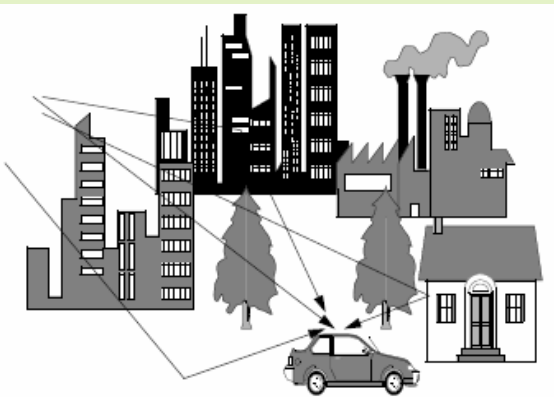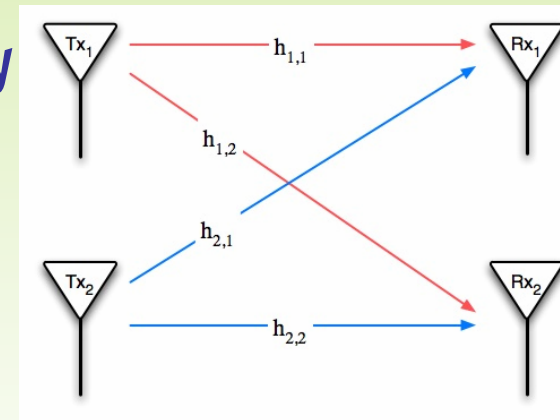# Introduction to Wireless MIMO – Theory and Applications

Multipath is not enemy but ally

IEEE LI, November 15, 2006

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Why MIMO

- Motivation: current wireless systems
  - Capacity constrained networks
  - Issues related to quality and coverage
- MIMO exploits the *space* dimension to improve wireless systems capacity, range and reliability
- MIMO-OFDM – the corner stone of future broadband wireless access
  - WiFi – 802.11n
  - WiMAX – 802.16e (a.k.a 802.16-2005)
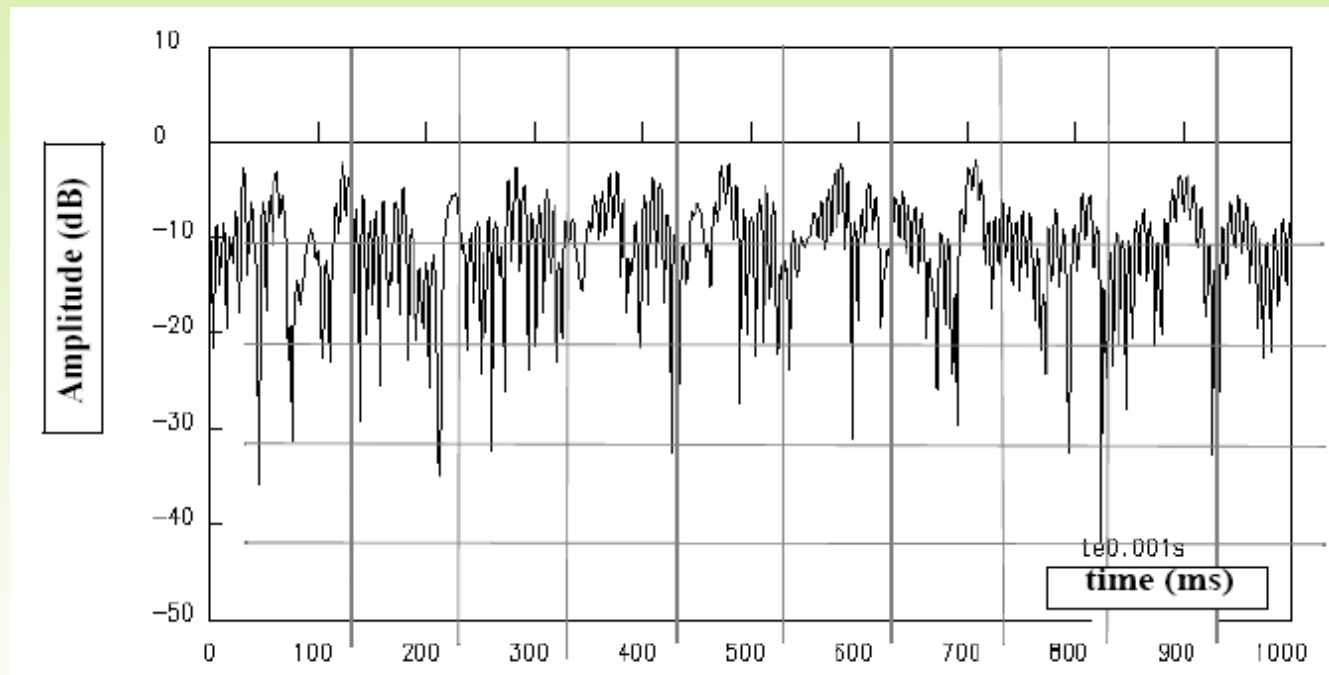 - 3G / 4G

Transmission on a multipath channel

In wireless communication the propagation channel is characterized by multipath propagation due to scattering on different obstacles
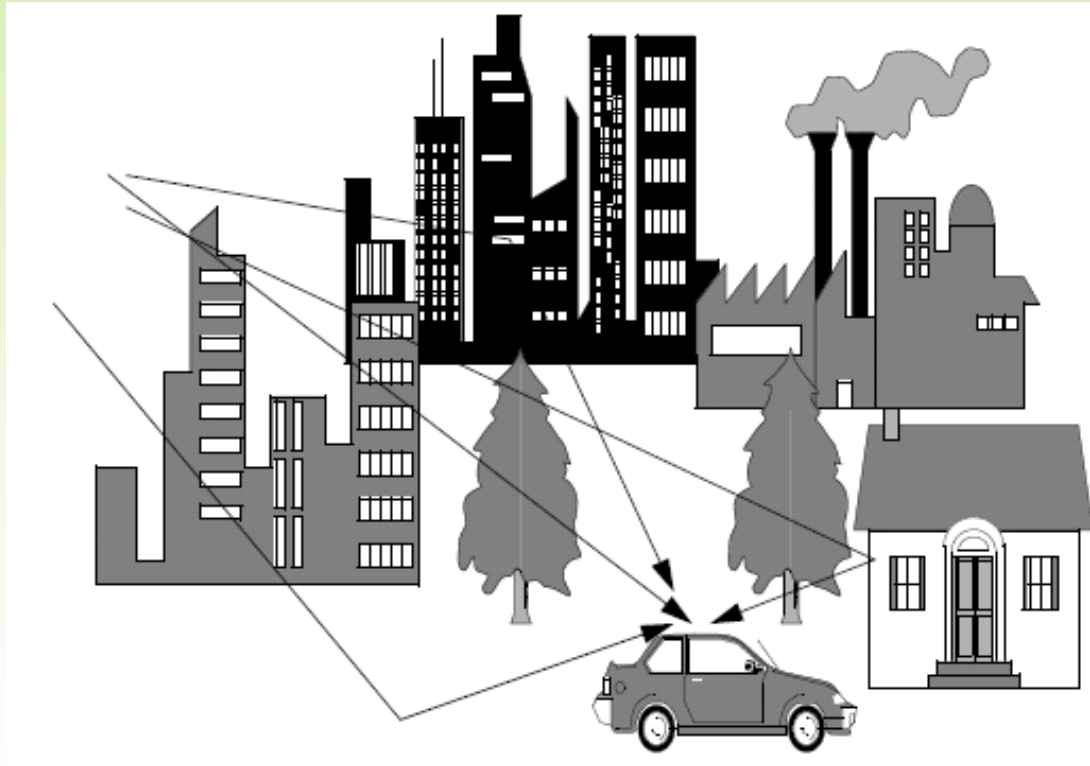
- Time variations: Fading => SNR variations
- Time spread => frequency selectivity
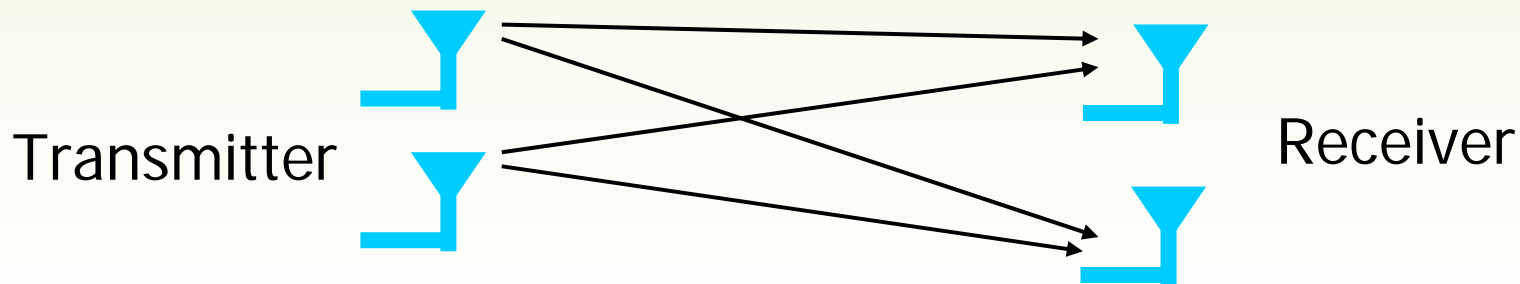
Transmission on a multipath channel

Fading:

- The received level variations result in SNR variations
- The received level is sensitive to the transmitter and receiver locations
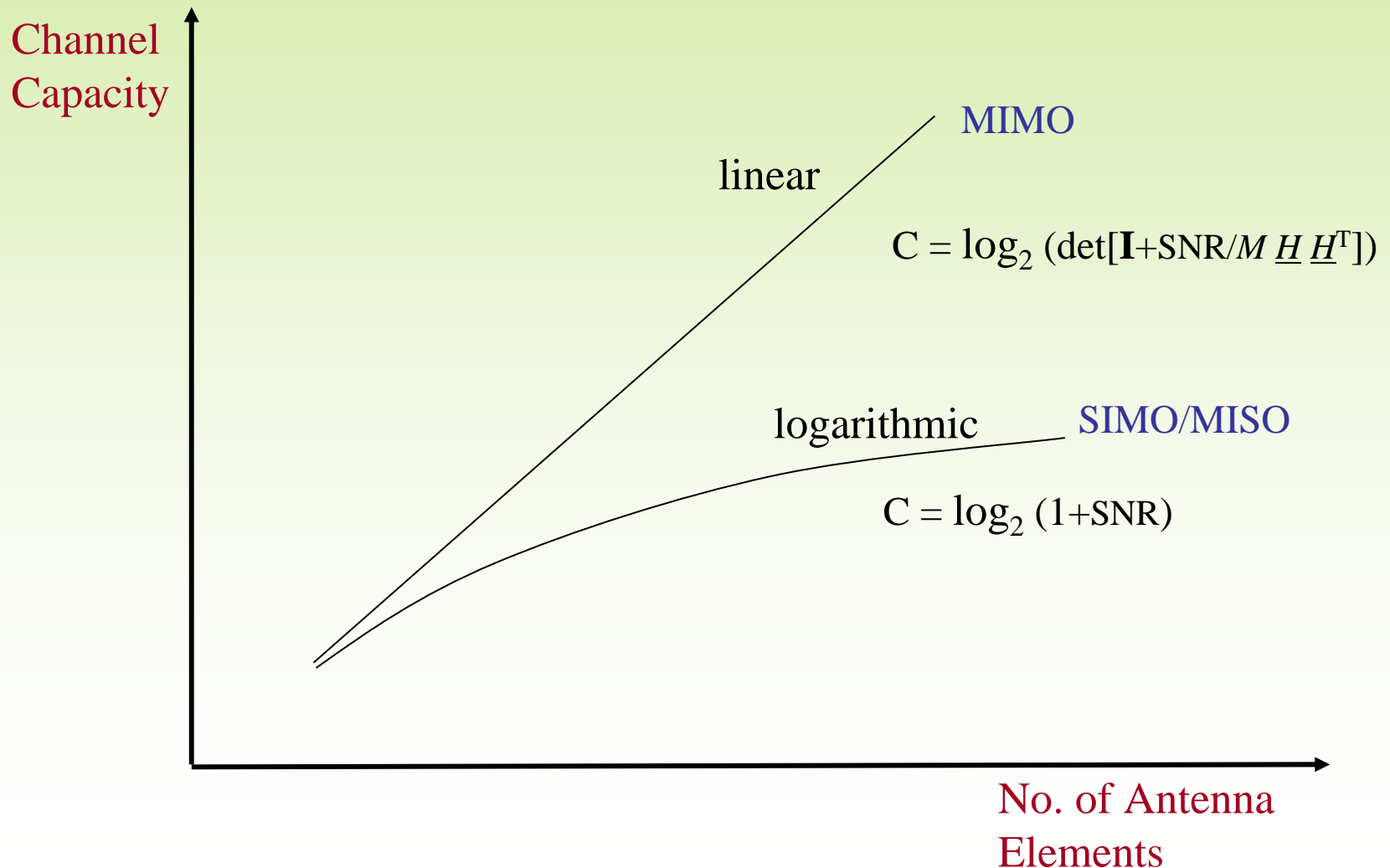
MIMO Defined

- MIMO is an acronym that stands for **M**ultiple **I**nput **M**ultiple **O**utput.

- It is an antenna technology that is used both in transmission and receiver equipment for wireless radio communication.

- There can be various MIMO configurations. For example, a 2x2 MIMO configuration is 2 antennas to transmit signals (from base station) and 2 antennas to receive signals (mobile terminal).

Transmitter

Receiver

MIMO vs. SIMO/MISO
(Linear vs. Logarithmic Improvement)

Channel
Capacity

MIMO

linear

$$C = \log_2 (\det[\mathbf{I}+\text{SNR}/M\ \underline{H}\ \underline{H}^{\text{T}}])$$

logarithmic   SIMO/MISO

$$C = \log_2 (1+\text{SNR})$$

No. of Antenna
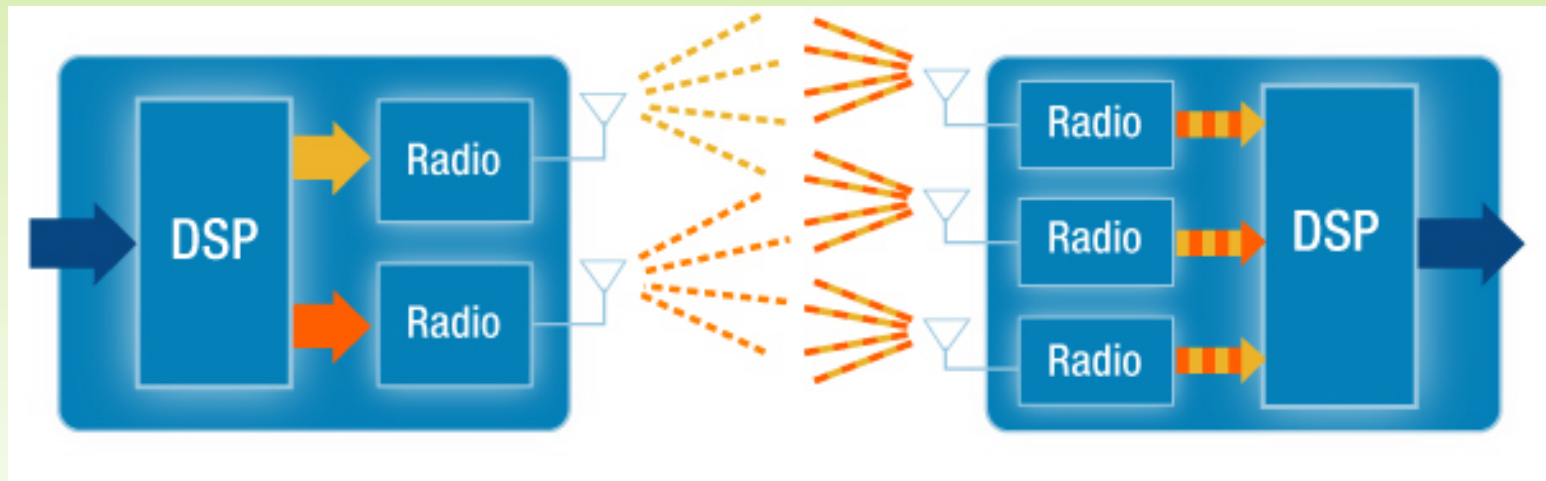Elements

How MIMO Works

- MIMO takes advantage of multi-path.

- MIMO uses multiple antennas to send multiple parallel signals (from transmitter).

- In an urban environment, these signals will bounce off trees, buildings, etc. and continue on their way to their destination (the receiver) but in different directions.

- "Multi-path" occurs when the different signals arrive at the receiver at various times.

- With MIMO, the receiving end uses an algorithm or special signal processing to sort out the multiple signals to produce one signal that has the originally transmitted data.

How MIMO Works (cont.)

Multiple data streams transmitted in a single channel at the same time

Multiple radios collect multipath signals

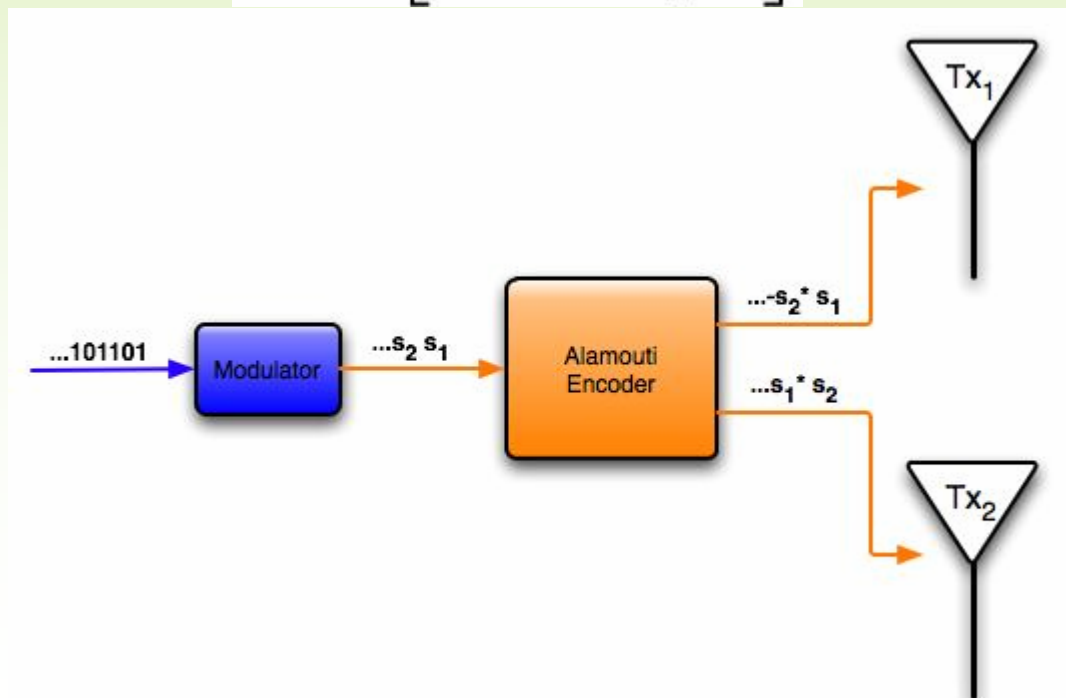Delivers simultaneous speed, coverage, and reliability improvements

Types of MIMO

- MIMO involves Space Time Transmit Diversity (STTD), Spatial Multiplexing (SM) and Uplink Collaborative MIMO.

- *Space Time Transmit Diversity (STTD)* - The same data is coded and transmitted through different antennas, which effectively doubles the power in the channel. This improves Signal Noise Ratio (SNR) for cell edge performance.

- *Spatial Multiplexing (SM)* - the "Secret Sauce" of MIMO. SM delivers parallel streams of data to CPE by exploiting multi-path. It can double (2x2 MIMO) or quadruple (4x4) capacity and throughput. SM gives higher capacity when RF conditions are favorable and users are closer to the BTS.

- *Uplink Collaborative MIMO Link* - Leverages conventional single Power Amplifier (PA) at device. Two devices can collaboratively transmit on the same sub-channel which can also double uplink capacity.
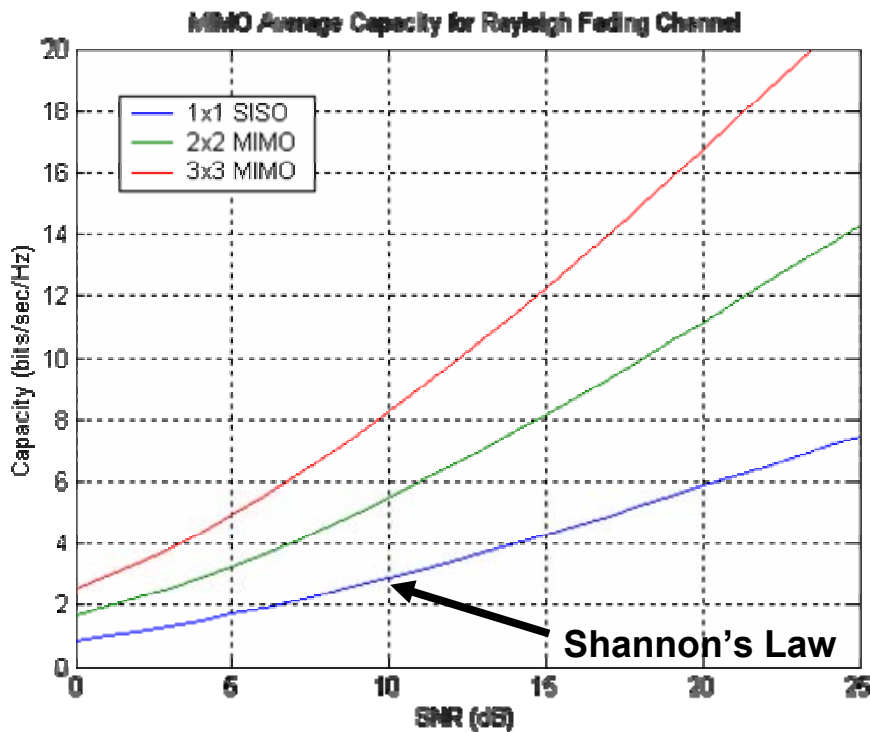
Space-Time Transmit Diversity
Alamouti Code

time

$$X = \begin{bmatrix} s_1 & -s_2^* \\ s_2 & s_1^* \end{bmatrix}$$

space

MIMO Increases Throughput
Spatial Multiplexing

MIMO Average Capacity for Rayleigh Fading Channel

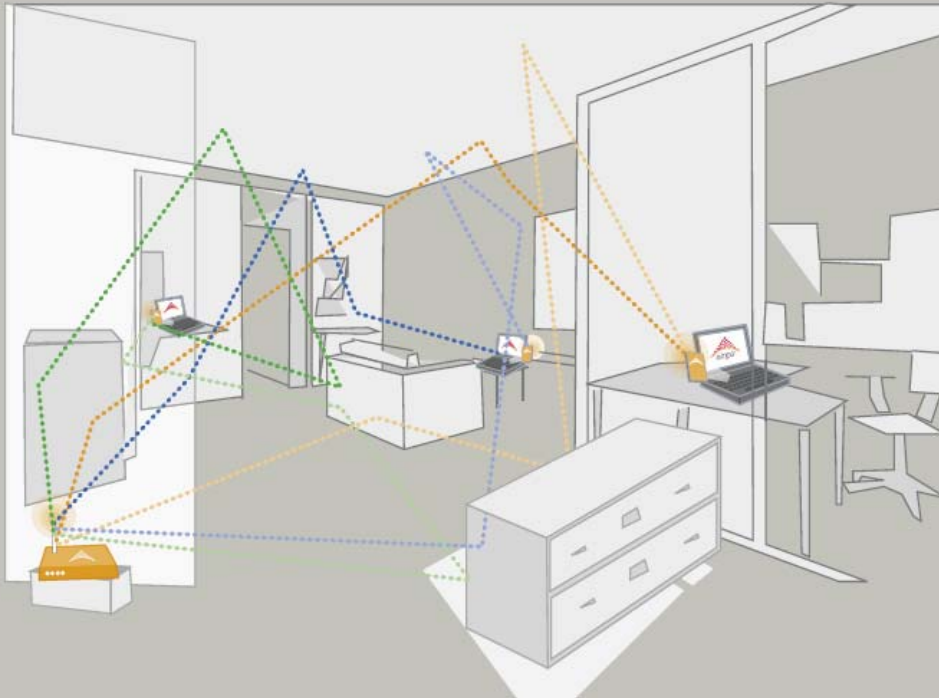- 1x1 SISO
- 2x2 MIMO
- 3x3 MIMO

Shannon's Law

Wireless throughput scales as more radio transmissions are added onto the same channel

Only baseband complexity, die size/cost, and power consumption limits the number of simultaneous transmissions (assuming good channel conditions)

MIMO Channel Capacity
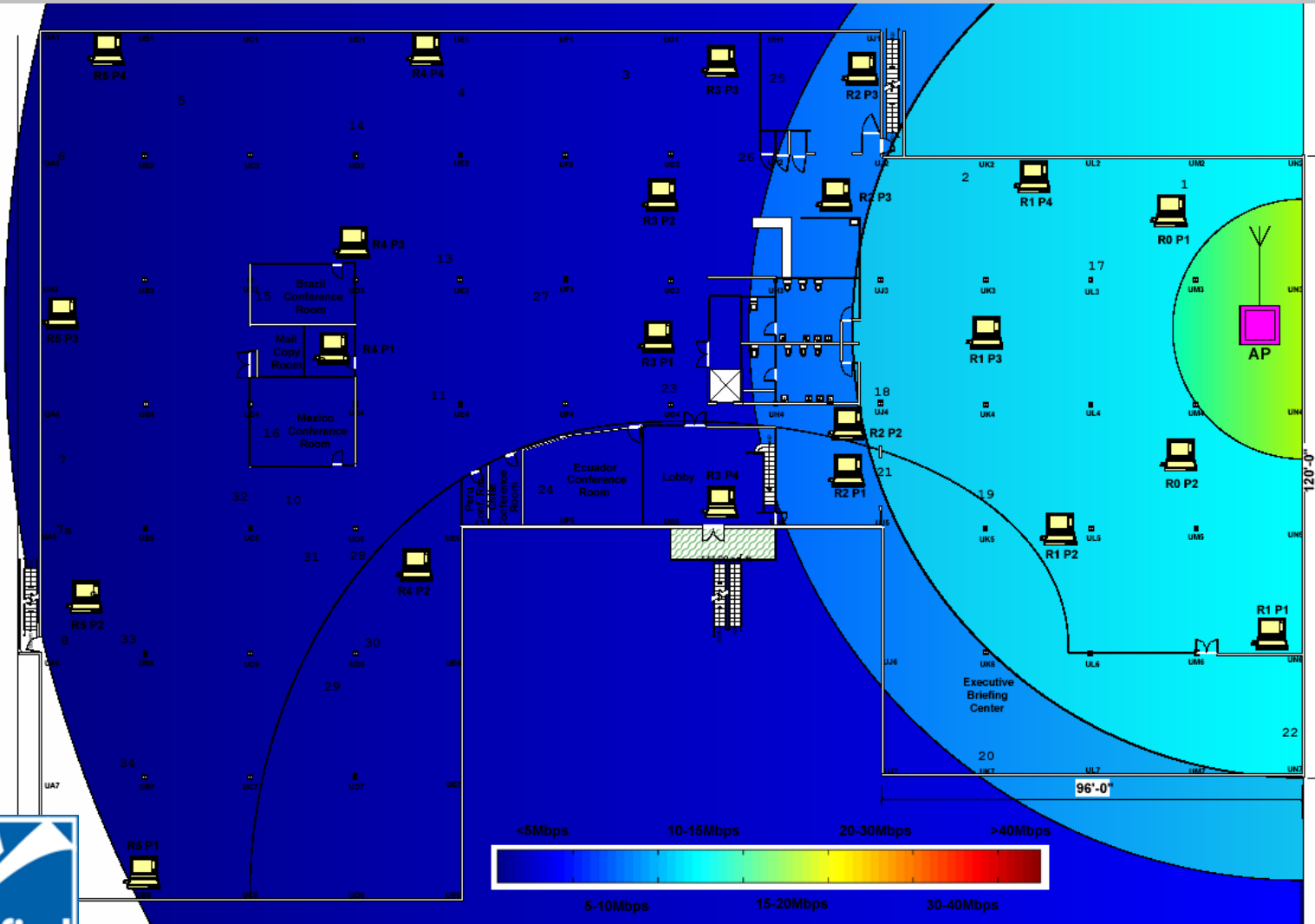

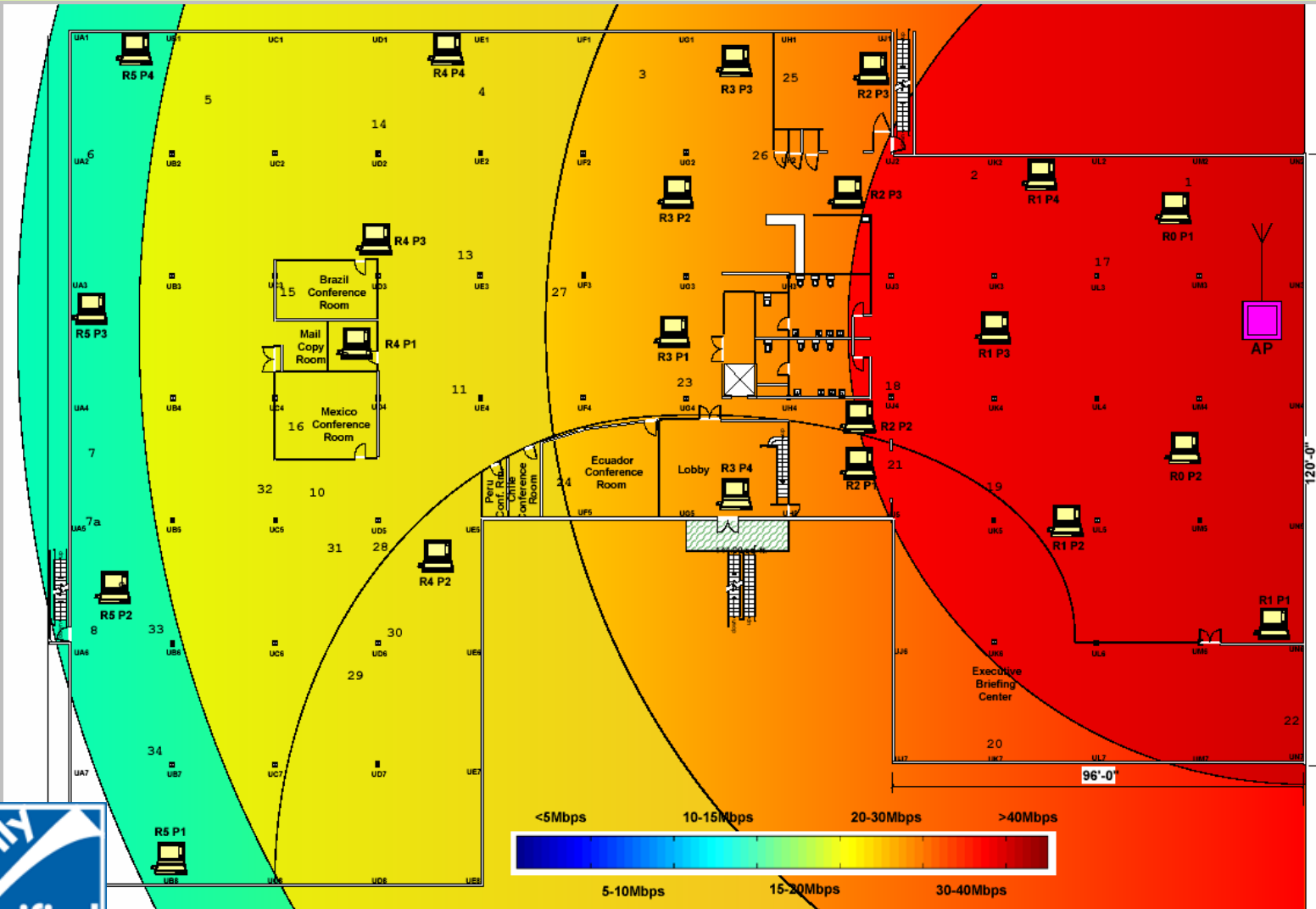Capacity CDF, MIMO, SNR=10,20 dB
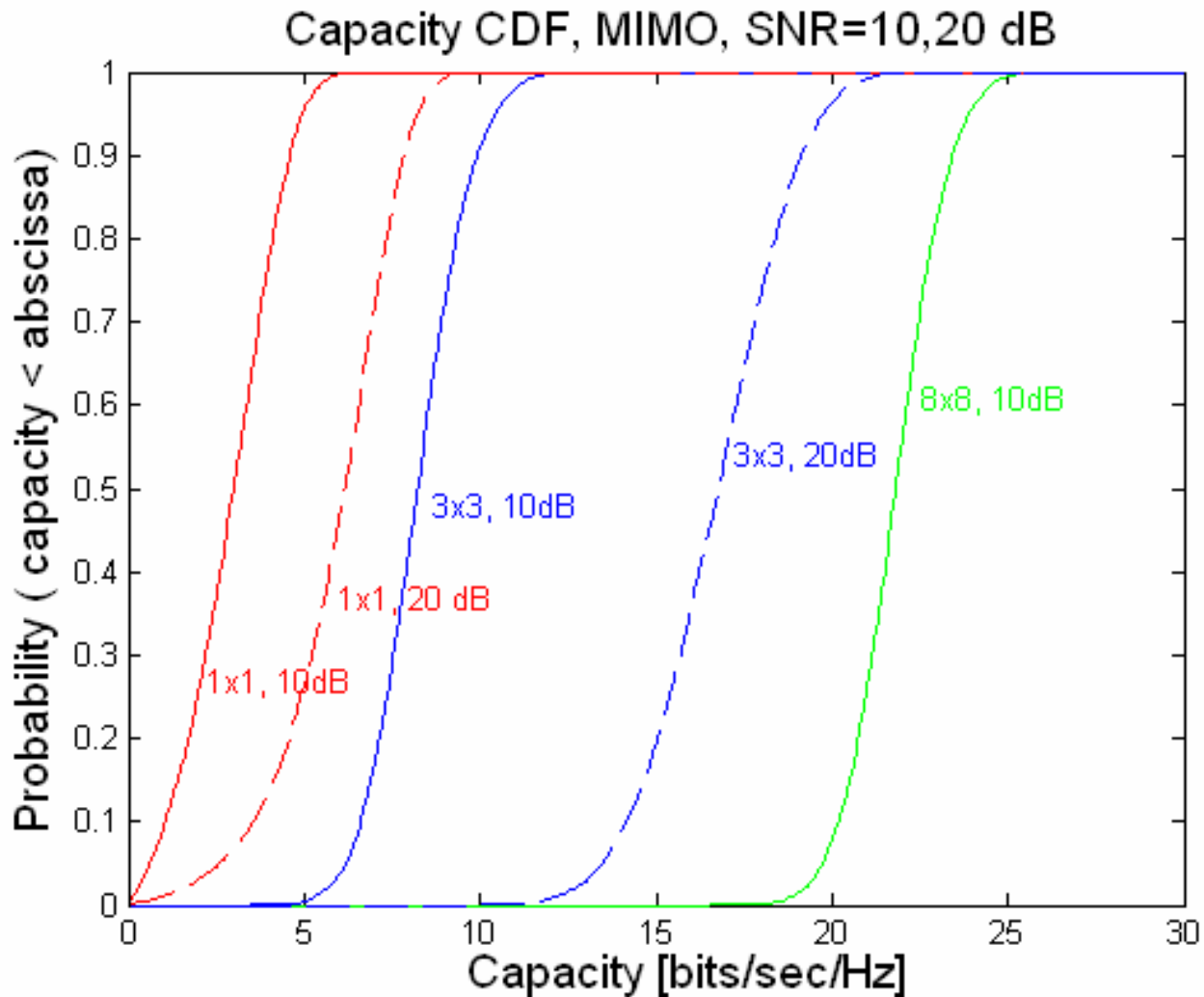
MIMO Increases Range

Each multipath route is treated as a separate channel, creating many "virtual wires" over which to transmit signals

Traditional radios are confused by this multipath, while MIMO takes advantage of these "echoes" to increase range and throughput
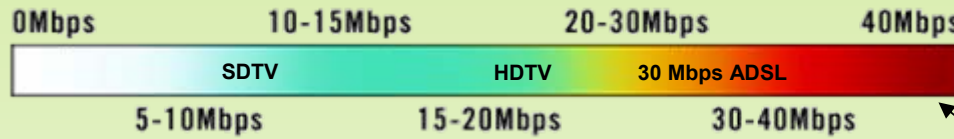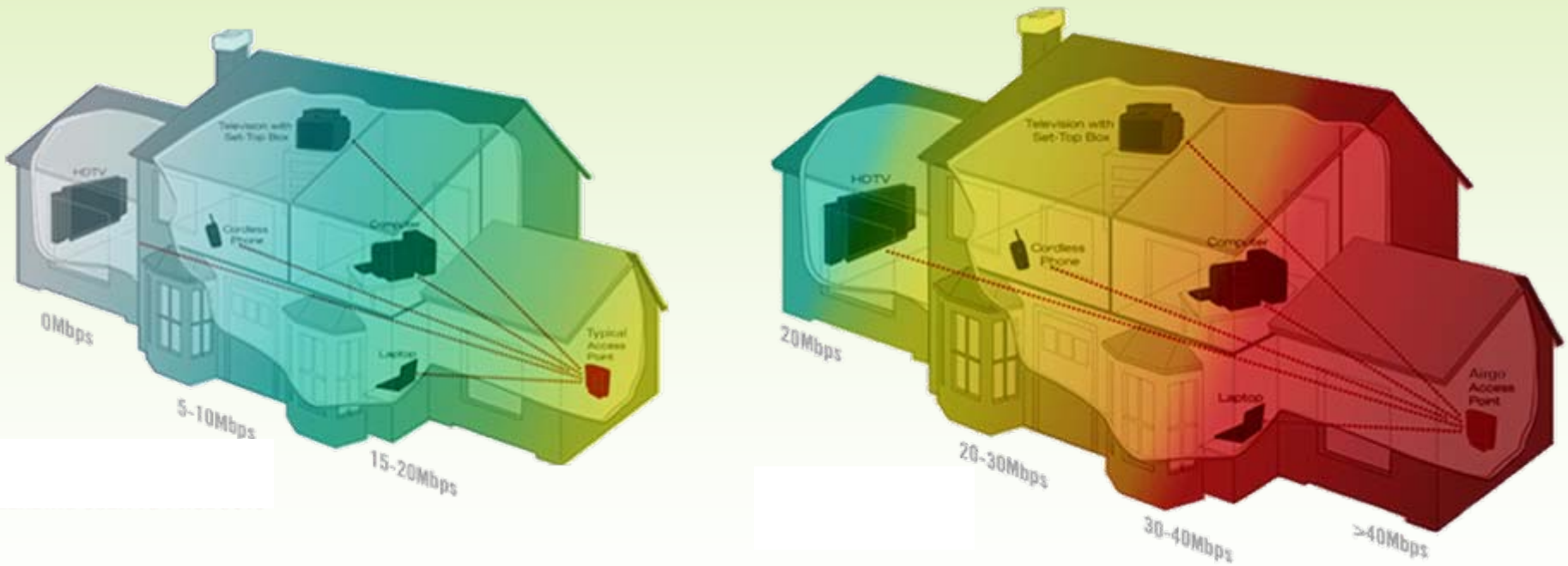
Single Radio Performance (Office)

MIMO Performance (Office)

Single Radio vs. MIMO Performance

Different from Traditional Multiple Access Techniques

- It is not FDMA – multiple users using the same frequency
- It is not TDMA – multiple users communicate simultaneously
- It is not CDMA/Spread Spectrum – frequency band occupied is similar to that of conventional QAM system
- It is not SDMA – there are no directed steered/switched beams in space (e.g., smart antennas)
- *It is ECDMA (Environmental CDMA): like CDMA <u>without</u> having to spread the signal through space-time coding; here the code is the imprint of the environment on the signal and it comes free…*

Exploiting Multipath Rather than Mitigating It

MIMO Channel

H(4x4)
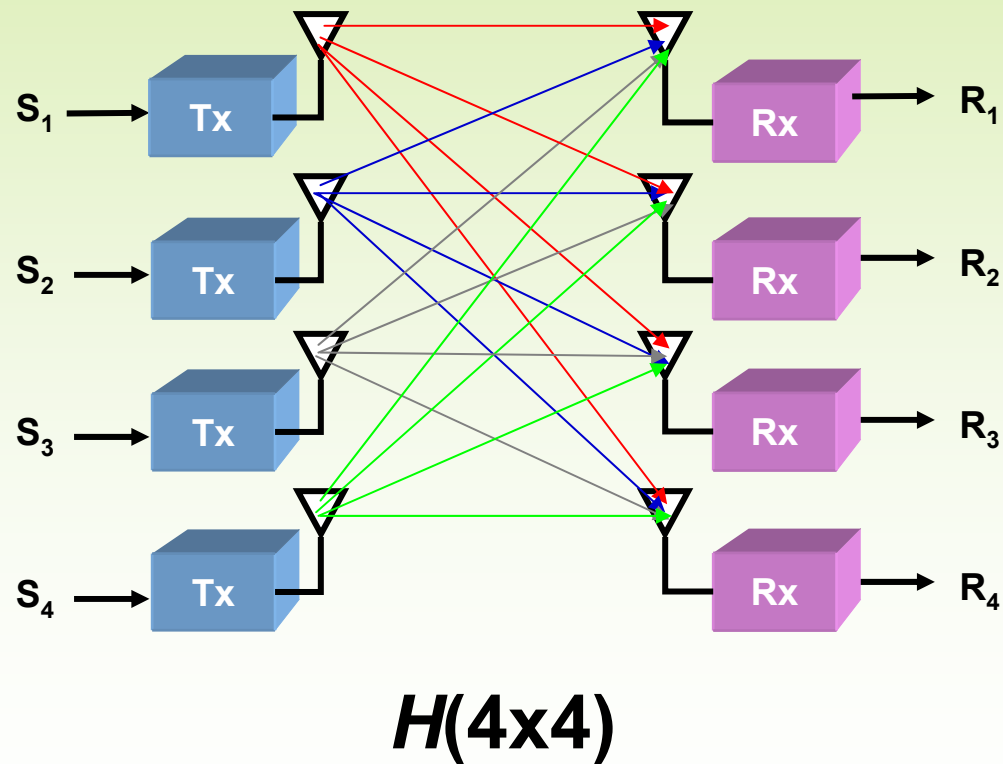
The "Magic":
Separating the self-coded signals
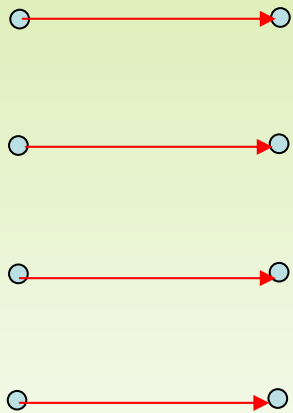
using laser diodes
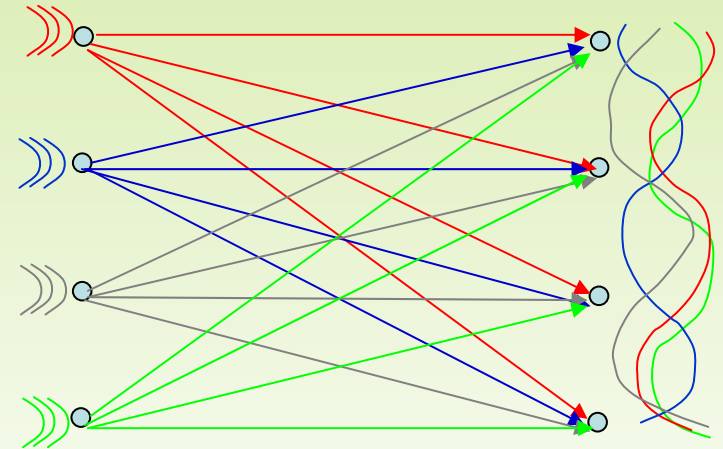
using radio frequency

MIMO Channel Matrix
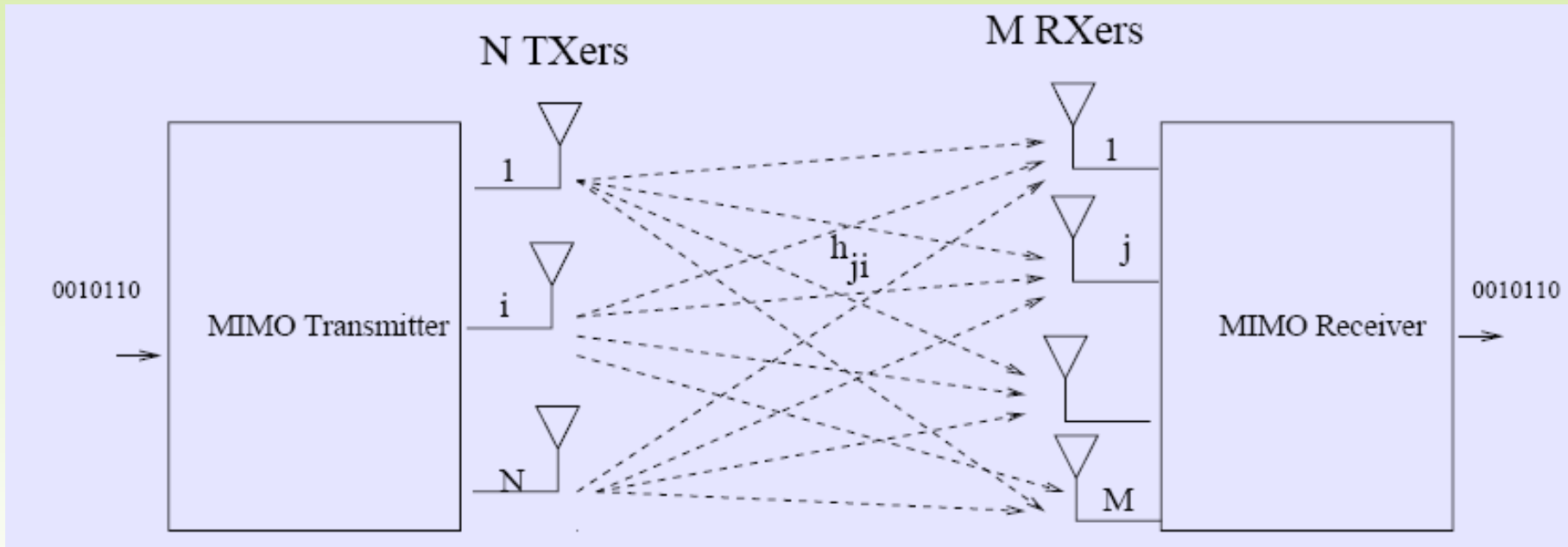
Example for 3 X 4 system:

Number of spatial streams equals rank(H) ≤ min(M, N)

$$H = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \\ h_{41} & h_{42} & h_{43} \end{bmatrix}$$

h_{ij} are complex numbers: a+jb (amplitude&phase) and frequency selective

OFDM
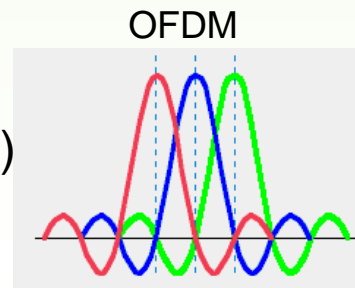
How It Works

Example for 3 X 3 system:

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \underbrace{\begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix}}_{\mathbf{H}} \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} + \mathbf{Noise}$$

$$\begin{bmatrix} \hat{b}_1 \\ \hat{b}_2 \\ \hat{b}_3 \end{bmatrix} = \mathbf{H}^{-1} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Impact of Channel Model

MIMO performance is very sensitive to channel matrix *invertibility*

The following degrades the conditioning of the channel matrix:
- Antenna correlation caused by:
- small antenna spacing, or
- small angle spread

Line of sight component compared with multipath fading component:
- multipath fading component, close to i.i.d. random, is well conditioned
- Line of sight component is very poorly conditioned.

MIMO-SM in Line-of-Site
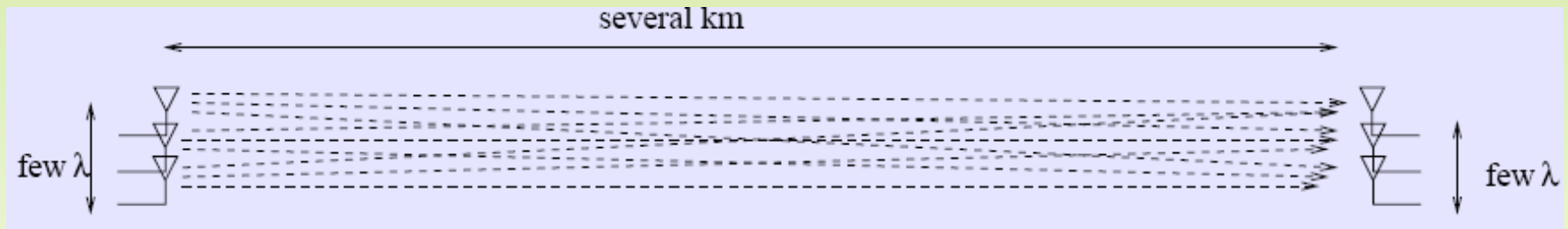
$$\mathbf{H} \approx \alpha \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

The system is near rank one (non invertible)!

Spatial multiplexing requires multipath to work!!!

Zero-Forcing Receiver

$$\begin{bmatrix} x_1 \\ x_2 \\ \vdots \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} & .. \\ h_{21} & h_{22} & .. \\ \vdots & \vdots & \vdots \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \\ \vdots \end{bmatrix} + \mathbf{n}$$

Zero Forcing implements matrix (pseudo)-inverse (ignores noise enhancement problems):

$$\hat{\mathbf{s}} = \mathbf{H}^{\#}\mathbf{x}$$

Where,

$$\mathbf{H}^{\#} = (\mathbf{H}^{*}\mathbf{H})^{-1}\mathbf{H}^{*}$$

Example

Simultaneous Transmission of 3 Different Bit-Streams
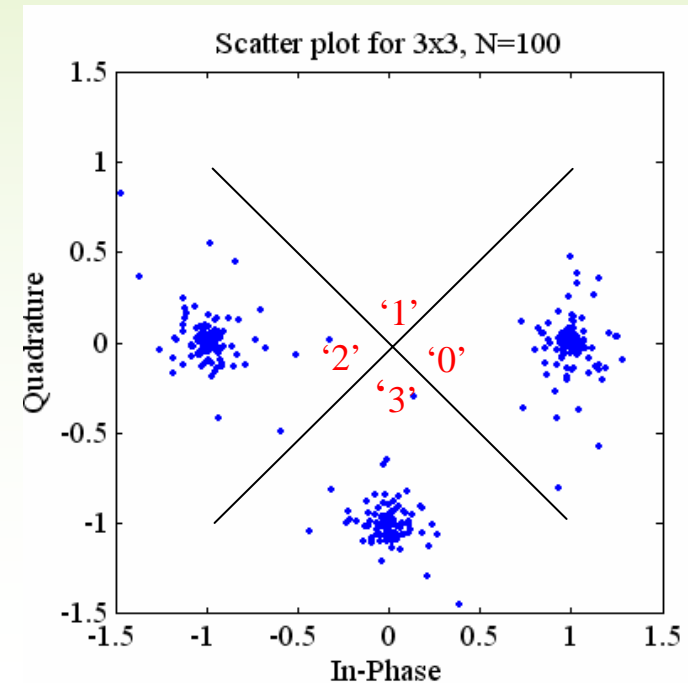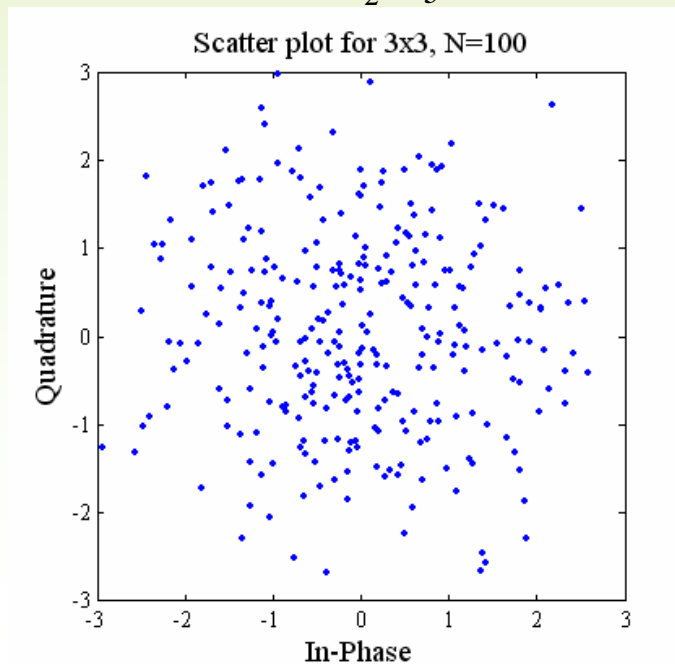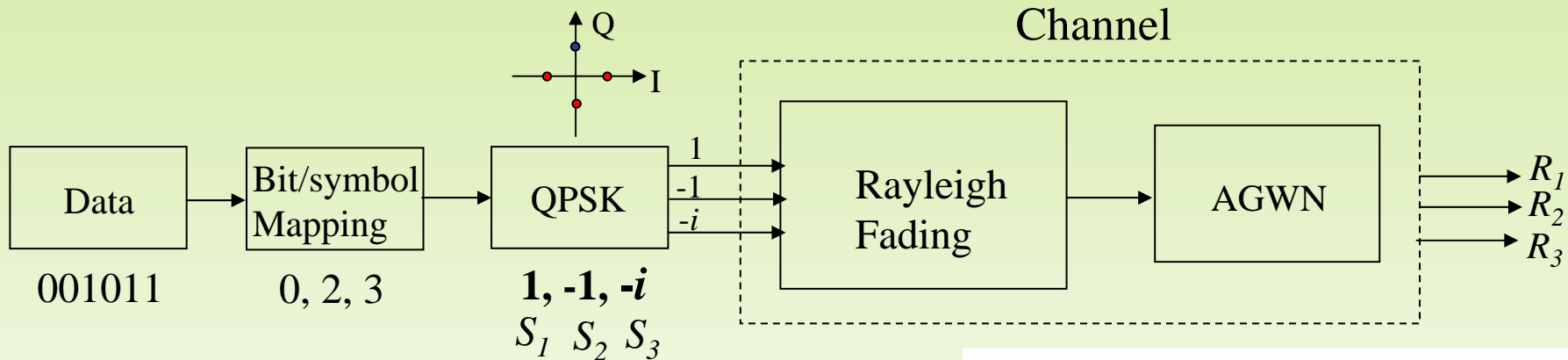
Downstream Signals
all signals sent at same frequency and same time
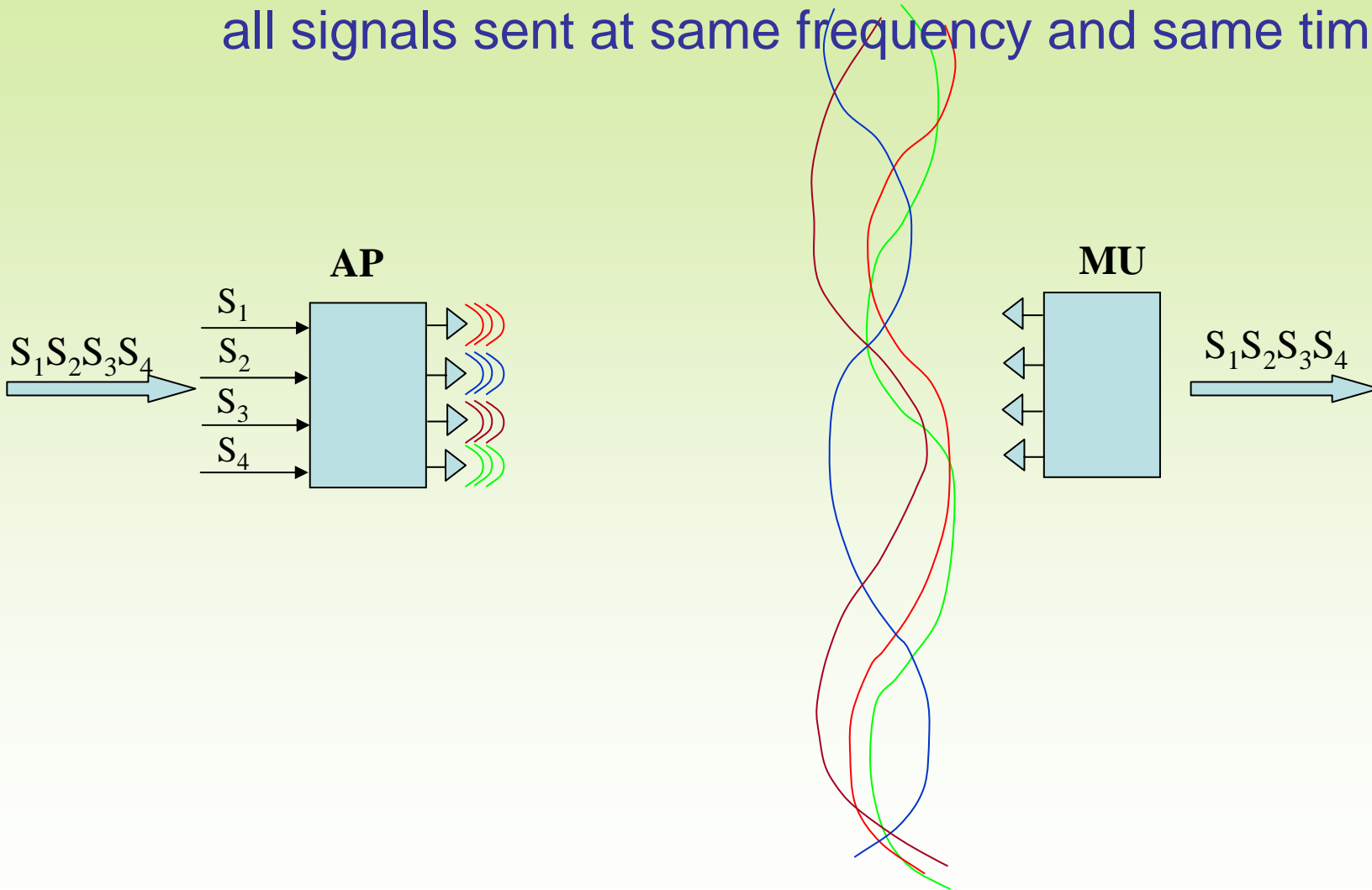
Mixed Signals
Downstream

AP   *MU*

S_1 ──h_{11}── R_1
h_{12}

h_{21}
S_2 ── R_2

S_3 ── R_3

h_{41}
S_4 ── R_4
h_{44}

channel mixing matrix

noise
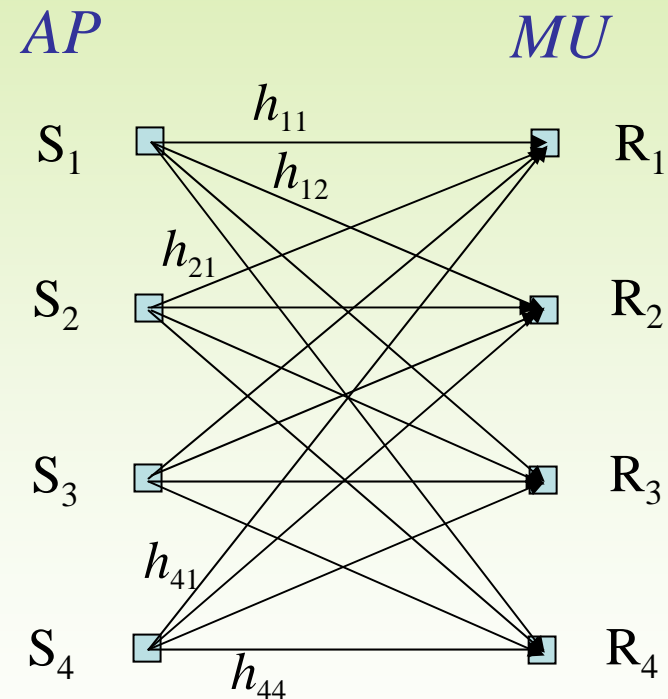
$$\underline{R} = \underline{H} \cdot \underline{S} + \underline{n}$$

e.g., $R_1 = h_{11}S_1 + h_{12}S_2 + h_{13}S_3 + h_{14}S_4 + n_1$

The Received Signals

$$R_1 = h_{11}S_1 + h_{12}S_2 + h_{13}S_3 + h_{14}S_4 + n$$
$$R_2 = h_{21}S_1 + h_{22}S_2 + h_{23}S_3 + h_{24}S_4 + n$$
$$R_3 = h_{31}S_1 + h_{32}S_2 + h_{33}S_3 + h_{34}S_4 + n$$
$$R_4 = h_{41}S_1 + h_{42}S_2 + h_{43}S_3 + h_{44}S_4 + n$$

$$\underline{R} = \underline{H} \cdot \underline{S} + \underline{n}$$

$$\underline{H} = \begin{pmatrix} h_{11} & h_{12} & h_{13} & h_{14} \\ h_{21} & h_{22} & h_{23} & h_{24} \\ h_{31} & h_{32} & h_{33} & h_{34} \\ h_{41} & h_{42} & h_{43} & h_{44} \end{pmatrix}$$

$$\underline{\hat{S}} \Leftarrow \underline{H^{-1}} \cdot \underline{R} \approx \underbrace{\underline{H^{-1}} \cdot \underline{H}}_{\underline{Y}} \cdot \underline{S}$$

If H is ill-conditioned (close to singular) Y will be far from the identity matrix Resulting in co-channel interference

Spatial Correlation
or how well the matrix H is conditioned
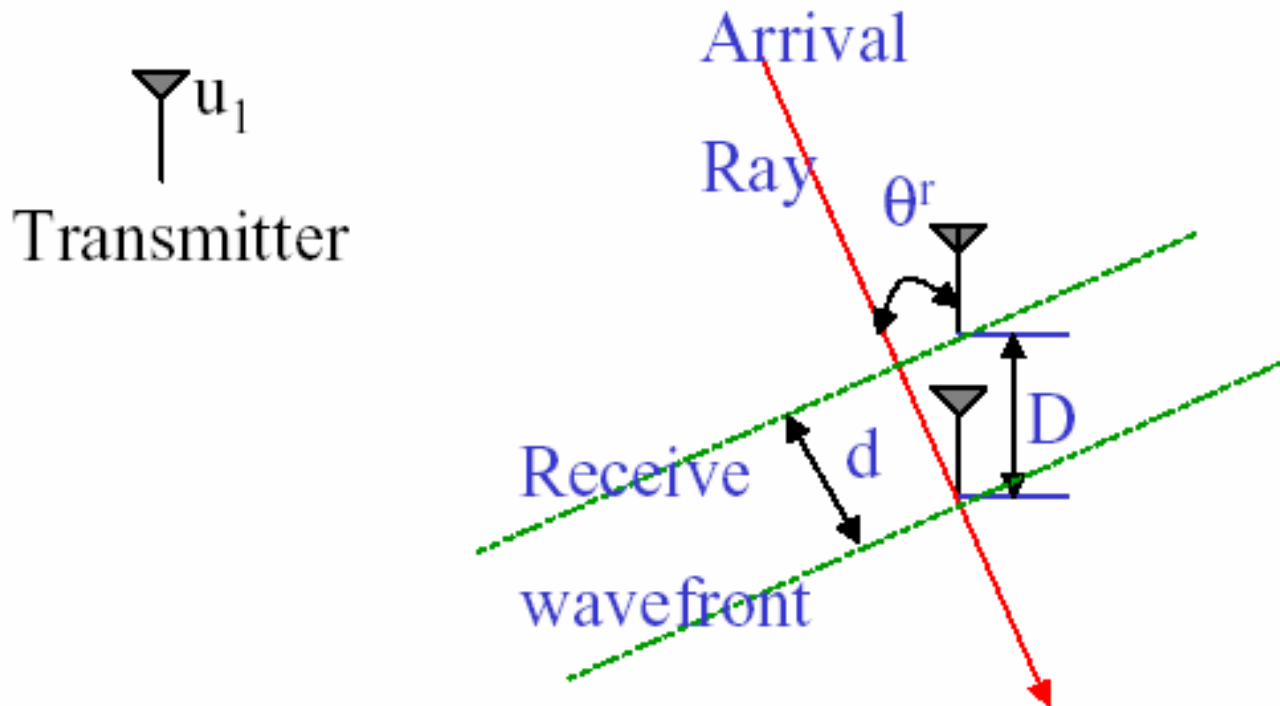
Spatial Correlation (cont.)

Correlation

$$\rho = \frac{E\left[(v_1 - E(v_1))(v_2 - E(v_2))\right]}{E\left[(v_1 - E(v_1))^2\right]E\left[(v_2 - E(v_2))^2\right]}$$

$Receive \quad Antenna \quad 1$

$$v_1 = \left(\sum_{j=1}^{L} \alpha_j \; e^{i\phi_j}\right) u_1$$

$Receive \quad Antenna \quad 2$

$$v_2 = \left(\sum_{j=1}^{L} \alpha_j \; e^{i\phi_j} e^{i\beta d_j}\right) u_1; \quad \beta = \omega/c; \quad d_j = D\cos\theta_j^r$$

Spatial Correlation (cont.)

Correlation Drops Significantly for D>λ When Angle Spread >65°
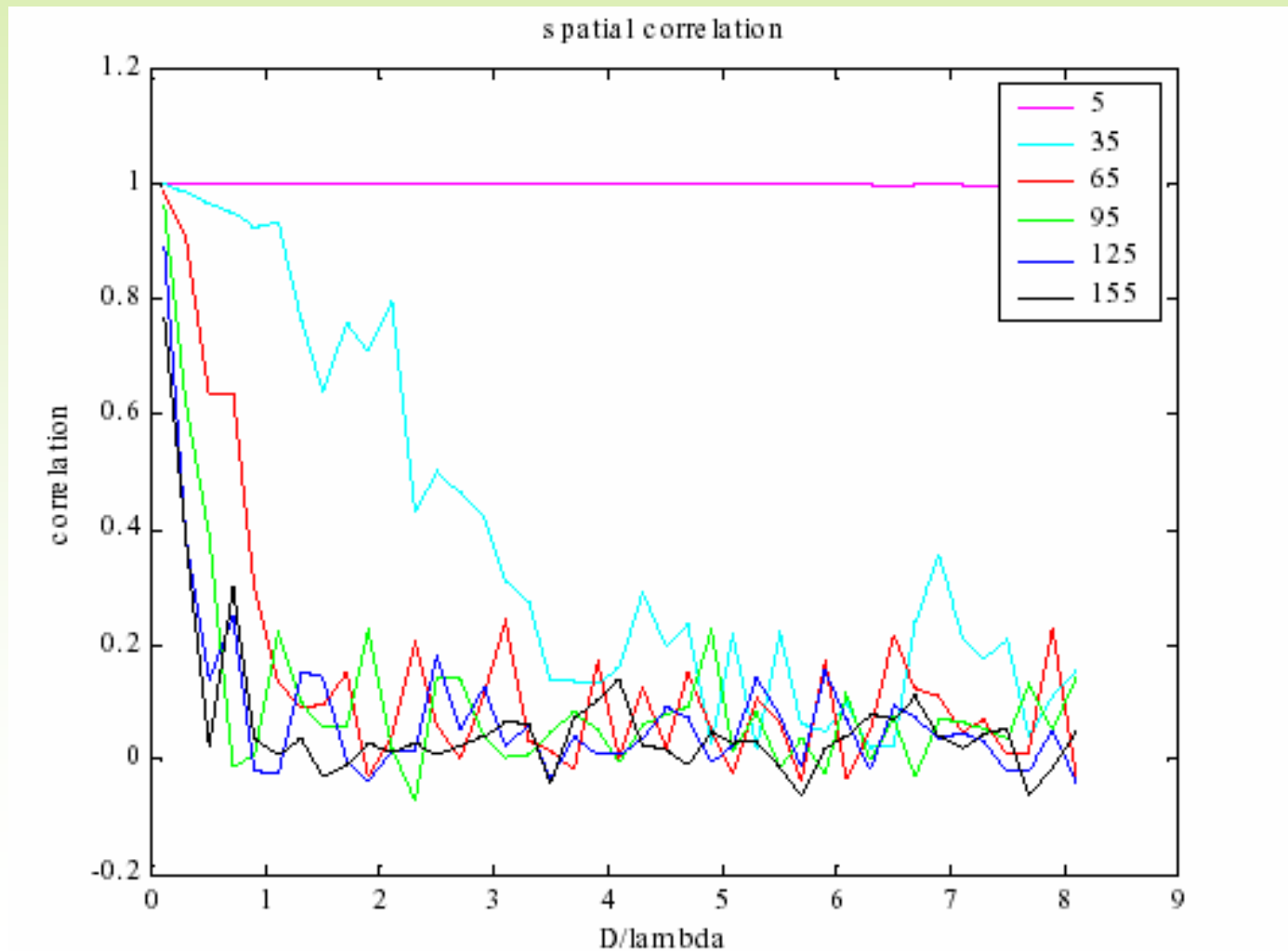
Co-Channel Interference

$$\underline{Y} = \underline{H_{est}}^{-1} \cdot \underline{H_{true}} \neq \underline{I}$$

$$\underline{\hat{S}} \Leftarrow \underline{Y} \cdot \underline{S}$$

$$\underline{Y} = \begin{pmatrix} y_{11} & y_{12} & y_{13} & y_{14} \\ y_{21} & y_{22} & y_{23} & y_{24} \\ y_{31} & y_{32} & y_{33} & y_{34} \\ y_{41} & y_{42} & y_{43} & y_{44} \end{pmatrix}$$

$$SINR_{S_k} = 20\log \frac{|y_{kk}|}{\left| \sum_{j \neq k} y_{kj} \right|}$$

Graceful Capacity Degradation in Partially Correlated Channels

Multi-path components do not need to be fully independent

3x3 MIMO Correlated Channel

SNR→∞

SNR=30 dB

SNR=20 dB

SNR=10 dB

$$\overline{C}_f = \log_2 (\det[\mathbf{I}+\text{SNR}/M\,\underline{H}\,\underline{H}^T]) / \log_2 (1+hh^T\text{SNR})$$
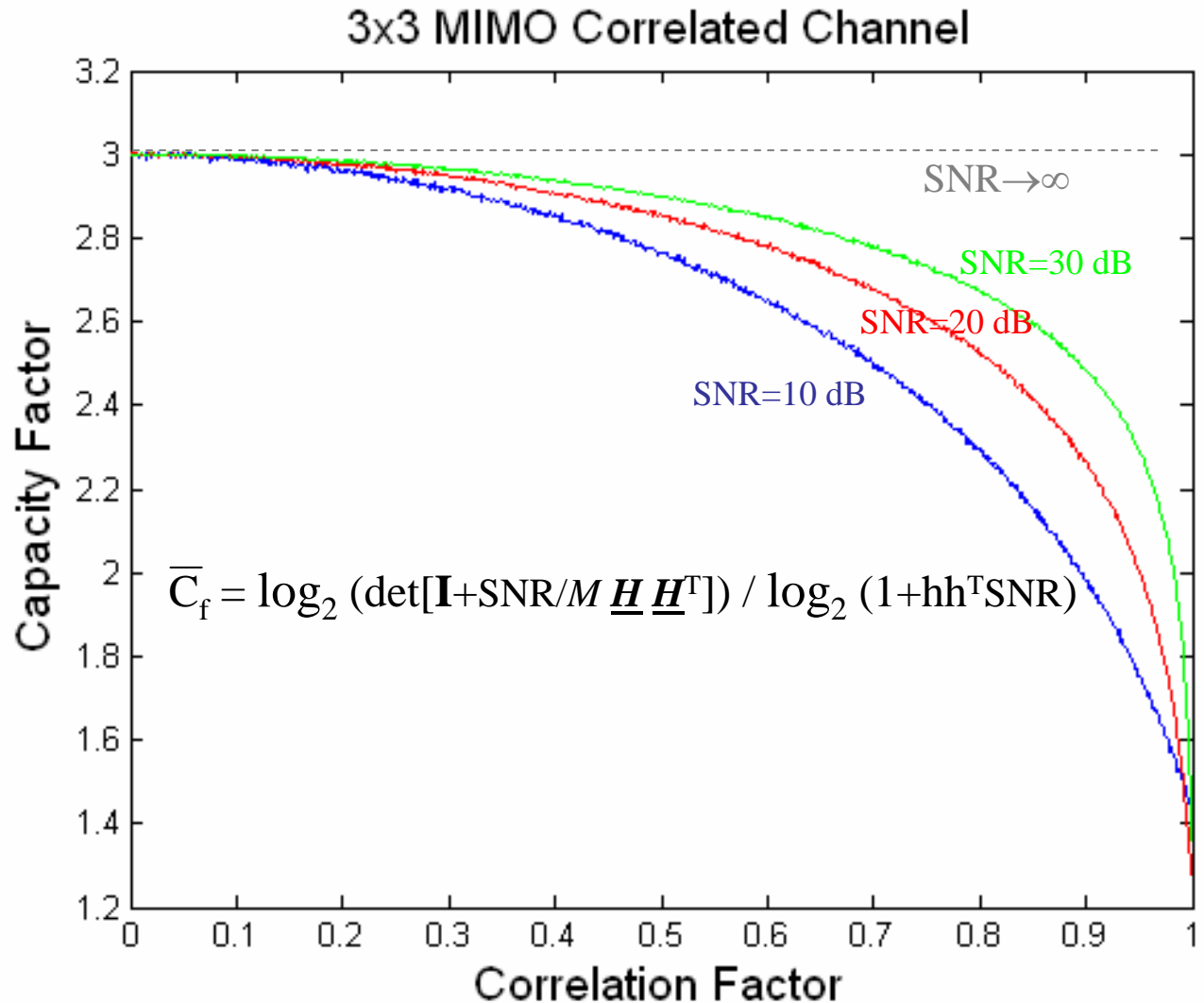
Random Capacity in MIMO Channels
Correlation Effect

Capacity CDF, 3x3 MIMO, SNR=10 dB

Collaborative MIMO

Downstream

MU

S_1 → $s_{11}\ s_{12}\ s_{13}\dots$

S_2 → $s_{21}\ s_{22}\ s_{23}\dots$

S_3 → $s_{31}\ s_{32}\ s_{33}\dots$

S_4 → $s_{41}\ s_{42}\ s_{43}\dots$

AP

$s_{11}\ s_{12}\ s_{13}\dots$ → S_1

$s_{21}\ s_{22}\ s_{23}\dots$ → S_2

$s_{31}\ s_{32}\ s_{33}\dots$ → S_3

$s_{41}\ s_{42}\ s_{43}\dots$ → S_4

S_1

S_2

S_3

S_4

R_1
R2
R3
R4
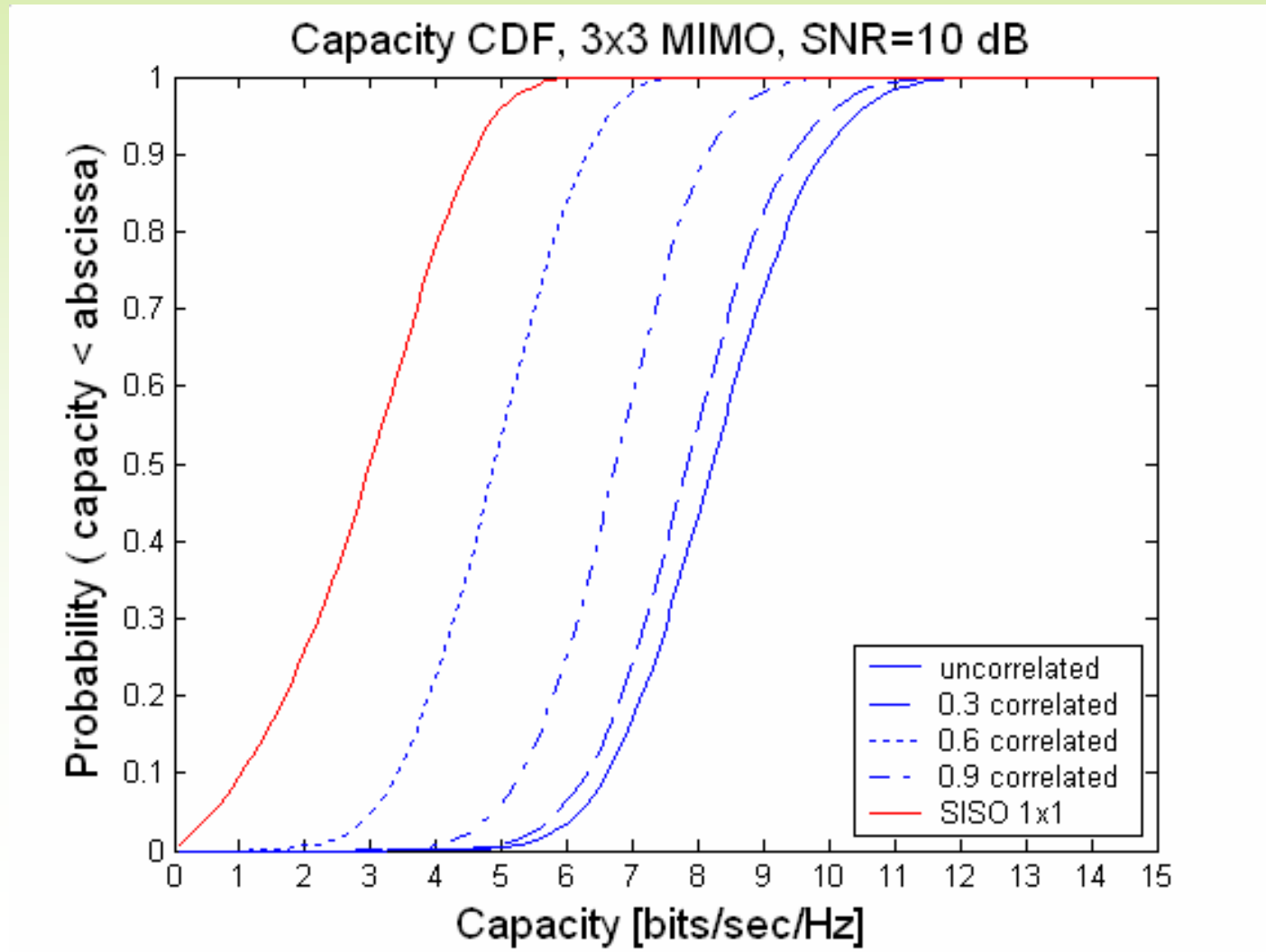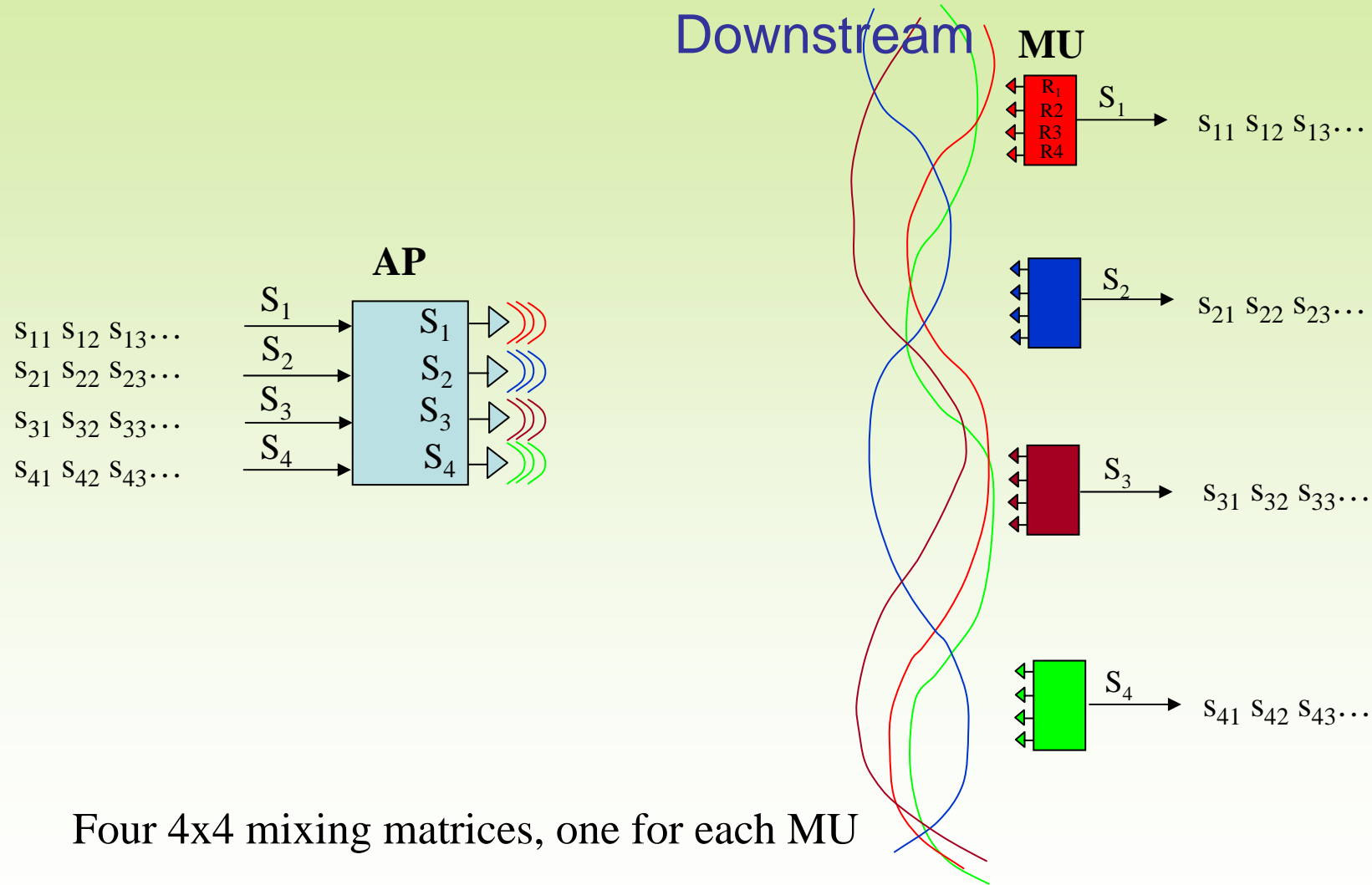
Four 4x4 mixing matrices, one for each MU

Collaborative MIMO

Upstream

MU

S_1 $s_{11}\ s_{12}\ s_{13}\ldots$

S_2 $s_{21}\ s_{22}\ s_{23}\ldots$

S_3 $s_{31}\ s_{32}\ s_{33}\ldots$

S_4 $s_{41}\ s_{42}\ s_{43}\ldots$

AP

S_1 R_1
S_2 R_2
S_3 R_3
S_4 R_4

$s_{11}\ s_{12}\ s_{13}\ldots$
$s_{21}\ s_{22}\ s_{23}\ldots$
$s_{31}\ s_{32}\ s_{33}\ldots$
$s_{41}\ s_{42}\ s_{43}\ldots$
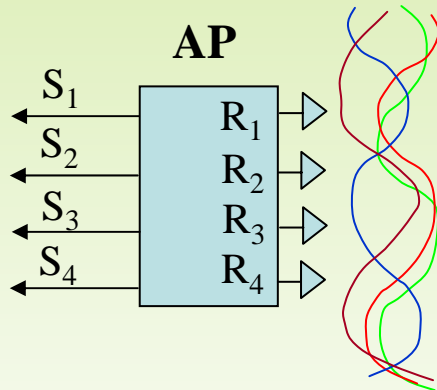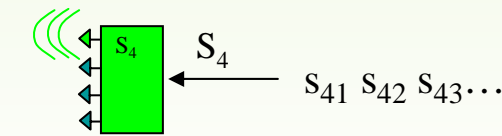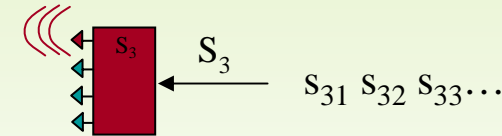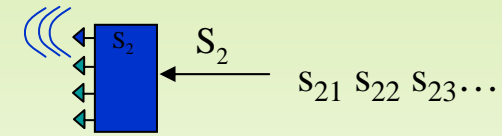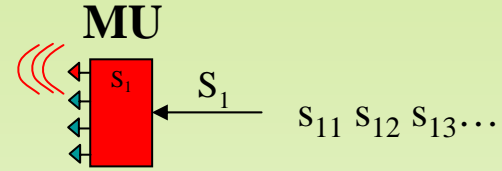
One 4x4 mixing matrix in the AP

Mixed Channels
Upstream

Four different MU's   *Single AP*

MU_1 S_1 h_{11} R_1

h_{12}

h_{21}

MU_2 S_2 R_2

channel mixing matrix

noise

MU_3 S_3 R_3

$$\underline{R} = \underline{H} \cdot \underline{S} + \underline{n}$$

h_{41}

MU_4 S_4 R_4

h_{44}

e.g., $R_1 = h_{11}S_1 + h_{12}S_2 + h_{13}S_3 + h_{14}S_4 + n_1$

MIMO Pre-Processing at the Transmitter
A single antenna at the mobile

- AP pre-processes the signals based on channel knowledge (CSI Tx)
- No MIMO processing in the mobile
- AP sends linear combination of all signals from each antenna such that when they all arrive at the mobile all undesired signals cancel out
- Effectively AP solves the equation to each mobile
- Benefits:
  - Mobile: lower cost, power and size
  - Scalability: more MIMO channels possible resulting in higher aggregate capacity
  - Strong physical-layer security, hard to break

MIMO Pre-Processing at the Transmitter
A single antenna at the mobile

MU

S_1

$s_{11} \; s_{12} \; s_{13}\ldots$

AP

$s_{11} \; s_{12} \; s_{13}\ldots$

$s_{21} \; s_{22} \; s_{23}\ldots$

$s_{31} \; s_{32} \; s_{33}\ldots$

$s_{41} \; s_{42} \; s_{43}\ldots$

S_1

S_2

S_3

S_4

S'_1

S'_2

S'_3

S'_4

S_2

$s_{21} \; s_{22} \; s_{23}\ldots$

S_3

$s_{31} \; s_{32} \; s_{33}\ldots$

S_4

$s_{41} \; s_{42} \; s_{43}\ldots$
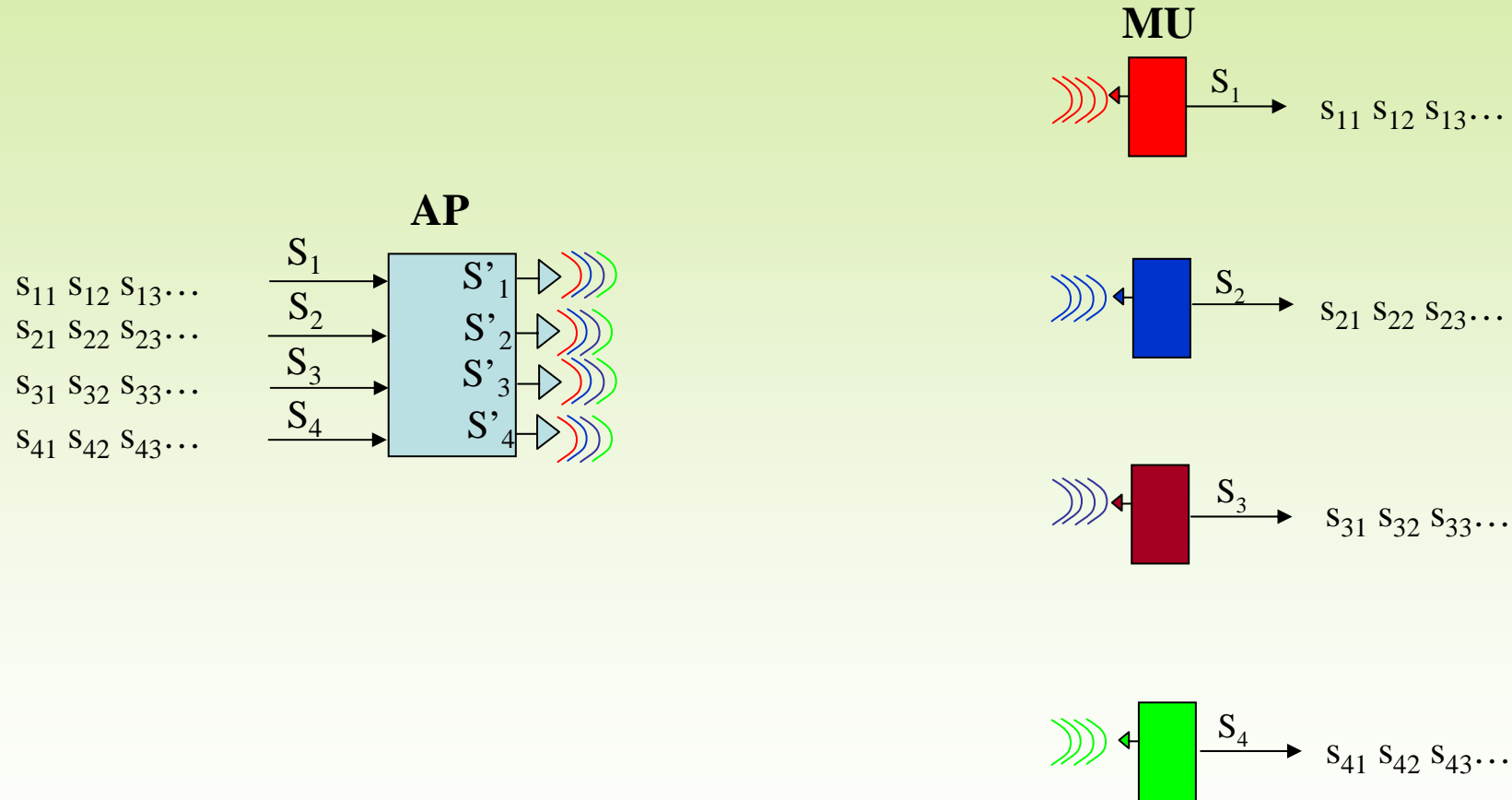
All undesired signals cancel out at the mobile

Mixed Channels
Downstream

Single AP

Single MU
Single antenna

channel mixing matrix

noise

$$\underline{R} = \underline{H} \cdot \underline{S}' + \underline{n}$$

h_{11}

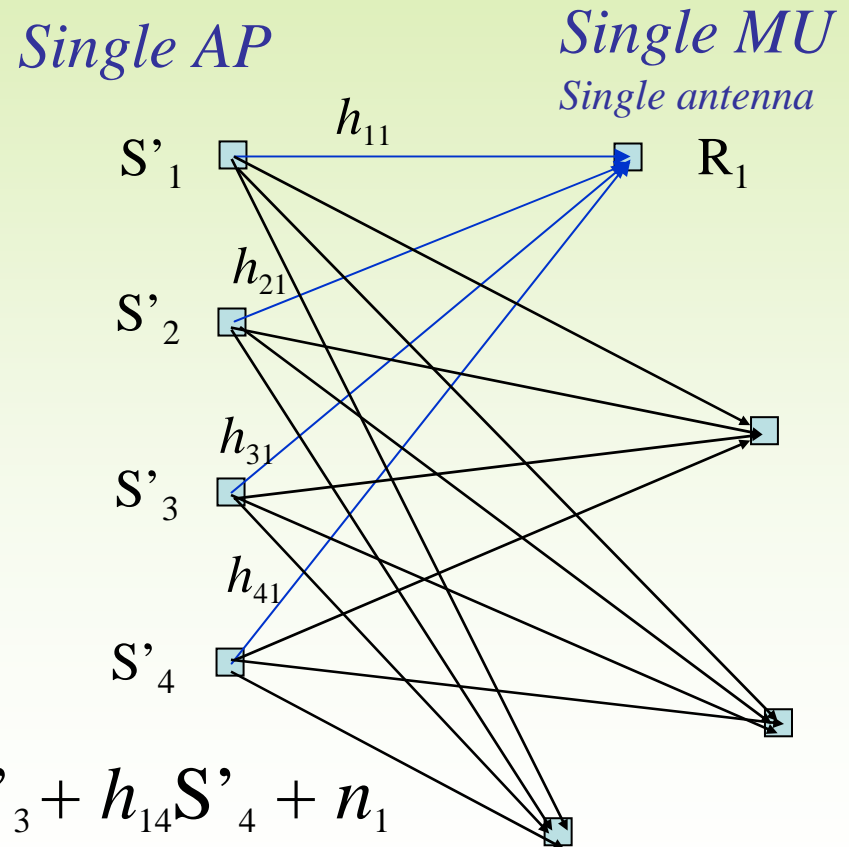S'_1 R_1

h_{21}

S'_2

h_{31}

S'_3

h_{41}

S'_4

e.g., $R_1 = h_{11}S'_1 + h_{12}S'_2 + h_{13}S'_3 + h_{14}S'_4 + n_1$

MIMO Pre-Processing at the Transmitter
Downlink

$$\underline{R} = \underline{H} \cdot \underline{S} + \underline{n}$$

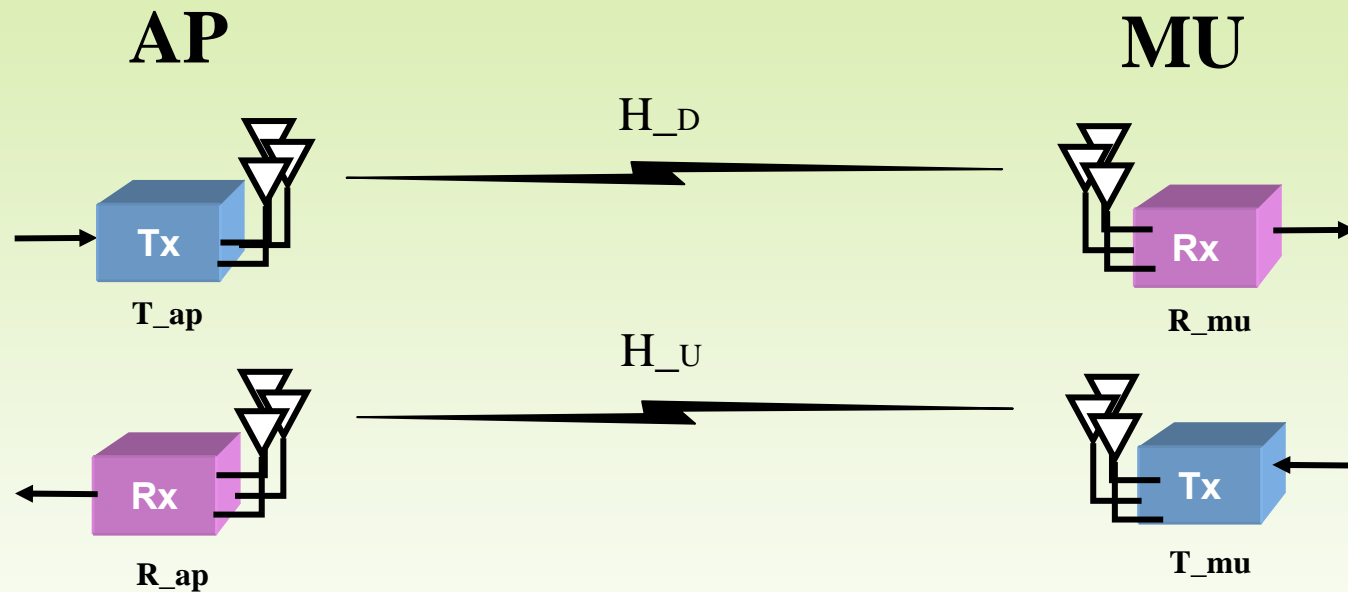$$\underline{S}' = \underline{W} \cdot \underline{S}$$

$$\underline{R}' = \underline{H} \cdot \underline{S}' + \underline{n} = \underbrace{\underline{H} \cdot \underline{W}}_{\underline{Y}} \cdot \underline{S} + \underline{n}$$

$$\underline{W} = \underline{H}^{-1}$$

$$\underline{Y} = \underline{H}_{true} \cdot \underline{H}_{est}^{-1} \neq \underline{I}$$

$$\hat{\underline{S}} \Leftarrow \underline{Y} \cdot \underline{S}$$

End-to-End Reciprocity

- Practically, downstream and upstream channel matrices are not reciprocal
- AP Tx/Rx chain mismatch could result in significant performance degradation

End-to-End Reciprocity (cont.)

$H'_D = R\_mu\ H_D\ T_{AP}$ end-to-end downstream

$H'_U = R_{AP}\ H_U\ T\_mu$ end-to-end upstream, estimated using training sequence

Note that R_mu, T_mu, R_{AP} and T_{AP} are diagonal matrices

H_D and H_U are the channel matrices (antenna-to-antenna) for downstream and upstream, respectively

$H_D = R^{-1}\_mu\ H'_D\ T^{-1}_{AP}$ antenna-to-antenna downstream

$H_U = R^{-1}_{AP}\ H'_U\ T^{-1}\_mu$ antenna-to-antenna upstream

$H_D = H^T\_U$ reciprocity from EM theory

$R^{-1}\_mu\ H'_D\ T^{-1}_{AP} = (R^{-1}_{AP}\ H'_U\ T^{-1}\_mu)^T$

$R^{-1}\_mu\ H'_D\ T^{-1}_{AP} = T^{-1}\_mu\ H'^T\_U\ R^{-1}_{AP}$

$$\boxed{H'_D = R\_mu\ T^{-1}\_mu\ H'^T\_U\ R^{-1}_{AP} T_{AP}}$$

Note that R_mu and T_mu are unknown

H'_U, T_{AP} and R_{AP} are known

Calibration at the AP

$P'_D = H'^T_U R^{-1}_{AP} T_{AP}$ matrix used for pre-processing

$Y = R_{mu} H_D T_{AP} P'^{-1}_D = R_{mu} H_D T_{AP} (H'^T_U R^{-1}_{AP} T_{AP})^{-1}$

$Y = R_{mu} H_D T_{AP} ((R_{AP} H_U T_{mu})^T R^{-1}_{AP} T_{AP})^{-1}$

$Y = R_{mu} H_D T_{AP} (T_{mu} H^T_U R_{AP} R^{-1}_{AP} T_{AP})^{-1}$

$Y = R_{mu} H_D T_{AP} (T_{mu} H_D R_{AP} R^{-1}_{AP} T_{AP})^{-1}$

$Y = R_{mu} H_D T_{AP} (T_{mu} H_D T_{AP})^{-1}$

$Y = R_{mu} H_D T_{AP} T^{-1}_{AP} H^{-1}_D T^{-1}_{mu}$

$$\boxed{Y = R_{mu} H_D H^{-1}_D T^{-1}_{mu}}$$ highly diagonal (low interference)

No Calibration at the AP

$P'\_D = H'^T\_U$ matrix used for pre-processing

$Y = R\_mu \, H\_D \, T\_{AP} \, P'^{-1}\_D = R\_mu \, H\_D \, T\_{AP} (H'^T\_U)^{-1}$

$Y = R\_mu \, H\_D \, T\_{AP}((R\_{AP} \, H\_U \, T\_mu)^T)^{-1}$

$Y = R\_mu \, H\_D \, T\_{AP}(T\_mu \, H^T\_U \, R\_{AP})^{-1}$

$Y = R\_mu \, H\_D \, T\_{AP}(T\_mu \, H\_D \, R\_{AP})^{-1}$

$$Y = R\_mu \, H\_D \, T\_{AP} \, R^{-1}\_{AP} \, H^{-1}\_D \, T^{-1}\_mu$$

diagonality could
be spoiled resulting
in interference

# End-to-End Reciprocity
Conclusions

- AP Tx/Rx mismatch could result in significant performance degradation
- MU Tx/Rx mismatch has relatively small effect on performance
- Calibration in the AP is *necessary* and *sufficient*

Mobility Effects

- Motions of mobiles change the channel matrix

- Since the packet length is very short, the change of channel matrix is supposed to be negligible. Estimation of channel matrix using header (preamble) only is considered as the channel responses for decoding the entire packet.

- The SINR results are much worse than what were expected originally. The reason is: when the condition number of H is very high, H^{-1} is very sensitive to small changes of H.

- SINR for some multiplexing channels may be less than 10dB even when the displacement of a Tx or Rx is less than 2% of the wavelength

- Better estimation of channel matrix is required.
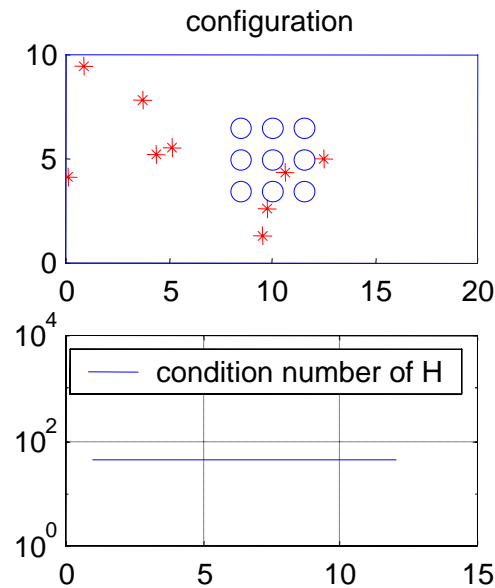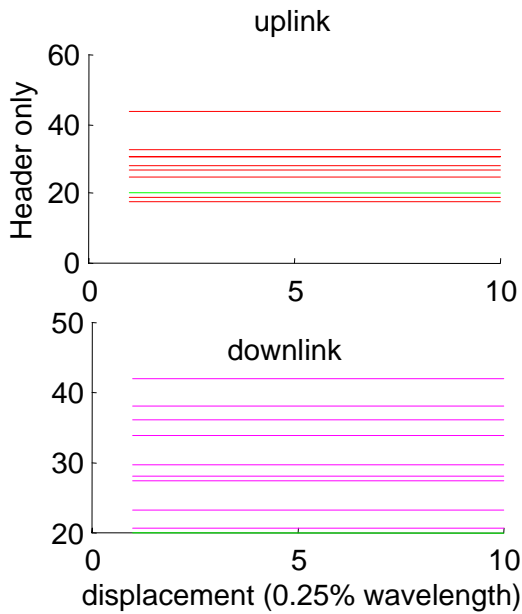
Effect of Mobilty

Statistical Model
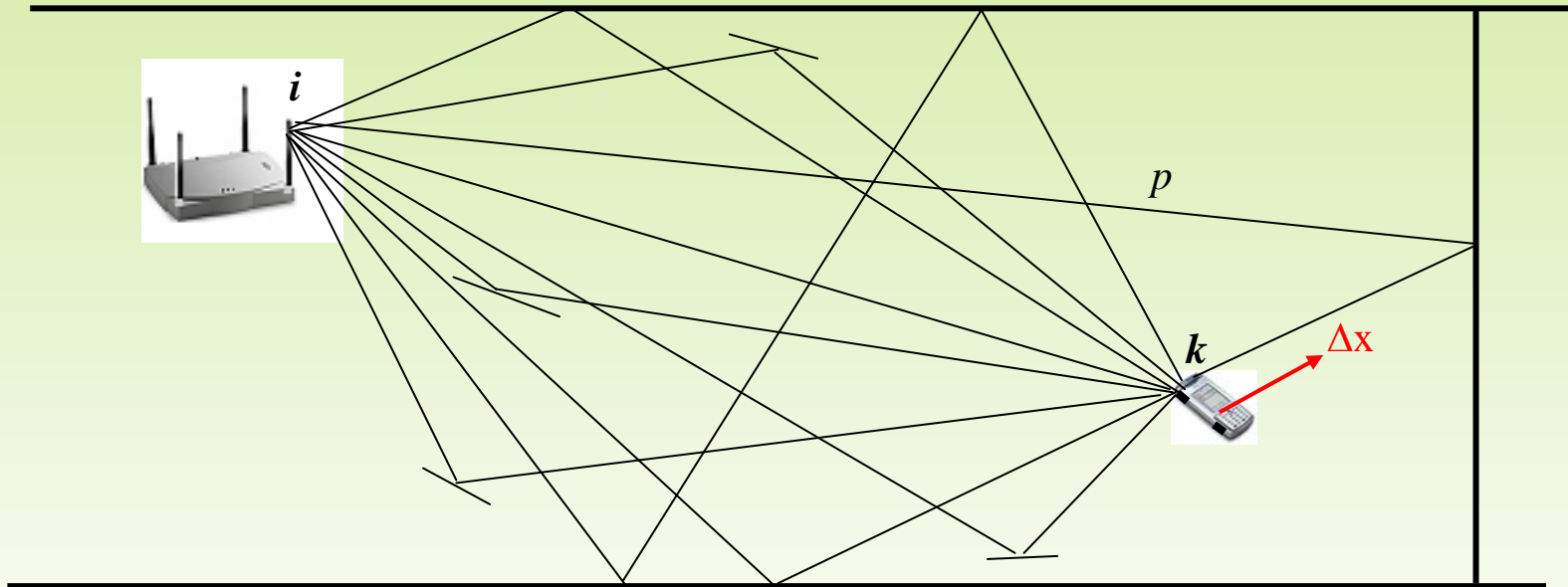
$$h_{ik} = \sum_p a_p e^{-j\theta_p} \longrightarrow \tilde{h}_{ik} = \sum_p a_p e^{-j(\theta_p + (2\pi/\lambda)\Delta x \cos\varphi_p)}$$

θ_p, φ_p iid uniformly $[0, 2\pi]$

9 Mobiles; None Move
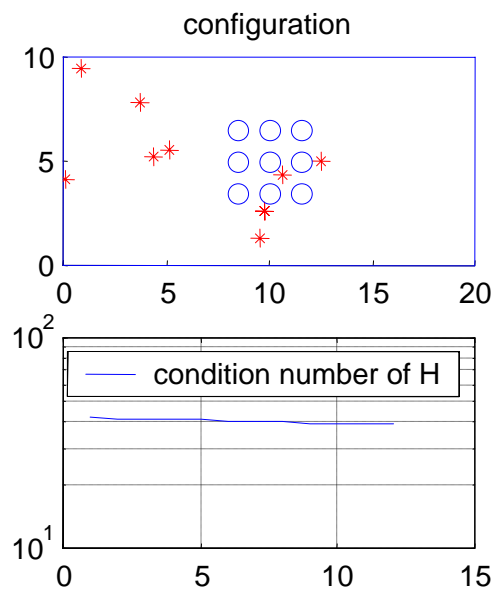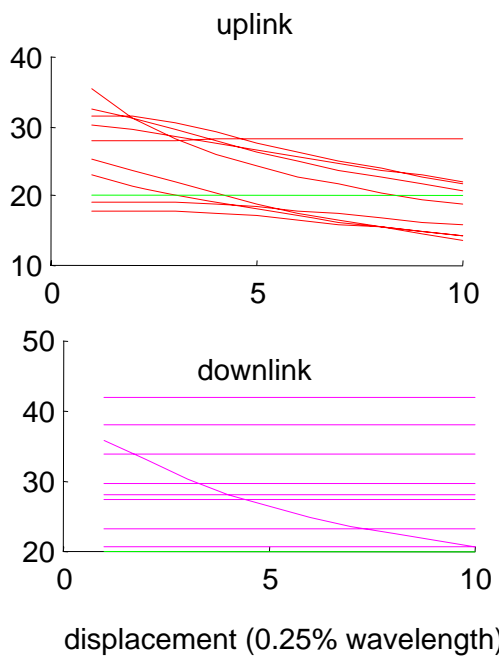## Input SNR=20dB

SINR's constants and finite (imperfect channel estimation due to noise)

9 Mobiles; 1 Moves
Input SNR=20dB

uplink

configuration

downlink

condition number of H

displacement (0.25% wavelength)

-Uplink: ALL SINR's are deteriorating as the displacement increases except that for the moving mobile

-Downlink: ALL SINR's remain unchanged except that for the moving mobile

Applications

- WLAN – WiFi 802.11n
- Mesh Networks (e.g., MuniWireless)
- WMAN – WiMAX 802.16e
- 4G
- RFID
- Digital Home

High Throughput WiFi - 802.11n
General

- Using the *space* dimension (MIMO) to boost data rates up to 600 Mbps through multiple antennas and signal processing

- Target applications include: large files backup, HD streams, online interactive gaming, home entertainment, etc.

- Backwards compatible with 802.11a/b/g
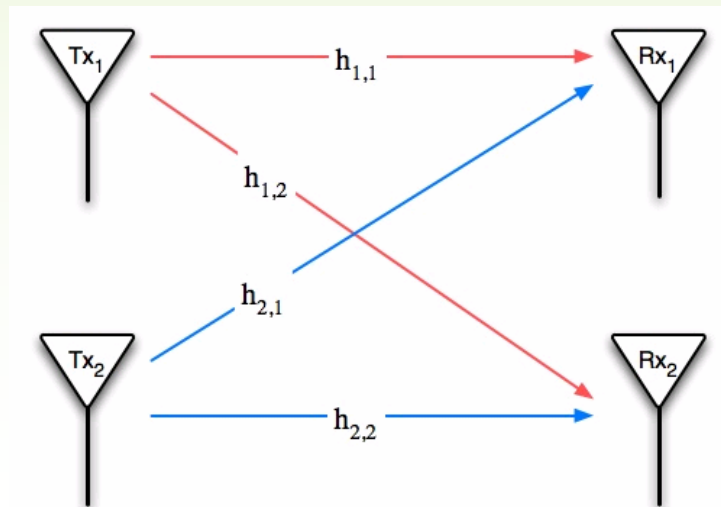
High Throughput WiFi - 802.11n
## Technology Overview

- 2.4 GHz and 5.8 GHz unlicensed bands

- Channel bandwidth of 20 MHz and 40 MHz

- Up to 4 spatial streams (e.g., 4x4)

- Current product offerings (pre-N) use only 2 spatial streams with 3Tx / 3Rx in the AP and 2Tx / 3Rx in the mobile supporting up to 300 Mbps

- Spatial diversity, spatial multiplexing, beamforming

- Enhancements in both PHY and MAC (e.g., frame aggregation, block-ACK, space-time coding, power save, green field mode, etc.)

MIMO in MuniWireless

- High capacity (MIMO) cross-links
- WiFi access

MIMO in Ad-Hoc Network

single-hop comm.
multi-hop comm.
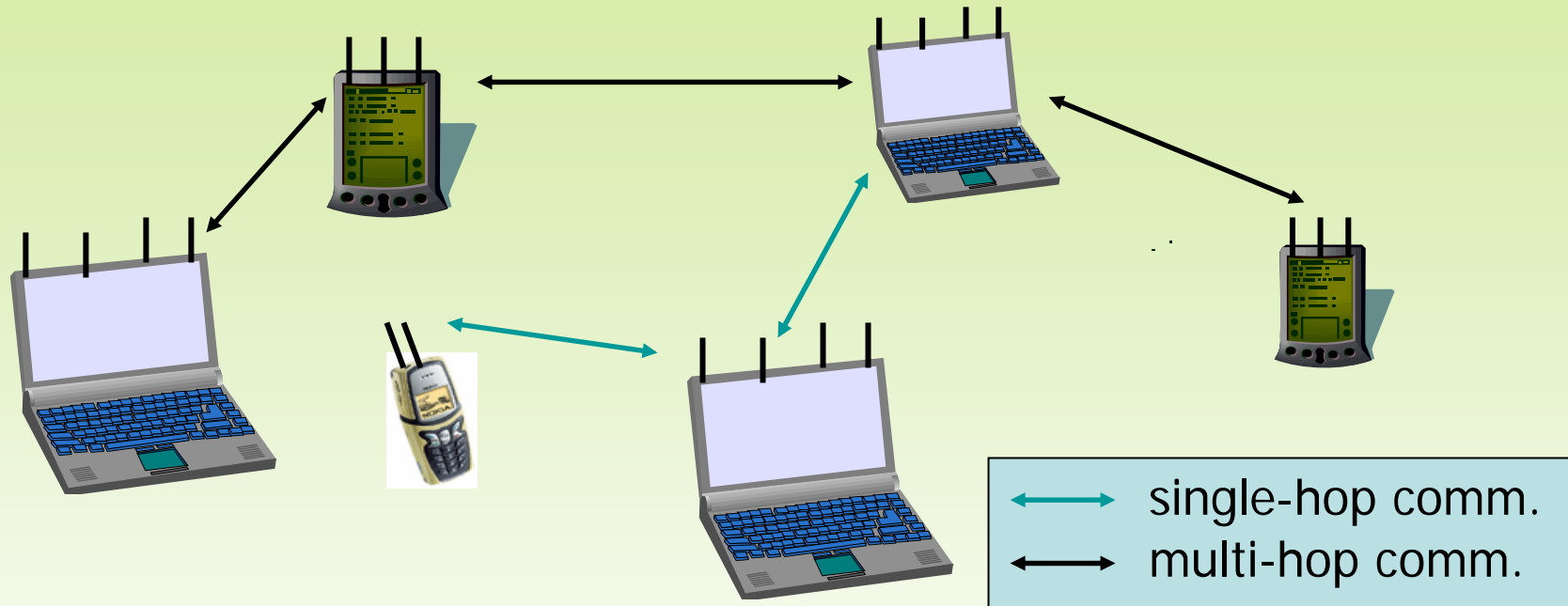
- A collection of wireless mobile nodes that self-configure to form a network (data rate + range)
- No fixed infrastructure is required
- Any two nodes can communicate with each other
- High capacity link are useful for scalability and multimedia services

Mobile-WiMAX 802.16e
Technology Overview

- Non line of site, up to 4-6 mbps per user for a few km

- 2.5 GHz (US) and 3.5 GHz licensed bands

- Channel bandwidth from 1.25 to 20 MHz

- QPSK, 16 QAM and 64 QAM modulation

- OFDMA access (orthogonal uplink)

- TDD for asymmetric traffic and flexible BW allocation

- Advanced Antenna Systems (AAS): Beamforming, spatial diversity, spatial multiplexing using MIMO (2x2)
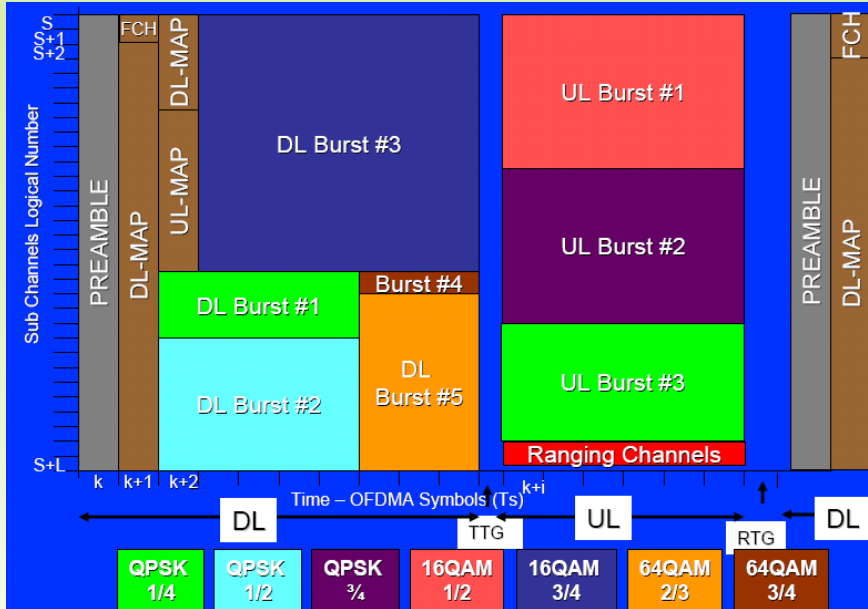
MIMO in WiMAX

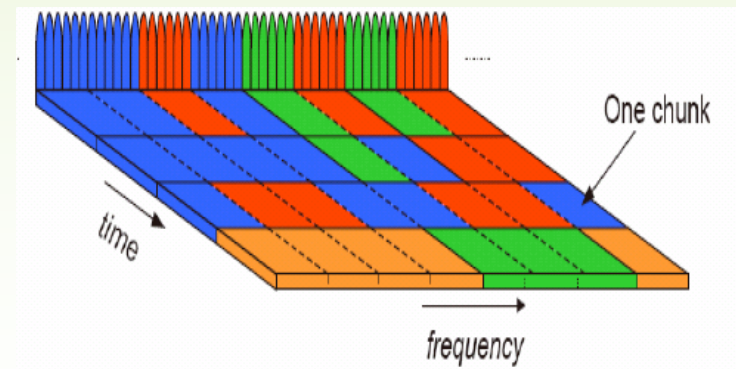## A 2x2 MIMO Configuration in 802.16e
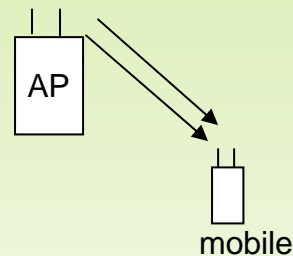
Downlink
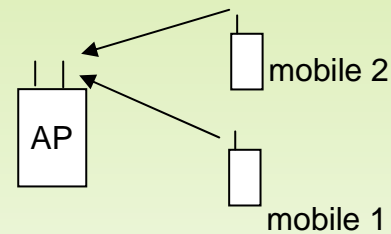
Uplink (collaborative)

- Increasing spectral efficiency (bps/Hz)
- Downlink – higher capacity and user peak rates
- Uplink – higher capacity only
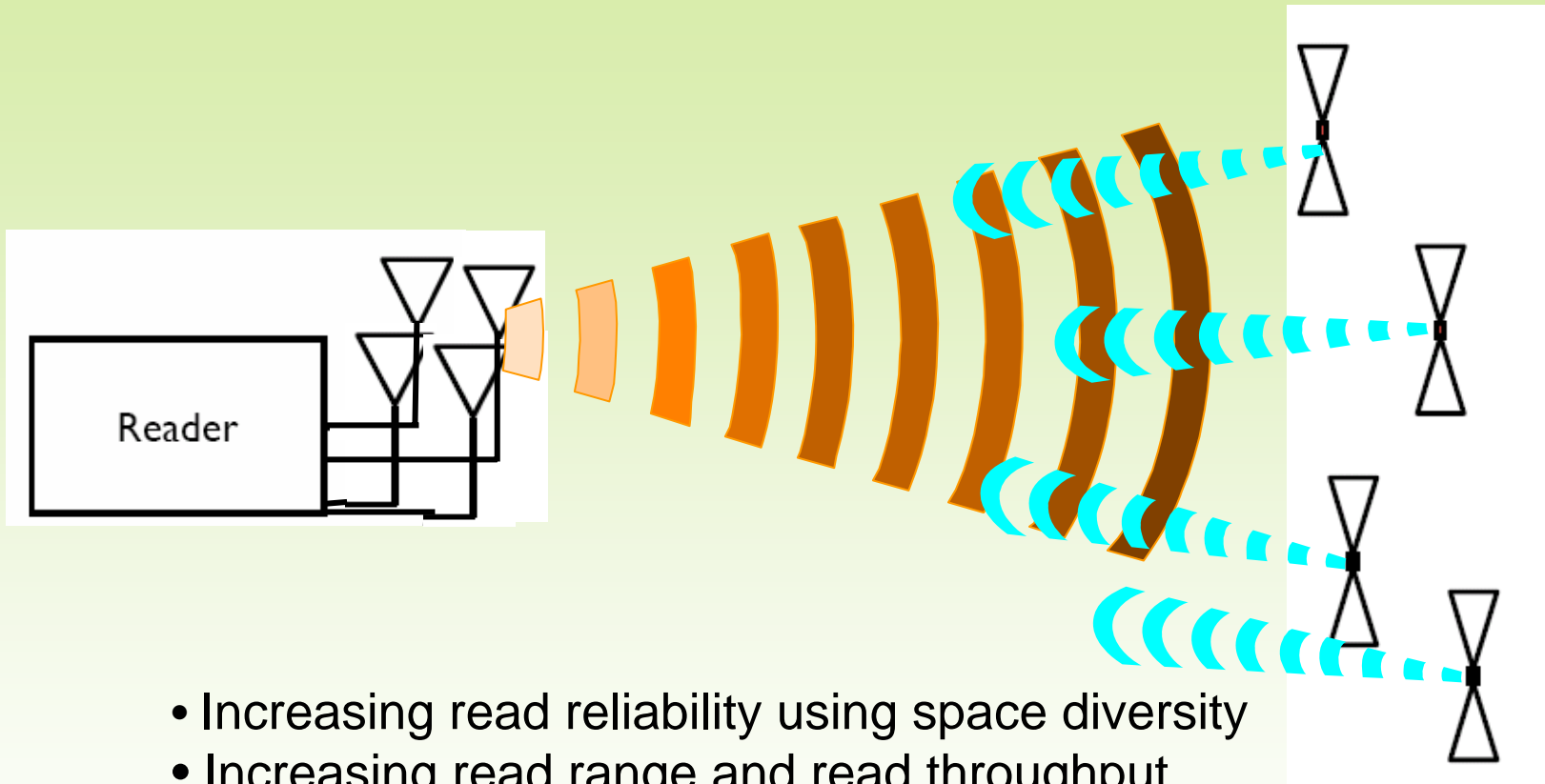
MIMO in WiMAX (cont.)

OFDMA TDD Frame Structure

Time/Frequency Multi-User Diversity

MIMO in WiMAX (cont.)
Layer 3 Throughput Comparison

technology	throughput per sector/per channel		
	downlink		uplink
	1 Rx	2 Rx	2 Rx
1XEVDO rev A 2.5 MHz	0.9 Mbps	1.3 Mbps	0.5 Mbps
HSPA 10 MHz	2.4 Mbps	3.6 Mbps	1.5 Mbps
Mobile WiMax 2:1 DL/UL 2x2 -10 MHz	NA	14 Mbps	5.3 Mbps

MIMO in RFID

- Increasing read reliability using space diversity
- Increasing read range and read throughput
- Full channel information at the reader
  comes for free (tag backscatter)

MIMO Enables the Digital Home

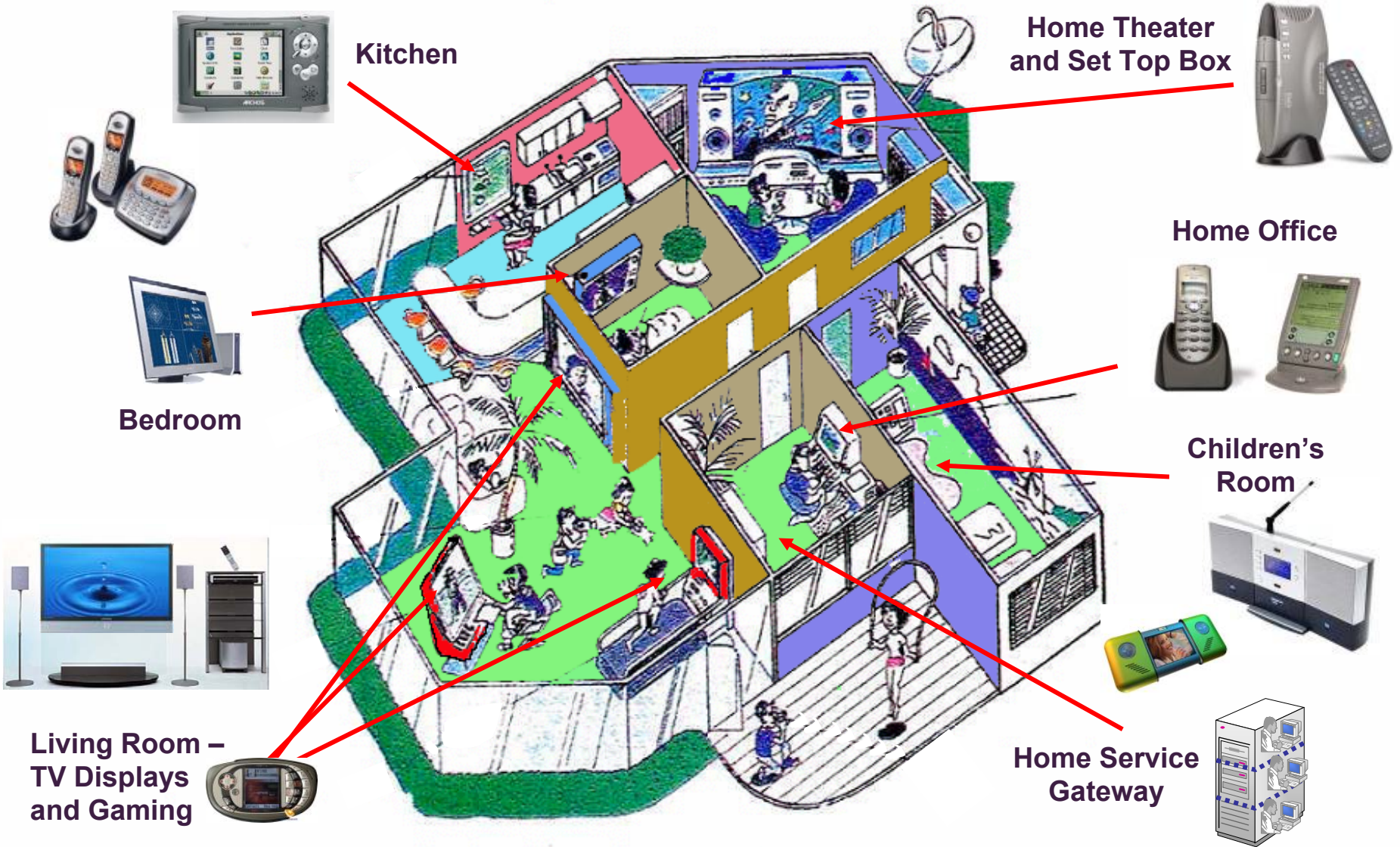**MIMO** delivers whole home coverage with the speed and reliability to stream multimedia applications

MIMO can reliably connect cabled video devices, computer networking devices, broadband connections, phone lines, music, storage devices, etc.

MIMO is interoperable and can leverage the installed based of 802.11 wireless that is already deployed: computers, PDAs, handheld gaming devices, cameras, VoIP Phones, etc.

The Ultimate Digital Home

WiFi 802.11n

Kitchen

Home Theater and Set Top Box

Home Office

Bedroom

Children's Room

Living Room – TV Displays and Gaming

Home Service Gateway

Questions ?

Contact Information:

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Thank You!