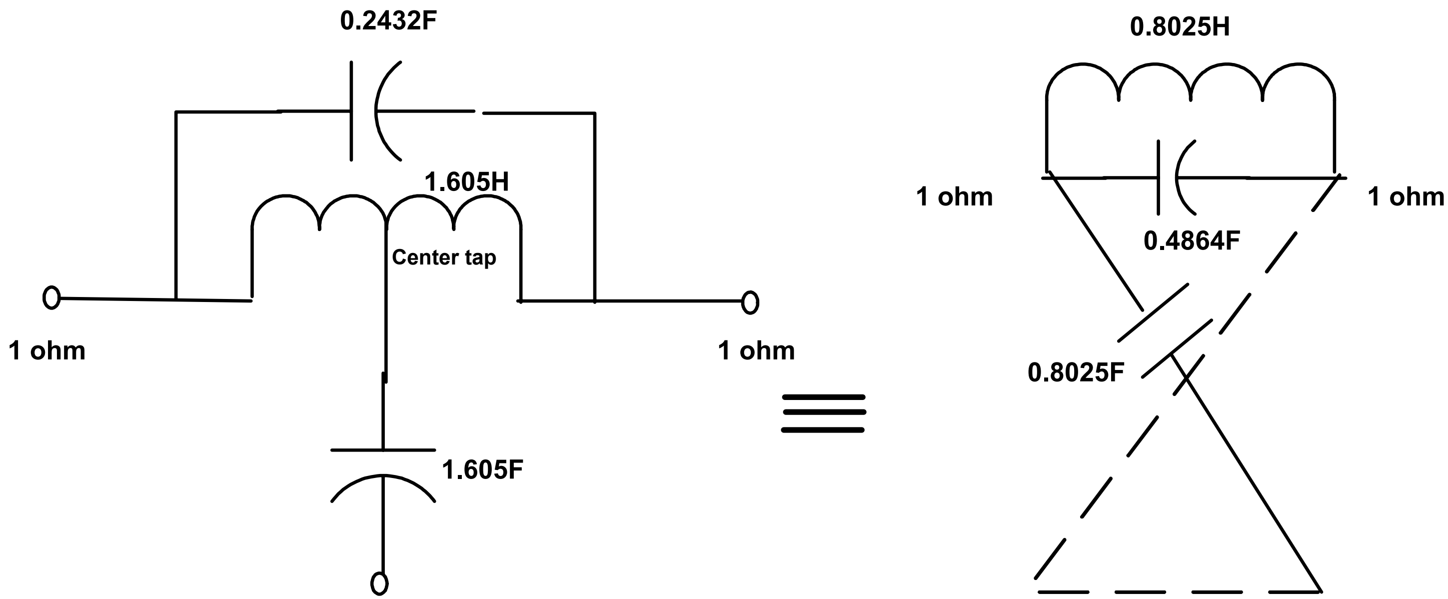
Method of exponentially tapering a network to a higher (or lower) load impedance value with minimal effect on response

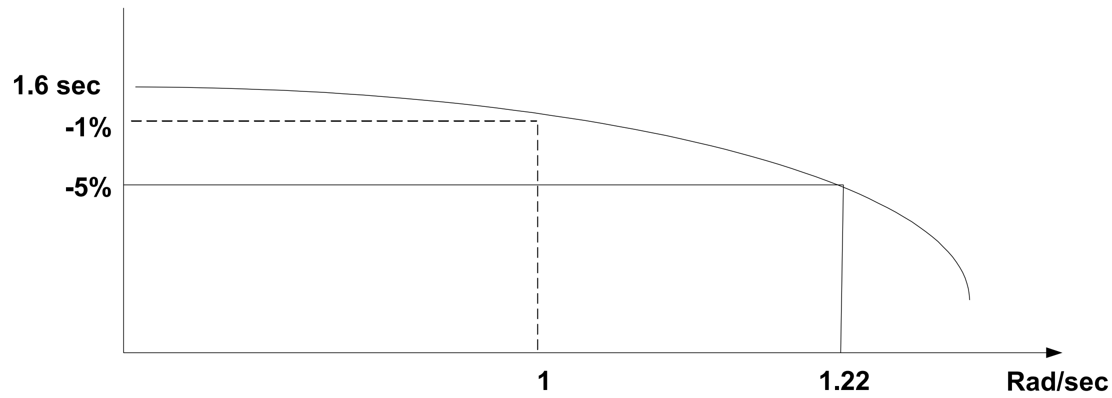
State Variable (Biquad) Bandpass Filter





All-PASS DELAY LINE SECTION





SINGLE TRANSFORMER HYBRID