Modeling Parallel Amplifiers in a System Chain Analysis

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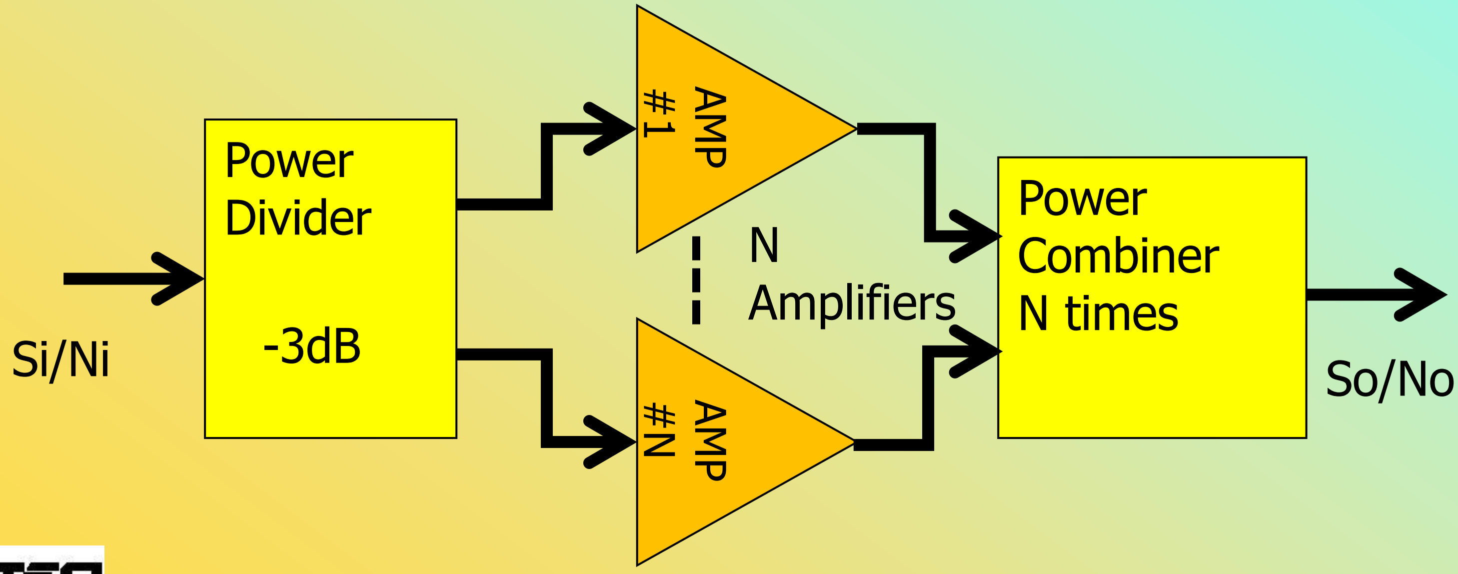
Topics

- Introduction
- Analyzing a Systems Dynamic Range
- Factors in Determining Dynamic
  - Noise Characteristics
  - Noise Figure
  - Harmonics and Intermodulation Interference
- Microwave Dividers & Combiners
- Parallel Amplifiers Combining Losses
- Noise Figure of Parallel Amplifiers
- Output Power & Intermodulation Distortion Improvement
- Conclusion -

Introduction

Parallel Amplifier Technology

- System designs often that require output powers above the capability of single devices
- Can be realized by paralleling multiple devices
    - Combining in the power amplifier outputs gives theoretically “N” times the power output
        - N is the number devices in parallel -



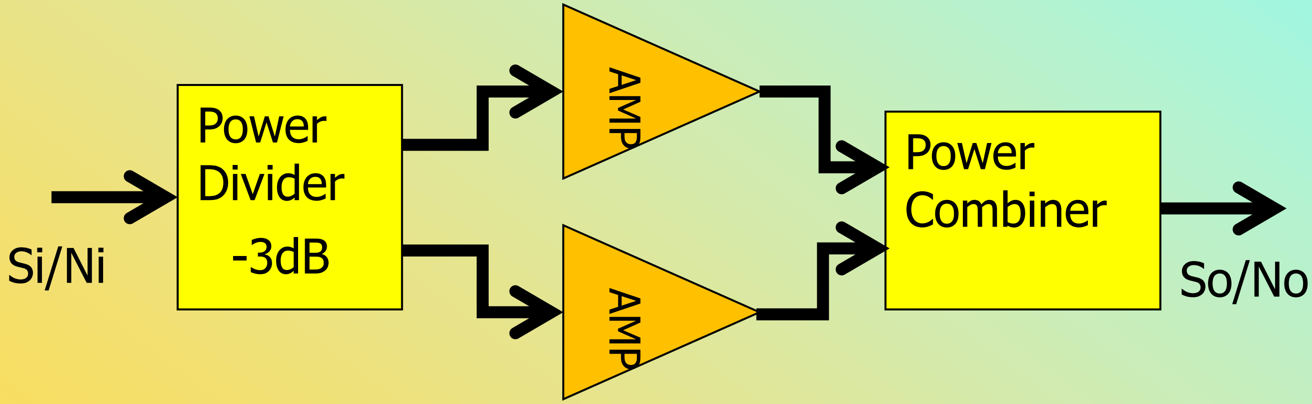
Advantages & Disadvantages

Advantages of using a Parallel Amplifier Topology

- Improving Dynamic Range (Output Power)
- Alternate configurations provide for
  - Input & Output Matching
  - Reflected power directed into a load
    - Prevents device destruction
  - Harmonic Rejection

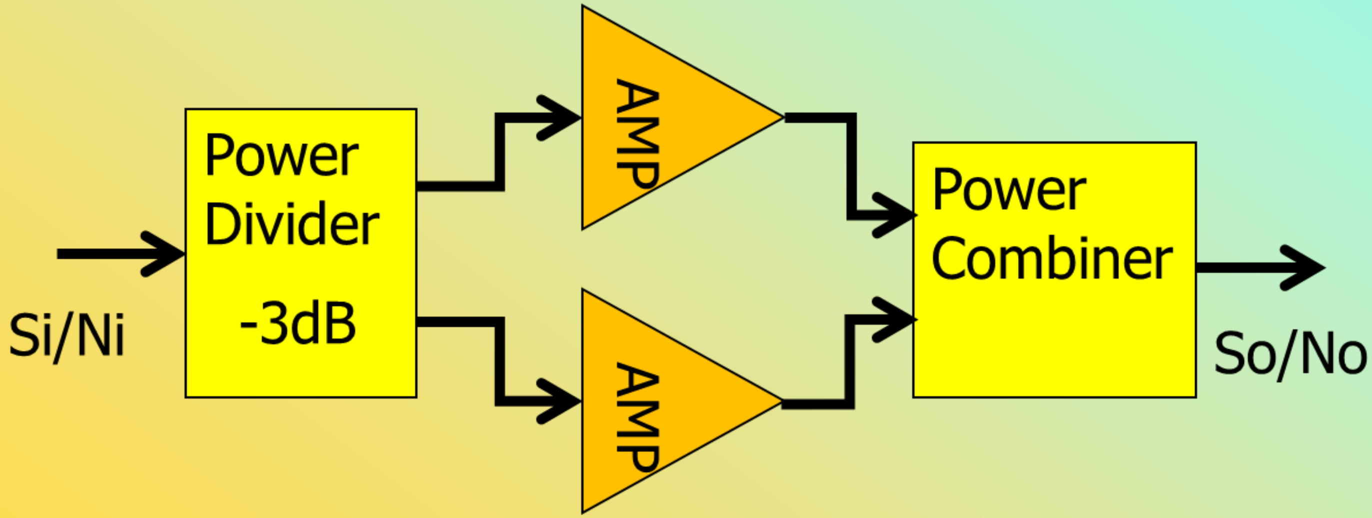
Disadvantages of using a Parallel Amplifier Topology

- Cost
- Complexity
- DC Power
- Size -

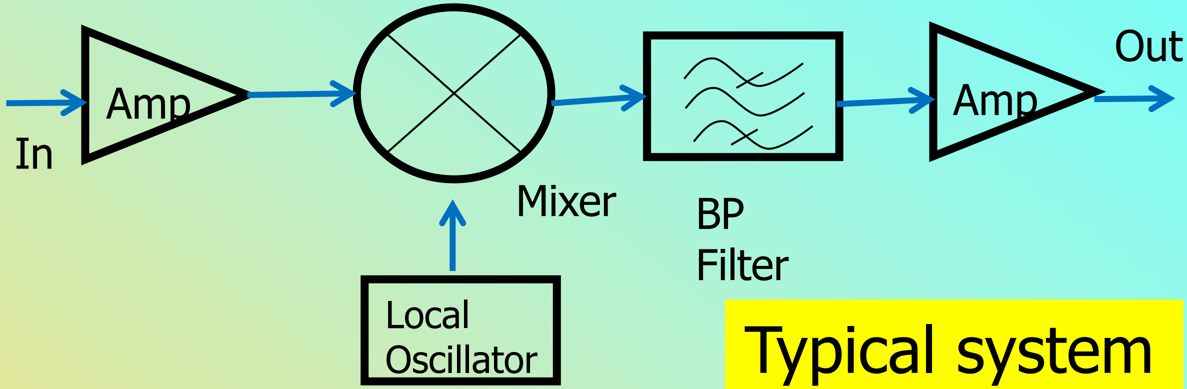


Objective

- Simulation programs seem to have a problem handling parallel amplifier topologies (NF & IP3)
- Discuss alternate methods of modeling this topology
- Find a model that can be successfully inserted into a systems dynamic range analysis -



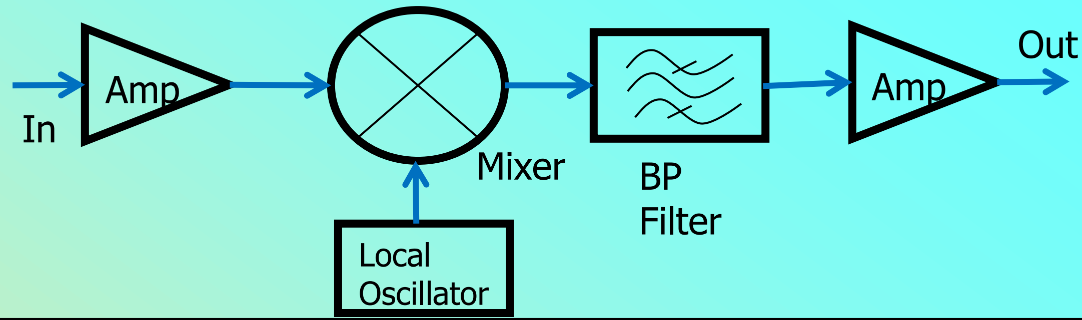
Analyzing a Systems Dynamic Range



Typical system

- Consider the effect of all the element characteristics on the complete system performance
- Example of system dynamic range requirements:
- Specification of 3rd Order Intermodulation Distortion (IM3)
  - -30 dBc (IM3) for two carriers each at an out of +37dBm
  - ➔ Output 3rd Order intermodulation Intercept Point (OIP3) ≥+52dBm
- Noise Figure (NF): 10 dB maximum
- Each system component adds to the Intermodulation Distortion & NF
  - System design requires a chain analysis
  - Determine each elements contribution to Distortion & NF -

Chain Analysis



Device Characteristics

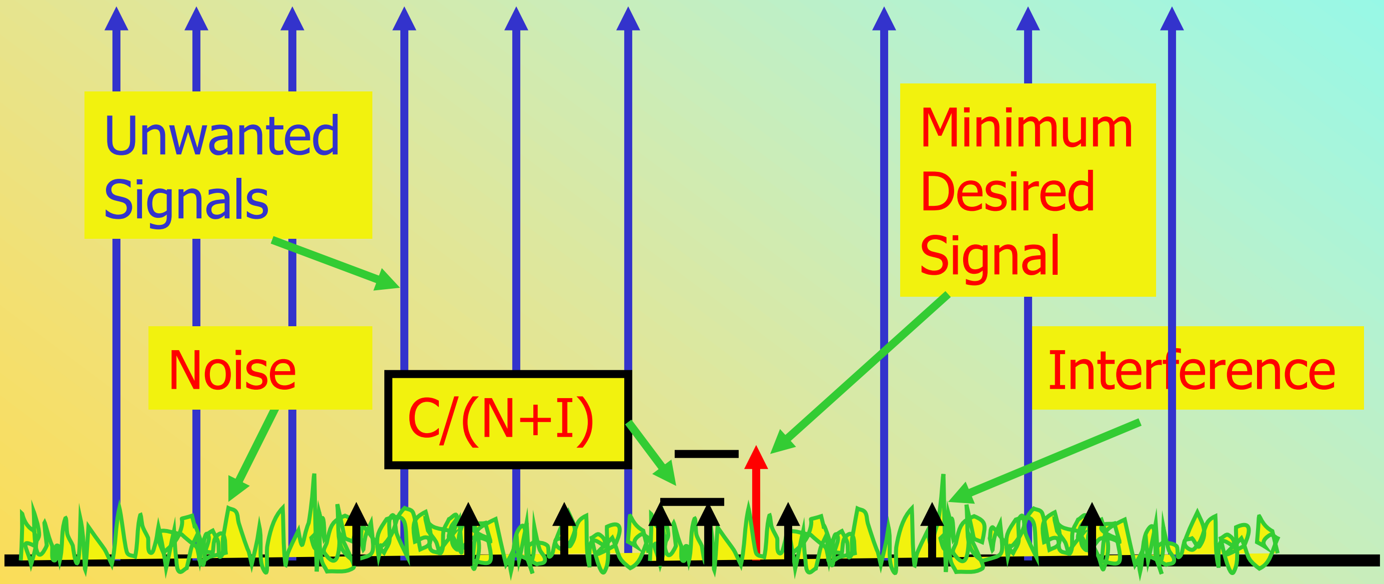
Spec				37	35	10	52		-30
				37.0	35.5	8.6	52.5	17.04	-31.1
DESCRIPTION	GAIN dB	dB NOISE FIGURE dB	Total IN-3rd dBm OUT	Carrier Power dBm	CUM GAIN dB	CUM NF dB	OIP3 3rd dBm	IIP3 3rd dBm	Intermod 2 Carriers dBc
INPUT LEVEL			100	1.5	0.0	0.00	100.0	100.0	-197.0
	0	0	100	1.5	0.0	0.00	97.0	97.0	-191.0
Amplifier	10	6.5	40	11.5	10.0	6.50	40.0	30.0	-57.0
Mixer	-9	9	23	2.5	1.0	7.13	22.4	21.4	-39.7
Filter	-0.5	0.5	100	2.0	0.5	7.21	21.9	21.4	-39.7
Amplifier	15	5	40	17.0	15.5	8.56	35.1	19.6	-36.3
Power Amplifier	20	5	56	37.0	35.5	8.60	52.5	17.0	-31.1

Factors in Determining Dynamic Range

- Dynamic range is defined by system requirements

Minimum Signal

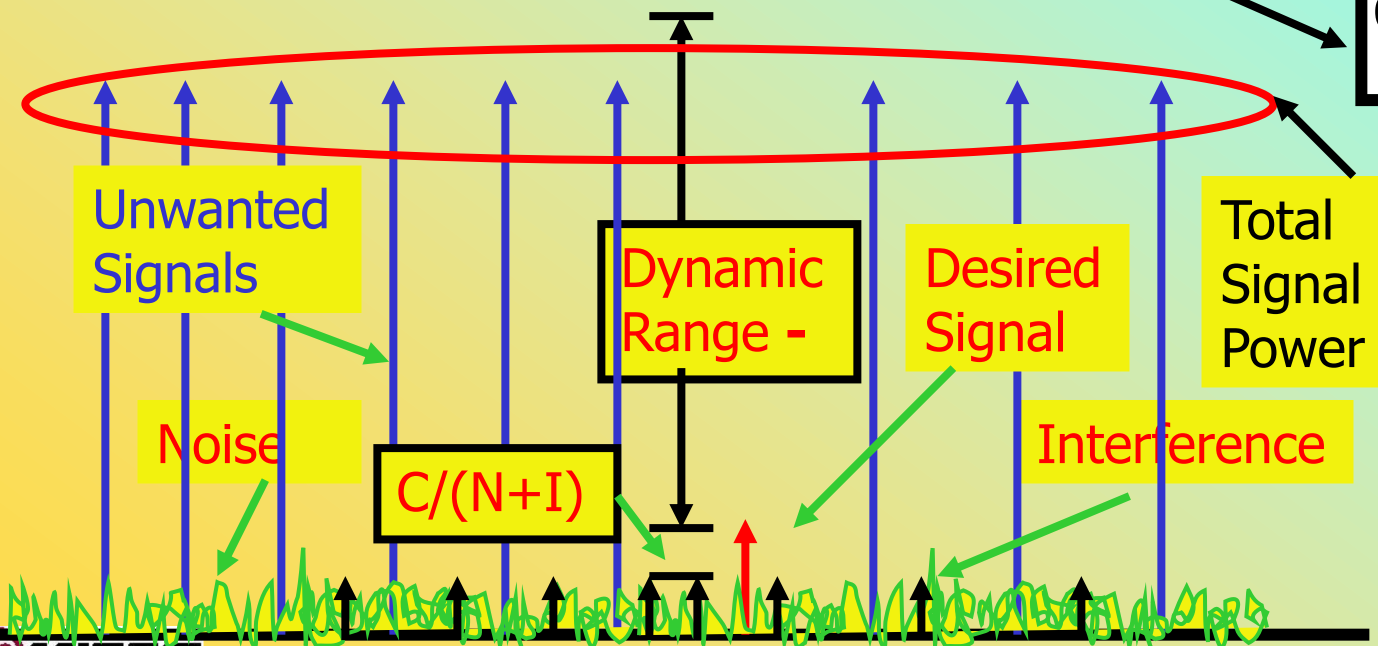
- Smallest Carrier (signal) level with respect to noise & interference (C/(N+I) that can be successfully recovered
- Limited by noise + interference



Maximum Signal – CW & Multiple Carrier Interference

- Single CW signal: Minimal harmonics
- Multiple signals: 3rd Order Intermodulation products are in the noise
- 3rd Order Intermodulation Interference (I3rd)
  - 2 equal carrier: I3rd (dBc) = -2(IP3-Carrier)
  - Carrier Triple Beats add +6dB / Multiple carriers

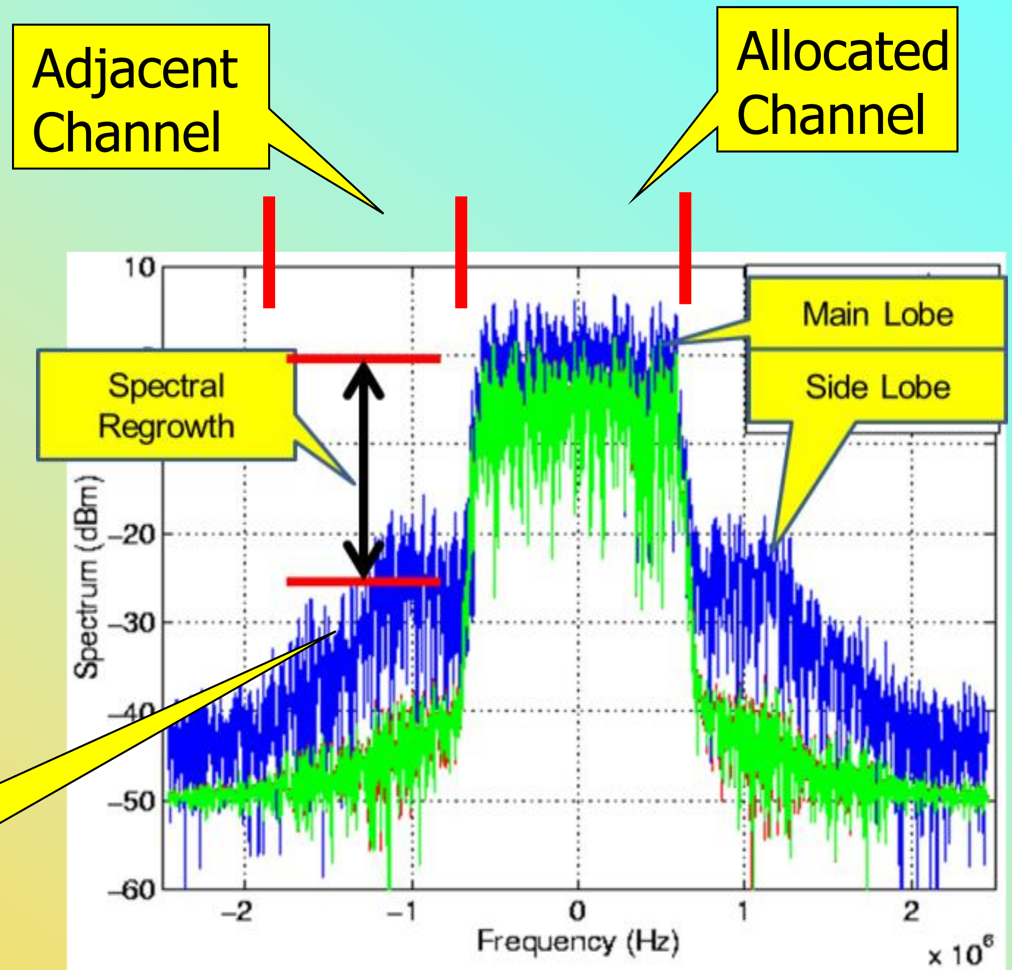
$$6 + 10 \cdot \log\left[\frac{3(N)^2}{8}\right]$$



Maximum Signal for a Modulated Single Carrier

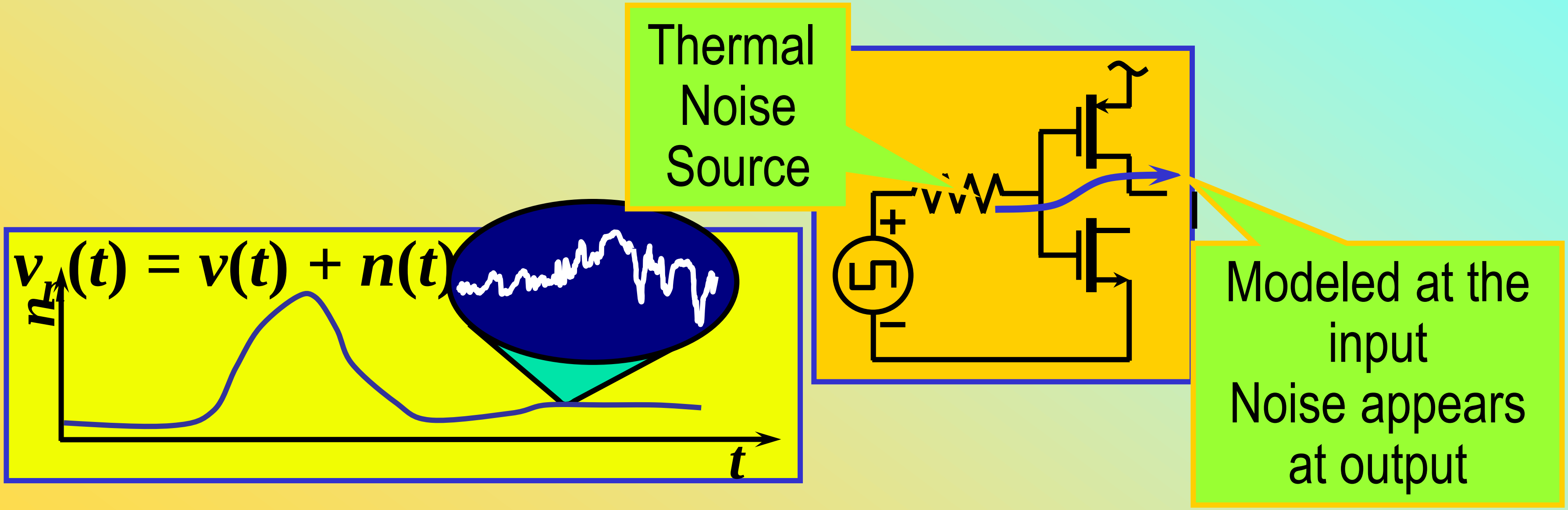
Spectral Regrowth – A Single Carrier Issue

- Modulated signal passing through a non-linear device (power amplifier)
- Green is the initial modulated signal
  - Sidebands are filtered and suppressed in the modulator
- In band modulation components mix & create in band and out of band spurious
- In band interference distorts the signal
- Out of band appears like side bands regrown (Blue Response
  - interferes with adjacent channels -



Noise Characteristics

- Noise is a Stationary Gaussian process characterized by its power spectrum
  - Thermal noise - constant amplitude over all frequencies
  - Flicker noise (1/f) - amplitude inversely proportional to frequency



Thermal Noise

- Thermal Noise is the random motion of electrons
- Noise level is unknown at any instant of time
- Precisely known over any long time period
  - Time >> 1/BW

$$V_n = \sqrt{\frac{4hfBR}{e^{\frac{hf}{kT}} - 1}}$$

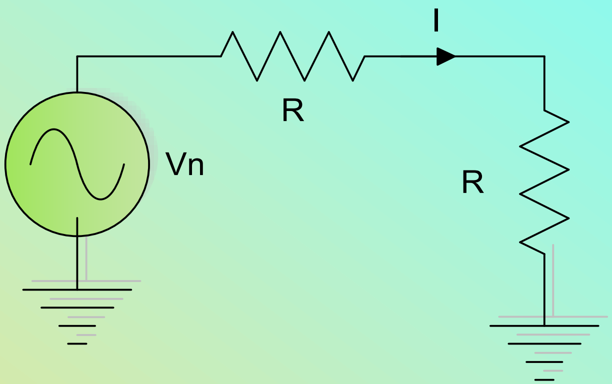
$$hf << kT$$

$$e^{\frac{hf}{kT}} - 1 \approx \frac{hf}{kT}$$

$$V_n = \sqrt{4kTBR}$$

$$I_n = \frac{Vn}{2R}$$

$$P_n = I^2 R = \left(\frac{V_n}{2R}\right)^2 R = \frac{(V_n)^2}{4R} = kTB$$



Thermal Noise Level

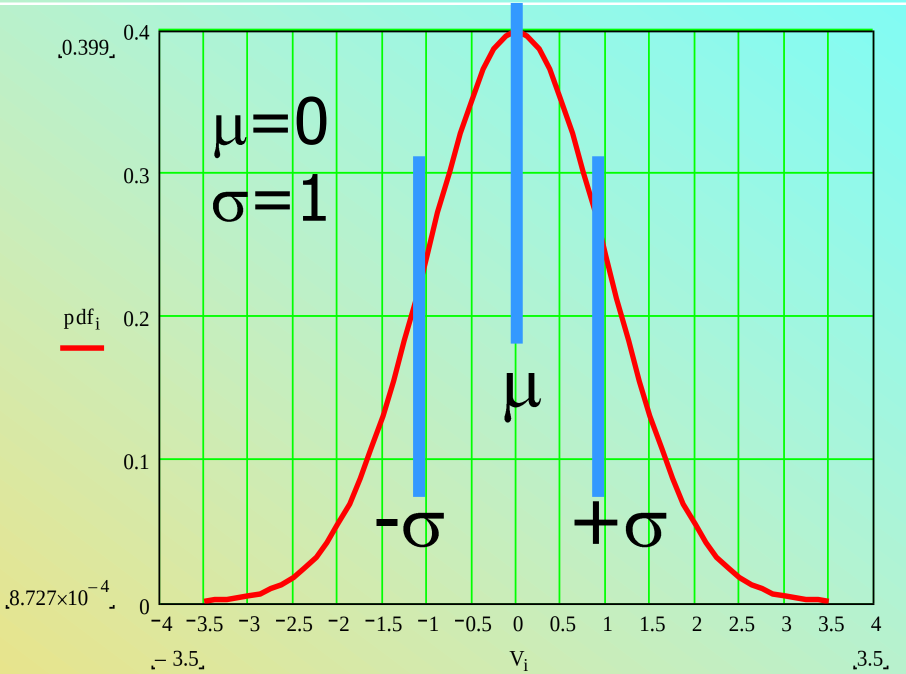
- Noise Power = KTB (Watts), Where,
- k= (-228.6 dB/ºK/Hz) [Boltzman's Constant]
- T= Equivalent Noise Temperature (ºK)
- B = Noise Bandwidth of a receiver
- At Room temperature 25°C → 298°K
- kTB = 4.11 x 10–18 milliWatts in a 1 Hertz Bandwidth → –173.859dBm/Hz (≈ -174dBm/Hz)

  → –203.859dBW/Hz (≈ -204dBW/Hz)

Noise and Error Probability (1 σ ← → RMS)

- Noise: Gaussian Function
- Well defined amplitude probability distribution function (pdf)
- Probability Density Function
  - μ is Average (Mean)
  - σ=standard deviation: Relates to the function spreading
  - σ ← → RMS noise
- Thermal Noise = kTB = -174dBm/Hz at 298 K = 1 standard deviation (1 σ)

$$pdf := \frac{1}{\sqrt{2\cdot\pi\cdot\sigma^2}}\cdot e^{\frac{-(V-\mu)^2}{2\cdot\sigma^2}}$$



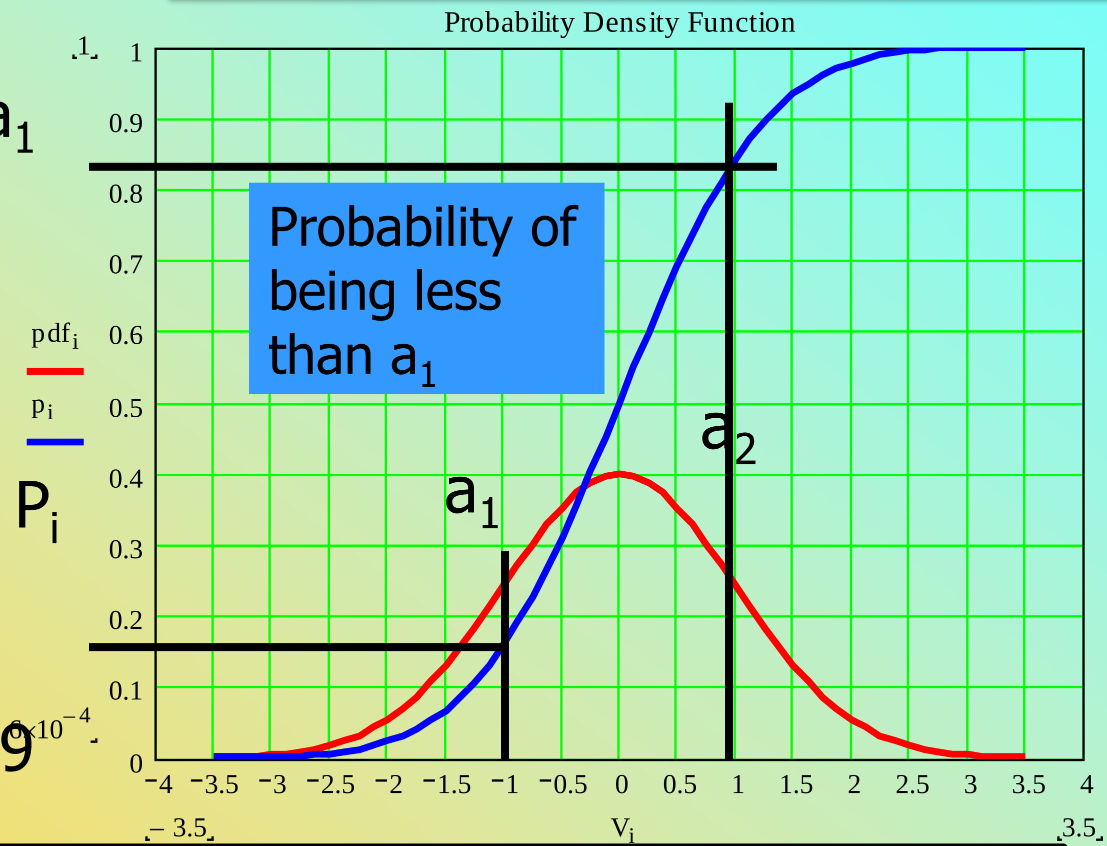
Integrating the Probability Distribution Function

$$p_i := \frac{1}{\sqrt{2\cdot\pi\cdot\sigma^2}}\cdot\int_{-\infty}^{a_i} e^{\frac{-(V-\mu)^2}{2\cdot\sigma^2}}\,dV$$

Probability of being in the range -∞ to a_1

- P_i = Area under the pdf curve from -∞ to a_1
- Probability of being between a1 & a2 is P(a2) – P(a1)
- V = 0 (Mean), probability = .5
- $P(V<-1\sigma)=.159$
- $P(V<+1\sigma)=.841$
- $P(V,>1\sigma)=1-.841=.159$

$P(-1\sigma <V<+1\sigma) =.682$

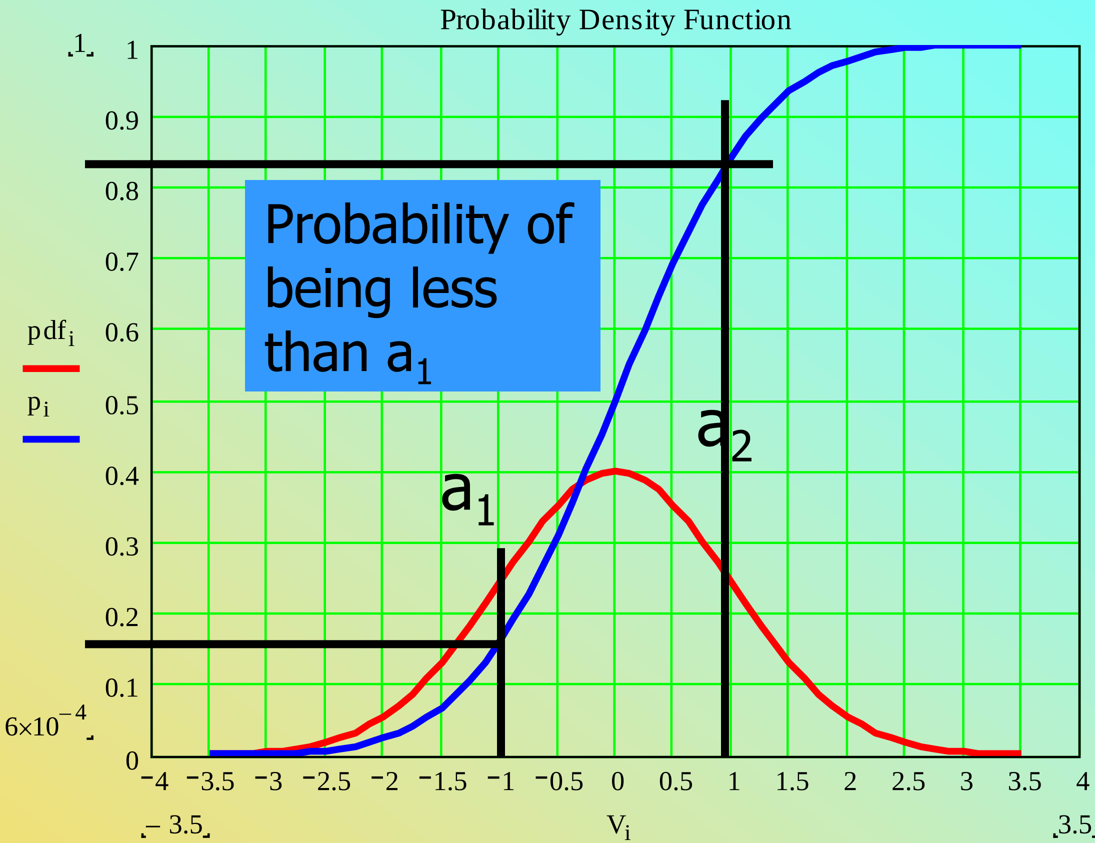


Noise at the RMS or less 68.2% of the time

Probability, Standard Deviation & RMS Noise

- $P(>|1\sigma|) = .318$
- $P(>|2\sigma|) = .046$
- $P(>|3\sigma|) = 2.7\times10^{-3}$
- $P(>|4\sigma|) = 6.3\times10^{-5}$
- $P(>|5\sigma|) = 5.7\times10^{-7}$
- Setting a noise threshold > $|5\sigma|$ from the mean has a statistical error rate of 0.57 errors per million bits of information

$$p_i := \frac{1}{\sqrt{2\cdot\pi\cdot\sigma^2}}\cdot\int_{-\infty}^{a_i} e^{\frac{-(V-\mu)^2}{2\cdot\sigma^2}}\,dV$$



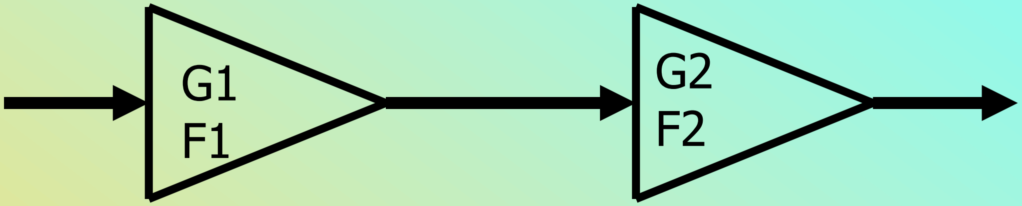
Noise Figure

- **Noise figure** is defined as a degradation in Signal to Noise Ratio (S/N)
- S/N degrades in every component (constant temperature and bandwidth)
  - F is the Noise Factor (Linear units)
  - Noise Figure = NF = 10 Log(F) in dB

$$F = \frac{S_i/N_i}{S_o/N_o} \geq 1,$$

- **Effective Input Noise Level (Ni) = kTBF**
  - kTB = -114dBm in a 1MHz BW
  - NF = 10 dB
  - B = 1MHz
  - Ni = KTB + NF = -114 dBm + 10 dB = -104 dBm in a 1 MHz Bandwidth -

Noise Figure of Two Cascaded Amplifiers



Total Noise

N4=kTB*F1*G1+ kTB* (F2-1)

Effective Noise figure Ftot

F_{eff}= N4/(kTB*G1)

F_{eff}=F1+(F2-1)/G1

NF_{eff}=10Log(F_{eff}) -

- Noise from each stage adds non-coherently
- Reflect total noise to input
- Compare to Thermal Noise ➔ (F) Noise Factor

1st Stage Out
N2=kTB*F1*G1

N2

G1

N4

Total Noise @
2nd Stage Input N4
N4=N2+N3

F1

N0

N1

N3

F2

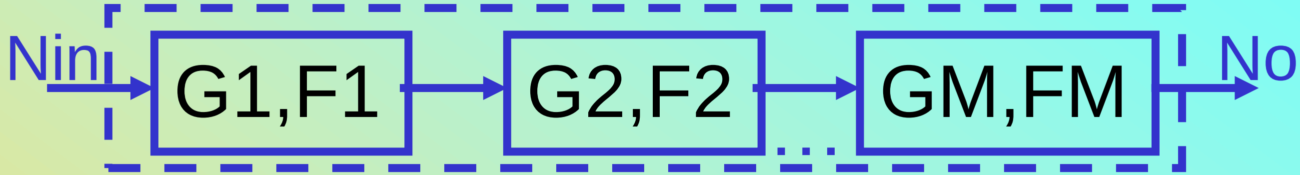
Input Noise
N0=kTB

Input Noise
+ NF
N1=kTB*(F1-1)

Added Noise
(Noise > kTB)
2nd Stage
N3=kTB*(F2-1)

Noise Figure of Cascaded Components

System of M components



- *Total Noise is the sum of the noise from each stage*

$$N_{\text{out}} = kT_o B_n \left\{ \prod_{m=1}^{M} G_m + (F_1 - 1) G_1 \prod_{m=2}^{M} G_m + \cdots + (F_M - 1) G_M \right\}$$

- Dividing No by the total gain G1G2..Gm results in the input & a effective system noise factor Fo

$$F_o = F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1 G_2} + \cdots + \frac{F_M - 1}{G_1 G_2 \cdots G_{M-1}}$$

$$Ft := F_1 + \sum_{i=2}^{N} \left[\frac{\left| F_i - 1 \right|}{\prod_{m=1}^{i-1} \left| g_m \right|} \right]$$

$\boxed{\text{NF=10Log(Fo)}}$

Noise Figure of a Passive Element

- Passive elements like an attenuator are purely resistive
- The output noise equals the input noise (kTB)
- Signal out is attenuated by the amount of loss
- $NF = [S_o/N_o] / [S_{in}/N_{in}] = S_o / S_{in} = Loss(dB)$

NF = 6 dB

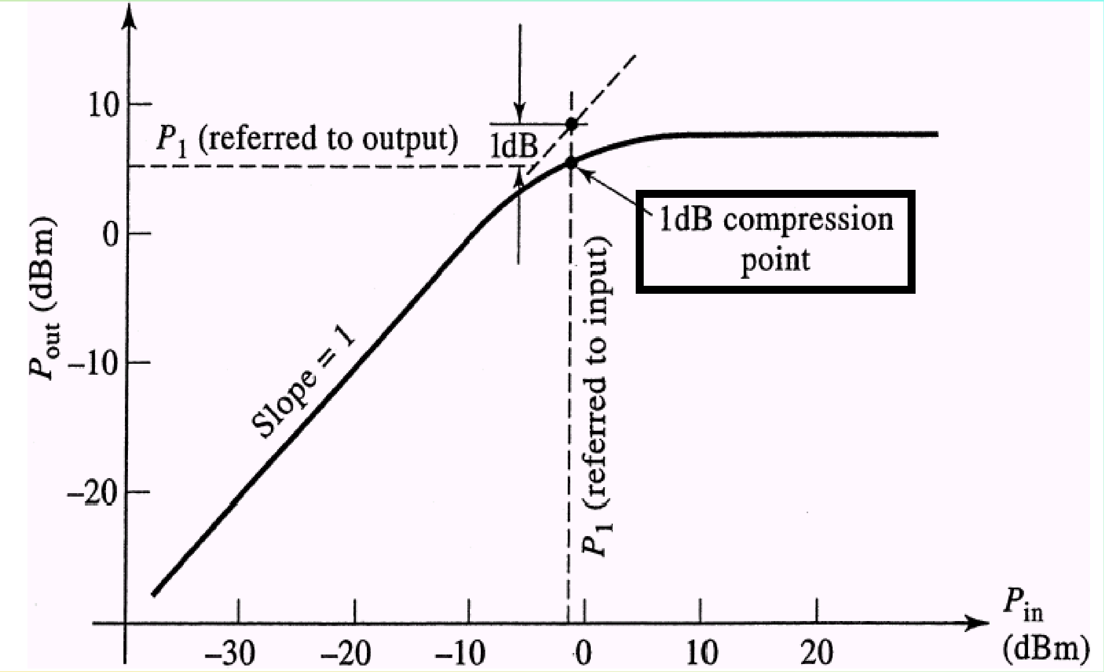
Harmonics and Intermodulation Interference

- Caused by Device Non-Linearity
- Everything is Non-Linear
- Linearity is an approximation of the Real World
- Active Devices are exhibit a higher degree of non-linearity than passive devices -

Non-Linear Devices

- Non-Linearity devices can be characterized by a Taylor series
  - $S_o = a_0 + a_1S_i + a_2S_i^2 + a_3S_i^3 + a_4S_i^4 + \ldots\ldots$
- Linear region is usually from small signal to the point where the small signal gain compresses 1 dB

- @ 1dB Compression 2nd Harmonic -20 to –26 dBc
- Saturation of most Solid State amplifiers occurs within 2dB of the 1 dB Compression Point

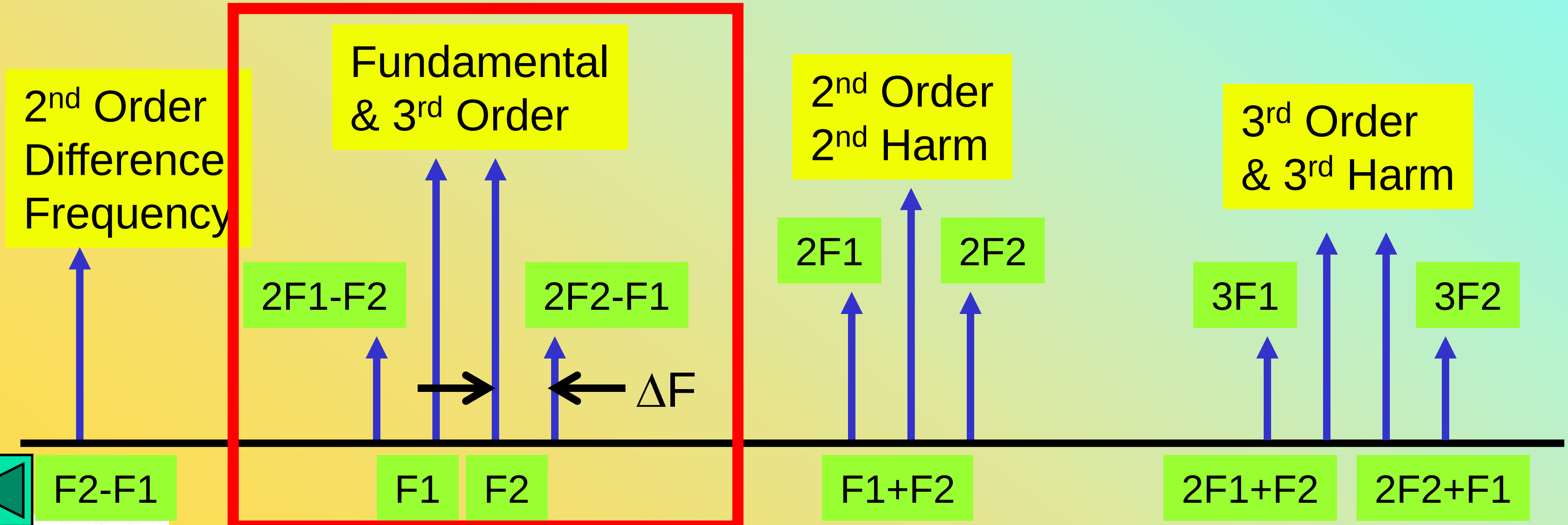


Intermodulation Products

- Caused by Multiple signals in a non-linear device (Taylor Series)
  - Results are the sum and difference frequency components of the fundamental and all of their harmonics
  - Well behaved Non-linear devices have the highest intermodulation levels at the 2nd and 3rd order terms of the a Taylor series
- $S_o = a_0 + a_1S_i + a_2S_i^2 + a_3S_i^3 + a_4S_i^4 + \ldots\ldots$
  - S_o = Output Signal
  - S_i = Input Signal
  - a_n = Coefficients of the device ($n = 0,1,2,3,4\ldots\ldots$)
- S_i, is two (2) signals: $S_i = E1 * \cos(\omega_1 t) + E1 * \cos(\omega_2 t)$
  - E1 is the peak amplitudes (equal) and ω_1 & ω_2 are the radian frequencies. -

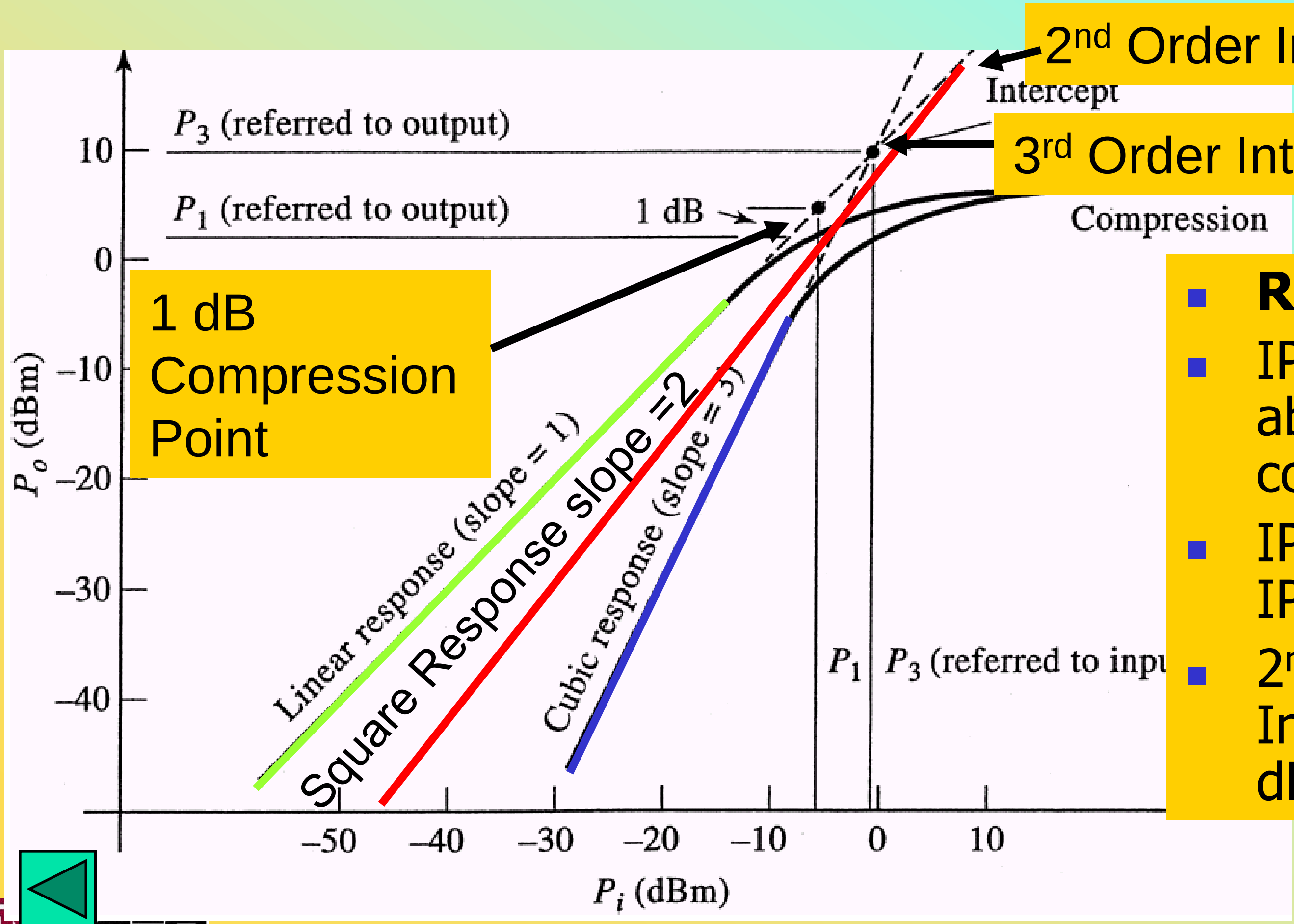
2^{nd} & 3^{rd} Order Intermodulation

- Input frequencies F1 & F2
- 2^{nd} order products occur at F1+F2 and |F1-F2|
  - Concern for BW greater than an octave
- 3^{rd} Order are the 2^{nd} harmonic of one signal mixing with the fundamental of a second signal
  - 2*F1-F2 or 2*F2-F1
- 3^{rd} order intermodulation is ΔF=F2-F1 away from the carrier -



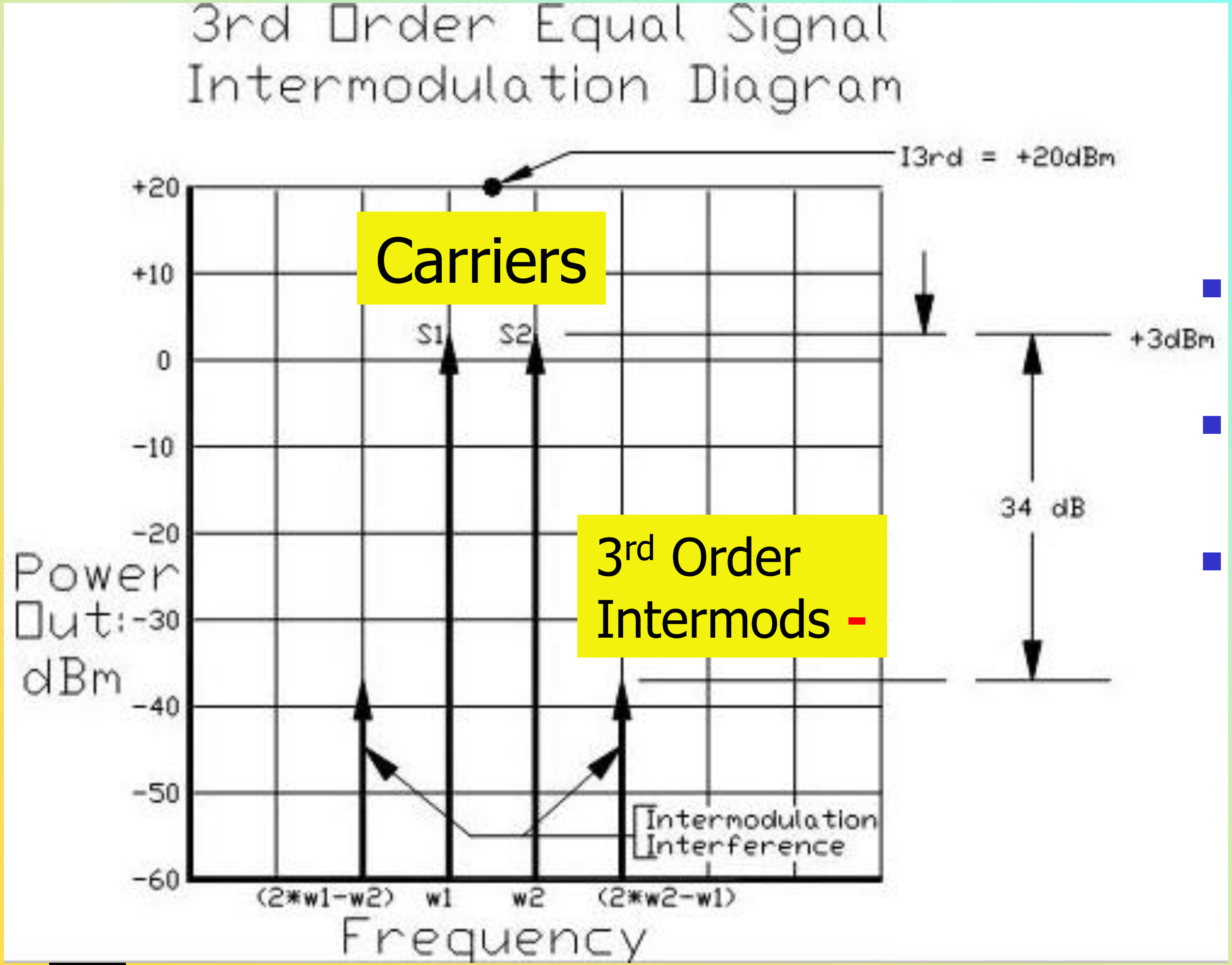
Intercept Point (IP) Determines Interference Level

- IP is an imaginary point where Slopes of the fundamental (|) & 2nd order IMD (|) or 3rd order IMD (|)meet.
- Used to calculate the level of multiple signal interference



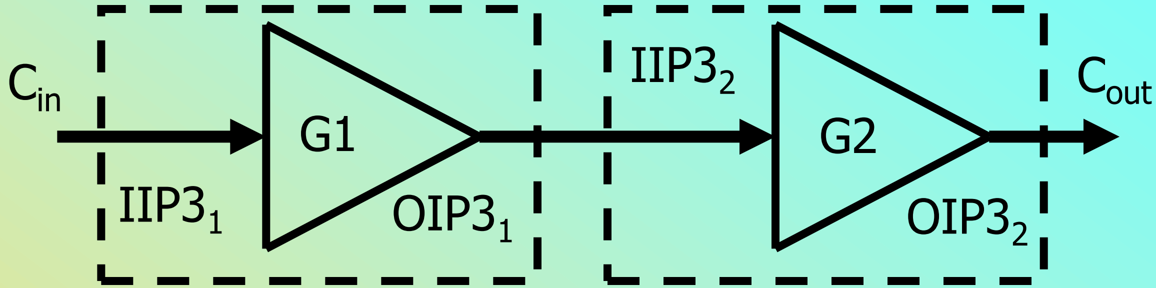
- **Rules of thumb:**
- IP3 is 7 to 10 dB above 1 dB compression point
- IP2 is 10 dB above IP3
- 2nd Harmonic Intercept point is 6 dB above IP2) -

Third-order Intermodulation Interference

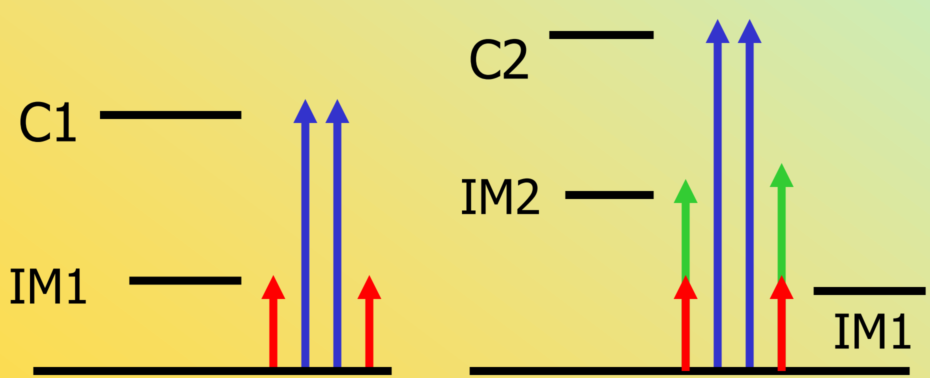


- $IM_{dBc} = -2(IP3-C)$
  - Intercept Point (IP3) in dBm
  - Carrier Level (C) in dBm
- Intercept Point (IP3) = +20 dBm
- Each Carrier C=+3 dBm
- 3rd Order intermodulation interference (IM_{dBc})= -2(IP3-C) = -34 dBc = IM_{dBc}

IP3 of Cascaded Components



- C_{in} = Carrier Level in & C_{out} = Carrier Level out
- Each component generates intermodulation products
- In general they are in phase and add coherently
- An equivalent system intercept point can be determined
- **Intermodulation Interference always gets worse**

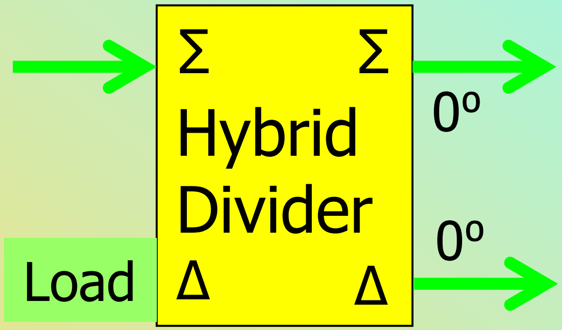


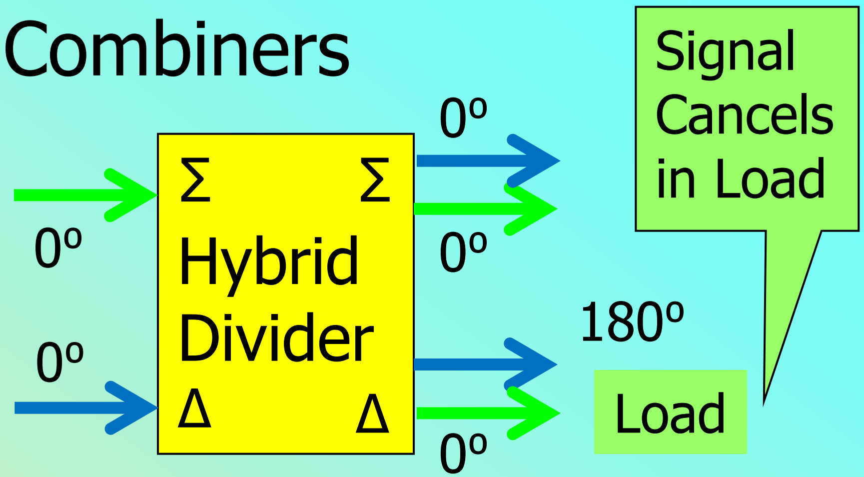
- ❑ Equation for the Intercept point of 2 stages
- ❑ Units are linear (not dB)

$$Int := \left[(I1 \cdot G2)^{-1} + I2^{-1}\right]^{-1}$$

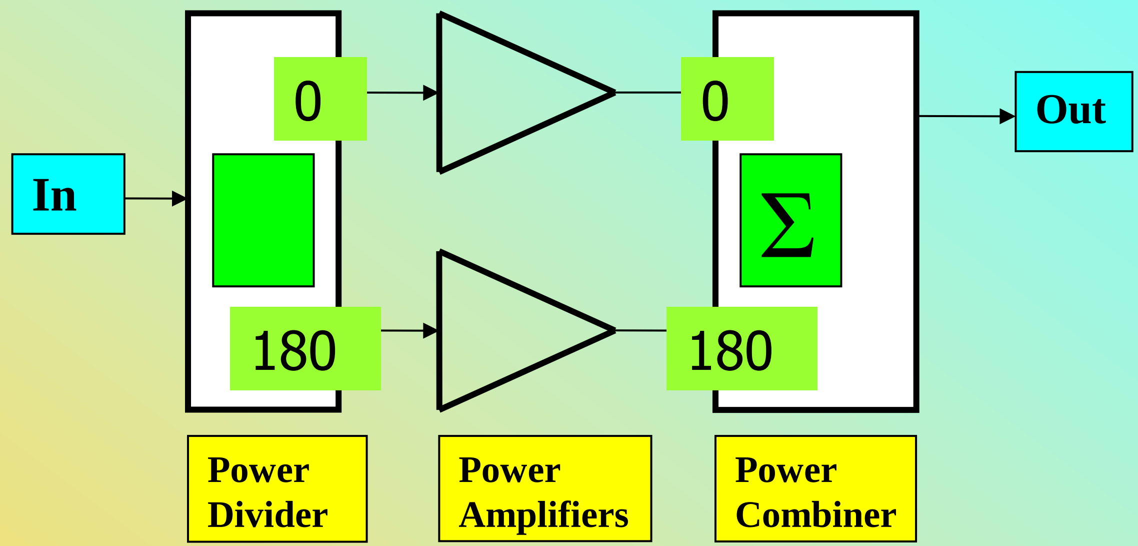
Microwave Dividers & Combiners

Hybrid Dividers & Combiners



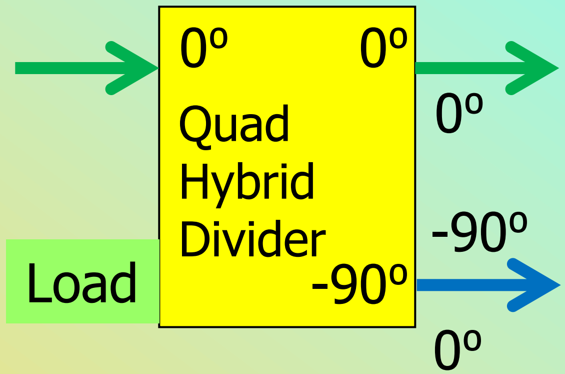


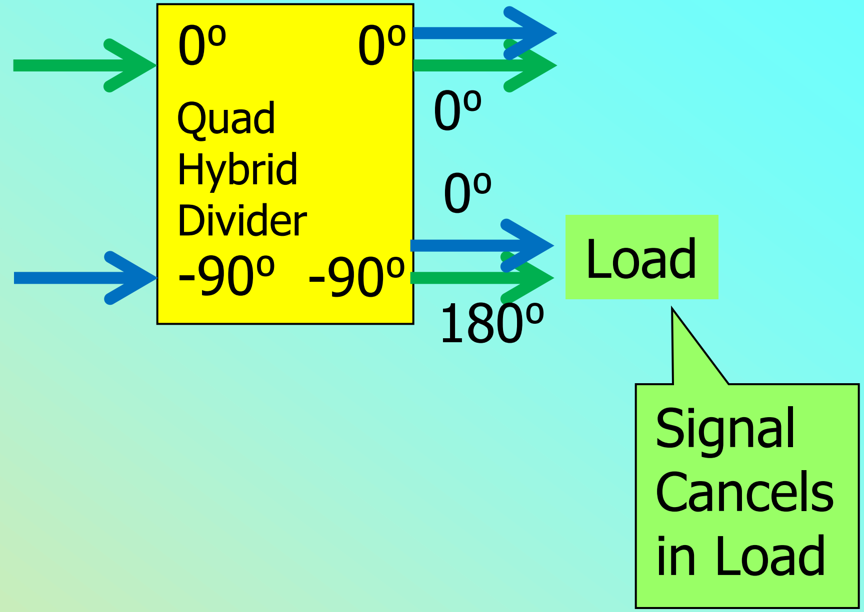
- Input signal ∑ ➔ 0^0 / 0^0
- Input signal Δ ➔ 0^0 (Σ) & 180^0 (Δ)
- In phase signals add in ∑ port
- Hybrid Dividers and combiners provide 2nd Harmonic cancellation



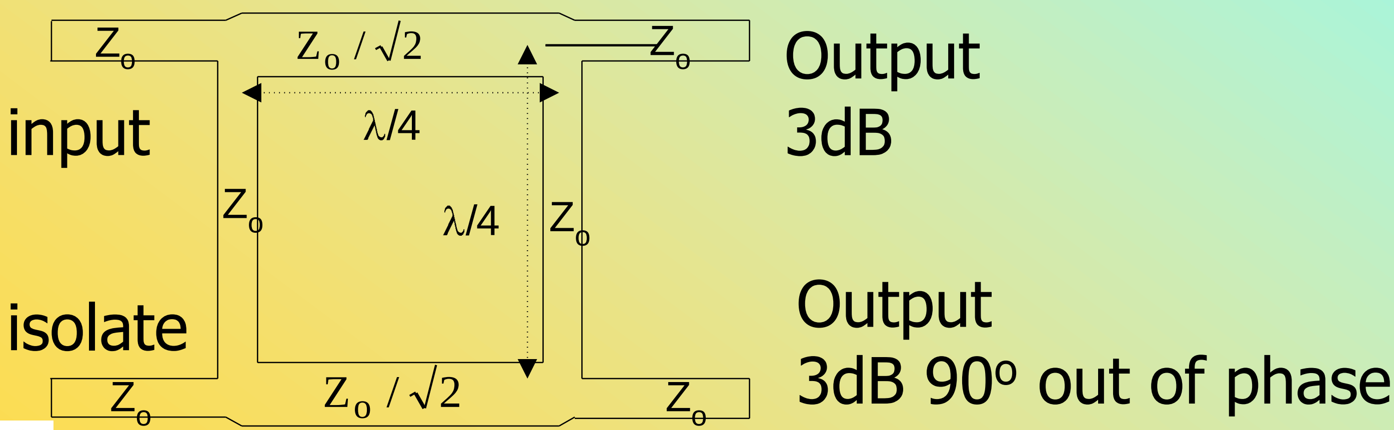
Distortion is created at the amplifier output and cancelled in the 180º Coupler -

Quadrature Dividers & Combiners

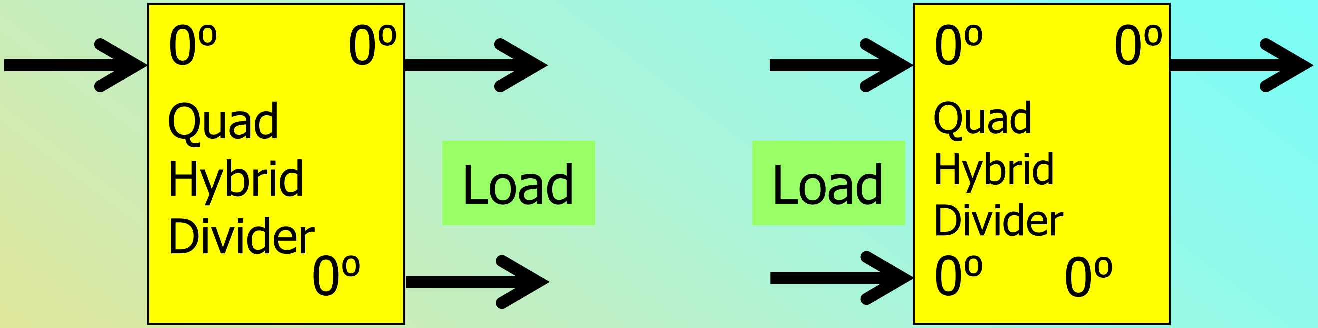




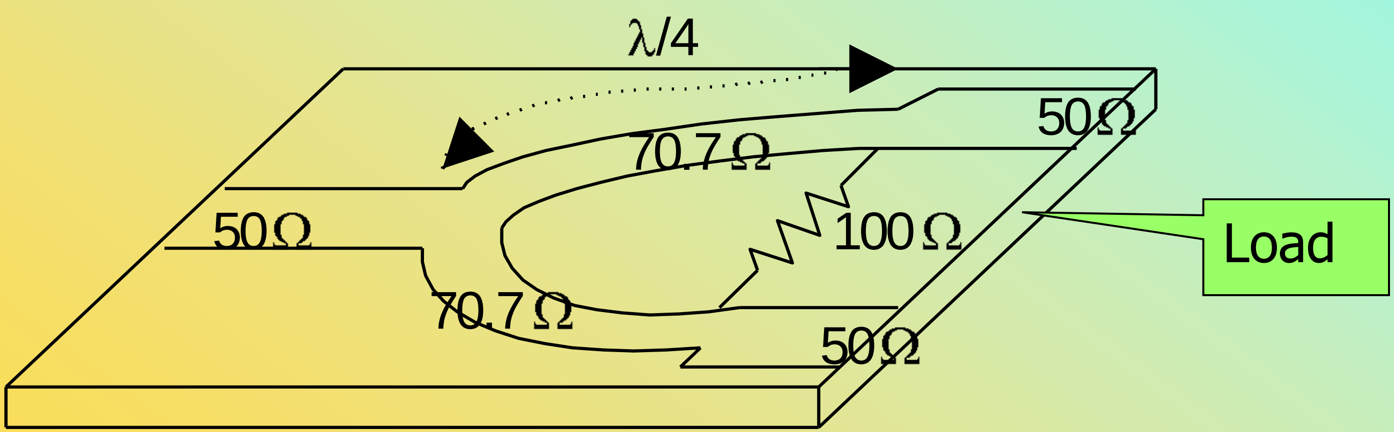
- Input signal: 0^0 / -90^0
- Signal add in (0^0) port
- Improved VSWR
  - Reflected signals ports are directed to the load



Wilkenson Dividers & Combiners



- Input & out signals are in phase
- Reflected signal go into the load & return to the input
- Excellent wide band performance



Coupler Described by a Matrix

S-Parameter Matrix describing a four port device

$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{21} & S_{22} & S_{23} & S_{24} \\ S_{31} & S_{32} & S_{33} & S_{34} \\ S_{41} & S_{42} & S_{43} & S_{44} \end{bmatrix}$$

Ex: S21, Output at Port 2 with respect to an input at port 1

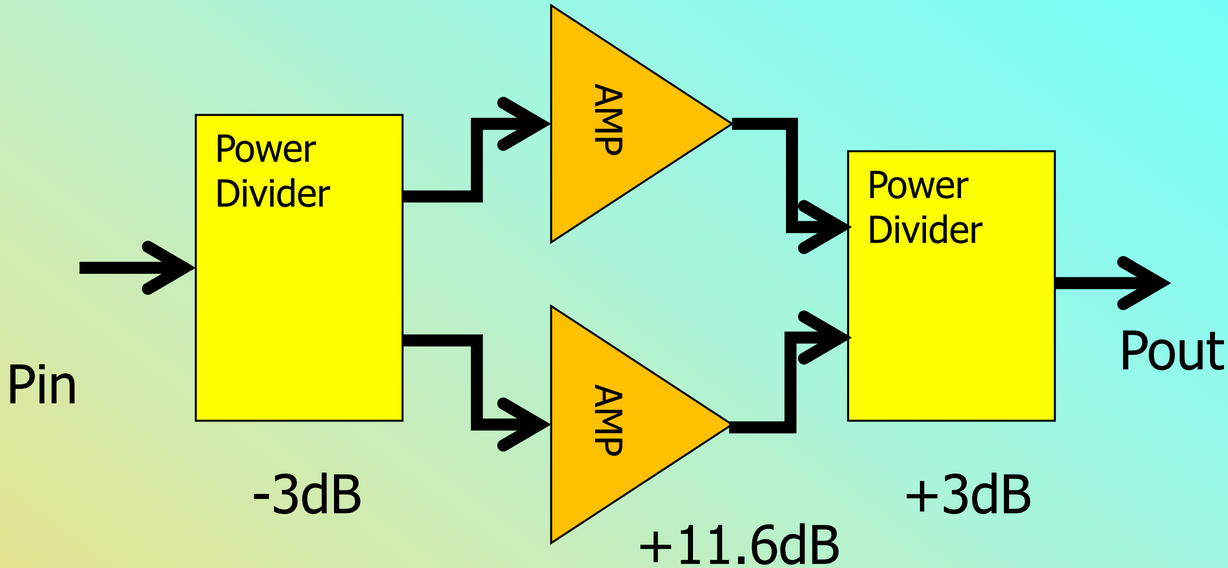
Matrix for Quadrature Coupler

$$[S] = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 & 1 & j & 0 \\ 1 & 0 & 0 & j \\ j & 0 & 0 & 1 \\ 0 & j & 1 & 0 \end{bmatrix}$$

Matrix for 0º / 180º Hybrid

$$[S] = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & -1 \\ 1 & 0 & 0 & 1 \\ 0 & -1 & 1 & 0 \end{bmatrix}$$

Parallel Amplifiers Combining Losses



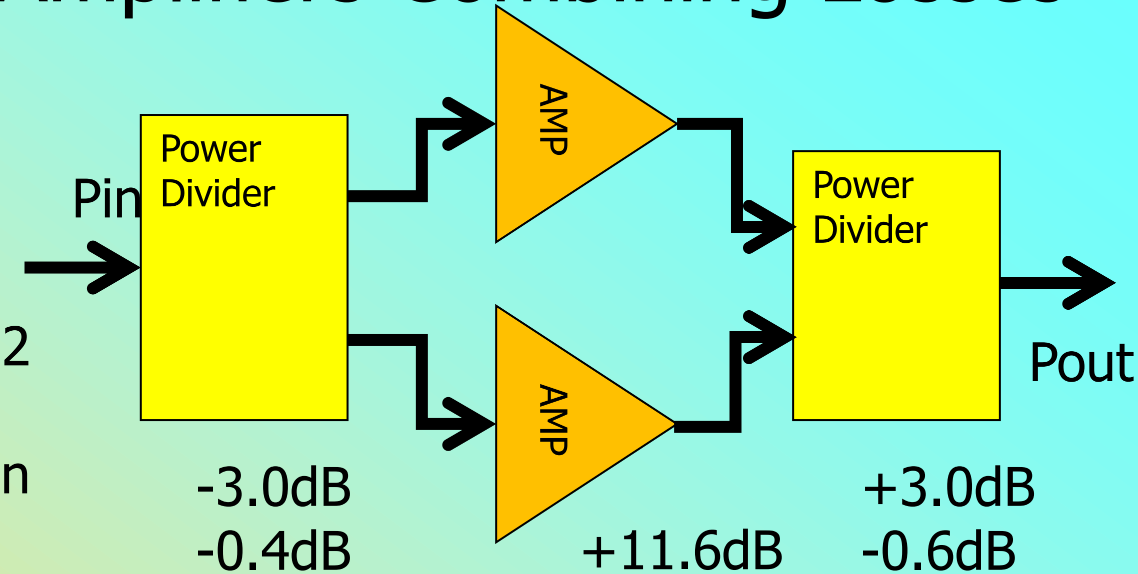
Ideal

- Signal: divided 2 ways (-3dB)
- Each path is identical
- Coherent signals are combined (result of 2 signals add (+3dB))
  - Net loss: 0dB
- Total Gain = Single Amplifier Gain = 11.6dB

DESCRIPTION	GAIN dB	GAIN dB
Input Gain	-3.0	-3.0
Single Amplifier	11.6	8.6
Output Gain	+3.0	11.6

(Coupler does not have gain)

Practical Parallel Amplifiers Combining Losses



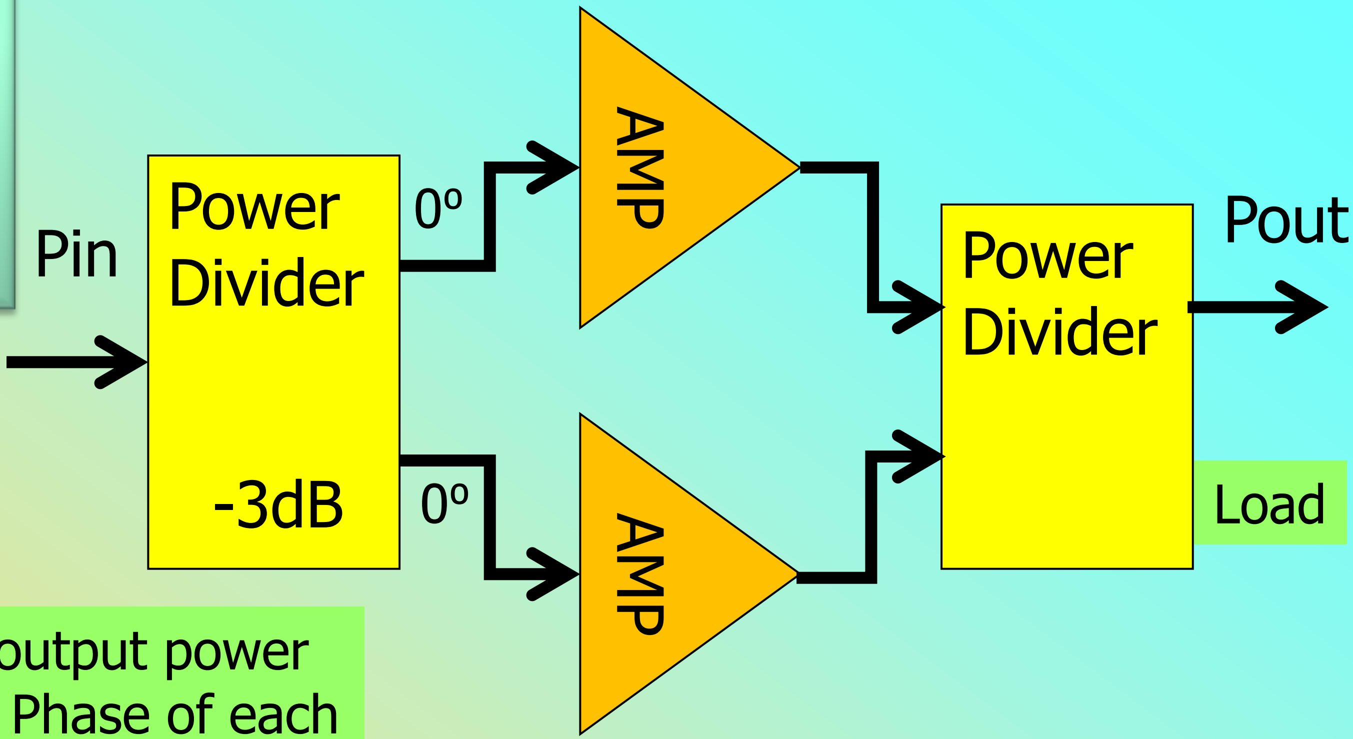
Practical Combiner

- Input Signal is divided to 2 outputs: -3.0dB
  - Input coupler insertion loss: Typically 0.4dB
  - Total input loss: -3.4dB
- Amplifier gain: 11.6dB
  - Cumulative gain: 8.2dB
- Signals are combined in the output coupler: +3dB (Coupler does not have gain)
  - Output combiner loss: 0.6dB ➔ +2.4dB
- Cumulative gain: 10.6dB

DESCRIPTION	GAIN dB	GAIN dB
Input Gain	-0.4	-0.4
Single Amplifier	11.6	11.2
Output Gain	-0.6	10.6

Signal dividing & Combining cancel

Combiner Mismatch Losses

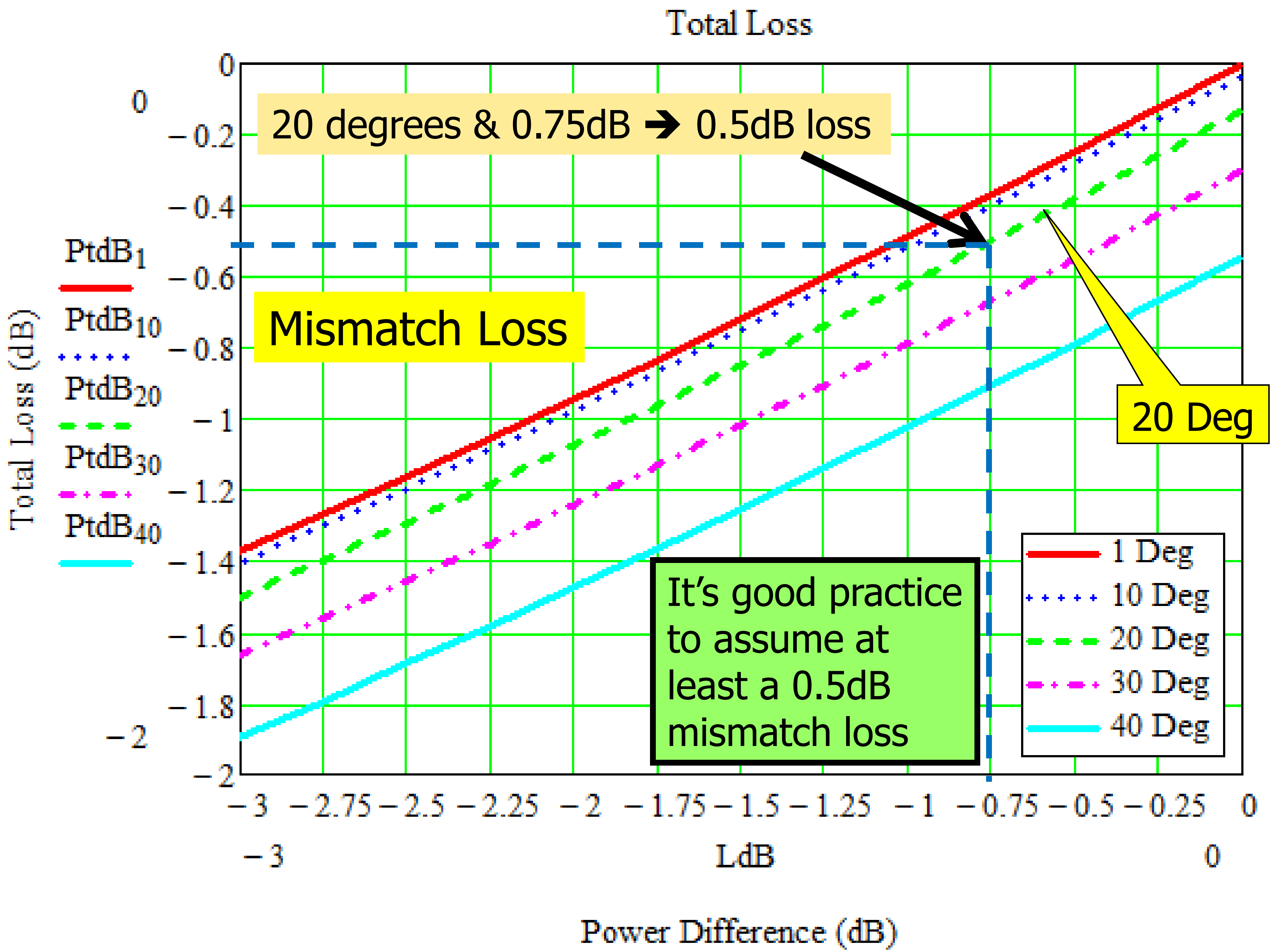


3dB increase in output power assumes Gain & Phase of each parallel path is identical

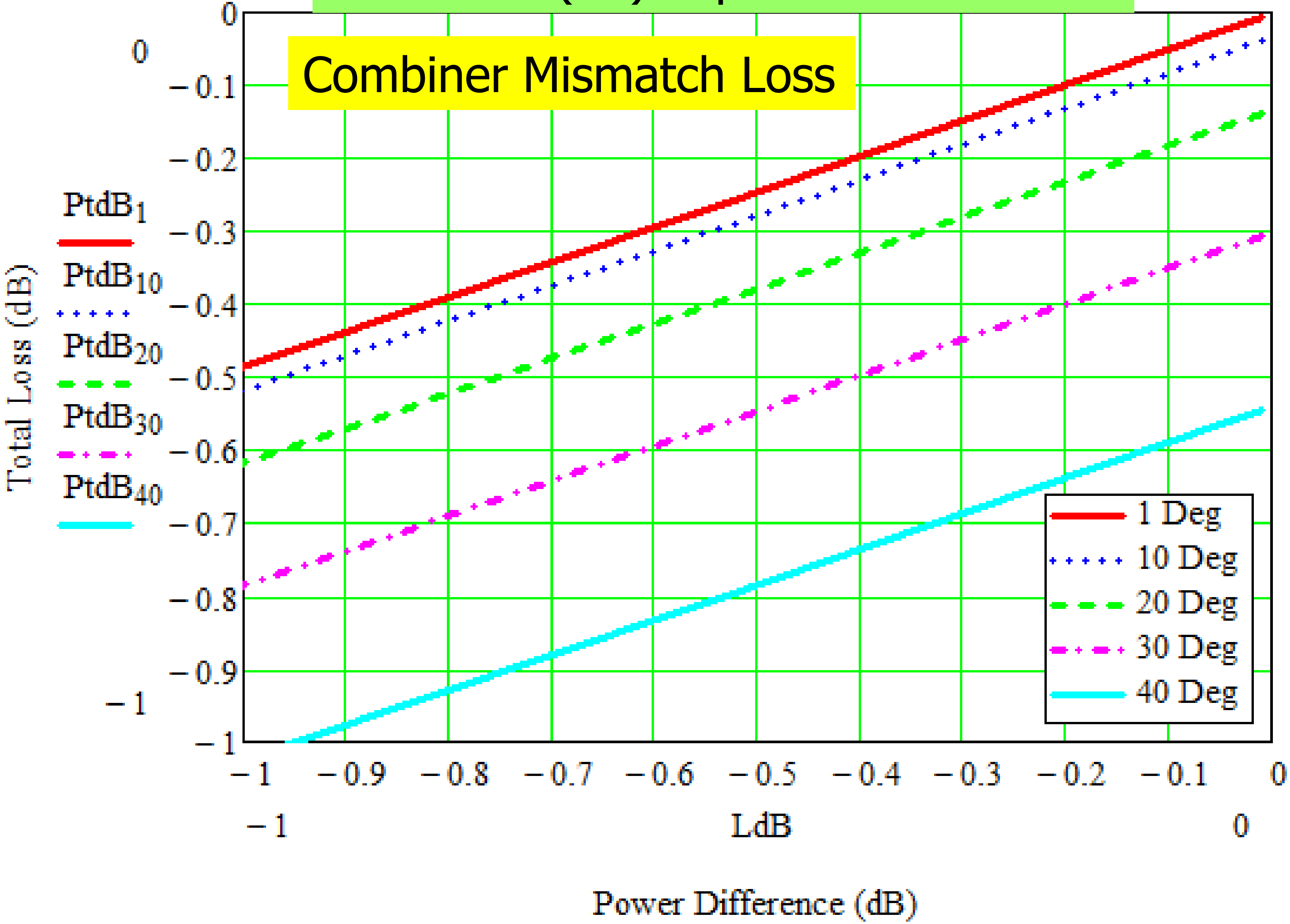
Typical Mismatch values

- Input coupler mismatch: 4º & 0.5dB each leg
- Amplifier mismatch: 10º & 0.75dB each leg
- Output combiner mismatch: 4º & 0.5dB each leg

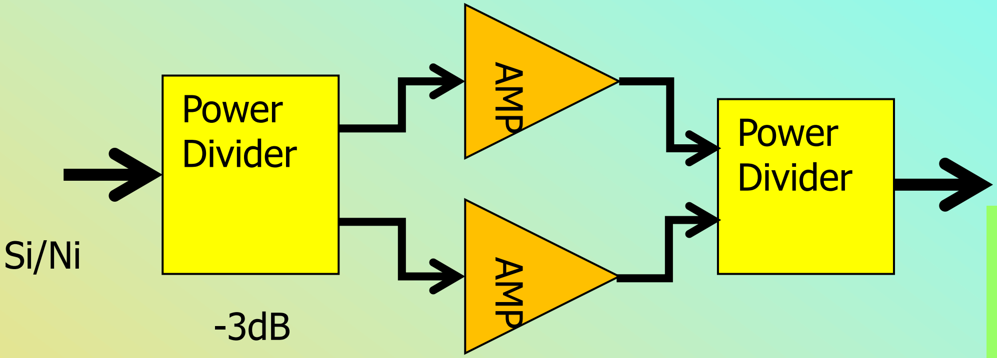
- Result non-ideal summing
- A portion of the signal goes to the "Load"


Total Loss
20 degrees & 0.75dB → 0.5dB loss
Mismatch Loss
20 Deg
It's good practice to assume at least a 0.5dB mismatch loss
PtdB1
PtdB10
PtdB20
PtdB30
PtdB40
1 Deg
10 Deg
20 Deg
30 Deg
40 Deg
Total Loss (dB)
0
−0.2
−0.4
−0.6
−0.8
−1
−1.2
−1.4
−1.6
−1.8
−2
−3 −2.75 −2.5 −2.25 −2 −1.75 −1.5 −1.25 −1 −0.75 −0.5 −0.25 0
LdB
Power Difference (dB)

Total Loss (dB) Expanded Resolution



Modeling Mismatch Losses



- Path has a Gain & Phase Mismatch
- A portion of the signal is directed to the load
- 20 degrees & 0.75dB ➔ 0.5dB loss

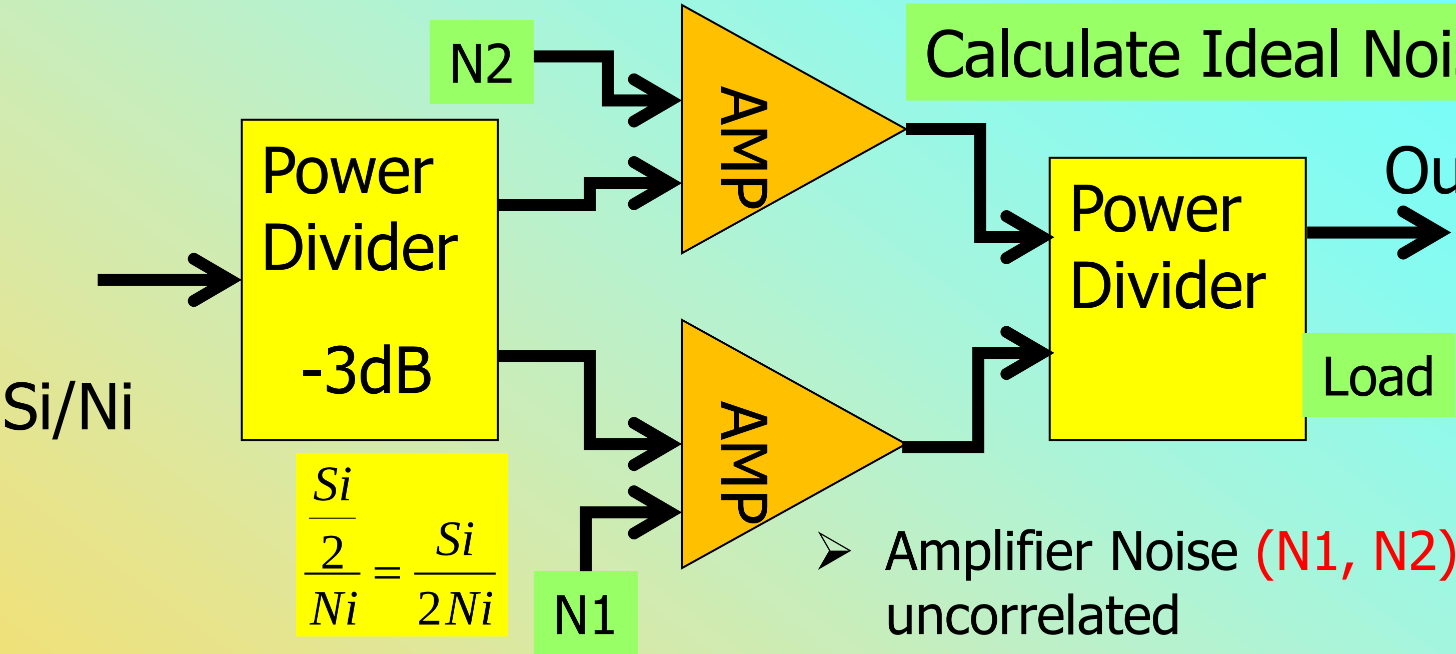
Mismatch Loss only shows Up in the output

Chain Analysis Modeling
(Dividing & combining cancels out)

DESCRIPTION	GAIN dB	GAIN dB
INPUT LEVEL		0.0
	0	0.0
Input Insertion Loss	-0.4	-0.4
Single Ended Amplifier	11.6	11.2
Output Insertion Loss	-0.6	10.6
Mismatch Loss	-0.5	10.1

Noise Figure of Parallel Amplifiers

Calculate Ideal Noise Figure



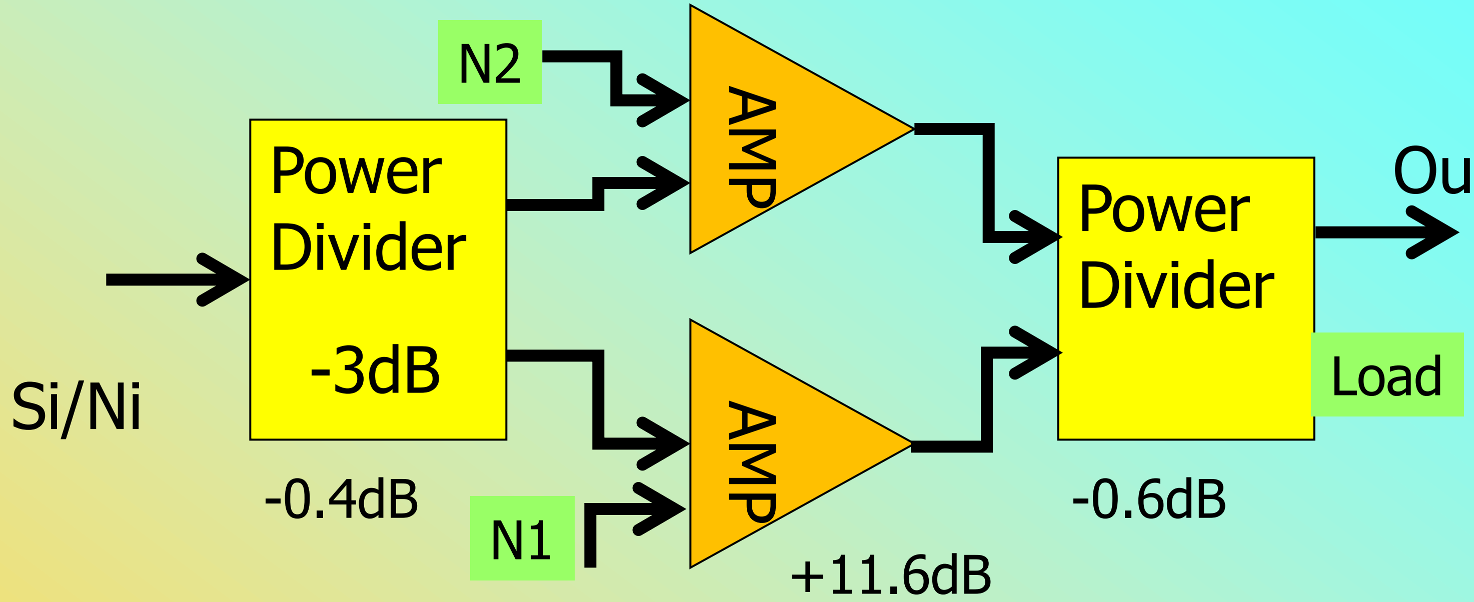
$$\frac{\frac{Si}{2}}{Ni} = \frac{Si}{2Ni}$$

- Signal attenuated 3dB at input
  - NF of each amp is degraded 3 dB

- **Ideally:** noise figure of combined amplifier is the same as each individual amplifier -

- Amplifier Noise (N1, N2) is uncorrelated
- Amplifier output noise
  - ½ noise goes into the output
  - ½ noise goes into the load
- Noise at "Out" = noise of a single amplifier
- Signal adds coherently
  - Output signal: increases 3 dB
  - 3 dB Noise Figure hit at input is recovered

Effect of Insertion loss on Noise Figure



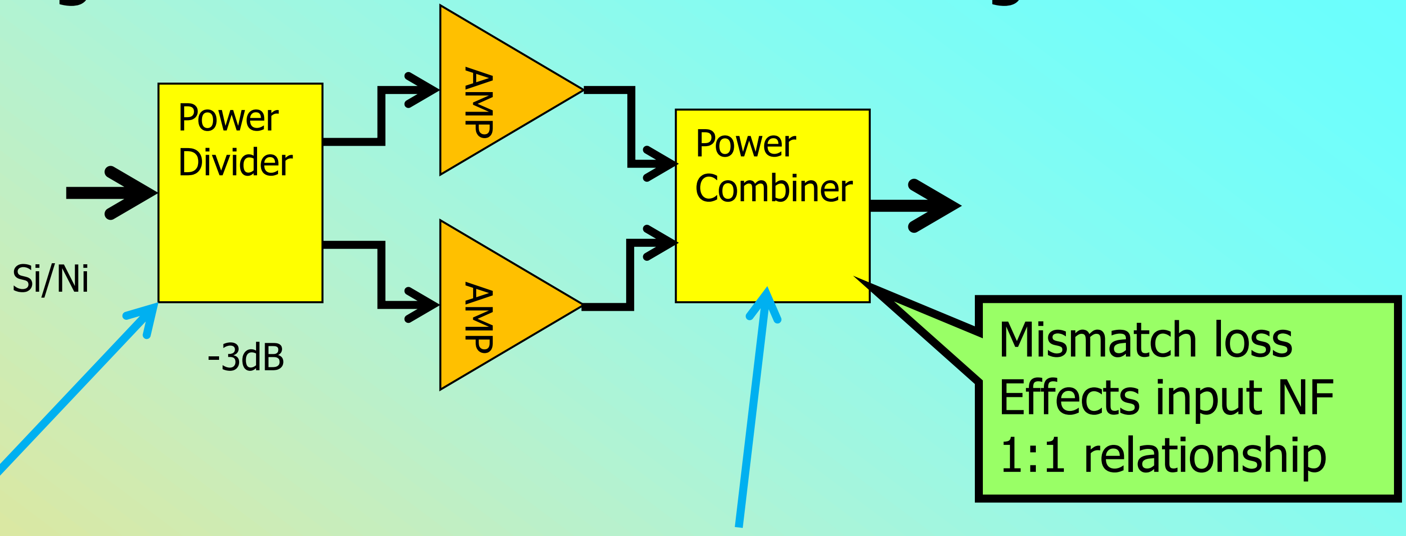
Input Circuit

- Insertion loss:
  - Not made up by the combining device
  - Adds to NF (0.4dB in our example)

Output Circuit

- Insertion loss (0.6dB)
  - No effect on NF
  - Lowers the gain -

Modeling Mismatch Loss on Noise Figure



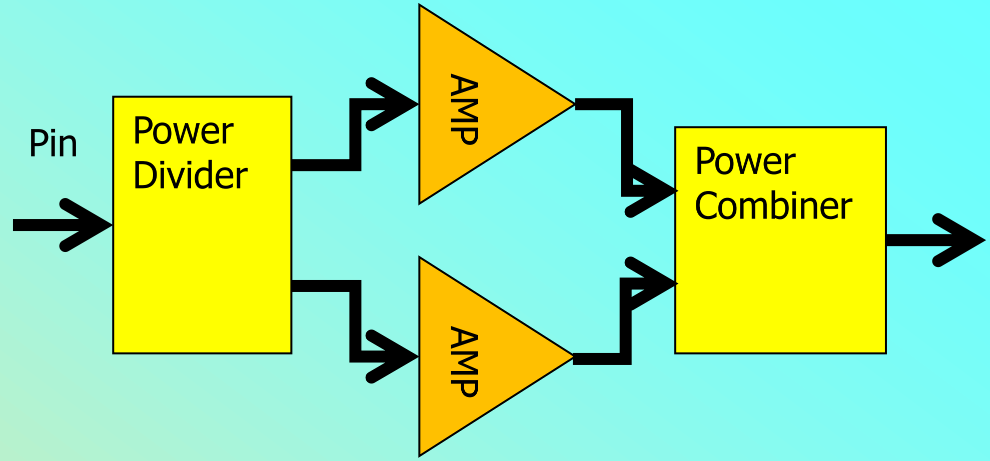
Input Circuit Mismatch Loss

- No effect on Noise
- Phase & Gain offsets are realized as an issue on the output

Output Circuit Loss: Effected by channel mismatch:

- Mismatch has little effect on Noise
  - Each leg is uncorrelated (-3dB)
- Mismatch causes signal combining loss
- S/N goes down ➔ NF goes up
- Effect is noticed at the output
- Mismatch loss has to be applied to the input with respect to NF
  - One for one increase in NF for signal loss at the output

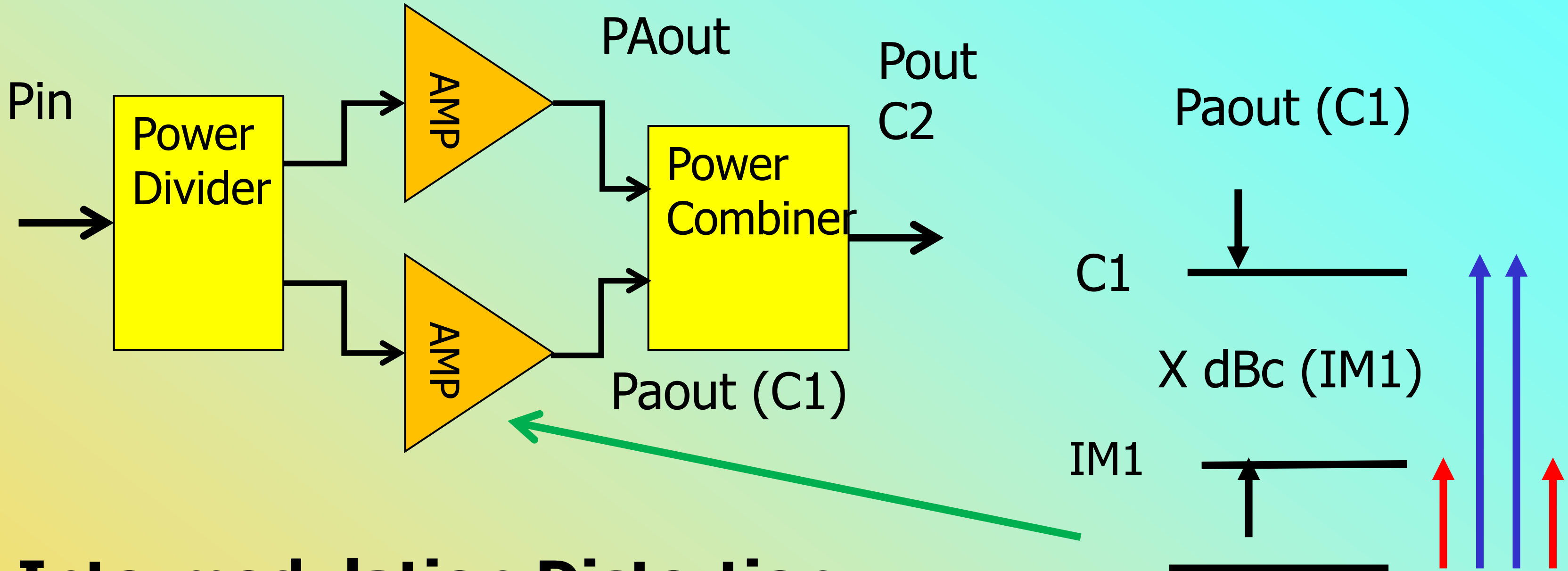
Calculate Effect of Insertion loss & Mismatch Loss on Noise Figure



- Input Insertion Loss
  - Same as a passive loss at input
- Mismatch Loss (from graph)
  - Input: Effects NF (not gain) at the input
  - Output: Effects gain not NF -

DESCRIPTION	GAIN dB	NOISE FIGURE dB	CUM GAIN dB	CUM NF dB
Input Insertion Loss	**-0.4**	**0.4**	-0.4	0.40
Mismatch Loss		**0.5**	-0.4	0.90
Single Ended Amplifier	**11.6**	**5**	11.2	5.56
Output Insertion Loss	**-0.6**	**0.6**	10.6	5.56
Mismatch Loss	**-0.5**		10.1	5.56

Output Power & Intermodulation Distortion Improvement



Out IM of Amp 1
IM1=-2(OIP3-C1)

Intermodulation Distortion produced in a single amplifier

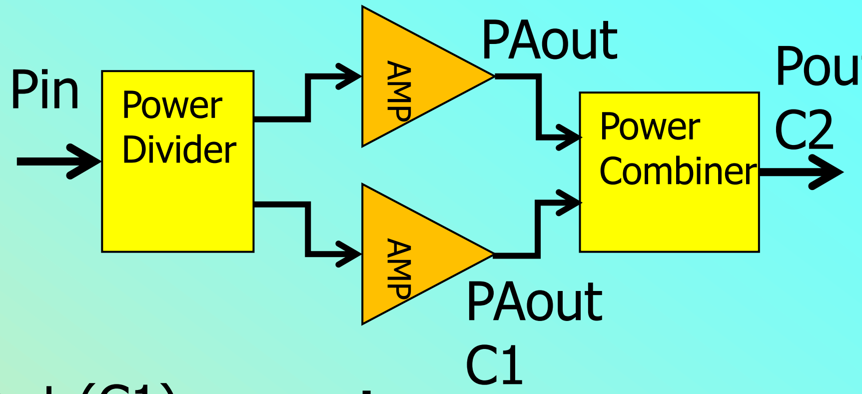
- IM1: Intermodulation distortion (dBc)
- C1: Single carrier output level
- OIP3: Intermodulation intercept point at the output of each amplifier (dBm) -

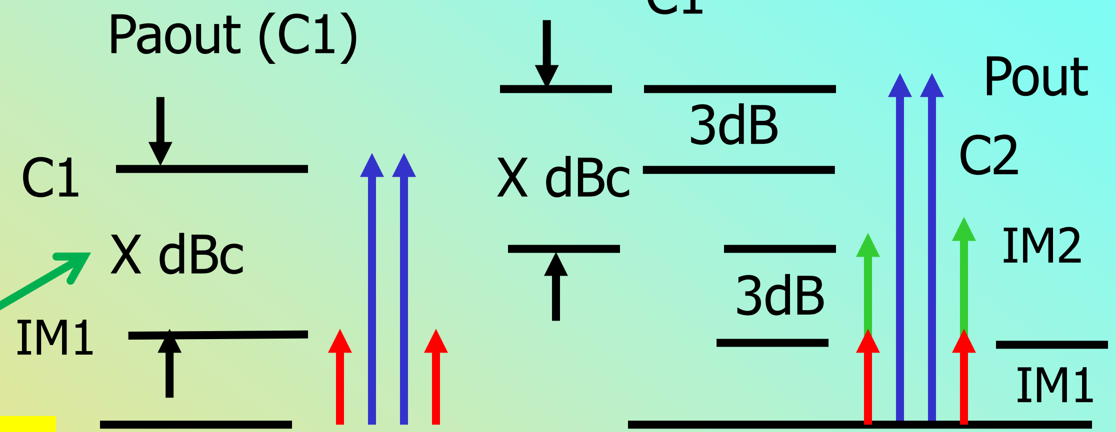
$$IM1_{DIS} = 2(OIP3 - C_1)$$

Intermodulation Improvement in Parallel Amplifiers

Ideal

- Combined output power increases 3 dB (C2-C1)
- Intermodulation interference
  - Distortion combines as signal (+3dB)
  - Relative intermodulation distortion remains the same ($IM1_{DIS}$ = X dBc)





Out IM of Amp 1
$IM1_{DIS}$=-2(OIP3-C1)

Out IM of Amp 2
$IM2_{DIS}$=-2(OIP3-C2)

$$IM1_{DIS} = 2(OIP3 - C_1)$$

$$OIP3 = \frac{IM1_{DIS}}{2} + C_1$$

- $IM1_{DIS}$ = X dBc is the same at C1 or C2
- C2 is 3dB greater than C1
- OIP3 @ C2 is 3dB higher than OIP3 @ C1 -

Model for Parallel Combined Amplifiers

- Input Coupler Insertion Loss: 0.4dB
- **Channel to Channel Mismatch Loss: 0.5dB**
- Power Amplifier
  - Gain: 11.6 dB
  - NF: 5dB
  - OIP3: +22dBm
- Input Coupler Insertion Loss: 0.4dB

DESCRIPTION	GAIN dB	NOISE FIGURE dB	IN-3rd dBm OUT
	0	0	100
Input Insertion Loss	-0.4	0.4	100
Mismatch Loss	**0**	**0.5**	100
Single Ended Amplifier	11.6	5	22+3
Output Insertion Loss	-0.6	0.6	100
Mismatch Loss	**-0.5**	**0**	100

Single Amp: IP3=+22dBm
Add 3 dB: Parallel Combining

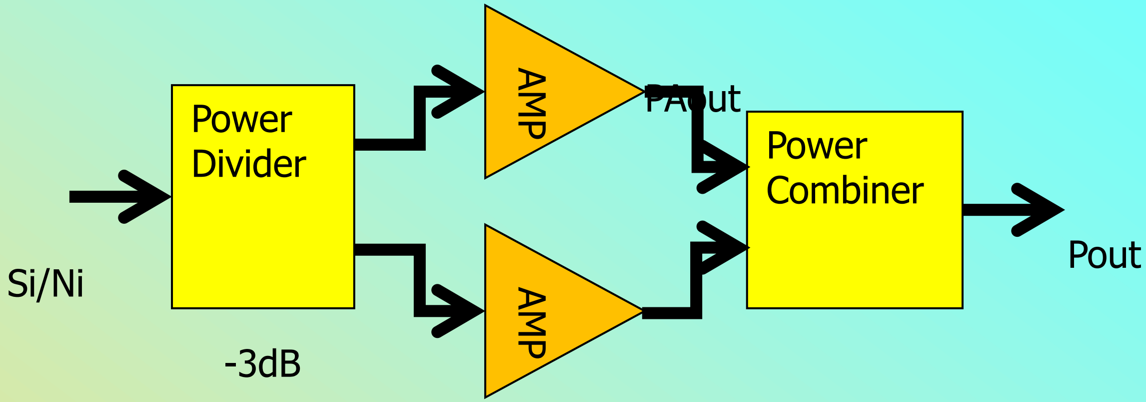
Proposed Chain Analysis Model for Parallel Combined Amplifiers

		dB	Total		10.1	5.9	23.9	13.8
		NOISE	IN-3rd		CUM	CUM	OIP3	IIP3
	GAIN	FIGURE	dBm		GAIN	NF	3rd	3rd
DESCRIPTION	dB	dB	OUT		dB	dB	dBm	dBm
	0	0	100		0.0	0.00	97.0	97.0
Input Insertion Loss	-0.4	0.4	100		-0.4	0.40	95.0	95.4
Mismatch Loss		0.5	100		-0.4	0.90	93.8	94.2
Single Ended Amplifier	11.6	5	25		11.2	5.90	25.0	13.8
Output Insertion Loss	-0.6	0.6	100		10.6	5.90	24.4	13.8
Mismatch Loss	-0.5		100		10.1	5.90	23.9	13.8

Single Amp IP3=+22dBm + 3dB

Gain & IP3

Practical Intermodulation Improvement



Actual IP3 Improvement

- P_{out} is degraded by:
  - Output combiner insertion loss
  - Mismatch loss
- OIP3 is single amplifier Intermodulation Intercept Point
- Intermodulation 3rd Order Intercept Point: $OIP3_{OUT}$ (@P_{OUT})
  - $OIP3_{OUT}$ = OIP3 + 3dB – Insertion loss – mismatch loss
  - **If output losses get too high this topology maybe impractical to implement -**

Summary

- Key parameters are PHASE & GAIN tracking of parallel paths
  - Couplers and amplifiers may have to be selected for their Gain & Phase tracking
  - Gain & Phase tracking are pertinent over;
    - Frequency
    - Temperature
- Good VSWR is important
  - VSWR is a gain & phase uncertainty factor not independently measurable
- <u>Combiner</u> Insertion loss is critical to the projects success
  - Survey the available combiner techniques to optimize the design
- 1 dB of loss (mismatch + combiner loss) is 20% less power
  - Input DC power is unaffected
  - Efficiency can significantly degrade -

Conclusion

- Many times a computer analysis doesn't readily handle parallel active circuits
- Always do a chain analysis
  - Analyze Subassemblies
  - Analyze Systems
  - The analysis reveals design weaknesses
  - Look at NF & IP3 reflected to the input
    - Design weaknesses show up independent of gain
- Don't model a passive circuit with gain
  - Coherent combining looks like it has gain, it's not
- **The more homework you do upfront the less heartache you have at the end -**