Phase Noise

Howard Hausman
MITEQ, Inc., Hauppauge, NY 11788
hhhausman@miteq.com

- Part 1
  The Fundamentals of Phase Noise
- Part 2
  Phase Noise Models & Digital Modulation Techniques
- Part 3
  Effects of Phase Noise on Signal Recovery
The Fundamentals of Phase Noise - Part 1

Topics

- Thermal Noise Characteristics
- Thermal Noise Effects on Threshold Performance
- Frequency / Phase Modulation
- Oscillator Basics
- Oscillator Stability
- Frequency Stability Related to Phase Noise
- Phase Noise Spectral Density
Applications Affected by Phase Noise

- Digital Communications
  - Causes Bit Errors
  - Not related to signal level
  - Causes timing errors

- Doppler RADAR
  - Limits the ability to identify slow moving objects

- Phase Tracking Systems
  - Causes tracking errors

- Phase Lock Loops
  - Trade off phase lock frequency tracking and noise compensation
  - Can limit phase lock loop acquisition/reacquisition

Phase Noise
Stationary Clutter Doppler Return
Thermal Noise Characteristics

- Thermal Noise is the random motion of electrons
  - At 0°K all motion stops - Zero Thermal Noise
  - Thermal Noise can be related to temperature
    - Excess thermal noise can be related to an increase in temperature: °K
- Thermal Noise level
  - Unknown at any instant of time
  - Statistically well behaved
    - Precisely known over a long time
      - Averaging time >> 1/BW
Deriving Thermal Noise

- Thermal Noise is only present in Real Elements, e.g. resistors, etc.
- Reactive elements have zero average thermal noise (L’s & C’s)
- Thermal noise in a real element, e.g. Resistor, is:

\[ V_n = \sqrt{\frac{4h f BR}{e^{h f / kT} - 1}} \]

- \( h \times f \) is momentum of an electromagnet particle
  - \( h = \text{Planck’s Constant: } h = 6.626 \times 10^{-34} \text{ J*S} \)
  - \( f = \text{frequency (Hz)} \)
- \( B \) is Bandwidth (Hz)
- \( R \) is Resistance (Ohms)
- \( k \) is Boltzmann’s constant
  - \( k = (-228.6 \text{ dB/°K/Hz}) \)
  - [Boltzman’s Constant (dB)]
- \( T \) is temperature in degrees Kelvin
Deriving Thermal Noise

\[ V_n = \sqrt{\frac{4h f BR}{e^{hf/kT} - 1}} \]

- \( h x f < < kT \)

\[ e^{hf/kT} - 1 \approx \frac{hf}{kT} \]

- \( h x f \) term cancels out
- Noise Voltage \( V_n = \sqrt{4kTBR} \)
Thermal Noise into a Load

- Noise into a matched load is: \( V_n / 2 \)

- Noise Voltage \( V_n = \sqrt{4kTBR} \)
- Noise Current \( I_n = V_n / (2R) \)
- Noise Power \( P_n = I_n^2 R \)

- Noise Power, \( P_n = \left( \frac{V_n}{2R} \right)^2 R \)
  \( \frac{V_n^2}{4R} = kTB \)
Deriving Thermal Noise

- $P_n$ = Thermal Noise Power = $kTB$ (Watts)
  - $k$ = Boltzman’s Constant
  - $k = (-228.6 \text{ dB/}^{\circ}\text{K}/\text{Hz})$ [Boltzman’s Constant (dB)]
  - $T$ = Temperature in Degrees Kelvin
  - $B$ is bandwidth in Hz
- At Room temperature $T= 25 \text{ C} \rightarrow 298 \text{ K}$
- $kTB = 4.11 \times 10^{-18}$ milliWatts in a 1 Hertz Bandwidth $\rightarrow -173.859\text{dBm/Hz (}\approx -174\text{dBm/Hz)}$
Signal to Noise Ratio

- Measure of relative signal power to noise power

Signal Level depends on usage
- This example is peak signal to RMS noise (Eb/No)
Noise Figure

- **Noise figure** is defined as a degradation in Signal to Noise Ratio

\[ F = \frac{\text{Si/Nil (input)}}{\text{So/No (output)}} \geq 1 \]

- Si/Nil is always greater than or equal to So/No
- F is the Noise Factor (Linear units)
- NF (dB) = \( S_{\text{in}}/N_{\text{in}} \) (dB) – \( S_{\text{o}}/N_{\text{o}} \) (dB)
- NF = 10 Log(F) in dB
- **Amplification doesn’t improve S/N**
- Ratio is constant

\[ F \geq 1 \]
\[ NF \geq 0 \]
Noise Figure Degradation

- Every Real Component adds Noise
- Low Noise systems
  - Amplify the input Signals & Noise
  - Minimizes the effects of other system noise generators

\[ \text{S/N is degrades in every real component} \]
  - At constant temperature and band-width
Noise Figure & Total Effective Input Noise

- At the input of a device
  - Signal Input ($S_{\text{in}}$)
  - Thermal Noise ($N_{\text{in}}$) + Device noise ($N_1$)
  - All add together & get amplified

- Example of Effective Input Noise Level ($N_{\text{in}}$) = $kTB_F$
  - $kTB \Rightarrow -174\text{dBm in a 1Hz BW}$
  - $F \Rightarrow NF = 10$ dB
  - $B = 5\text{MHz} \Rightarrow 10\log(5\text{MHz}/1\text{Hz}) = 67\text{dB}$
  - $N_{\text{in}} = KTB(\text{dB}) + NF = -174\text{ dBm} + 10\text{ dB} + 67\text{dB} = -97\text{ dBm in a 5 MHz Bandwidth}$

- Noise can be reflected to the input or output
  - Output Noise ($N_o$) is Input Noise times device gain ($A_1$) -
Noise Figure of a Passive Element

- Thermal noise does not add
  - Noise at the output of a resistor is the same as the input of a resistor
  - Signal decreases therefore S/N degrades

- Ideal reactive elements have no loss
  - Reactive Networks store power, don’t dissipate power
  - Noise figure is 0dB if the device has no loss

\[ N_{in} = kTB \]
\[ N_0 = kTB \]
First Stage Output Noise

- Noise at the output of the 1st stage
  - $N_{in} = kTB$
  - Noise Factor = Input device noise above $kTB$
  - $N_1 = F1 \times kTB - kTB = (F1 - 1) \times kTB$, $F1$ is the factor above $kTB$
  - Total input noise = $kTB + (F1 - 1) \times kTB = F1kTB$
  - Total Output Noise = $N_{O1} = kTB \times F1 \times G1$
Multistage (cascaded) System

- Noise at the input of the 2\textsuperscript{nd} stage (including 2\textsuperscript{nd} stage noise)
  - \( N_{i2} = N_{o1} + kTB - kTB \) (can’t add thermal noise twice)
  - \( N_{i2} = N_{o1} + kTB \times (F_2 - 1) \)
  - \( N_{i2} = kTB \times F_1 \times G_1 + kTB \times (F_2 - 1) \)

- Effective input noise = \( N_{eff} \)
  - \( N_{eff} = \frac{N_{i2}}{G_1} = \frac{kTB \times F_1}{G_1} + \frac{kTB \times (F_2 - 1)}{G_1} \)

- Amp #2 is noiseless when you consider the input noise = \( N_{eff} \)
Noise Figure of a Multistage (cascaded) System

- $N_{eff} = kTB(F_1+[F_2-1]/G_1) = kTB F_{eff}$
- Effective Input Noise factor $F_{eff} = F_1+[F_2-1]/G_1$
- $NF_{eff} = 10\log(F_{eff}) \Rightarrow$ Effective input Noise Figure
- Applying this formula to many stages

$$F = F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1G_2} + \cdots + \frac{F_n - 1}{G_1G_2\cdots G_{n-1}}$$
Cascaded Noise Figure Example

Mixer noise figures can be greater than loss.

Gain of last Amp doesn’t affect NF.

Component data (from datasheets)

<table>
<thead>
<tr>
<th>Component</th>
<th>Preselector filter</th>
<th>LNA</th>
<th>image reject filter</th>
<th>mixer</th>
<th>IF amp</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gain (dB)</td>
<td>-0.5</td>
<td>20</td>
<td>-1</td>
<td>-6</td>
<td>30</td>
</tr>
<tr>
<td>NF (dB)</td>
<td>0.5</td>
<td>3</td>
<td>1</td>
<td>6</td>
<td>5</td>
</tr>
</tbody>
</table>

Cascaded gain and noise figure (convert back to dB)

| NF dB                      | 0.5 | 3.5  | 3.505632 | 3.586457 | 3.811745 |
| Gain (dB)                  | -0.5 | 19.5 | 18.5     | 12.5    | 42.5    |
AM / FM Comparison
De-Modulated Signal to Noise

- AM & FM S/N do not have the same performance through a demodulator
- FM: S/N Improvement
  - Input S/N must be above threshold
- Phase Lock demodulator has no Threshold effect

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Carrier to Noise Ratio

- Assume the carrier is CW with $P_{\text{ave}} = C$, for simplicity
- $C/N = C$, Carrier level divided by the noise spectral density function integrated over the spectrum $-\frac{BW}{2}$ to $+\frac{BW}{2}$
Detected Noise

- Noise through a non-linear device (diode) produces a unique characteristics in frequency & amplitude

- Detected output has three components
  - Noise mixing with Noise
  - Noise mixing with signal
  - Recovered Signal (S)

- Linear Slope due to convolution of a two rectangles

- Rectangle due to convolution of an impulse & a rectangles
Detected Signal + Noise

- The S x N is negligible at High S/N
- At low S/N the S x N term identifies signal presence but does little in decoding the signal, usually cannot be processed
  - In Radar MDS is usually the S x N term

- SxN is a Rectangle due to convolution of an impulse & a noise rectangles

![Diagram showing signal and noise frequency bands and detector output](image.png)
Detection at Low S/N

- Detected terms
  - S, Detected Signal
  - N x N, Noise times Noise Term
  - S x N, Signal times noise term
- The most basic Radar function is detecting signal presence
- Minimum Detectable Signal (MDS)
  - Signal presence is detected
  - Signal recovery is doubtful
  - \( (S + SxN)/N \) because SxN is only present with Signal
Decoding Signal information

- Signal (S) must be greater than, \( N \times N \) term + \( S \times N \) term
  - For determination of Signal quality or Signal recovery
- Bit Error Rate (BER) the ratio \( S / (N + S \times N) \) must be considered

- Tangential Sensitivity is an ambiguous term
- \( 6 \text{dB} < S/N < 9 \text{dB} \)
Signal vs. Noise Expressions

- **C/N**: Carrier to Noise Ratio
  - Pre-detection Signal over noise

- **S/N**: Signal to Noise Ratio
  - Post detection Signal over noise

- **Eb/No**: Bit Energy to Noise Power
  - \( \text{Eb/No} = \text{So/No} \times (\text{BW}/\text{Rb}) \)
    - \( \text{BW} \) is IF bandwidth (Hz)
    - \( \text{BW} \) is related to symbol rate
    - \( \text{Rb} = \text{Bit Rate} \) (Bits/Second)
  - Assuming Signal to Noise ratio with an optimized bandwidth
Noise as a Probability Density Function

- Noise: Gaussian Function
- Well defined amplitude probability distribution (pdf)
  - $\mu$ is Average (Mean)
  - Signal Level or Zero
  - $\sigma$ = standard deviation: Relates to the function spreading
  - $\sigma \leftrightarrow$ RMS Noise
- Thermal Noise = $kT\beta$
  - $= -174\text{dBm at } 298 \text{ K}$

$$pdf := \frac{1}{\sqrt{2\pi\sigma^2}} \cdot e^{\frac{-(V-\mu)^2}{2\sigma^2}}$$

![Graph showing probability density function with $\mu = 0$, $\sigma = 1$]
Gaussian Noise

- Total Area under the probability curve is 1
- Probability of being in any sector of the function is the area under the function

Integrating the Gaussian Function from $-\infty \rightarrow +V$ is a probability density function.

The probability of being from $-\infty$ to $V$ is given on the Y-Axis (Blue Curve).

The probability of being between $a_1$ and $a_2$ is the value of the pdf at $a_2$ minus the value at $a_1$

\[
\{ P(a_2) - P(a_1) \}
\]
Probability, Standard Deviation & RMS Noise

- \( P(V<-1\sigma) = 0.159 \)
- \( P(V>1\sigma) = 1 - 0.841 = 0.159 \)
- Probability of being greater 1\( \sigma \) (1 standard deviation)
  - \( P(V<-1\sigma \& V>1\sigma) = 0.318 \rightarrow 31.8\% \)
- Probability of being less than 1\( \sigma \) from the mean
  - \( P(|V|<1\sigma) = 0.682 \rightarrow 68.2\% \)
  - \( P(|V|<2\sigma) = 0.046 \rightarrow 95.4\% \)
  - \( P(|V|<3\sigma) = 2.7 \times 10^{-3} \rightarrow 99.7\% \)
  - \( P(|V|<4\sigma) = 6.3 \times 10^{-5} \rightarrow 99.994\% \)
  - \( P(|V|<5\sigma) = 5.7 \times 10^{-7} \rightarrow 99.99994\% \)
Probabilities in a Gaussian Function

- One **standard deviation** from the **mean** (dark blue) accounts for about 68% of the set
- Two standard deviations from the mean (medium and dark blue) account for about 95%
- Three standard deviations (light, medium, and dark blue) account for about 99.7%
**Thermal Noise Effects on Threshold Performance**

**Signal-to-noise (S/N)**

- Noise added to signal and causes a fluctuation
- S/N is the ratio of average Signal Power to average Noise Power
- Average Signal Power
- Average Noise power is RMS Noise

\[
S = \frac{\sqrt{2}}{2} S_p
\]

\[
S = 6.15 \\
N = 0.90 \\
S/N = 6.8 \\
S/N_{dB} = 8.33 dB
\]
Noise Effecting Bit Error Rates (BER) in the Time Domain

- Noise is a Probability Density Function
  - \( \mu \) = Average noise level
  - \( \sigma \) = Standard Deviation = RMS Noise
  - RMS Noise = \( 1\sigma \) (Standard Deviation)
- BER is the probability of Noise exceeding the threshold
- Probability of Error is related to the number of \( \sigma \)'s to the boundary

\[
\begin{align*}
P(>|1\sigma|) &= 0.318 \\
P(>|2\sigma|) &= 0.046 \\
P(>|3\sigma|) &= 2.7 \times 10^{-3} \\
P(>|4\sigma|) &= 6.3 \times 10^{-5} \\
P(>|5\sigma|) &= 5.7 \times 10^{-7}
\end{align*}
\]
Minimum Input Signal Level – Single Signal

- System Information Example
  - \( C/N_{\text{MIN}} \) for successful signal reproduction (\( C/N = 10 \text{dB} \))
  - System Noise Figure (\( \text{NF}=3 \text{dB} \))
  - Signal Band Width (\( \text{BW}=10 \text{MHz} \))

- Minimum Signal level is \( S_{\text{MIN}} \)
  \[ S_{\text{MIN}} = -174 \text{dBm/Hz} + 10 \log(\text{BW}) + \text{NF} + C/N \]
  - Noise Level = -101 dBm
  - \( S_{\text{MIN}} = -91 \text{ dBm} \)

- Digital Signals are based on a Bit Error Rate
- Analog signals are based on a visual or audio quality standard
Threshold Detection Probabilities

- Threshold can be varied
- Probability can be skewed

Probability of a “1” detected as a “0”

Probability of a “0” detected as a “1” -
RADAR - Average False Alarm Rate vs Threshold to Noise Ratio

PRF: 1kHz
S/N = 11.8 dB
1 False Alarm/Hr
Detection Probability & False Alarm Rate

Example
Pd = 0.98
PFA = 0.003
Required S/N = 12 dB

Trade Off is probability of detection vs. probability of false alarms
Modulation

Generalized Modulated Carrier

\[ X_c(t) := \text{Re} \left| Ac \cdot e^{j \cdot \Theta c(t)} \right| \]

\[ X_c(t) := Ac \cdot \cos \left( \Theta c(t) \right) \]

\[ \Theta c(t) := 2 \cdot \pi \cdot Fc \cdot t + \phi(t) \]

Note: No Information in Amplitude
:. Power Amplifier can be Non-Linear

- \( X_c(t) = \) Modulated carrier
- \( Ac = \) carrier amplitude
- \( \Theta c(t) = \) Instantaneous phase
- \( Fc = \) average carrier frequency
- \( \Phi(t) = \) instantaneous phase around the average frequency Fc
- Instantaneous Frequency = \( d \Phi(t) / dt \)
AM Modulation

- Translation of Baseband spectrum to a carrier frequency
- $A_c$ is function of time

$$X_c(t) := A_c \cdot \cos \theta_c(t)$$
Frequency / Phase Modulation
Phase/Frequency (Exponential) Modulation

\[ X_c(t) := A_c \cdot \cos(\theta_c(t)) \]
\[ \theta_c(t) := 2 \cdot \pi \cdot F_c \cdot t + \phi(t) \]

- Ac is constant
- Information is contained in \( \phi(t) \)
**FM Modulation Index (β)**

\[ \Phi(t) = \text{Instantaneous Phase variation around carrier } F_c \]

\[ X_c(t) := A_c \cdot \cos \left( \theta_c(t) \right) \]

\[ \theta_c(t) := 2 \cdot \pi \cdot F_c \cdot t + \phi(t) \]

\[ X_c(t) = A_c \cos \left[ 2\pi F_c t + \phi(t) \right] \]

\[ \phi(t) := 2 \cdot \pi \cdot k_f \cdot \int_{-\infty}^{t} m(t) \, d\tau \]

- \[ m(t) = \text{Information waveform} \]
- \[ F_i = \frac{d \Phi(t)}{dt} = \text{Instantaneous Frequency around carrier } F_c \]
- \[ F_i = K_f \cdot m(t) \]
- \[ K_f = \text{Gain Constant} \]
  - \[ m(t) \text{ is normalized to } \pm 1 \]
  - \[ K_f = \Delta F \]
- \[ \Delta F = \text{Peak One sided Frequency Deviation} \]
FM Modulation Index ($\beta$)

$$X_c(t) = Ac \cos [2\pi Fc \ t + \varphi (t)]$$

$$\Phi(t) = \text{Instantaneous Phase variation around carrier } Fc$$

$$\phi(t) := 2\cdot\pi \cdot k_f \int_{-\infty}^{t} m(\tau) \ d\tau$$

- $K_f = \Delta F$
- If $m(\tau) = \cos(2\pi Fm \ \tau)$ \ [sinusoidal modulation]
- Integrating $m(t)$
- $\Phi(t) = [(2\pi \Delta F) / (2\pi Fm)] \ * \ sin \ (2\pi Fm \ \tau)$
- $\Phi(t) = (\Delta F / Fm) \ * \ sin \ (2\pi Fm \ \tau)$
- $\beta = \Delta F / Fm = \text{modulation index (Radians)}$
- $\Phi(t) = \beta \ * \ sin \ (2\pi Fm \ \tau)$
FM Spectral Analysis

- \( X_c(t) = A_c \cos (2 \pi f_c t + 2\pi k_f \int m(\tau) \, d\tau) \)
- For sinusoidal modulation: \( m(t) = \cos(2\pi f_m t) \)
- \( X_c(f) \) is the Fourier Transform of \( X_c(t) \)
- \( X_c(f) \) sequence of \( \delta \) functions at multiples of \( f_m \) from \( f_c \)
  - \( \delta \) functions at \( f_c \pm nf_m \)
- Amplitudes are Bessel Coefficients of the first kind, Order \( n \) and independent variable \( \beta \) \( [J_n(\beta)] \)

\[ B \approx 2\Delta f \]
Frequency / Phase Modulation Side Bands

- \( J_n(\beta) \) = Bessel Function of the First kind, order \( n \), Argument \( \beta \)
- \( n \) = side band number from carrier
- \( \beta \) = Modulation index in Radians
- Sideband Levels \( J_n(\beta) \) (Linear units)
- Levels in dBc = \( 20 \log_{10} [J_n(\beta)] \)

Bessel Function Solution

\[
\ln(\beta) := \frac{1}{2 \pi} \int_{-\pi}^{\pi} e^{\beta \cos(\theta)} \cos n \theta \, d\theta
\]
Bessel Function (Side Band) Levels

- Note for Low Beta, Higher order sidebands are not significant.

\[ J_n(\beta) = \ln(\beta) \]
Frequency Modulation - Low Beta

- Bessel Function of the First kind, N order, Argument $\beta$
- Low Beta ($\beta < 1$) has only 2 significant sidebands

$$J_n(\beta) := \frac{1}{2\pi} \int_{-\pi}^{\pi} e^{\beta \cos(\theta)} \cos(n\theta) \, d\theta$$

- Sideband Level = dBc = $20 \log(\beta/2)$
- AM sidebands are in phase
- FM sidebands are out of phase
Phase Modulation

\[ X_c(t) := Ac \cdot \cos \left( \theta_c(t) \right) \quad \theta_c(t) := 2 \cdot \pi \cdot F_c \cdot t + \phi(t) \]

- Phase Modulation: \( \Phi(t) \)
- \( \Phi(t) = \beta \cdot m(t) \): \( \beta \) = peak phase deviation
  - \( \beta \) = Modulation Index in Radians, same as FM
  - \( m(t) \) = information normalized to 6 1
- \( X_c(t) = Ac \cdot \cos(2 \cdot \pi \cdot F_c \cdot t + \beta \cdot m(t)) \)
- \( \beta \) is the same for PM or FM
- For small \( \beta \)
- Sideband Level = \( \text{dBc} = 20 \log(\beta/2) \)
Oscillator Basics

- Negative Resistance Oscillators
- Feedback Oscillators

**Negative Resistance Oscillators - Basic Configuration**

**Resonator Circuit**
- Resonator: LC, Stub, Varactor Tuned Circuit, YIG, etc.

**Active Circuit**
- Transistor, Tunnel Diode, Gunn Diode, etc.

**Output Network**
- Passive Matching Ckt & Buffer Amplifier -
Theory of Negative Resistance Oscillators

- at Resonance (Fo)
  - ZL is real only at the resonant frequency (ZL(Fo))
  - ZL(Fo) = -Zo
  - Result: Reflected voltage without an incident voltage (oscillates)

- An Emitter Follower is a classic negative resistance device
- Technique used at microwave frequencies
  - Spacing between components often precludes the establishment of a well defined feedback path.

Resonator is a One port network

\[ Z_L = R_L @ \omega_0 \]
\[ Z_o = -R_L \]

Reflection coefficient

\[ \rho := \frac{V_r}{V_i} \]
\[ \rho := \frac{Z_L - Z_o}{Z_L + Z_o} \]
Feedback Oscillators (Two port networks)

- \((V_1 + V_0 \cdot H_2) \cdot A \cdot H_1 = V_0\)
- \(V_1 \cdot A \cdot H_1 = V_0(1 - A \cdot H_1 \cdot H_2)\)

\[
\frac{V_0}{V_1} := \frac{(A \cdot H_1(s))}{1 - A \cdot H_1(s) \cdot H_2(s)}
\]

- \(A \cdot H_1(s) \cdot H_2(s) = \) open loop gain = \(AL(s)\)
Barkhausen Criteria

- Barkhausen criteria for a feedback oscillator
  - open loop gain = 1
  - open loop phase = 0
  - $|A*H1(s)*H2(s)| = |AL(s)| = 1$
  - Angle $(A*H1(s)*H2(s)) = 0$
  - $s = \omega_0$ (for sinusoidal signals)
  - $\text{Re } AL(\omega_0) = 1$
  - $\text{Im } AL(\omega_0) = 0$
  - Transfer function blows up (Output with no Input) - Oscillation
    - Vo is finite when $V1 = 0$ -

\[
\frac{Vo}{V1} := \frac{(A \cdot H1(s))}{1 - A \cdot H1(s) \cdot H2(s)}
\]
Starting an Oscillator

- To start an oscillator it must be triggered
  - Trigger mechanism: Noise or a Turn-On transient
- Open loop gain must be greater than unity
- Phase is zero degrees (exponentially rising function)

- Poles in the Right Half Plane
- Exponentially Rising Function

\[ x_n := e^{\alpha t_n} \cdot \cos(2\pi \omega t_n) \]

\[ \alpha = \text{Real Part of } A*H1(s)*H2(s), \quad > 1 \]
Amplitude Stabilization

- As amplitude increases, gain decreases, the effective $g_m$ (transconductance gain) is reduced.
- Poles move toward the Imaginary axis.
- Oscillation amplitude stabilizes when the poles are on the imaginary axis.
- Self-correcting feedback (variable $g_m$) maintains the poles on the axis and stabilizes the amplitude.
Frequency Stability Analysis

- Conditions for Oscillation
  - Sufficient gain in the 3 dB bandwidth (Open Loop Gain > 1)
  - At Fo; Sum of all components around the loop are real (Resistive, Zero Phase)
  - Circuit oscillates at resonance \( \omega_0 = \frac{1}{(LC)^{1/2}} = 2\pi F_0 \)

\[
F = \frac{1}{2\pi \sqrt{LC}}
\]

\[
BW_{3dB} = \frac{F_0}{Q}
\]
Coarse & Fine Frequency Stability

- **Coarse frequency** of oscillation is determined by the resonant frequency - **Amplitude**
- **Fine Frequency** of oscillation is determined by **PHASE**
  - Loop phase shift is automatically compensated
  - Phase changes forces frequency off of $F_0$
- 3 dB bandwidth provides +/-45° compensating phase

\[
F_0 = \frac{1}{2\pi \sqrt{LC}}
\]

\[
\text{BW}_{3\text{dB}} = \frac{F_0}{Q}
\]
Oscillator Stability

- Factors Affecting Oscillator Stability
  - Stability of the Resonator
  - Q of the resonator
- Causes of Oscillator Frequency Drift
  - Change in resonant frequency
  - Change of Open Loop Phase
  - Amplitude Changes
    - Oscillators operate in a non-linear mode
    - Changes in Amplitude changes phase -

\[ F_0 = \frac{1}{2\pi \sqrt{LC}} \]

\[ \text{BW}_{3dB} = \frac{F_0}{Q} \]
Parasitic Phase Shifts vs Frequency Stability

- $Q = \frac{F_0}{BW_{3dB}} \Rightarrow BW_{3dB} = \frac{F_0}{Q}$
- 1 Pole Resonant Circuit
  - 3 dB bandwidth shifts +/- 45°
- Phase change
  - If maximum $\Delta F_0 = BW_{3dB}$
  - $(\Delta F_0 / \Delta \phi)$ sensitivity of the frequency to phase changes
  - $(\Delta F_0 / \Delta \phi) \approx BW_{3dB} / 90°$
  - $\Delta F_0 = BW_{3dB} = \frac{F_0}{Q}$
  - $\Delta F_0 / \Delta \phi \approx [F_0/Q]/90°$ (Hz/Deg)
  - $\Delta F_0 / \Delta \phi \approx F_0/(Q*90°)$ (Hz/Deg)
- Higher Q Smaller $\Delta F_0 / \Delta \phi$ (phase)
Parasitic Phase Shifts vs Frequency Stability

- Frequency stability vs Phase is proportional to Q
  - Phase changes around the loop
  - Loop Self Corrects Phase Variations
- Parasitic Phase shifts have less effect on frequency in Higher Q circuits -
Frequency Stability – Resonator Dependent

- Center Frequency Resonator (Fo)
- Q of the Resonator
  - Phase Stability (A function of Q=\(F_0/BW_{3dB}\))
- \(\Delta F_0 / \Delta \phi \text{ (Hz/Deg)} \approx F_0 / [90 \degree Q]\)

<table>
<thead>
<tr>
<th></th>
<th>Q Min</th>
<th>Q Max</th>
<th>Stability PPM/C</th>
</tr>
</thead>
<tbody>
<tr>
<td>LC Resonators:</td>
<td>50</td>
<td>150</td>
<td>100</td>
</tr>
<tr>
<td>Cavity resonators</td>
<td>500</td>
<td>1000</td>
<td>10</td>
</tr>
<tr>
<td>Dielectric resonators:</td>
<td>2,000</td>
<td>10000</td>
<td>1</td>
</tr>
<tr>
<td>SAW devices:</td>
<td>300</td>
<td>10000</td>
<td>0.1</td>
</tr>
<tr>
<td>Crystals</td>
<td>50000</td>
<td>100000</td>
<td>0.01</td>
</tr>
</tbody>
</table>
Oscillator Stability

- **Long Term Frequency Stability**
  - Usually a function of the Resonator Center Frequency Stability
  - Change in Frequency ($\Delta F$) with respect to center frequency ($F_0$)
  - Stated as $\Delta F/F_0$ in Parts Per Million (PPM)
  - Time frame: Typically hours to years
  - Stability over Temperature

- **Short Term Frequency Stability**
  - Usually a function of noise perturbations
  - Residual FM
  - Allen Variance
  - Phase Noise -
Residual FM
Slow Moving Frequency Variations

- Change in frequency $\Delta F$ is much greater than the rate of frequency change, $f_m$ ($\Delta F/f_m = \beta >> 1$)
- Spectrum has a flat top
  - Peak to Peak change in frequency is the Residual FM
- Typically measured 6dB down from the peak -
Allen Variance

Phase / Frequency Noise Variations >1 Second

- Defines accuracy of clocks
- One half of the time average over the sum of the squares of the differences between successive readings of the frequency deviation sampled over the sampling period.
- Allen variance is function of the time period used between samples
- Measure frequency at time interval T2-T1
- \((F_2 - F_1) / F_1\) is the fractional change in frequency over time interval T2-T1

Pulse width
Is constant - Tau

\[ F_1 \quad \text{T1} \quad F_2 \quad \text{T2} \quad F_3 \quad \text{T3} \]
Two-point Allen variance - $\sigma_y(\tau)$

- Time domain measure of oscillator instability.
- It can be directly measured using a frequency counter
  - Repetitively measure the oscillator frequency over a time period $\tau_a u$.
- Allen variance is the expected value of the RMS change in frequency with each sample normalized by the oscillator frequency.
- Data is in Parts per Million or Parts per Billion
Allen Variance Computation

- Typical specification might be frequency variation in 100 seconds
- Take two samples of frequency a 100 seconds apart
- Repeat the measurement
- Allen variance is the $\frac{1}{2}$ the square root of the sum of the squares of all the samples taken -

![Graph showing Sample Time (seconds) vs Sq. Root Variance]

- Inverse of time between samples time is carrier offset
- 0.001 Hz to 1kHz
Frequency Stability and its Effect on Phase Noise

- Resonators Stability – Does Not Effect Phase Noise
- Phase Sensitivity Effects Phase Noise
Phase Noise - Short Term Stability

- Measures oscillator Stability over short periods of time
  - Typically 0.1 Seconds to 0.1 microseconds
- Noise varies the oscillator phase/frequency
  - **Not amplitude related**
- Noise level increases close to the carrier
  - Typical offset frequencies of interest: 10Hz to 10MHz
  - Stability closer to the carrier is measured using Allen Variance
  - Noise further from the carrier is usually masked by AM thermal noise
- Phase Noise cannot be eliminated or affected by filtering
- Phase & Frequency are related:
  - Frequency is the change in phase with respect to time
  - $\Delta \varphi / \Delta t \rightarrow d\varphi/dt$ as $t \rightarrow 0$
Short Time Phase / Frequency Noise (<1 Second)

- Specified and measured as a spectral density function typically in a 1 Hz bandwidth
- Normalized to dBc/Hz at a given offset from the carrier
- Level relates phase noise in degrees

- Modulation index ($\beta$) of noise in a 1 Hz bandwidth
- Level in dB = 20 Log ($\beta$/2) where $\beta$ is in radians
Phase Noise Measurement

- Measurement at a frequency offset from the carrier (fm) is the time interval of phase variation
  - 1 kHz offset is phase variation in 1 millisecond
  - Resolution Bandwidth is the dwell time of the measurement

- 1 Hz resolution bandwidth is a 1 second measurement time
- A 1 Hz resolution bandwidth at 1 kHz from the carrier
  - Measuring phase variation in 1 millisecond averaged 1000 times (1 Hz) -
Measurement Data

- Data is normalized to a 1Hz resolution bandwidth
- Data is actually taken at much faster rates
- In automated test equipment
  - Rates are shortened as the analyzer gets further from the carrier
  - Accurate measurement don’t require averaging 1000 times

\[ \text{dBc/Hz} \]
Conclusion

- Thermal Noise can be thought of as a vector with a Gaussian amplitude at any phase
- This vector adds to the desired signal and creates an uncertainty in the signal characteristic
- If thermal noise changes the phase characteristic of the device it has to be evaluated as phase modulation
- This phase modulation has a Gaussian phase distribution which adds to the phase characteristic of the desired signal
- Phase Noise is dominant close to the carrier (greater than thermal noise)
- Demodulation close to the carrier must consider Phase Noise levels as well as amplitude related thermal noise levels
- Part 2 will focus:
  - Phase Noise Generation
  - Phase Noise Models
  - Effects on Digital Modulation