INTRODUCTION TO ACTIVE AND PASSIVE ANALOG FILTER DESIGN INCLUDING SOME INTERESTING AND UNIQUE CONFIGURATIONS

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TOPICS

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- Basic Filter Polynomial types
- Frequency and Impedance Scaling
- Active Low-Pass Filters
- Design of D-Element Active Low-Pass Filters
- High-Pass Filters
- Band-Pass Filters
- Band-Reject Filters
- High-Q Notch Filters
- Q-Multiplier Active Band-Pass Filters
- Some Useful Passive Filter Transformations
- Attenuators
- Power Splitters
- Miscellaneous Circuits
- Questions and Answers
Introduction

Generalized Passive Filter

\[ T(s) = \frac{E_L}{E_s} = \frac{N(s)}{D(s)} \]

\[ T(s) = \frac{1}{s^3 + 2s^2 + 2s + 1} \]

N=3 Butterworth
Normalized 3dB at 1 Rad/s

Pole Zero Plot on j-Omega Axis
Most Popular Filter Polynomial Types

Butterworth
Chebyshev
Linear Phase
Elliptic Function
Butterworth

The Butterworth approximation based on the assumption that a flat response at zero frequency is more important than the response at other frequencies. Normalized transfer function is an all-pole type. Roots all fall on a unit circle. The attenuation is 3 dB at 1 rad/s.

N=5 Butterworth LPF and its Dual
Butterworth LC Element Values (Representative Table, Extensive tables in reference)

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<th>$C_1$</th>
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<th>$C_3$</th>
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## Butterworth Normalized Pole Locations

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<th>Order $n$</th>
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Chebyshev

If the poles of a normalized Butterworth low-pass transfer function were moved to the right by multiplying the real parts of the pole position by a constant $k_r$ and the imaginary parts by a constant $k_i$ where both $k$'s are $<1$, the poles would lie on an ellipse instead of a unit circle.

The frequency response would ripple evenly. The resulting response is called the Chebyshev or Equiripple function.

Chebyshev filters are categorized in terms of ripple (in dB) and order N.
Chebyshev Low-Pass Filter
Linear Phase Low Pass Filters

Butterworth filters - good amplitude and transient characteristics
Chebyshev family of filters - increased selectivity but poor transient behavior
**Bessel** transfer function - optimized to obtain a linear phase, i.e., a maximally flat delay

Frequency response - Much less selective than other filter types

The low-pass approximation to a constant delay can be expressed as the following general Bessel transfer function:

\[
T(s) = \frac{1}{\sinh s + \cosh s}
\]
Linear Phase Low-Pass Filters

- Other Linear Phase Filter Families
  - Gaussian
  - Gaussian to 6dB
  - Gaussian to 12dB
  - Linear Phase with Equiripple Error (0.05° and 0.5°)
  - Maximally Flat Delay with Chebyshev Stop-Band

Extensive tables are available in the reference
Effects of Non-Linear Phase

Amplitude and Phase Response of N=3 Butterworth Low-Pass Filter

Group Delay of N=3 Butterworth Low-Pass Filter

Square Wave containing Fourier Series

\[ A(t) = A \left( \frac{1}{2} + \frac{2}{\pi} \cos \omega_1 \tau - \frac{2}{3\pi} \cos 3\omega_1 \tau + \frac{2}{5\pi} \cos 5\omega_1 \tau + \ldots \right) \]

a) Equally delayed components
b) Unequally delayed components
Elliptic Function

All filter types previously discussed are all-pole networks. Infinite rejection occurs only at the extremes of the stop-band. Elliptic-function filters have zeros as well as poles at finite frequencies. Introduction of transmission zeros allows the steepest rate of descent theoretically possible for a given number of poles. However the highly non-linear phase response results in poor transient performance.
Normalized Elliptic Function Low-Pass Filter

\[ R_{dB} = \text{Ripple in dB up to cut-off (1 Rad/sec)} \]

\[ A_{\text{min}} = \text{Minimum Stop-Band Attenuation in dB} \]

\[ \Omega_S = \text{Normalized Frequency (Rad/sec) to achieve } A_{\text{min}} \text{ (Steepness Factor)} \]
Frequency and Impedance Scaling from Normalized Circuit

**Frequency Scaling**

\[
FSF = \frac{\text{desired reference frequency}}{\text{existing reference frequency}}
\]

(a) LC filter; (b) active filter; (c) frequency response

Normalized \( N = 3 \) Butterworth low-pass filter normalized to 1 rad/sec:

(a) LC filter; (b) active filter; (c) frequency response
Denormalized low-pass filter scaled to 1000Hz: (a) LC filter; (b) active filter; (c) frequency response.

All Ls and Cs of the normalized Low-Pass filter are divided by $2 \pi F_C$ where $F_C=1,000$ Hz

Note Impractical Values
Impedance Scaling

Rule

A transfer function of a network remains unchanged if all impedances are multiplied (or divided) by the same factor.

This factor can be a fixed number or a variable, as long as every impedance element that appears in the transfer function is multiplied (or divided) by the same factor.
Impedance scaling can be mathematically expressed as

\[ R' = ZxR \]

\[ L' = ZxL \]

\[ C' = \frac{C}{Z} \]

Frequency and impedance scaling are normally combined into one step rather than performed sequentially. The denormalized values are then given by

\[ L' = \frac{ZxL}{FSF} \]

\[ C' = \frac{C}{Z \times FSF} \]

Impedance-scaled filters using Z=1K: (a) LC filter; (b) active filter.
Bartlett’s Bisection Theorem

A passive network designed to operate between two equal terminations can be modified to work between two unequal terminations and still have the same Transfer Function (except for a constant multiplier) if the network is symmetrical. It can then be bisected and either half scaled in impedance.

(a) Normalized N=3 LPF
(b) Bisected LPF
(c) Right half impedance scaled by 1.5
(d) Re-combined LPF

Resulting filter frequency and impedance scaled to 200Hz, 1K Source, 1.5K Load
Active Low-Pass Filters

Unity gain Active Low-Pass N=2 and N=3

**N=2 Section**

\[
T(s) = \frac{1}{C_1 C_2 s^2 + 2C_2 s + 1}
\]

\[
T(s) = \frac{1}{\alpha^2 + \beta^2} \left( \frac{2\alpha}{\alpha^2 + \beta^2} s + 1 \right)
\]

\[
C_1 = \frac{1}{\alpha} \quad C_2 = \frac{\alpha}{\alpha^2 + \beta^2}
\]

**N=3 Section**

\[
T(S) = \frac{1}{s^3 A + s^2 B + sC + 1}
\]

\[
A = C_1 C_2 C_3 \\
B = 2C_3(C_1 + C_2) \\
C = C_2 + 3C_3
\]

Values can be frequency and Impedance scaled
Design of D-Element Active Low-Pass Filters and a Bi-Directional Impedance Converter for Resistive Loads

Generalized Impedance Converters (GIC)

\[ Z_{11} = \frac{Z_1 Z_3 Z_5}{Z_2 Z_4} \]

By substituting RC combinations for \( Z_1 \) through \( Z_5 \), a variety of impedances can be realized.
If $Z_4$ consists of a capacitor having an impedance $1/sC$ where $s=j\omega$ and all other elements are resistors, the driving point impedance becomes:

$$Z_{11} = \frac{sCR_1 R_3 R_5}{R_2}$$

The impedance is proportional to frequency and is therefore identical to an inductor having an inductance of:

$$L = \frac{CR_1 R_3 R_5}{R_2}$$

Note: If $R_1$ and $R_2$ and part of a digital potentiometer the value of $L$ can be digitally programmable.
D Element

If both $Z_1$ and $Z_3$ are capacitors $C$ and $Z_2, Z_4$ and $Z_5$ are resistors, the resulting driving point impedance becomes:

$$Z_{11} = \frac{R_5}{s^2 C^2 R_2 R_4}$$

An impedance proportional to $1/s^2$ is called a D Element.

$$Z_{11} = \frac{1}{s^2 D}$$

where:

$$D = \frac{C^2 R_2 R_4}{R_5}$$

If we let $C=1 \text{F}, R_2=R_5=1 \ \Omega$ and $R_4=R$ we get $D=R$ so:

$$Z_{11} = \frac{1}{s^2 R}$$

If we let $s=j\omega$ the result is a Frequency Dependant Negative Resistor FDNR

$$Z_{11} = \frac{1}{-\omega^2 R}$$
D Element Circuit
A transfer function of a network remains unchanged if all impedances are multiplied (or divided) by the same factor. This factor can be a fixed number or a variable, as long as every impedance element that appears in the transfer function is multiplied (or divided) by the same factor.

The 1/S transformation involves multiplying all impedances in a network by 1/S.
### The 1/S Transformation

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<tr>
<th>Element</th>
<th>Impedance</th>
<th>Transformed Element</th>
<th>Transformed Impedance</th>
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<td>$R$</td>
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<td>$\frac{R}{s}$</td>
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Design of Active Low-Pass filter with 3dB point at 400Hz using D Elements

Filter is Linear Phase ±0.5° Type
Elliptic Function Low-Pass filter using GICs

Requirements: 0.5dB Maximum at 260Hz
60dB Minimum at 270Hz
Steepness factor=1.0385

Normalized Elliptic Function Filter
C11 20 θ=75°
N=11 R_{db}=0.18dB Ωs=1.0353 60.8dB
Frequency and Impedance Scaled Final Circuit

Note: 1 meg termination resistor is needed to provide DC return path.
Bi-Directional Impedance Converter for Matching D Element Filters Requiring Capacitive Loads to Resistive Terminations

Value of R Arbitrary
Rs is source and load resistive terminations
CGIC is D Element Circuit Capacitive Terminations
High-Pass Filters

Normalized Reciprocal Low-Pass High-Pass Relationship
Passive High-Pass Filters

Low-Pass to High-Pass Transformation for Normalized Values

\[ C_{hp} = \frac{1}{L_{lp}} \quad L_{hp} = \frac{1}{C_{lp}} \]

Replace Low-Pass Values by Reciprocal Components
Then frequency and impedance scale to desired cut-off
Active High-Pass Filters

\[ C_{hp} = \frac{1}{R_{lp}} \]
\[ R_{hp} = \frac{1}{C_{lp}} \]

To convert a normalized active low-pass filter into an active high-pass filter replace each resistor by a capacitor having the reciprocal value and vice versa. The filter can then be scaled to the desired cut-off and impedance level.

This conversion does not apply to feedback resistors that determine an amplifiers gain which applies to some active configurations.
Band-Pass Filters

This figure shows the relationship of a low-pass filter when transformed into a band-pass filter. The response at frequencies of the low-pass filter results in the same attenuation at corresponding bandwidths of the band-pass filter.
Band-Pass Filters

Band-Pass Transformation Procedure for LC Filters

1) Design a low-pass filter having the desired bandwidth of the band-pass filter and impedance level.

2) Resonate each inductor with a series capacitor and resonate each capacitor with a parallel inductor. The resonant frequency should be the desired center frequency of the band-pass filter.

\[ F_o = \frac{1}{2\pi\sqrt{LC}} \]

Normalized N=3 Butterworth Low-Pass Filter

Scaled to 3dB at 100Hz and 600-ohms

Resulting Band-Pass Transformation
Wide Band Band-Pass Filters

Cascade of Low-Pass and High-Pass

Example of Wide Band Band-Pass Filter

Effect of Interaction for Less Than an Octave of Separation of Cut-Offs

To prevent impedance interaction between a passive low-pass filter and high-pass filter, a 3dB Attenuator between filters is helpful.
Band Reject Filters

This figure shows the relationship of a high-pass filter when transformed into a band-reject filter. The response at frequencies of the high-pass filter results in the same attenuation at corresponding bandwidths of the band-reject filter.
Band-Reject Filters

Band-Reject Transformation Procedure for LC Filters
1) Design a high-pass filter having the desired Bandwidth of the band-reject filter and impedance level.
2) Resonate each inductor with a parallel capacitor and resonate each capacitor with a series inductor. The resonant frequency should be the desired center frequency of the band-reject filter.

- a) N=3 Normalized 1dB Chebyshev LPF
- b) Transformed HPF
- c) Frequency and Impedance Scaled HPF (500Hz BW)
- d) Resonate inductors and capacitors
- e) Resulting Response
This circuit is in the form of a bridge where a signal is applied across terminals’ 1 and 2 the output is measured across terminals’ 3 and 4. At $\omega=1$ all branches have equal impedances of $0.707 \angle -45^\circ$ so a null occurs across the output.
The circuit is redrawn in figure B in the form of a lattice. Circuit C is the identical circuit shown as two lattices in parallel.
There is a theorem which states that any branch in series with both the \( Z_A \) and \( Z_B \) branches of a lattice can be extracted and placed outside the lattice. The branch is replaced by a short. This is shown in figure D above. The resulting circuit is known as a Twin-T. This circuit has a null at 1 radian for the normalized values shown.
To calculate values for this circuit pick a convenient value for C. Then

\[ R_1 = \frac{1}{2\pi f_0 C} \]

The Twin-T has a Q \((f_0/BW_{3dB})\) of only \(\frac{1}{4}\) which is far from selective.

Note: R-source <<R1      R-load >>R1
Circuit A above illustrates bootstrapping a network $\beta$ with a factor $K$. If $\beta$ is a twin-T the resulting $Q$ becomes:

$$Q = \frac{1}{4(1-K)}$$

If we select a positive $K < 1$, and sufficiently close to 1, the circuit $Q$ can be dramatically increased. The resulting circuit is shown in figure B.
Impedance of a center-tapped parallel resonant circuit at resonance is $\omega_r L Q$ total and $\omega_r L Q/4$ from end to center tap (due to $N^2$ relationship). Hence a phantom negative resistor of $-\omega_r L Q/4$ appears in the equivalent circuit which can be cancelled by a positive resistor of $\omega_r L Q/4$ resulting in a very deep null at resonance (60dB or more).
$T(s)$ can be any band-pass circuit having properties of unity gain at $f_r$, adjustable $Q$ and adjustable $f_r$. 

Adjustable $Q$ and Frequency Null Network
If $T(s)$ in circuit A corresponds to a band-pass transfer function of:

$$T(s) = \frac{\omega_r s}{s^2 + \frac{\omega_r}{Q} s + \omega_r^2}$$

The overall circuit transfer function becomes:

$$\frac{\text{Out}}{\text{In}} = \frac{\omega_r s}{s^2 + \frac{\omega_r}{Q} s + \omega_r^2} \frac{Q}{1 - \beta}$$

The middle term of the denominator has been modified so the circuit Q is given by $Q/(1-\beta)$ where $0 < \beta < 1$. The Q can then be increased by the factor $1/(1-\beta)$. Note that the circuit gain is increased by the same factor.
A simple implementation of this circuit is shown in figure B. The design equations are:

First calculate $\beta$ from $\beta = 1 - \frac{Q_r}{Q_{\text{eff}}}$

where $Q_{\text{eff}}$ is the overall circuit Q and $Q_r$ is the design Q of the bandpass section.

The component values can be computed from:

\[
\begin{align*}
R_3 &= \frac{R}{\beta} \\
R_4 &= R \\
R_5 &= \frac{R}{(1-\beta)A_r}
\end{align*}
\]

\[
\begin{align*}
R_2 &= \frac{Q_r}{\pi f C} \\
R_{1a} &= \frac{R_2}{2} \\
R_{1b} &= \frac{R_{1a}}{2Q_r^2-1}
\end{align*}
\]

Where $R$ and $C$ can be conveniently chosen.
Some Useful Passive Filter Transformations to Improve Realizability

Advantages:
Reduces value of $L$
Allows for parasitic capacity across inductor

Advantages:
Increases value of $L$
This transformation can be used to reduce the value of a terminating resistor and yet maintain the narrow-band response.

where the restrictions \( R_2 < R_1 \) and \( (R_1 - R_2)/(R_1 R_2) < \omega_0 C_1 \) apply.
The following example illustrates how this approximation can reduce the source impedance of a filter.
An inductor can be used as an auto-transformer by adding a tap

Resonant circuit capacitor values can be reduced
Intermediate branches can be scaled in impedance.

Leakage inductance can wreak havoc.
Effect of leakage inductance can be minimized by splitting capacitors which adds additional poles.
State Variable Bandpass Filter

\[ i = \frac{E_{\text{OUT}}}{\text{SCR}_2 R_3} \]

\[ L = CR_2 R_3 \]

\[ Q = \frac{F_0}{3 \text{dbBW}} \]

\[ R_1 = 2\pi F_0 LQ \]

\[ \text{Gain@} F_0 = -\frac{R_1}{R_4} \]
Attenuators

Minimum Loss Resistive Pad For Impedance Matching

\[ R_1 = R_s \sqrt{1 - \frac{R_L}{R_s}} \]

\[ R_2 = \sqrt{\frac{R_L}{1 - \frac{R_L}{R_s}}} \]

Voltage Loss dB = \(20 \log_{10} \left( \frac{R_1(R_2 + R_L)}{R_2 R_L} \right) + 1\)

Power Loss dB = Voltage Loss dB - 10 \(\log_{10} \left( \frac{R_s}{R_L} \right)\)
Symmetrical $T$ and $\pi$ Attenuators

$K = 10^{\text{dB}/20}$

For a Symmetrical $T$ Attenuator

$R_1 = Z \frac{K-1}{K+1}$

$R_3 = \frac{2ZK}{K^2 - 1}$

Unbalanced

Balanced
For a Symmetrical \( \pi \) Attenuator

\[
R_1 = Z \frac{K+1}{K-1}
\]

\[
R_3 = Z \frac{K^2-1}{2K}
\]

(a) Unbalanced

(b) Balanced
T and PI Attenuators at 500-ohms Impedance Level Values can be scaled to other impedances

<table>
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<th>dB</th>
<th>$R_1$</th>
<th>$R_2$</th>
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<tr>
<td>10</td>
<td>260</td>
<td>351</td>
<td>963</td>
<td>712</td>
</tr>
</tbody>
</table>
Bridged T Attenuator

\[ R_1 = \frac{R_0}{K-1} \quad \quad \quad R_2 = R_0(K-1) \]

where \( K = 10^{\text{dB}/20} \)

Only \( R_1 \) and \( R_2 \) change to vary attenuation and they change inversely.
Return Loss

\[ A_\rho = 20 \log \left| \frac{Z_s + Z_x}{Z_s - Z_x} \right| \]

\( Z_s = \) standard or Ref Impedance
\( Z_x = \) Impedance being Measured

If \( Z_s = R_s \), then a symmetrical attenuator of X dB designed for an impedance of \( R_s \) preceding any network insures a minimum return loss of 2X dB no matter what the impedance of the network, including zero or infinity (short or open).

For example a 3dB symmetrical attenuator insures a minimum Return Loss of 6dB even if terminated with a short or open.
Resistive Power Splitter

\[ N = \text{total number of ports} - 1 \quad (N=K-1) \]

\[ R = R_0 \frac{N-1}{N+1} \]

where \( R_0 \) is the impedance at all ports.

Power Loss dB = \( 10 \log_2 \frac{1}{N^2} \)
Miscellaneous Circuits and Topics
Constant Delay High Pass Filter

Delay of N=3 Butterworth High-Pass Filter 3dB at 100Hz

Delay peaks near 3dB Cutoff and approaches zero at higher frequencies
Not acceptable if constant delay is desired in the Pass-Band
Solution

Low-Pass Filter has unity gain in stop-band

All-Pass Delay Line has unity gain

In pass-band of Low-Pass Filter signals are cancelled in summer by subtraction

In stop-band of Low-Pass Filter the signal path is through the delay line
Simple Active Shunt Inductor

Let \( R_2 \gg R_1 \)

\[ L = R_1 R_2 C \]

\[ F_{Q_{\text{max}}} = \frac{1}{2 \pi C \sqrt{R_1 R_2}} \]

\[ Q_{\text{max}} = \frac{1}{2} \sqrt{\frac{R_2}{R_1}} \]
All-Pass Delay Line Section

Flat delay of 1.60 Sec within 1% to 1 Rad/S

$$N = \frac{2 \pi F T_{\text{total}}}{1.6}$$

Round off to nearest higher $N$
Use $N$ sections each scaled to delay of $T_{\text{total}}/N$
Impedance scale to practical values

Use high-Q Inductors to avoid dips at parallel resonant frequency
Any ladder network has a dual which has the same transfer function. In order to transform a network into its dual:

- Inductors are transformed into capacitors and vice versa having the same element values (henrys into farads and vice versa)
- Resistors are transformed into conductances (ohms into mhos)
- Open circuit becomes a short circuit and vice versa
- Voltage sources become current sources and vice versa
- Series branches become shunt branches and vice versa
- Elements in parallel become elements in series and vice versa

Note that the following table has schematics on both top and bottom which are duals of each other.
Out of Band Impedance of Low-Pass and High-Pass Filters to Allow Combining

• The input and/or output impedance of a low-pass and high-pass filter is determined to a great extent by the last element.

• For example a low-pass filter having a series inductor at the load end has a rising impedance at that end in the stop band (for rising frequencies).

• A high-pass filter having a series capacitor at the load end has a rising impedance in the stop band (for lower frequencies).

• This property allows the paralleling of low-pass and high-pass filters with minimal interaction as long as the pass-bands don’t overlap. By selecting odd or even order filters and/or by using a circuit’s dual, the appropriate series terminating element can be forced.
Simplified Low-Pass High-Pass Combining at Output

Practical Implementation in POTS Splitter
Wide Band Band-Reject Filter by Low-Pass High-Pass Filters in Parallel

The technique of connecting LC filters in parallel that have rising out-of-band impedance can be used to generate a wide-band band-reject filter. Wide-band is generally defined as at least one octave between cut-offs.

Example having sufficient separation of cut-offs

Individual Responses

Combined Effect of Insufficient Separation
This is the fourth edition of the *Electronic Filter Design Handbook*. This book was first published in 1981. It was expanded in 1988 to include five additional chapters on digital filters and updated in 1995. This revised edition contains new material on both analog and digital filters. A CD-ROM has been included containing a number of programs which allow rapid design of analog filters from input requirements without the tedious mathematical computations normally encountered. The digital filter chapters are all integrated with a profusion of MATLAB examples.