Matching Network Design Using Non-Foster Impedances

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Outline

- Introduction
- Stability
- Laboratory Measurements: High-Q Negative Elements for Receive
- Laboratory Measurements: Non-Foster Monopole and Dipole
- Technology Development

Introduction

Motivation

- **Requirement:** broadband, efficient, *electrically-small* antennas ($\ell \ll \lambda$)
- **Electrically-short monopole:** characterized by ***large reactance*** and ***low radiation resistance*** (varies rapidly with frequency)
- **Wheeler and Chu (1947):**
  - Electrically-small antennas have almost the same directivity as larger antennas and can in principle perform as well as the larger ones
  - Problem is in *transferring power* to and from small antennas, which are very difficult to impedance match
- **With conventional matching technology, small antennas suffer from:**
  - ***Poor gain*** due to mismatch loss or lossy impedance matching (when attempting a broadband match)
  - ***Narrow bandwidth*** due to the high Q of the antenna (when attempting to maximize gain)

Noise Consequences of Low Antenna Gain

Consider:

- An electrically-small broadband VHF antenna with typical gain 20-30 dB below isotropic
- A MIL receiver, designed for best compromise in sensitivity & dynamic range (noise figures of 6-8 dB)

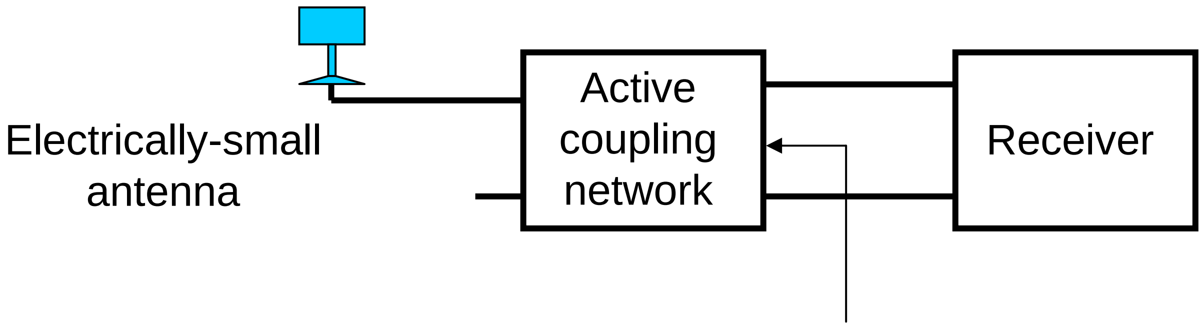
In such a system:

- It is receiver noise *–not external noise–* that limits sensitivity
- ***Greater sensitivity will result from increasing antenna gain via non-Foster matching***
- Passive matching limited by gain-bandwidth constraints

Early Work on Negative Elements with Small Antennas

1971: First mention of using *negative inductance* for bandwidth extension of dipole antennas (Poggio and Mayes)

1977: First use of an active coupling network with *negative resistance* to improve noise figure (Bahr)



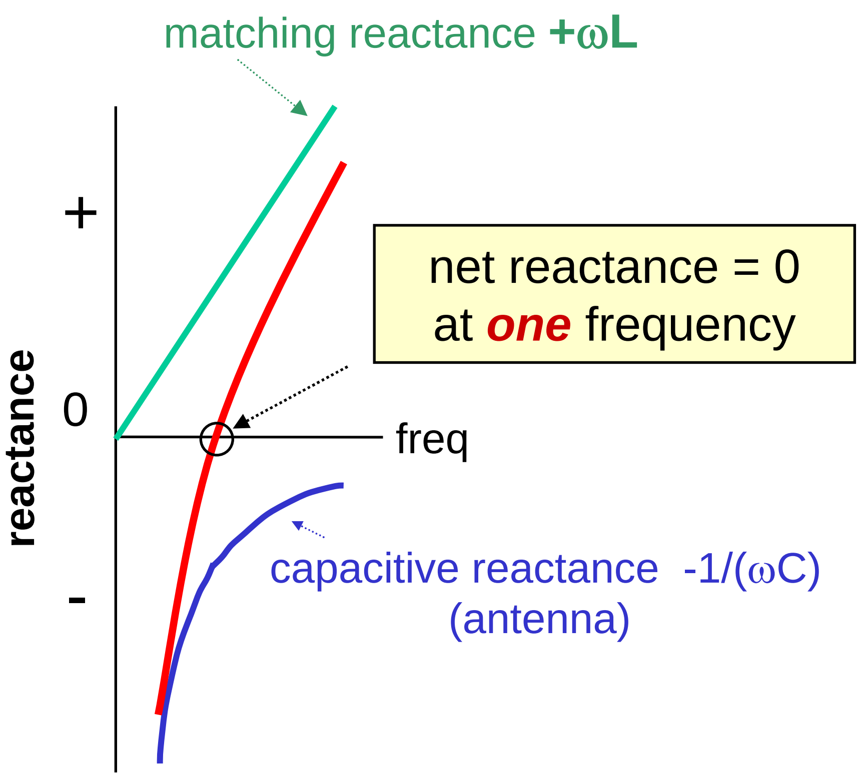
$$Z_C = -|r| + jx$$

Non-Foster impedance matching employs negative reactive elements (–L, –C)

Fundamental Limits of Passive Matching

- **The Fano-Youla gain-bandwidth theory:**
  - for matching networks containing positive RLC or distributed elements
  - gives limits on the achievable bandwidth
  - implies that certain sources and loads (e.g. electrically-small antennas) cannot achieve a good match, regardless of circuit complexity
- **Circuits containing *negative elements* ("non-Foster" networks):**
  - are not constrained by gain-bandwidth theory
  - can achieve wide matching bandwidths with "difficult" loads arising from electrically-short antennas

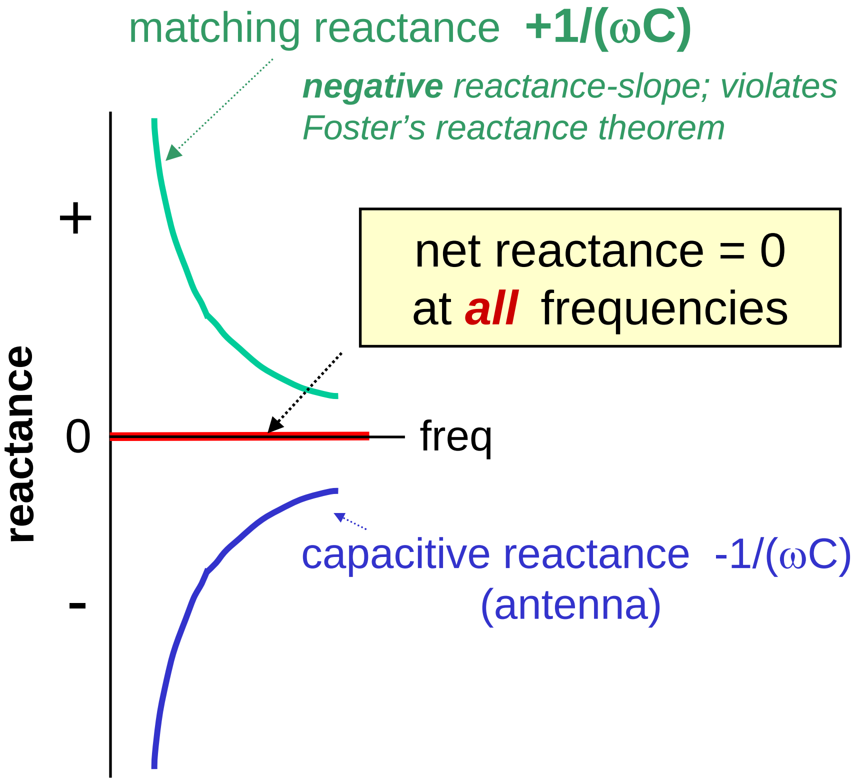
Conventional vs. Negative Impedance Matching



matching reactance **+ωL**

net reactance = 0
at ***one*** frequency

capacitive reactance -1/(ωC)
(antenna)



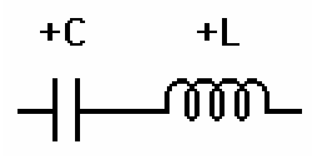
matching reactance **+1/(ωC)**

negative *reactance-slope; violates Foster's reactance theorem*

net reactance = 0
at ***all*** frequencies

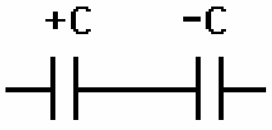
capacitive reactance -1/(ωC)
(antenna)

CONVENTIONAL



Resonate +C with a ***positive inductor***

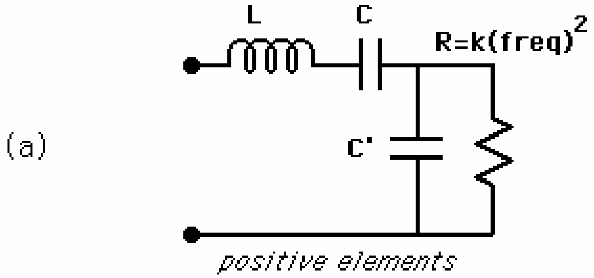
NEGATIVE IMPEDANCE



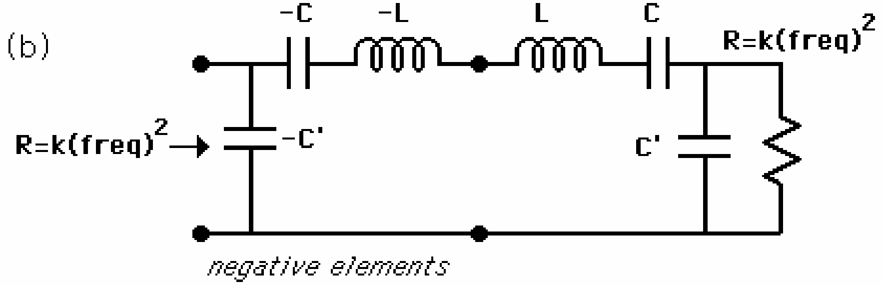
Resonate +C with a ***negative capacitor***

Canonical Approach to Negative-Element Matching

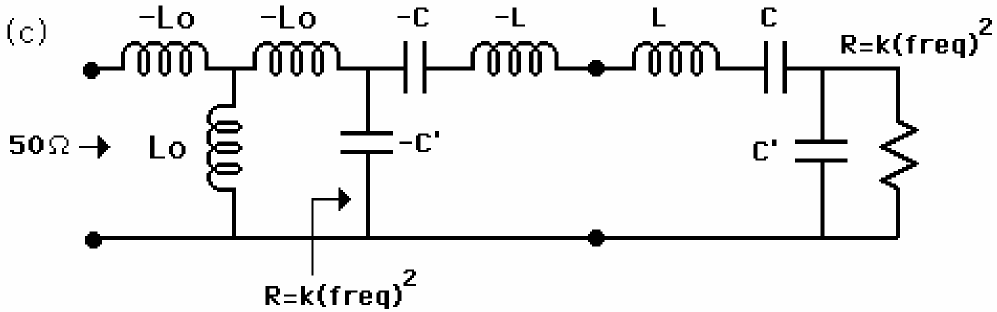
Antenna Model



Antenna model with L, C, and C' canceled by –L, –C, and –C'



Inductive-T completes the match to 50Ω



Dualizer

- For the tee and pi L networks shown

$$Z_{in} = \omega^2 L_o^2 / Z_L$$

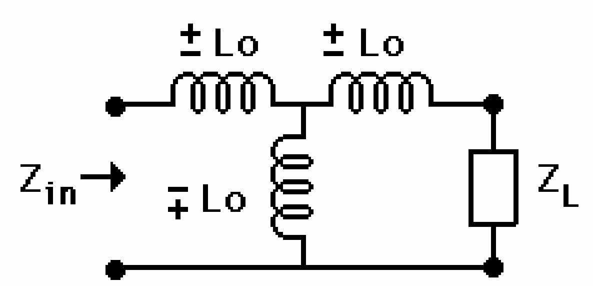
- If Z_L is the frequency-dependent resistance

$$Z_L = k\omega^2$$

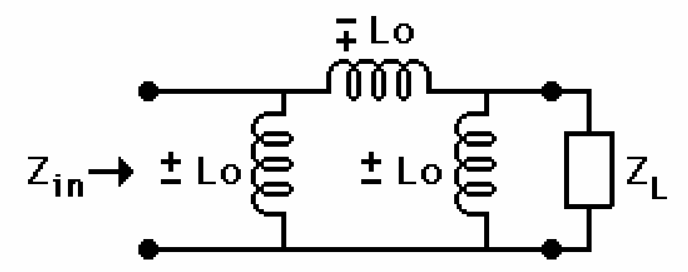
and if we want the input impedance Z_{in} to be the real and constant value R_o, we choose

$$L_o^2 = kR_o$$

(also used in coupled-resonator filter design)



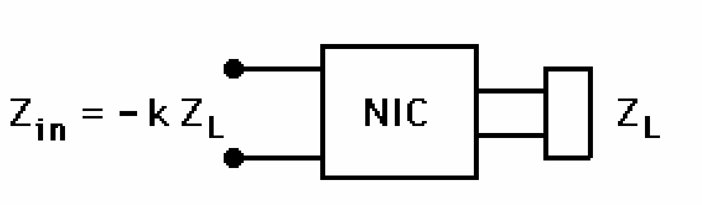
Inductor-T Dualizer



Inductor-Pi Dualizer

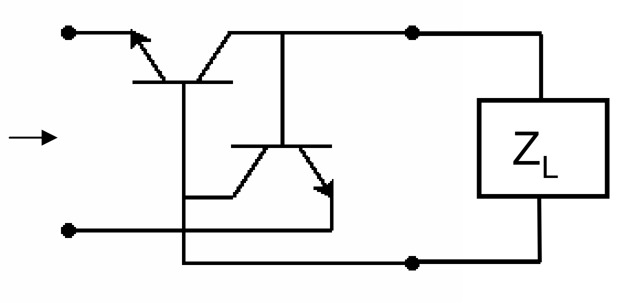
Realizing Negative Elements

A *negative* element is produced by terminating a *negative impedance converter* (NIC) with a corresponding *positive* element



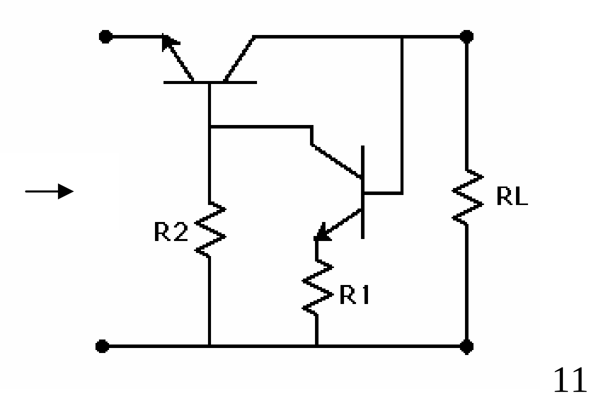
Floating negative impedance
(Linvill, 1953)

$Z_{in} = -Z_L$ →

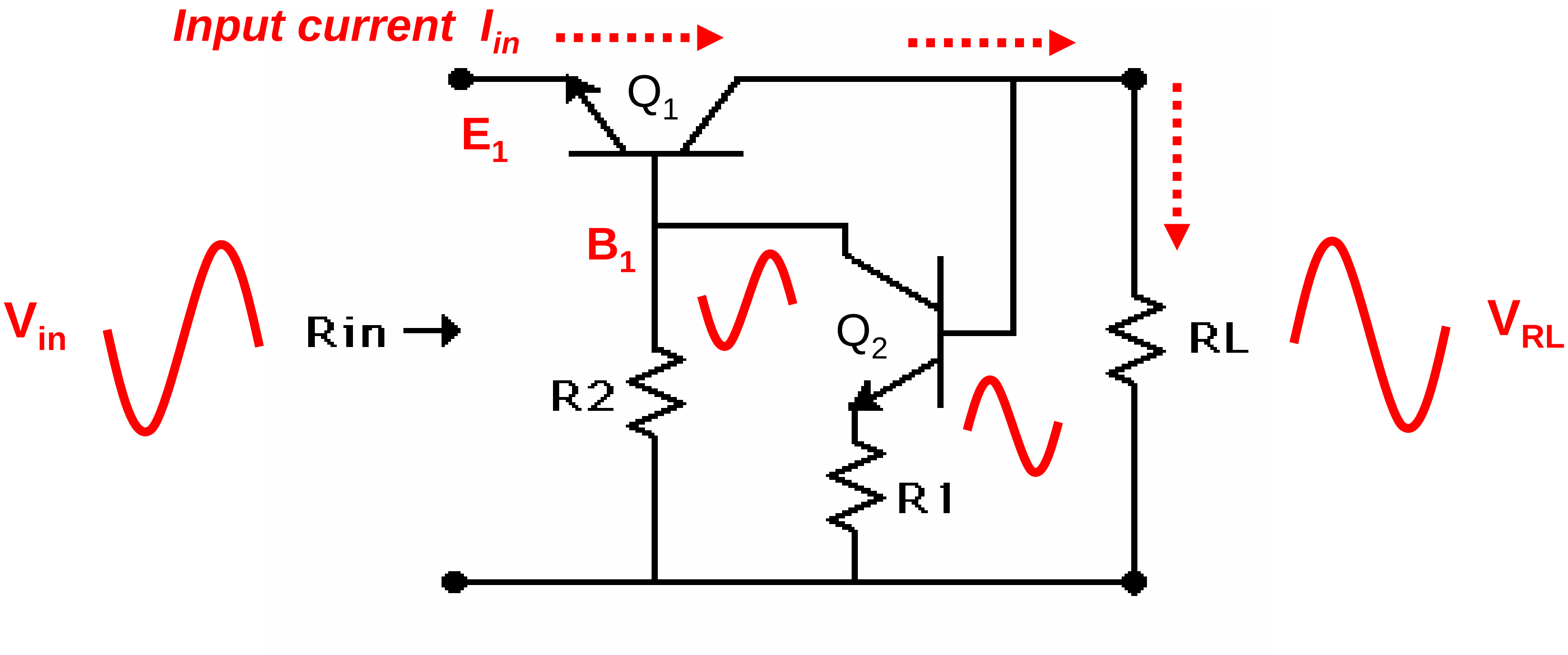


Grounded negative resistance
(Linvill, 1953)

$R_{in} = -(R_2/R_1)R_L$ →



How a Voltage-Inversion NIC Works

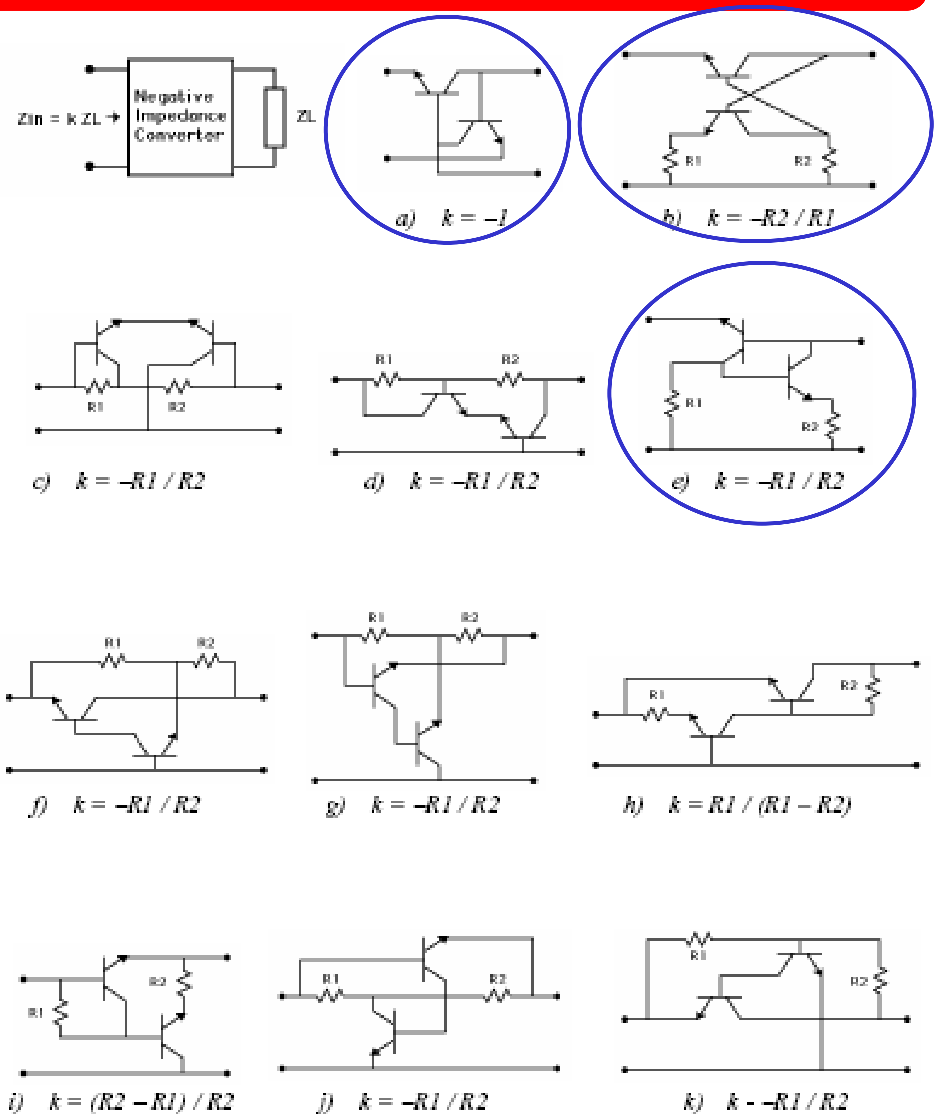


- Input current flows through Q_1 producing **V_{RL}** across **R_L**
- **V**$_{RL}$ fed back through CE stage Q_2 producing 180° phase inversion at **B_1**
- Voltage at **E_1**, **V_{in}**, appears in phase with voltage at **B_1**
- **$R_{in} = V_{in} / I_{in}$ seen to be negative of R_L: because current is same, but voltages are inverted**

Practical NICs - *1*

- Of the many NICs that have been proposed, only the *Linvill* and *Yanigisawa* circuits have been built and tested

- Some other circuits can be shown to possess inconsistent phasing with practical devices

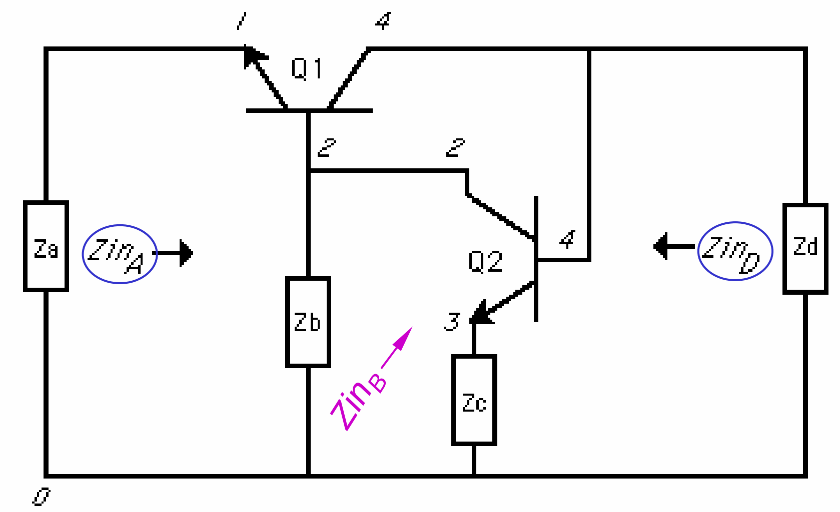


a) $k = -1$
b) $k = -R2 / R1$
c) $k = -R1 / R2$
d) $k = -R1 / R2$
e) $k = -R1 / R2$
f) $k = -R1 / R2$
g) $k = -R1 / R2$
h) $k = R1 / (R1 - R2)$
i) $k = (R2 - R1) / R2$
j) $k = -R1 / R2$
k) $k - -R1 / R2$

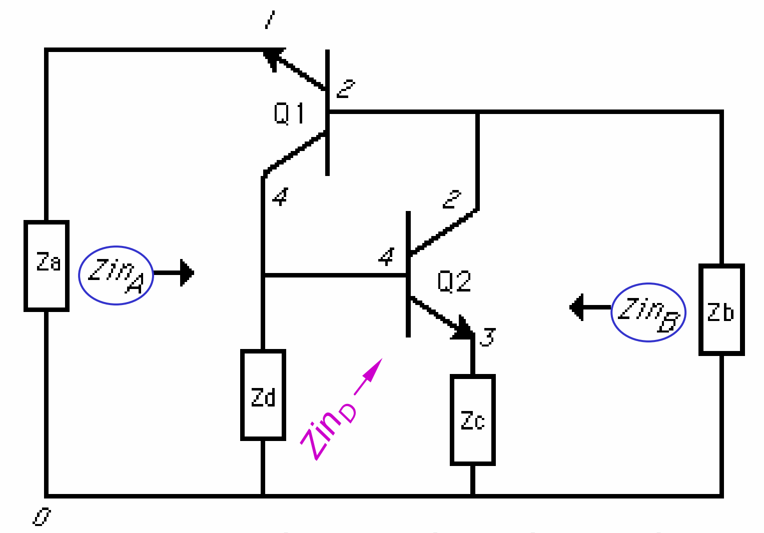
Negative Impedance Converter Networks

Practical NICs - *2*

Both the Linvill and Yanagisawa NICs are derived from the same terminated or “augmented network”



Augmented network



Augmented network, redrawn, but otherwise identical

Linvill OCS: $Zin_A = -(Z_b / Z_c)\, Z_d$

Linvill SCS: $Zin_D = -(Z_c / Z_b)\, Z_a$

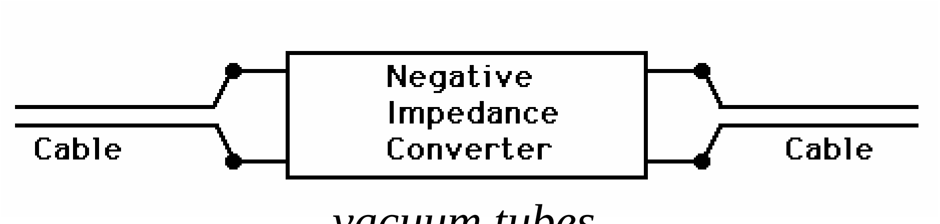
Yana OCS: $Zin_A = -(Z_d / Z_c)\, Z_b$

Yana SCS: $Zin_B = -(Z_c / Z_d)\, Z_a$

The Linvill and Yanagisawa OCS configurations are, in fact, the same circuit

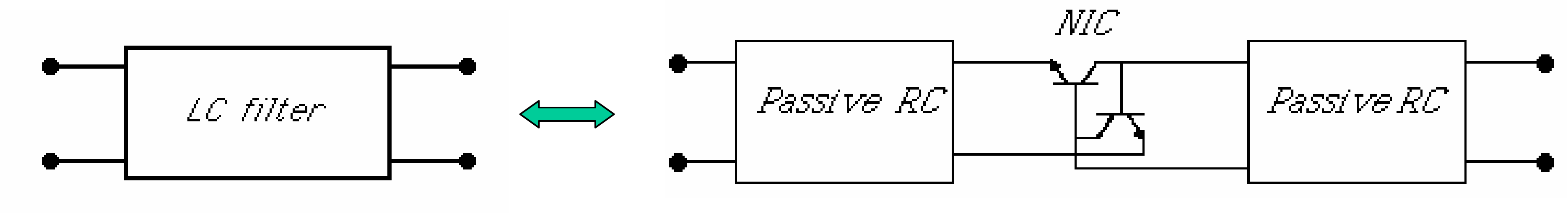
Early Use of NICs

G. Crisson (1931): Developed *negative impedance repeaters* to reduce loss on telephone lines



vacuum tubes

J. G. Linvill (1954): First active-RC filter



The transfer function of any LC filter is realizable via an *RC*-***NIC***-*RC* structure

Modern Uses of Negative Elements and NICs

- Active RC filters *(NICs, gyrators, FDNRs are fundamental elements)*

- Q-enhancement of passive resonators in active filters

- **Broadband matching of electrically-small antennas**

Stability

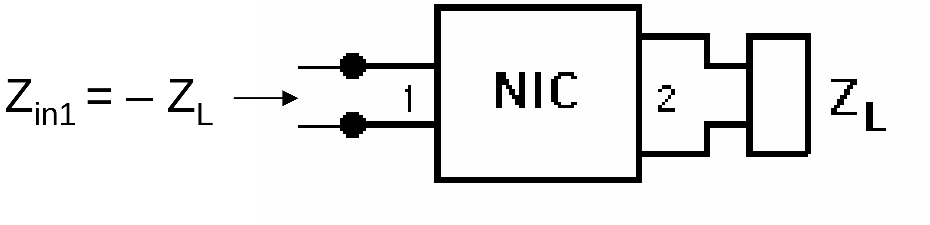
Stability of NICs - *1*

Theorem: Negative Impedance Converters are *open circuit stable* at one port and *short-circuit stable* at the other port

Brownlie, 1965; Hoskins, 1966

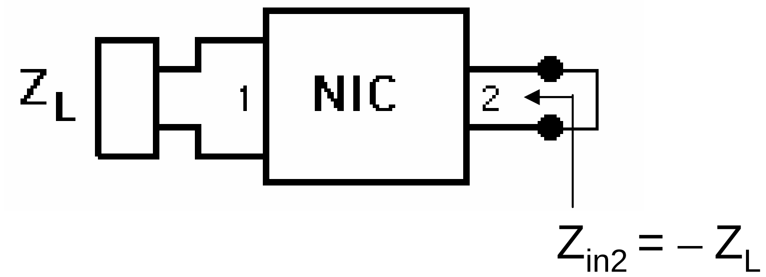
Open-circuit stable:

For any passive impedance Z_L at port 2, the network defined by *open-circuiting port 1* is stable



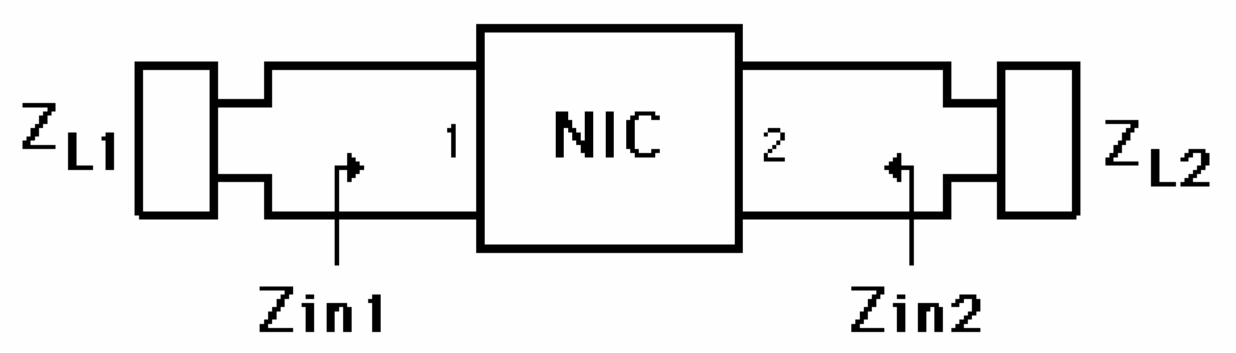
Short-circuit stable:

For any passive impedance Z_L at port 1, the network defined by *short-circuiting port 2* is stable



Stability of NICs - *2*

The inherent conditional stability of an NIC constrains the *magnitude of the impedances* that can be connected to the *open*-circuit-stable port and to the *short*-circuit-stable port



***Open-circuit*-**stable port:

requires $|Z_{L1}| \geq |Z_{in1}|$

***Short-circuit*-**stable port:

requires $|Z_{L2}| \leq |Z_{in2}|$

By what margins? – depends upon nature of Z_{L1} *and* Z_{L2}

Stability of Non-Foster Networks

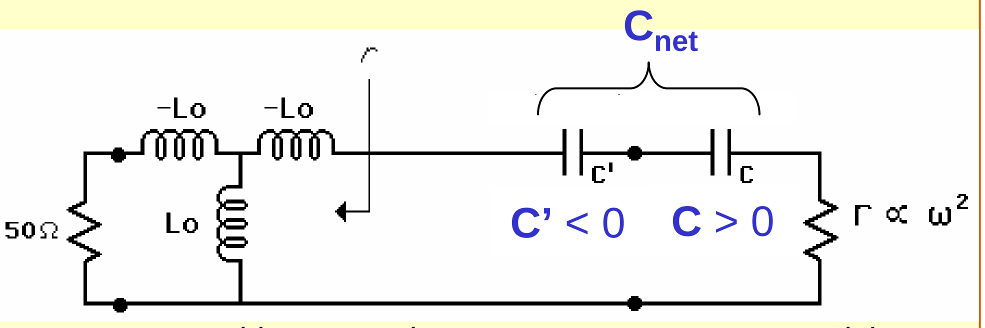
- NICs must be terminated properly for stability
- In addition, the natural frequencies of any network containing NICs must reside in left-half s-plane
- *In practice, the natural frequencies cannot be allowed to get very close to the j-axis*

For *network* stability, loop impedances must be positive

e.g. **C** is positive : **C' is** negative

but $\mathbf{C}_{\mathbf{net}}$ = (**C** *in series with* **C'**)

must be positive



Matching Network *antenna model*

Predicting Stability in NIC Circuits

- Transfer function of an NIC: $T = A / (1 + A\beta)$

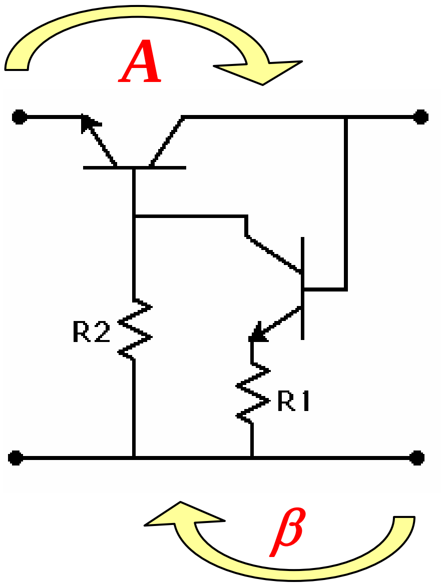
- The feedback loop in an NIC must provide *gain and/or phase margin* for stability:

  $$|A\beta| < 1 \qquad \angle A\beta < 180^{o}$$

- Middlebrook’s technique permits accurate evaluation of $A\beta$ with all loading effects

- *Idea:* break feedback loop; perform current-gain *and* voltage-gain analyses, combine results to yield $A\beta$

- With adequate component simulation ***models,*** the technique is an <u>excellent predictor of stability</u> for both the NICs and the overall network



Linvill Grounded NIC

Laboratory Measurements:

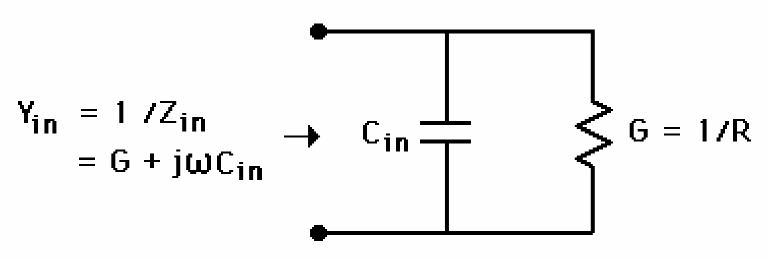
High-Q Negative Elements for Receive

Design of High-Q Negative Elements

- Historical results for negative-R and active filters: ***good***
- Results for negative L,C: ***poor*** (low-Q elements)
- **EDO has developed broadband, stabilized NICs and high-Q negative L, C elements**
- Experimental results follow for representative circuits

Grounded Negative Capacitors and Inductors

Capacitor modeled as an ideal C_{in} in parallel with a conductance G

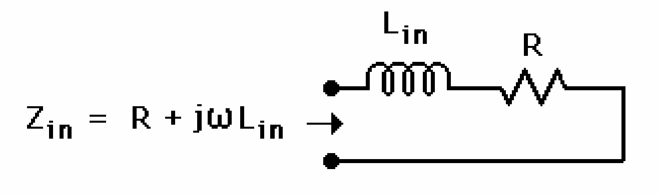


Capacitor Q: *magnitude* of $\frac{\omega C_{in}}{G}$

C_{in} is negative

G may be positive *or* negative

Inductor modeled as an ideal L_{in} in series with a resistance R

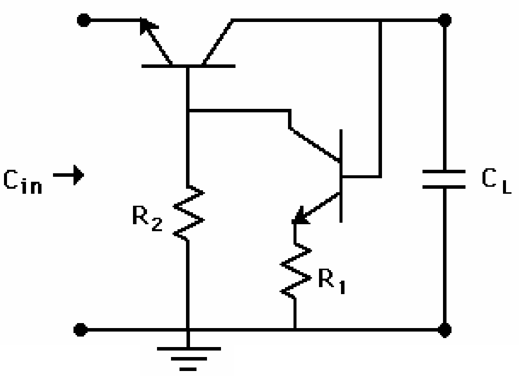


Inductor Q: *magnitude* of $\frac{\omega L_{in}}{R}$

L_{in} is negative

R may be positive *or* negative

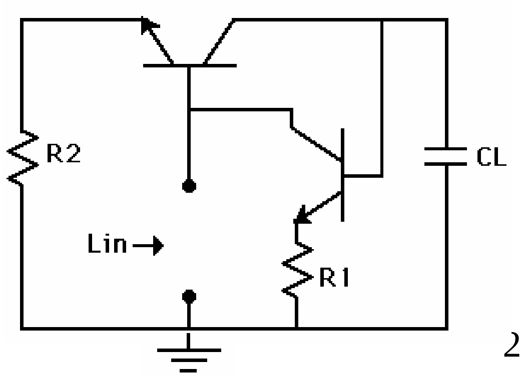
Linvill OCS Negative Capacitor



$C_{in} = -(R_1/R_2)C_L$

Linvill SCS Negative Inductor

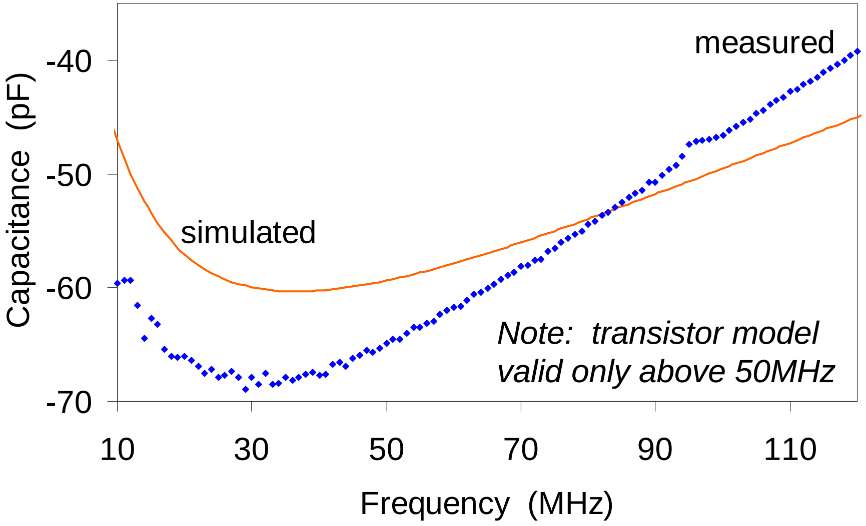
w/capacitive inversion



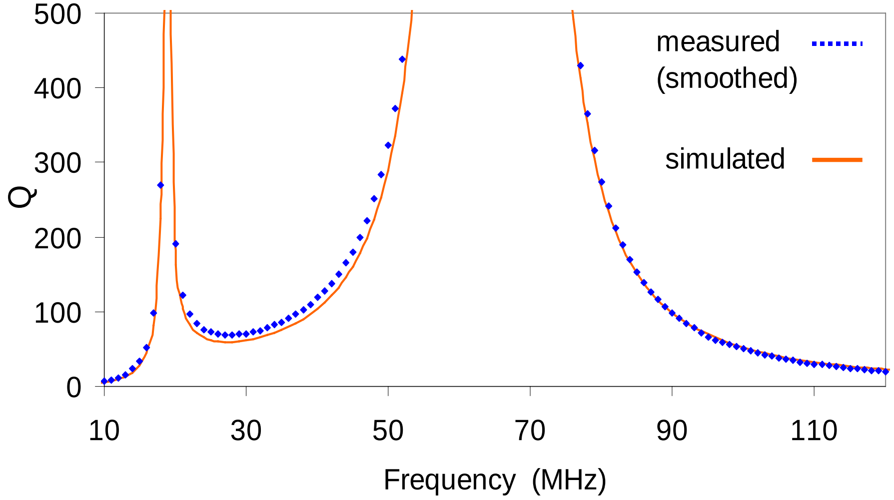
$L_{in} = -R_1R_2C_L$

Experimental Results for Negative Capacitor

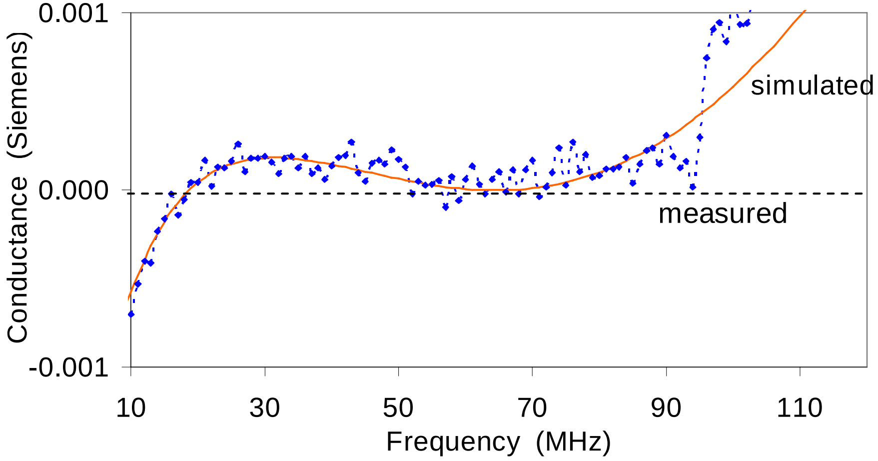
Capacitance C_{in}



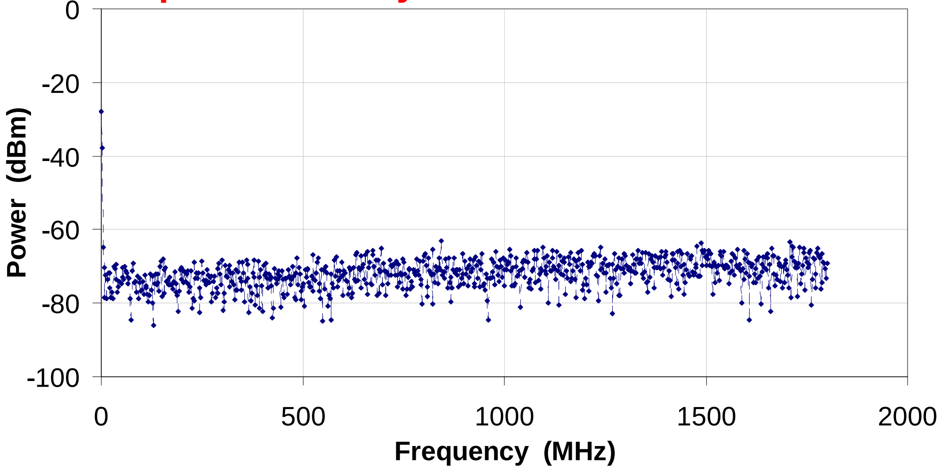
Q



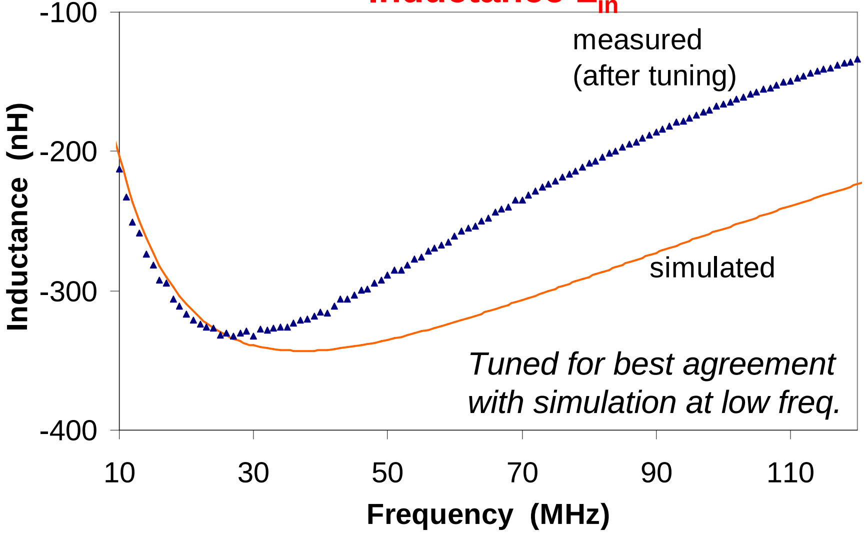
Conductance G



Spectrum Analyzer Meas. of Noise Power

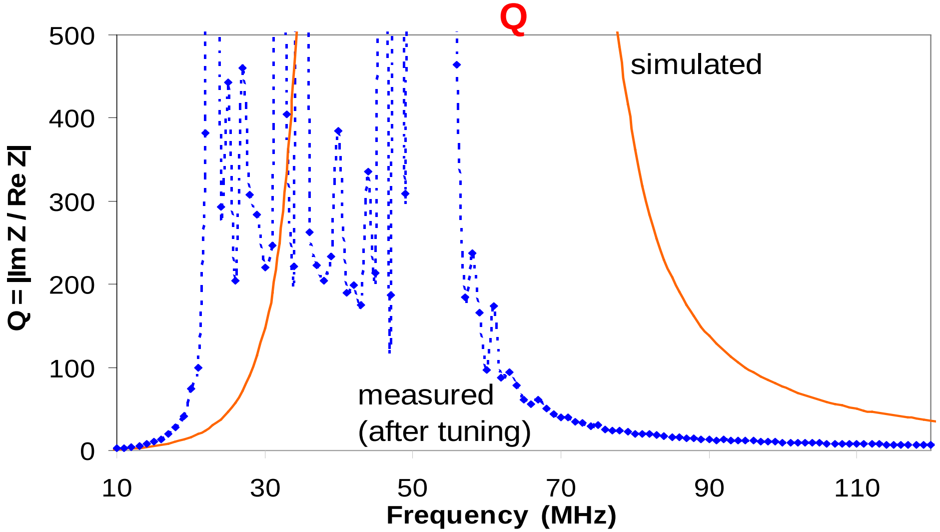


Experimental Results for Negative Inductor

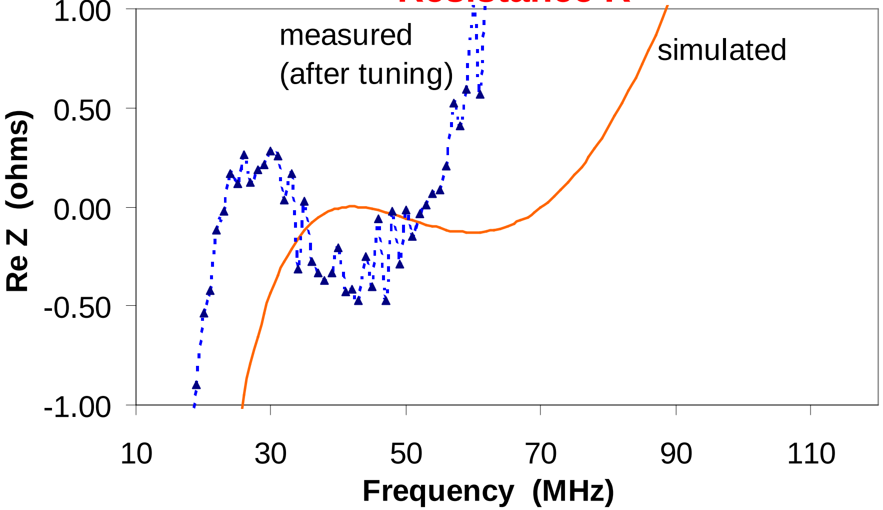


Inductance L_{in}

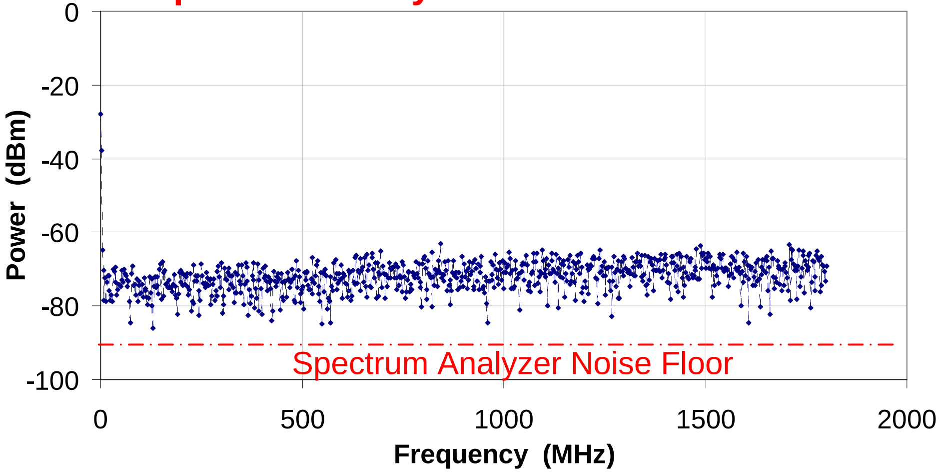
Tuned for best agreement with simulation at low freq.



Q

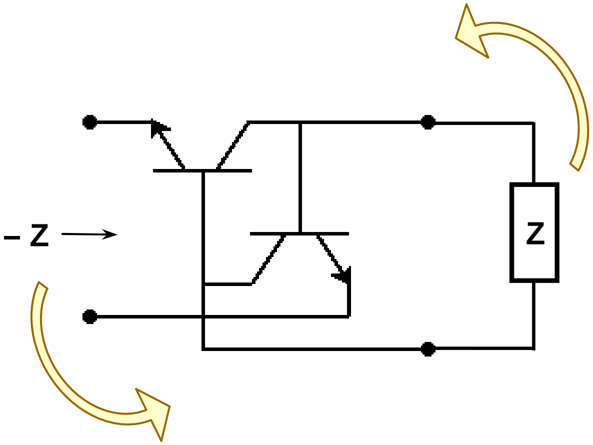


Resistance R

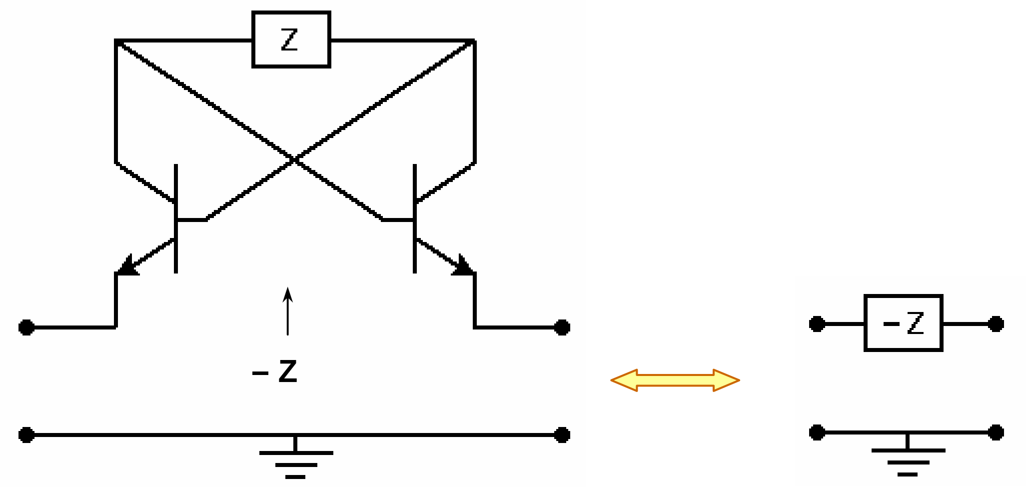


Spectrum Analyzer Meas. of Noise Power

Floating Negative Elements



Linvill Floating *Negative Impedance Converter* Terminated in Z



Linvill Floating NIC Terminated in Z used as *Series Negative Element*

Series negative *capacitor* used in impedance matching of electrically-short monopole and dipole

Laboratory Measurements: *Non-Foster Monopole and Dipole*[*]

[*] work performed for US Army I2WD (CECOM)

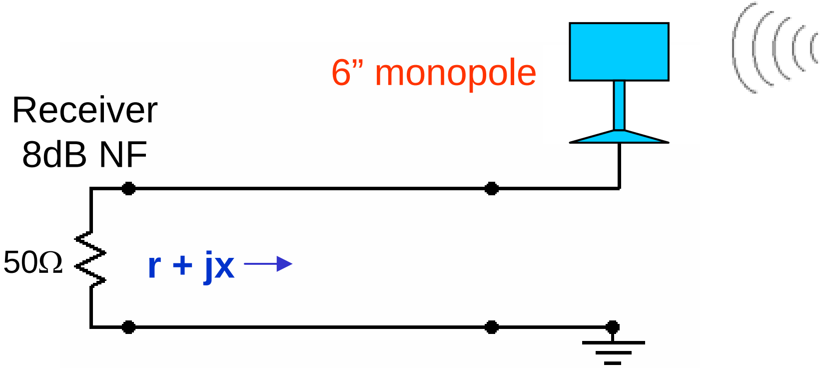
Monopole – Experimental Results

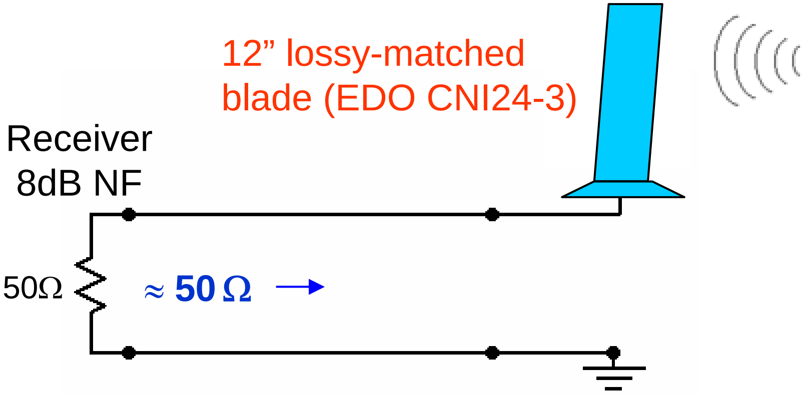
Experimental demonstration of partial non-Foster impedance matching with a *monopole* antenna:

- Negative capacitor cancels the reactance of a electrically-short 6” monopole (partial matching)
- Measured improvement in signal-to-noise ratio with non-Foster-matched electrically-short monopole: up to **9 dB** at 30 MHz (as compared to lossy-matched blade antenna of *twice* the size; receiver NF 8 dB)

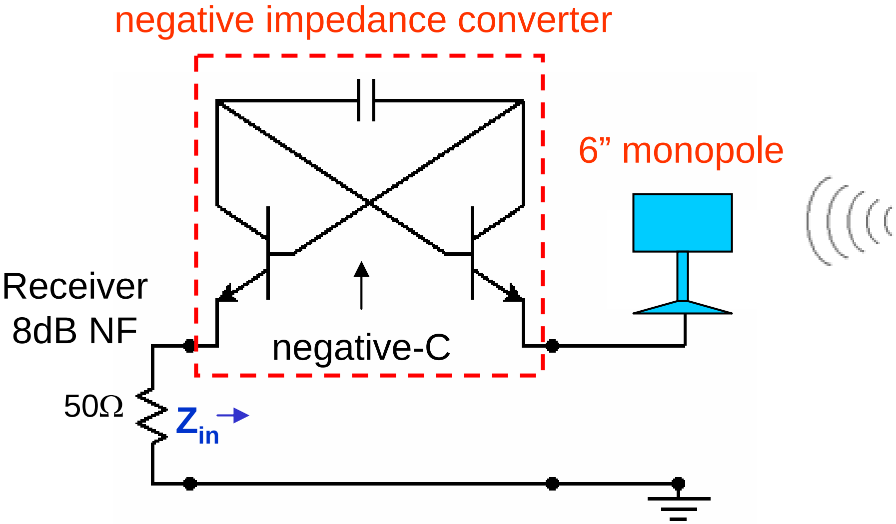
Measurement of Signal-to-Noise Ratio on the Antenna Range

6" and lossy 12" reference antennas: behaved almost identically





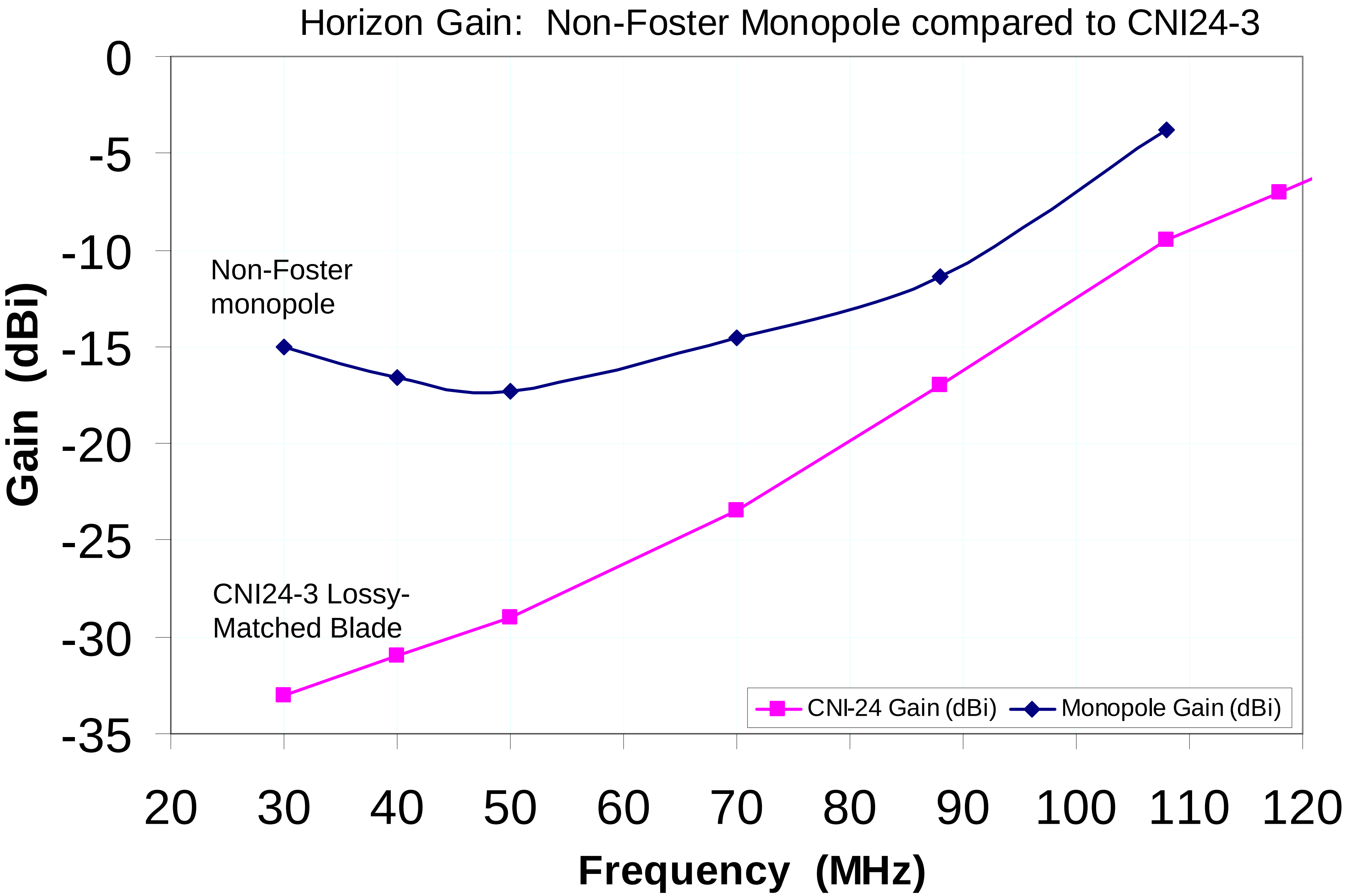
6" monopole with non-Foster matching improves signal-to-noise ratio



Z_{in} = r + jx'

|x'| << |x|

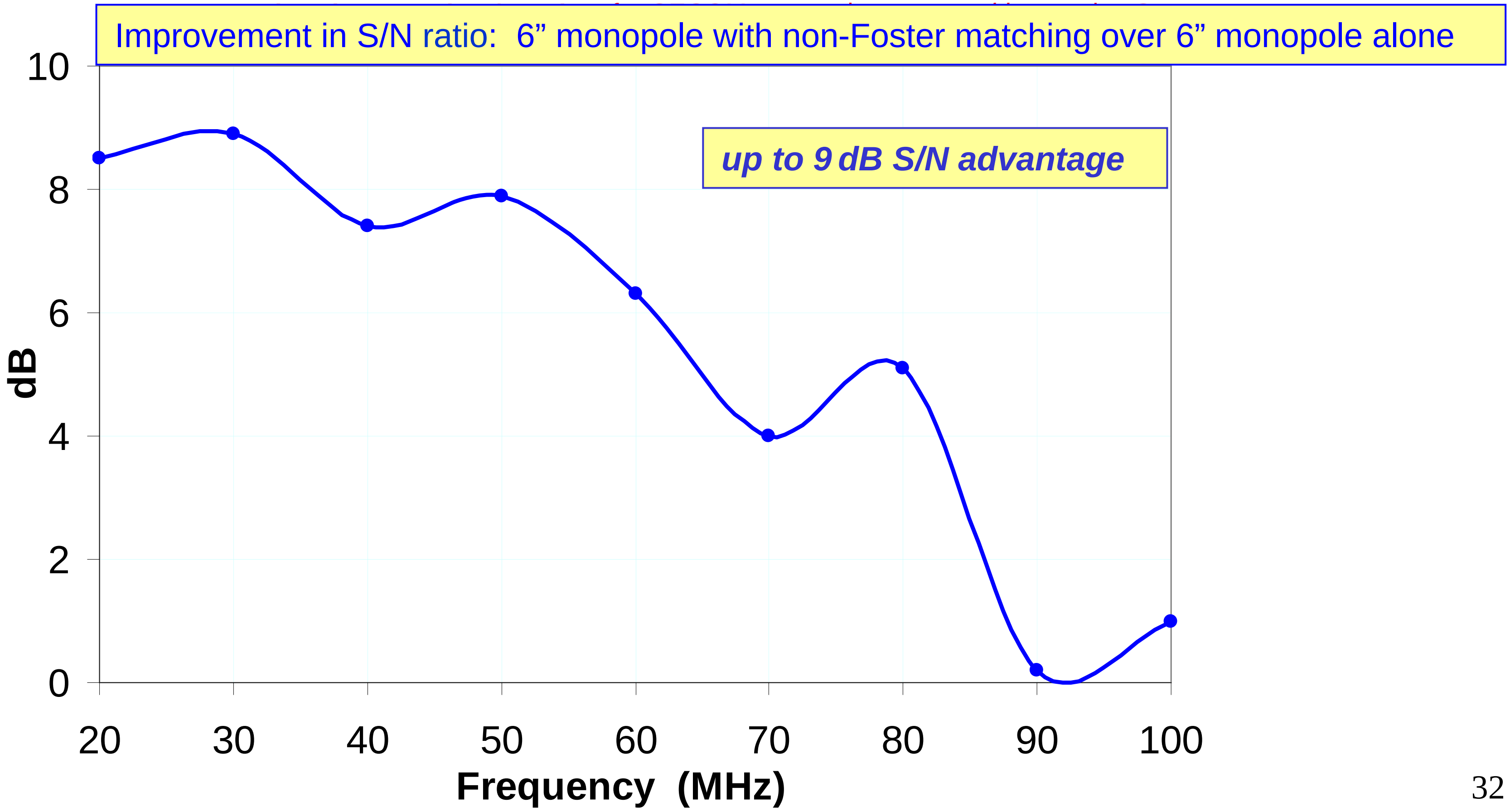
Measured Improvement in Horizon Gain



Measured Improvement in Signal-to-Noise Ratio

Low noise receiver: 8 dB noise figure

Measurements taken at discrete10 MHz intervals; Excel curve-fit produces plot



Improvement in S/N ratio: 6” monopole with non-Foster matching over 6” monopole alone

up to 9 dB S/N advantage

Dipole – Experimental Results

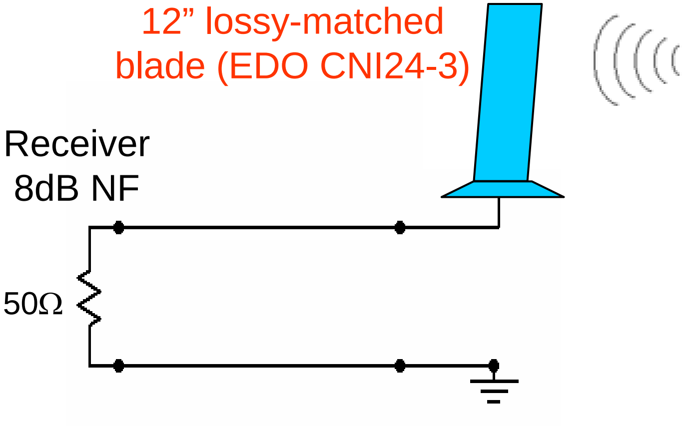
Experimental demonstration of partial non-Foster impedance matching with a *dipole* antenna:

- Negative capacitor cancels the large portion of reactance of an electrically-short dipole, 12” total length
- Measured improvement in signal-to-noise ratio with non-Foster-matched electrically-short dipole: up to **20 dB** (see graphs) as compared to 12” lossy-matched blade monopole antenna

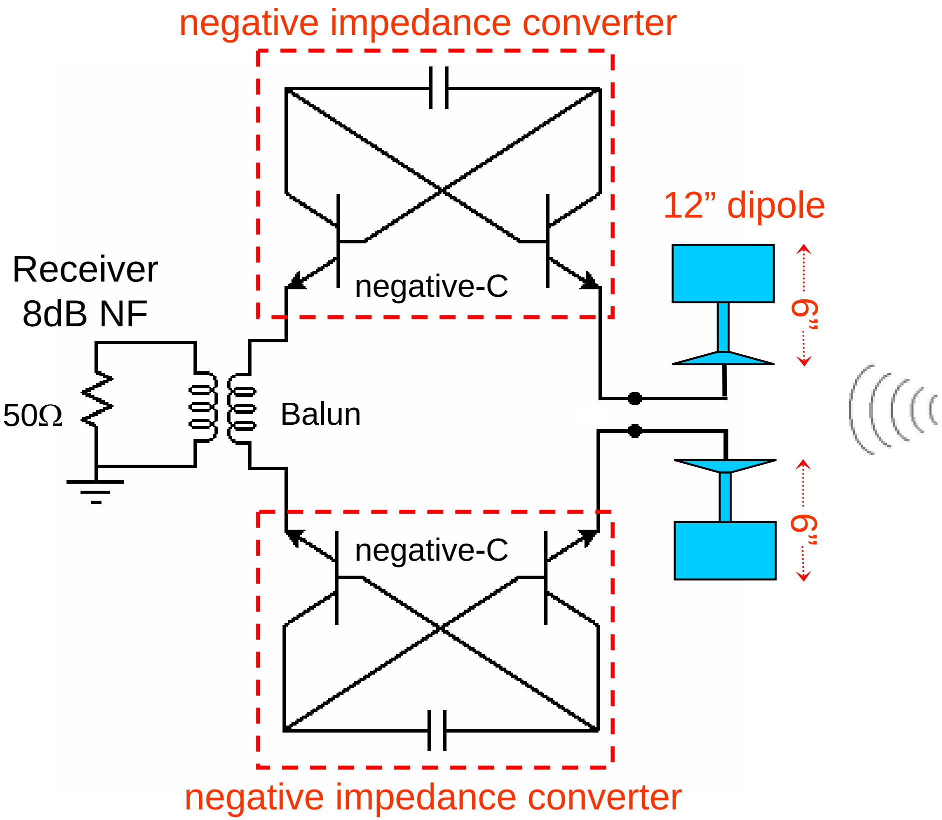
Measurement of Signal-to-Noise Ratio on the Antenna Range

12” monopole reference antenna

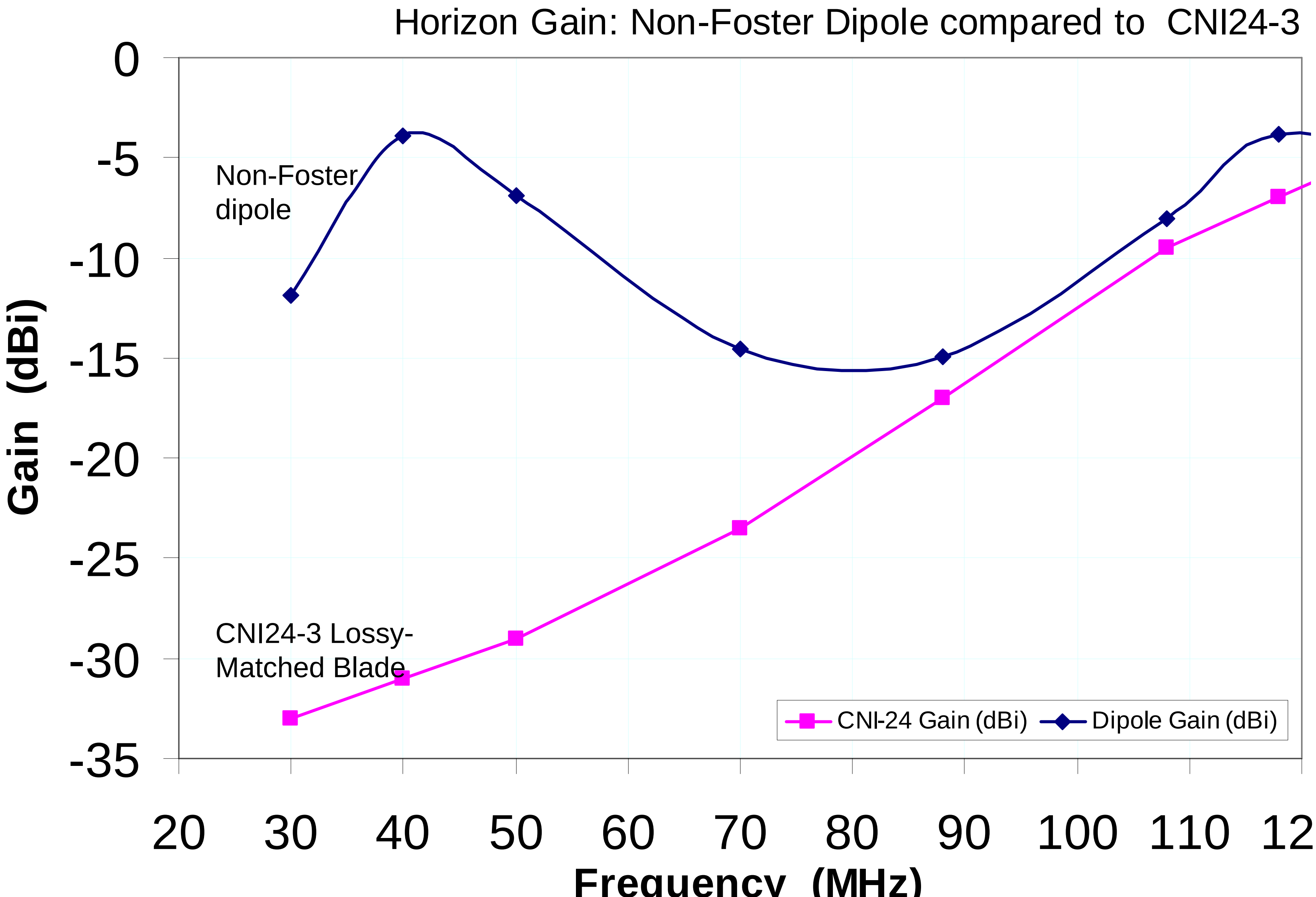
***12” dipole antenna** (6” per arm)*
with non-Foster matching improves signal-to-noise ratio



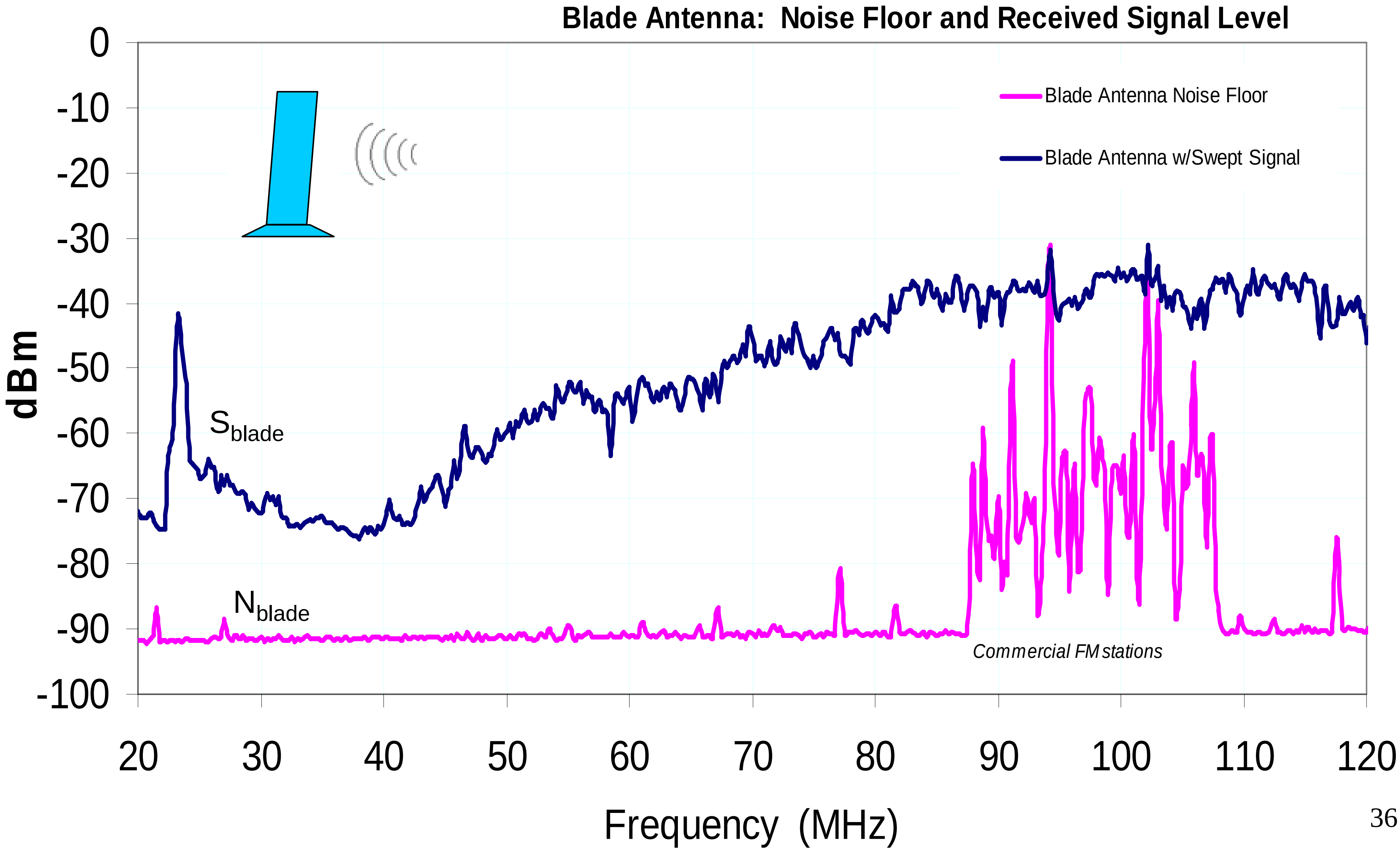
Swept-frequency measurements



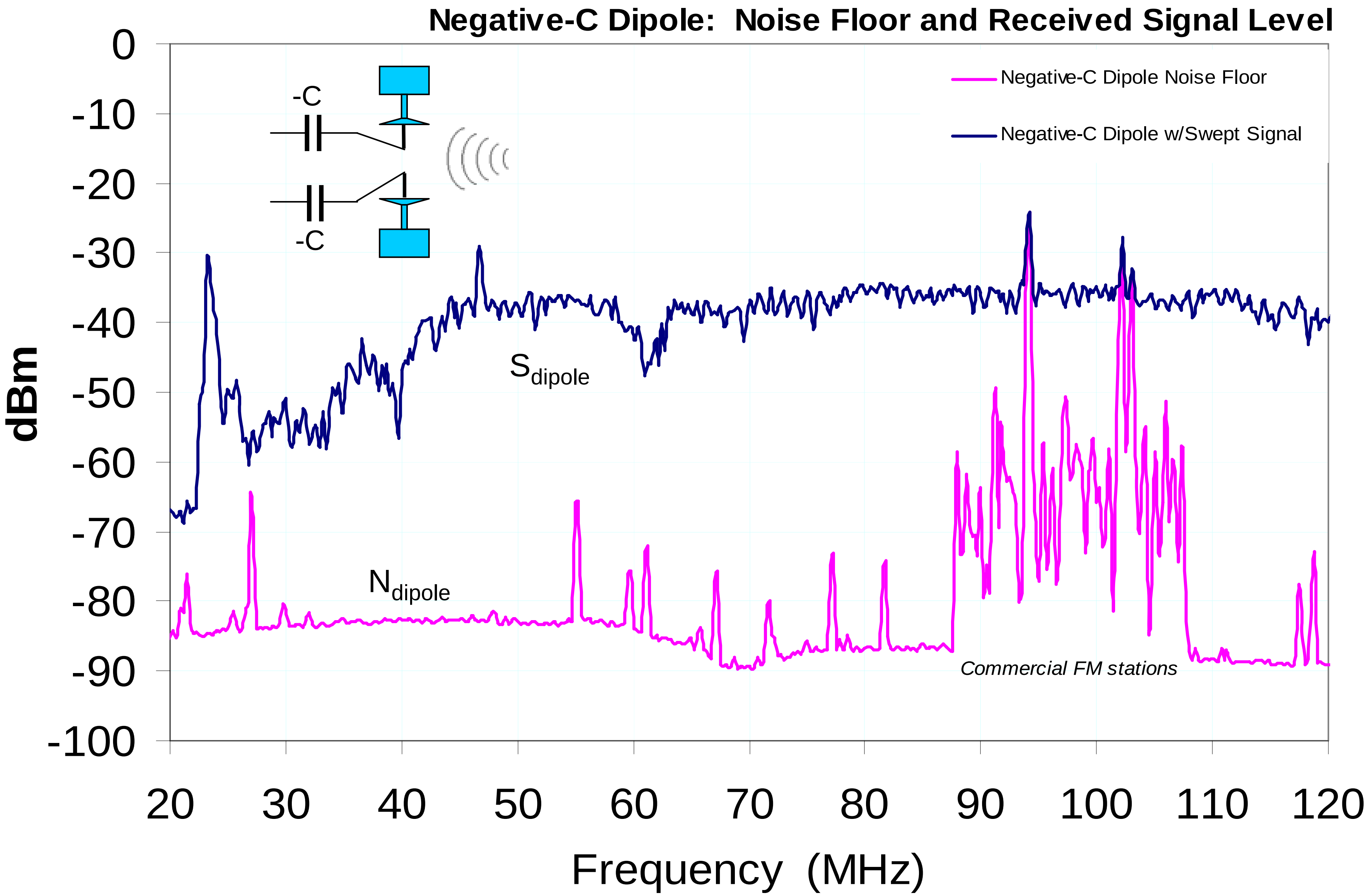
Measured Improvement in Horizon Gain



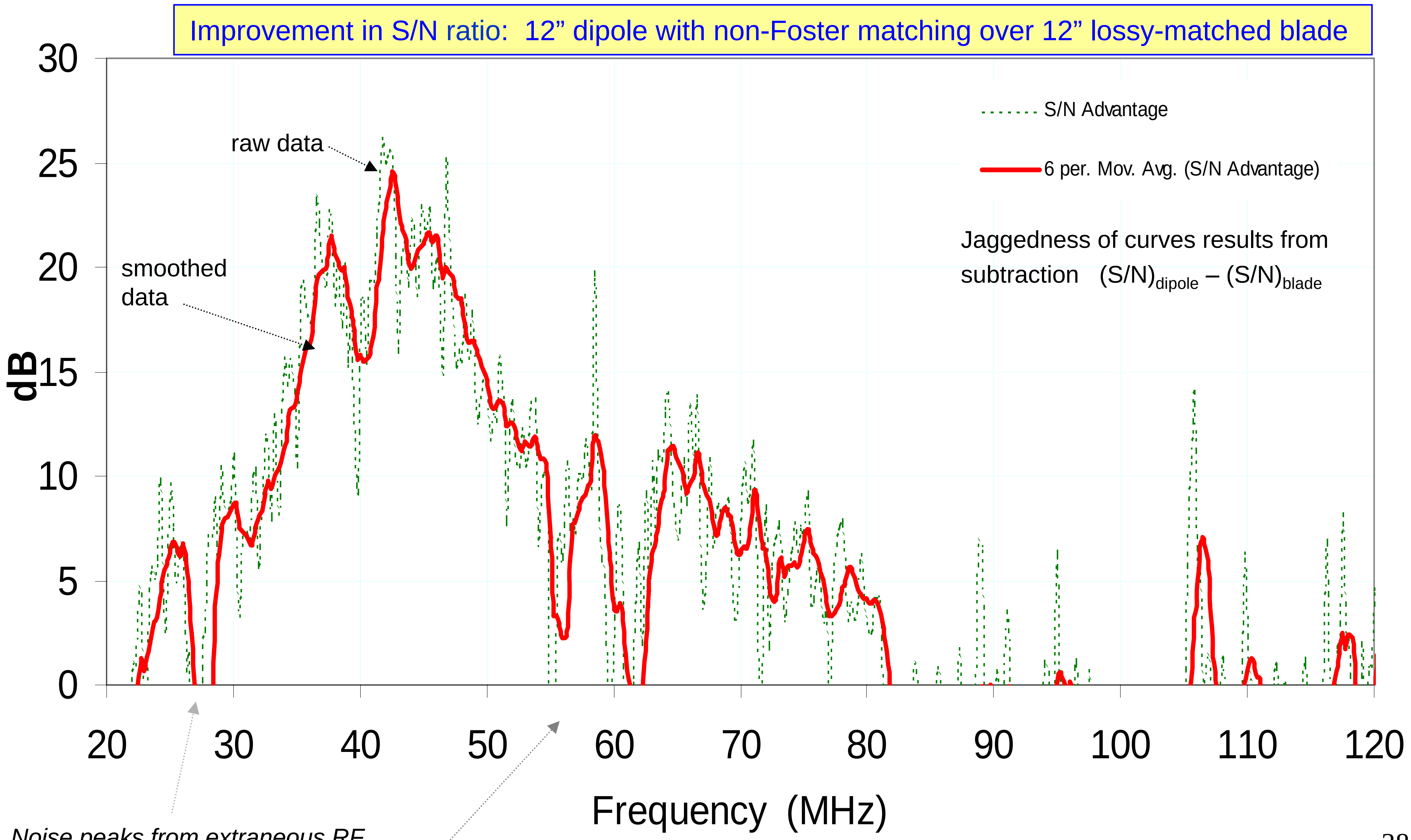
Blade Antenna: Noise Floor and Received Signal



Negative-C Dipole: Noise Floor and Received Signal



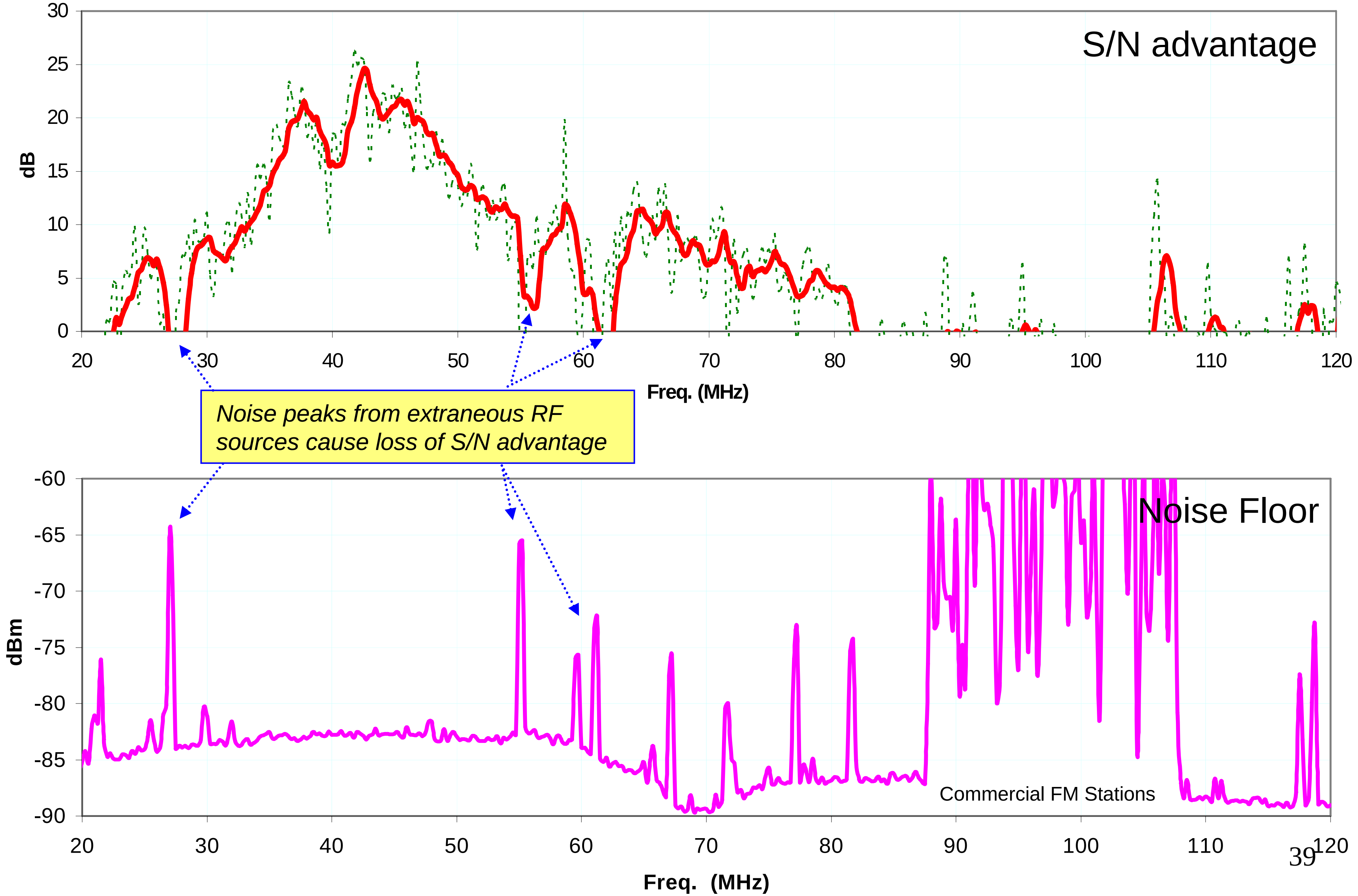
Signal-to-Noise Advantage, 12” Negative-C Dipole over 12” Lossy-Matched Blade Monopole Antenna



Jaggedness of curves results from subtraction $(S/N)_{dipole} - (S/N)_{blade}$

Noise peaks from extraneous RF sources cause loss of S/N advantage

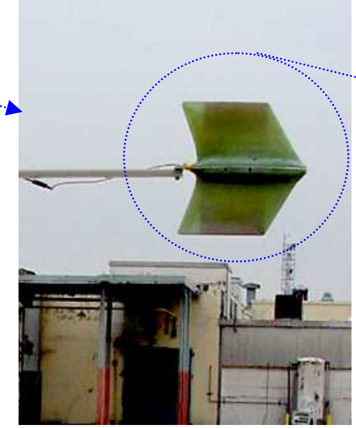
No S/N Advantage When External Noise Dominates



Noise peaks from extraneous RF sources cause loss of S/N advantage

CECOM Dipole on Test Range

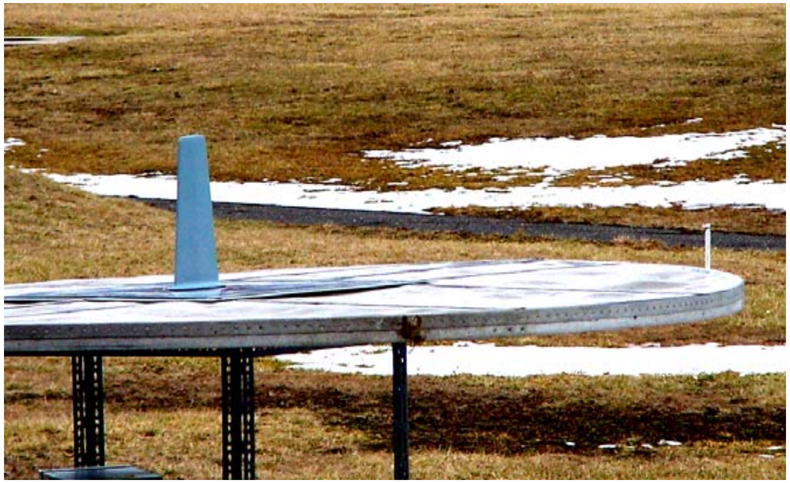
Side view



Head-on view

CECOM Dipole Compared to CNI-24 Monopole Blade on 8 ft. Ground Plane

Blade on ground plane



Technology Development

Technology Development

1. Investigate using additional negative elements in a non-Foster matching circuits
2. Determine the optimal tradeoff among the design parameters to obtain the largest improvement in signal-to-noise ratio over the broadest bandwidth
3. Develop additional types of negative circuit elements, especially negative inductors for electrically-small loop and flush cavity antennas
4. Acquire or develop accurate device models to design low-noise FET NICs
5. Auto-tuning / self-adjusting circuitry
6. Investigate alternative matching network topologies – *NIC bracketing* vs. *individual negation of elements*
7. Transmit applications – a special problem