How Far is the Far Field?

Some New Findings

by

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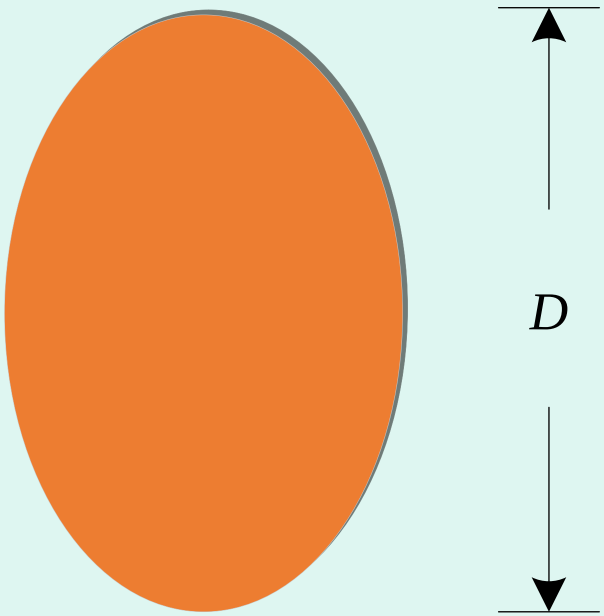
BACKGROUND - 1

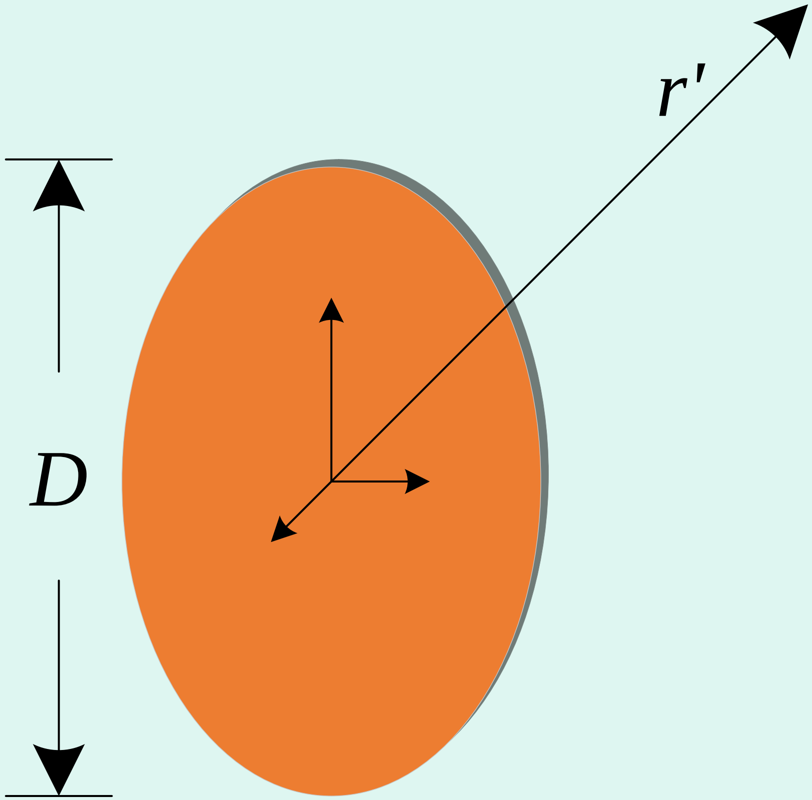


- A perfectly conducting (PEC) object scatters an incident electromagnetic (EM) wave of wavelength λ.
- The scattered fields can be represented as integrals over the scattering surface.
- These integrals become much simpler when the observer is in the Far Field.
- Knowing where the Far Field begins is essential both for computational and measurements reasons.
- The question then is: how far from the scatterer is the Far Field?

NOTES:

1. Instead of a scatterer, we may have a radiator, in which case we talk about the radiated fields.
2. D is the characteristic dimension of the object: the largest distance between any two points on the object.

BACKGROUND - 2



This formula comes about by considering the phase error in Green's function OR the discrepancy between a spherical and a planar wave front.

- The most frequently quoted formula for determining the Far Field region is

$$\boxed{r' > \frac{2D^2}{\lambda}}$$

- We call this the traditional formula (TF).
- This formula has worked very well on many occasions.
- But not well at all on others
  - In the next few slides we show cases where it has not worked and attempts at improving it.

BACKGROUND - 3

While measuring the scattering cross section of cylindrical bodies, Knott and Senior* observed that "Large errors can be incurred in measurements at the standard Far Field distance**, especially of bodies characterized by strong edge scattering…Errors as great as 6 dB can occur and it may be necessary to exceed the usual Far Field distance** by a factor of 5 or more to reduce the error to 1 dB".

*E. F. Knott and T. B. A. Senior, "How Far is Far?", *IEEE Trans. Antennas Propagat*., Vol. AP-22, No. 5, 1974, pp. 363-369.

**As given by the TF.

BACKGROUND - 4

Carver and Newell* state that the TF "is inadequate for pattern measurements in connection with space-borne SAR antennas".

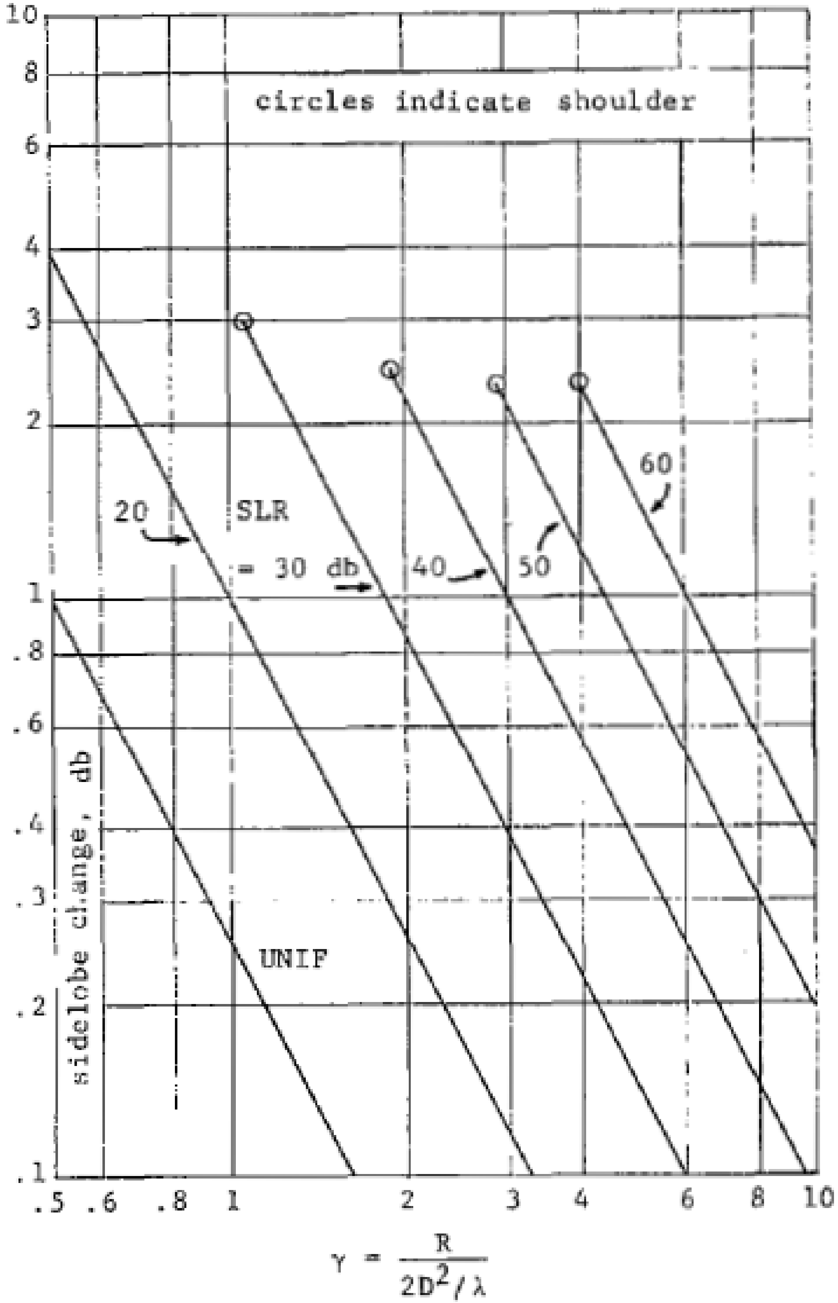
*K. R. Carver and A. C. Newell, "SAR Antenna Calibration Techniques", Proc. of the 1978 Syn. Aperture Radar Technol. Conf. (NASA Document ID 19780022509, downloadable from http://ntrs.nasa.gov/search.jsp?R=19780022509).

BACKGROUND - 5

- Hansen* studied the error produced by the TF in calculating the sidelobe level of an antenna array, and how this error reduces by using a factor larger than the factor 2 in the TF.

- In the next slide we show the principal results he obtained using a line source with a Taylor distribution.

* R. C. Hansen, "Measurement Distance Effects on Low Sidelobe Patterns", *IEEE Trans. Antennas Propagat.*, vol. AP-32, No. 6, pp. 591-594, 1984.



1. Sidelobe change versus normalized measurement distance for Taylor $\bar{n}$ line source.

BACKGROUND - 6

- Graph of sidelobe change (dB) as a function of multiples of the TF.
- SLR: Sidelobe Level Ratio (main to first sidelobe).
- Example: for a 40 dB SLR, we need to go almost 10 times farther out than what the TF predicts in order to have a sidelobe change of only 0.1 dB.

BACKGROUND - 7

- Yaghjian* suggests the formula

$$\boxed{r' > \frac{2D^2}{\lambda} + \lambda}$$

commenting that "the added λ covers the possibility of the maximum dimension D of the antenna being smaller than a wavelength".

- No other justification is provided.

* A. D. Yaghjian, "An Overview of Near-Field Antenna Measurements", *IEEE Trans. Antennas Propagat.*, vol. AP-34, No. 1, pp. 30-45, 1986.

BACKGROUND - 9

Laybros and Combes* studied dipoles of length up to one wavelength and their results indicate that the Far Field begins at a distance much greater than the TF dictates.

*S. Laybros and P. F. Combes, "On Radiating-Zone Boundaries of Short, λ / 2, and λ Dipoles", *IEEE Antennas Propagat. Mag.,* Vol. 46, No. 5, Oct. 2004.

BACKGROUND - 10

Abdallah *et al.** conducted numerical experiments using thin-wire dipoles of three different sizes. They concluded that the dipole must be at least five wavelengths long ($D \geq 5l$) for the TF to hold.

*M. N. Abdallah, T. K. Sarkar, M. Salazar-Palma and V. Monebhurrun, "Where Does the Far Field of an Antenna Start?" IEEE Trans. Antennas Propagat. Mag., vol. 58, No. 5, pp. 115-124, Oct. 2016.

BACKGROUND - 11

TABLE 1–DEFINITIONS OF THE NEAR-FIELD/FAR-FIELD BOUNDARY

Definition for shielding	Remarks	Reference
$\lambda/2\pi$	1/r terms dominant	Ott, White
$5\lambda/2\pi$	Wave impedance=377Ω	Kaiser
For antennas		
$\lambda/2\pi$	1/r terms dominant	Krause
3λ	D not >>λ	Fricitti, White, Mil-STD-449C
$\lambda/16$	Measurement error<0.1 dB	Krause, White
$\lambda/8$	Measurement error<0.3 dB	Krause, White
$\lambda/4$	Measurement error<1 dB	Krause, White
$\lambda/2\pi$	Satisfies the Rayleigh criteria	Berkowitz
$\lambda/2\pi$	For antennas with D<<λ and printed-wiring-board traces	White, Mardiguian
$2D^2/\lambda$	For antennas with D>>λ	White, Mardiguian
$2D^2/\lambda$	If transmitting antenna has less than 0.4D of the receiving antenna	MIL-STD 462
$(d+D)^2/\lambda$	If d>0.4D	MIL-STD 462
$4D^2/\lambda$	For high-accuracy antennas	Kaiser
$50D^2/\lambda$	For high-accuracy antennas	Kaiser
$3\lambda/16$	For dipoles	White
$(D^2+d^2)/\lambda$	If transmitting antenna is 10 times more powerful than receiving antenna, D	MIL-STD-449D

More definitions of the Far Field from:

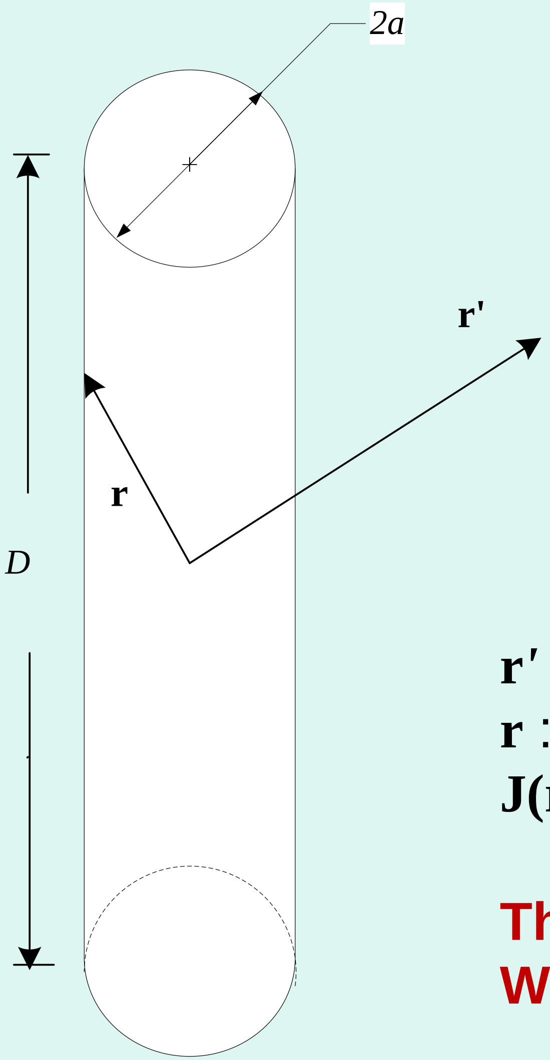
C. Capps, "*Near Field or Far Field?*", EDN Magazine, pp. 95-102, August 16, 2001. (http://www.edn.com/design/communications-networking/4340588/Near-field-or-far-field-)

BACKGROUND - 9

- It appears then that there is a degree of uncertainty as to where the Far Field begins.

- In what follows, we will use a mathematical approach to examine the issues involved and reach a new set of formulas.

BASIC SETUP



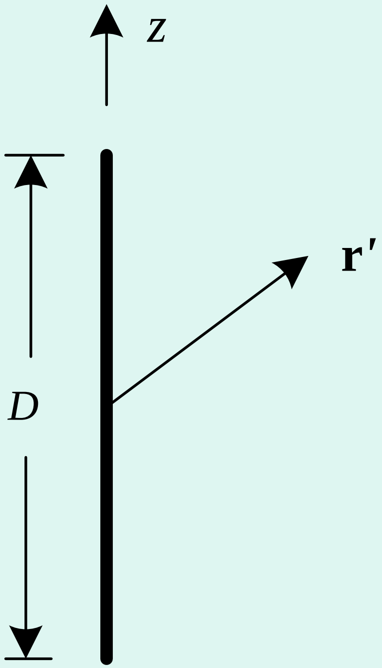
- A PEC cylinder of radius ***a*** scatters an EM wave.
- The magnetic intensity of the scattered field is given by

$$\mathbf{H}^s(\mathbf{r}') = \int_S \mathbf{J}(\mathbf{r}) \times \nabla g(R)\, dS\,, \quad g(R) = \frac{\mathrm{e}^{-ikR}}{4\pi R}\,, \quad R = |\mathbf{r} - \mathbf{r}'|$$

$\mathbf{r}'$: field or observation point
$\mathbf{r}$: source or integration point on surface S of cylinder
$\mathbf{J}(\mathbf{r})$: total linear current density on surface of cylinder

This expression is unnecessarily complicated. Without loss of generality, we will use a simpler one.

BASIC SETUP SIMPLIFIED



- We let the radius of the cylinder go to zero, ending up with a thin wire.
- In the process, the source point becomes

$$\mathbf{r} = z\hat{z}\,, \quad -\frac{D}{2} \le z \le \frac{D}{2}$$

- while, for the linear current density, we write

$$\mathbf{J}(\mathbf{r}) = \frac{I(z)}{2\pi a}\hat{z}$$

- and for the distance function

$$R = \sqrt{r'^2 - 2\left(\hat{z}\cdot\hat{r}'\right)zr' + z^2}\,, \quad r' = \left|\mathbf{r}'\right|, \quad \hat{r}' = \mathbf{r}'/r'$$

OBJECTIVES

- The expression for the magnetic intensity becomes

$$\mathbf{H}^{s}\left(\mathbf{r}'\right) = -\int_{-D/2}^{D/2} \left(\hat{z}\times\hat{R}\right) I(z) g(R) \left(ik + \frac{1}{R}\right) dz\,, \quad \hat{R} = \left(z\hat{z} - \mathbf{r}'\right)/R$$

$$g(R) = \frac{\mathrm{e}^{-ikR}}{4\pi R}\,, \quad R = \sqrt{r'^{2} - 2\left(\hat{z}\cdot\hat{r}'\right)zr' + z^{2}}$$

- In the Far Field, we drop the 1 / R term in the integrand, and we approximate Green's function in amplitude and phase by the function

$$\tilde{g}\left(z,r'\right) = \frac{\mathrm{e}^{ikr'\left[1-\left(\hat{z}\cdot\hat{r}'\right)\frac{z}{r'}\right]}}{4\pi r'} = \frac{\mathrm{e}^{ikr'}\mathrm{e}^{-ik\left(\hat{z}\cdot\hat{r}'\right)z}}{4\pi r'}$$

- Below, we will define the Far Field region in terms of the penalties we are willing to accept in making these three simplifications.

GREEN'S FUNCTION: ERROR IN AMPLITUDE

- The amplitude of Green's function is 1 / R
- We define the relative error in amplitude, η , by

$$\eta = \frac{\left|(1/R)-(1/R_a)\right|}{1/R} = \left|1-\frac{R}{R_a}\right|, \quad R_a = r'$$

- On which we impose the error bound

$$0 \le \eta < \alpha , \quad \alpha > 0$$

- Which ultimately implies that

$$\frac{D}{2r'} < \alpha \quad \text{OR} \quad \boxed{r^a > \frac{D}{2\alpha}}$$

- This is the definition of the Far Field region for the relative amplitude error being smaller than α.

GREEN'S FUNCTION: ERROR IN PHASE - 1

- The exact phase is kR.
- The approximate phase is $kr'\left[1-\left(\hat{z}\cdot\hat{r}'\right)\left(\dfrac{z}{r'}\right)\right]$
- In effect, we have replaced the distance function by its two-term Taylor series expansion about $u = (z / r') = 0$.
- What is the error we commit? It is the remainder in *Taylor's Theorem with a Remainder*. For our case

$$\sqrt{1-2\left(\hat{z}\cdot\hat{r}'\right)u+u^2} = 1-\left(\hat{z}\cdot\hat{r}'\right)u+R_1$$

- This is an ***exact*** expression with R_1 the remainder.

GREEN'S FUNCTION: ERROR IN PHASE - 2

- The remainder is given by

$$\boxed{R_1 = \frac{u^2}{2}\frac{d^2\left(\sqrt{1-2\left(\hat{z}\cdot\hat{r}'\right)u_1 + u_1^{\,2}}\right)}{du_1^{\,2}} = \frac{1-\left(\hat{z}\cdot\hat{r}'\right)^2}{\left[1-2\left(\hat{z}\cdot\hat{r}'\right)u_1 + u_1^{\,2}\right]^{3/2}}\frac{u^2}{2}}$$

$$\boxed{0 < |u_1| < |u|}$$

- STRATEGY: Maximize the remainder and then bound it to get an expression for the Far Field region.

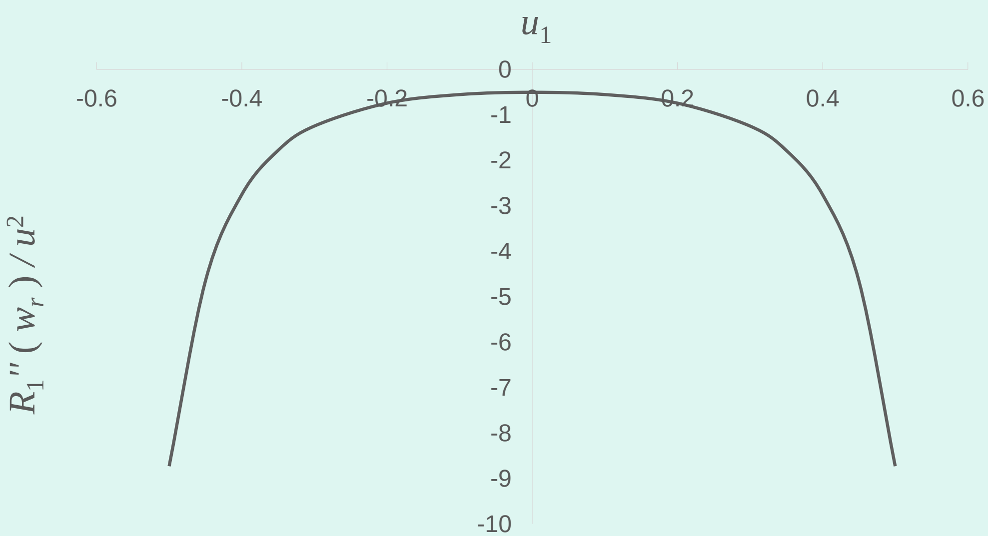
- First maximize with respect to (wrt) $w = \hat{z}\cdot\hat{r}'$

  by taking the first derivative of R_1 and setting it to zero.

GREEN'S FUNCTION: ERROR IN PHASE - 3

- The relevant zeros are given by $$w_r = \frac{1+u_1^2 - \sqrt{\left(1+u_1^2\right)^2 - 3u_1^2}}{u_1}$$
- At these values, the second derivative is

$$R_1''\left(w_r\right) = -u^2 \frac{\sqrt{\left(1+u_1^2\right)^2 - 3u_1^2}}{\left\{-\left(1+u_1^2\right) + 2\sqrt{\left(1+u_1^2\right)^2 - 3u_1^2}\right\}^{5/2}} < 0$$

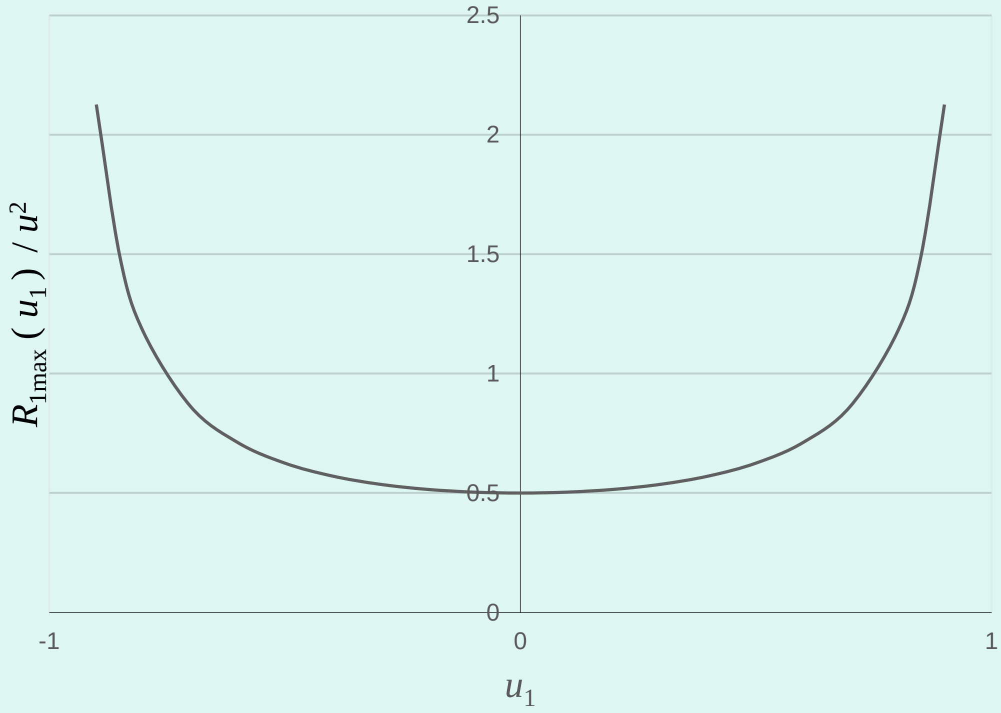


w_r correspond to a maximum

GREEN'S FUNCTION: ERROR IN PHASE - 4

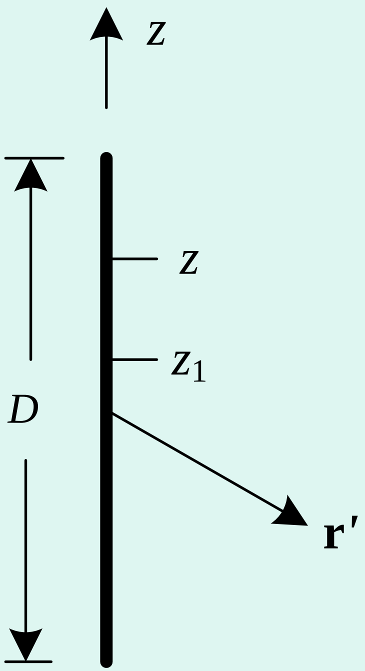
- The remainder maximized wrt w is

$$R_{1\max}(u_1) = \frac{\left[\left(1+u_1^2\right) + 2\sqrt{\left(1+u_1^2\right)^2 - 3u_1^2}\right]^{1/2}}{\sqrt{3}\left(1-u_1^2\right)\left[1+u_1^2+\sqrt{\left(1+u_1^2\right)^2 - 3u_1^2}\right]} u^2$$



- The next step is to maximize the remainder wrt u_1.
- Note that this is a strictly increasing function of u_1^2.

GREEN'S FUNCTION: ERROR IN PHASE - 5



- We recall that

$$u_1^2 = \left(\frac{z_1}{r'}\right)^2 < u^2 = \left(\frac{z}{r'}\right)^2 \leq \left(\frac{D}{2r'}\right)^2$$

- But, from amplitude condition, $\dfrac{D}{2r'} < \alpha$

- Therefore, $u_1^2 < \left(\dfrac{D}{2r'}\right)^2 < \alpha^2$

- And

$$R_{1\max}(u_1) < \underbrace{\frac{\left[1+\alpha^2+2\sqrt{\left(1+\alpha^2\right)^2-3\alpha^2}\right]^{1/2}}{\sqrt{3}\left(1-\alpha^2\right)\left[1+\alpha^2+\sqrt{\left(1+\alpha^2\right)^2-3\alpha^2}\right]}}_{B(\alpha)} u^2 = B(\alpha)u^2 \leq B(\alpha)\left(\frac{D}{2r'}\right)^2$$

GREEN'S FUNCTION: ERROR IN PHASE - 6

- We impose the phase error bound

$$\frac{2\pi}{\lambda} r' R_{1\max}\left(u_1\right) < \frac{\pi}{\beta}$$

- Which, in combination with the previous statement, yields

$$\boxed{r^p > \frac{\beta B\left(\alpha\right)}{2} \frac{D^2}{\lambda}}$$

- This is the definition of the far-field region for the phase error being smaller than π/β.

- Note that if we set $\alpha = 0$, $\beta = 8$, we recover the TF since $B(0) = 0.5$.

THE SECOND TERM IN THE INTEGRAL REPRESENTATION OF THE MAGNETIC FIELD - 1

- From above, the scattered field is

$$\mathbf{H}^s(\mathbf{r}') = -\int_{-D/2}^{D/2} \left(\hat{z} \times \hat{R}\right) I(z) g(R) \left(ik + \frac{1}{R} \right) dz$$

- In the Far Field, we keep this term and we drop this term.
- We can then impose the amplitude condition

$$\frac{\frac{1}{R}}{k} < 10^{-\gamma} \; , \quad \gamma > 0$$

- We can show that

$$\frac{\frac{1}{R}}{k} \leq \frac{1}{kr'\left(1 - \frac{D}{2r'}\right)} < 10^{-\gamma}$$

THE SECOND TERM IN THE INTEGRAL REPRESENTATION OF THE MAGNETIC FIELD - 2

- From the last statement, we get

$$\boxed{r^{s,a} > \frac{10^{\gamma}}{k} + \frac{D}{2}}$$

- This is the magnitude condition on the Far Field for omitting the second term in the last factor of (2.6) in favor of the first. The superscript *s,a* signifies this omission.

- It has the unique property that the right-hand side does NOT go to zero with D.

THE SECOND TERM IN THE INTEGRAL REPRESENTATION OF THE MAGNETIC FIELD - 3

- We re-examine the two terms for phase error

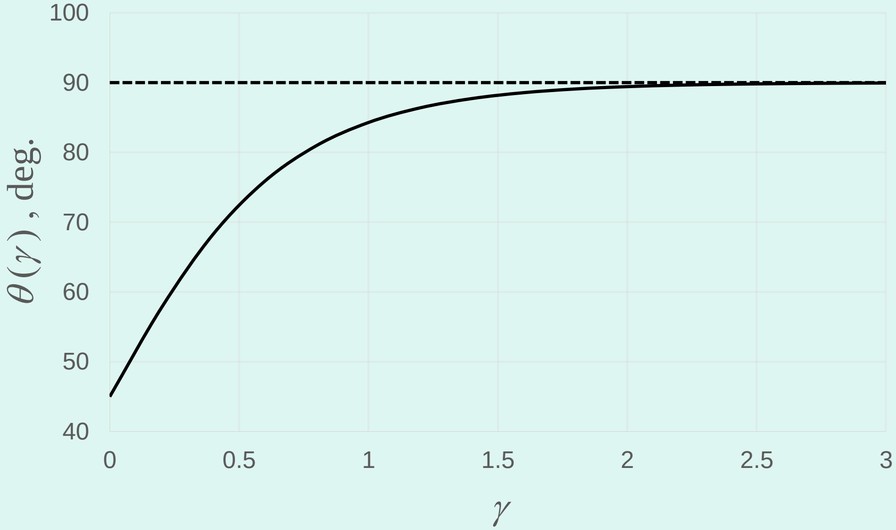
$$\frac{1}{R}+ik=\sqrt{\left(\frac{1}{R}\right)^2+k^2}\,\mathrm{e}^{i\tan^{-1}(kR)}$$

- Dropping the first term results in a phase of 90 deg. How close is it to the actual phase?
- We recall that $kR>10^{\gamma}$
- Then the actual phase (in degrees) is between

$$\theta=\frac{180}{\pi}\tan^{-1}\left(10^{\gamma}\right)$$

and 90 deg. Graphically (next slide)

THE SECOND TERM IN THE INTEGRAL REPRESENTATION OF THE MAGNETIC FIELD - 4



- The exact phase is somewhere between the two curves.
- If $\gamma = 1$, the phase error is less than 5.7 deg.
- If $\gamma = 2$ (more likely), the phase error is less than 0.6 deg.
- Despite the smallness of the error, we can derive a formula based on it (next slide).

THE SECOND TERM IN THE INTEGRAL REPRESENTATION OF THE MAGNETIC FIELD - 5

We bound the difference between the approximate and exact phase

$$\frac{\pi}{2} - \tan^{-1}\left(kR\right) < \frac{\pi}{\delta}, \quad \delta > 0$$

By expanding the arctangent in inverse powers of *kR*, we can show that

$$\frac{\pi}{2} - \tan^{-1}\left(kR\right) < \frac{1}{kR} \leq \frac{1}{kr'\left(1 - \dfrac{D}{2r'}\right)}$$

From the two, we get the condition

$$\boxed{r^{s,p} > \frac{\delta}{\pi k} + \frac{D}{2}}$$

SUMMARY OF THE FORMULAS

- We first measure distance in wavelengths by writing

$$r_{ff} = \frac{r'}{\lambda}, \quad d = \frac{D}{\lambda}$$

- Then we have

$$r_{ff}^{a} > \frac{d}{2\alpha}, \quad r_{ff}^{p} > \frac{\beta}{4} d^{2}, \quad r_{ff}^{s,a} > \frac{10^{\gamma}}{2\pi} + \frac{d}{2}, \quad r_{ff}^{s,p} > \frac{\delta}{2\pi^{2}} + \frac{d}{2}$$

- And

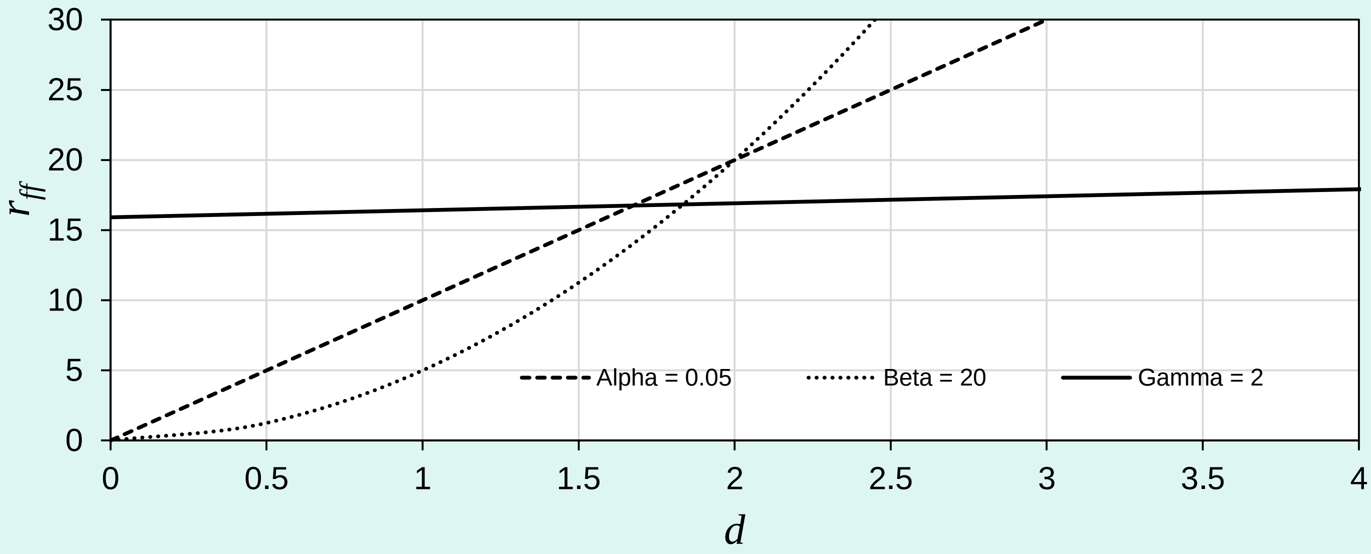
$$r_{ff} = \max\left\{ r_{ff}^{a}, r_{ff}^{p}, r_{ff}^{s,a}, r_{ff}^{s,p} \right\}$$

- The values of α, β, γ and δ are application dependent.

➢ We will refer to the four formulas above as α, β, γ, δ in the graphs that follow.

EXAMPLE 1: TYPICAL VALUES

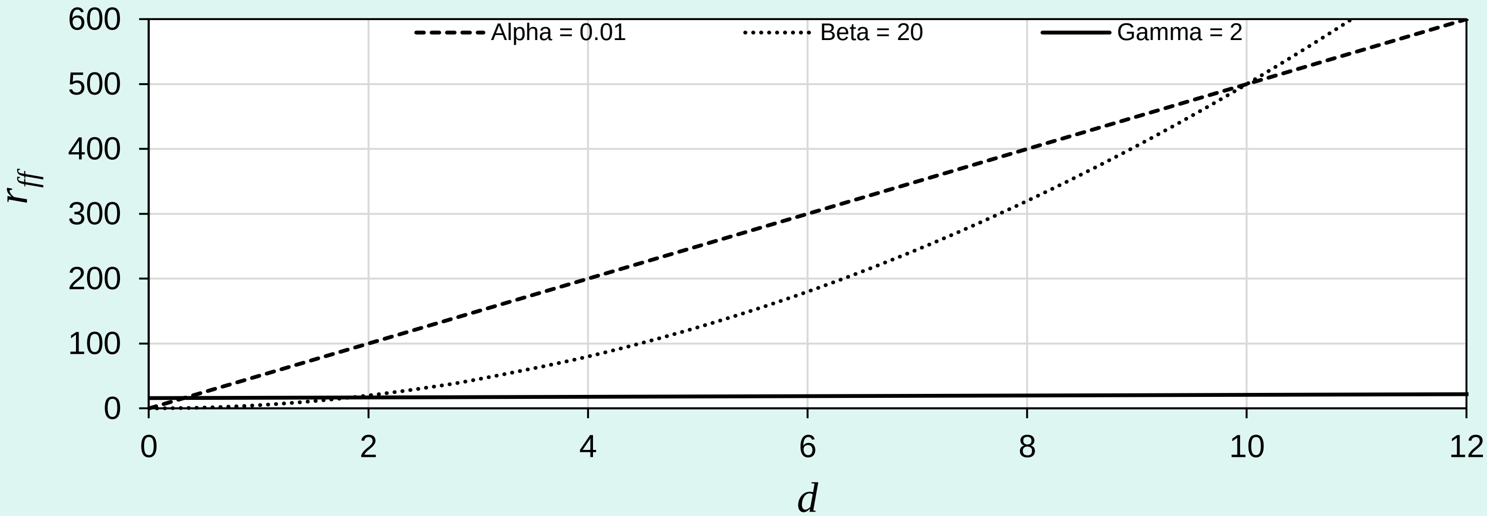
- α = 0.05, a 5% relative error in amplitude.
- γ = 2, a 20-dB drop of the $1/R$-term relative to the k-term.
- β = 20, which keeps the phase error to below 9 degrees.



We see that γ dominates for wire lengths up to 1.68 lambda, with α taking over up to 2 lambda. After that, the phase error formula dominates.

EXAMPLE 2: SMALL $\alpha = 0.01$ (1 % rel. error in ampl.)

$\alpha = 0.01$, $\beta = 20$, $\gamma = 2$

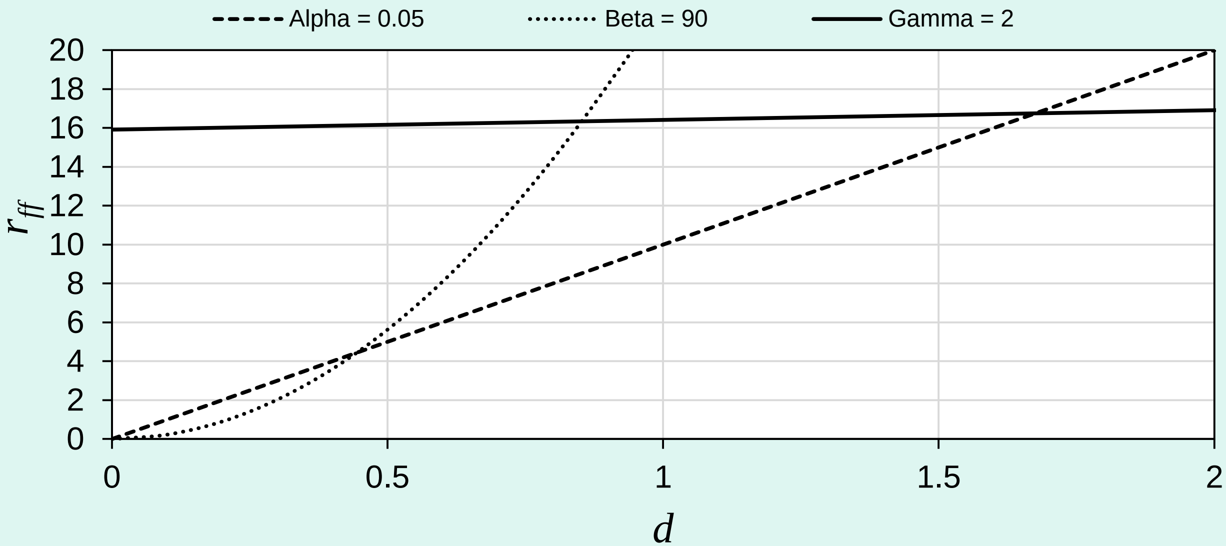


We see that γ dominates for wire lengths up to 0.3215 lambda, with α taking over up to 10 lambda. After that, the phase error formula dominates.

Note that for a 10-lambda wire, the Far Field begins at 500 lambda!

EXAMPLE 3: LARGE $\beta = 90$ (2-deg. phase error)

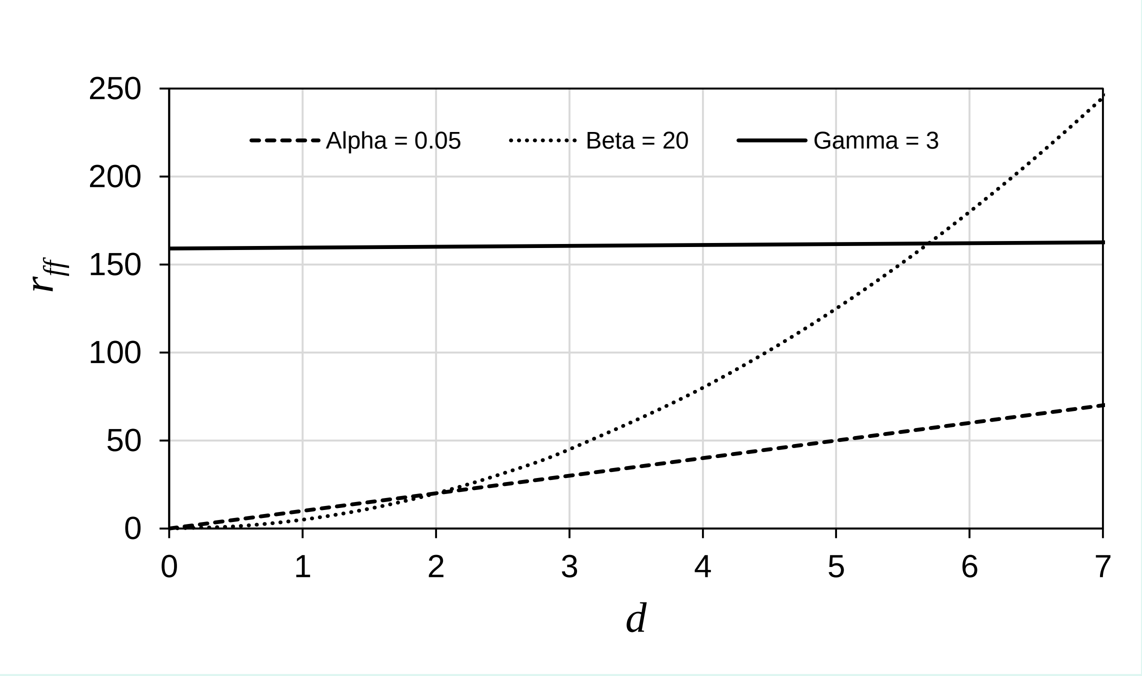
$\alpha = 0.05$, $\beta = 90$, $\gamma = 2$



We see that γ dominates for wire lengths up to 0.8522 lambda, with the phase error formula taking over after that. Graph α plays no role here.

EXAMPLE 4: LARGE $\gamma = 3$, (implies that $kR > 1{,}000$)

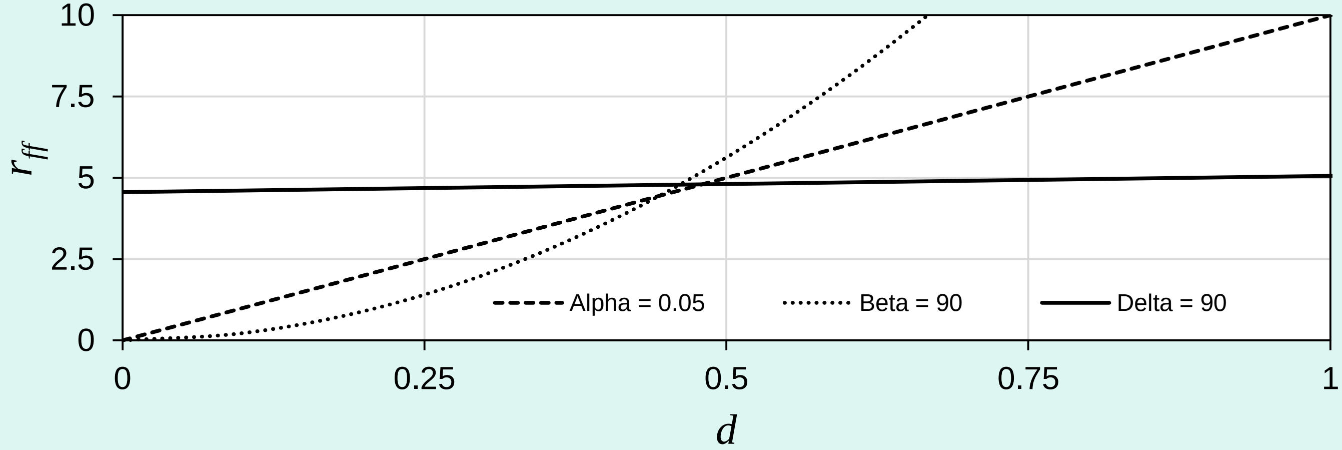
$\alpha = 0.05$, $\beta = 20$, $\gamma = 3$



We see that γ dominates for wire lengths up to 5.6921 lambda, with the phase error formula taking over after that. Graph α plays no role here.

EXAMPLE 5: TIGHT CONTROL OF PHASE

$\alpha = 0.05$, $\beta = 20$, $\delta = 90$



We see that δ dominates for wire lengths up to 0.4614 lambda, with the phase error formula taking over after that. Graph α plays no role here.

A NOTE

The approach we describe above is similar to that in Stutzman and Thiele*. The analysis, however, is quite different and does not require the distinction between short and long dipoles (or wires in our case). We also make the important distinction of including the error constraints in our formulas thus stressing the point that these formulas are not hardwired but depend on the case at hand.

* W. L. Stutzman and G. A. Thiele, *Antenna Theory and Design*, 3rd ed. New York: John Wiley & Sons, 2012.

CONCLUDING REMARKS

- We developed formulas for locating the Far Field boundary. They
  - are based on an analysis of the integral representation for the scattered/radiated magnetic field.
  - are good for back-of-the-envelope calculations and, maybe, more.

- A similar analysis may be feasible in the presence of an infinite, PEC plane.

- Usually, planes are neither infinite nor perfect, and articles of interest (*e.g.*, antennas) may not be PEC and may be attached to some complex platform.
  - A safe, quick and inexpensive way to proceed in such situations is to use computer simulations rather than formulas as above.

- Thank you for your attendance and attention!