Phase Noise

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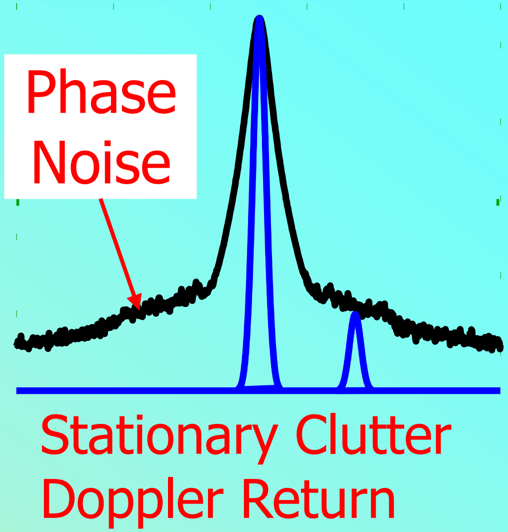
- Part 1
  The Fundamentals of Phase Noise
- Part 2
  Phase Noise Models & Digital Modulation Techniques
- Part 3
  Effects of Phase Noise on Signal Recovery

The Fundamentals of Phase Noise - Part 1

Topics

- Thermal Noise Characteristics
- Thermal Noise Effects on Threshold Performance
- Frequency / Phase Modulation
- Oscillator Basics
- Oscillator Stability
- Frequency Stability Related to Phase Noise
- Phase Noise Spectral Density -

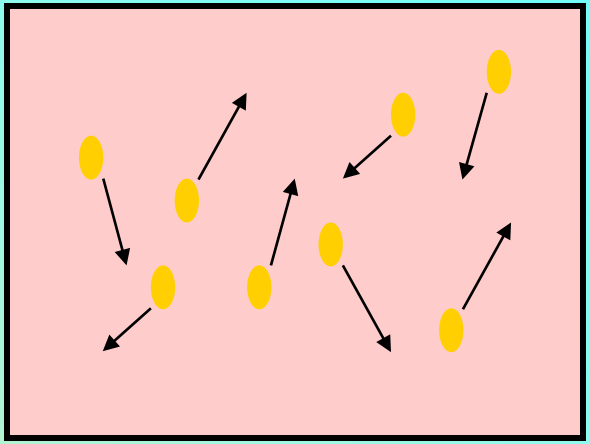
Applications Affected by Phase Noise

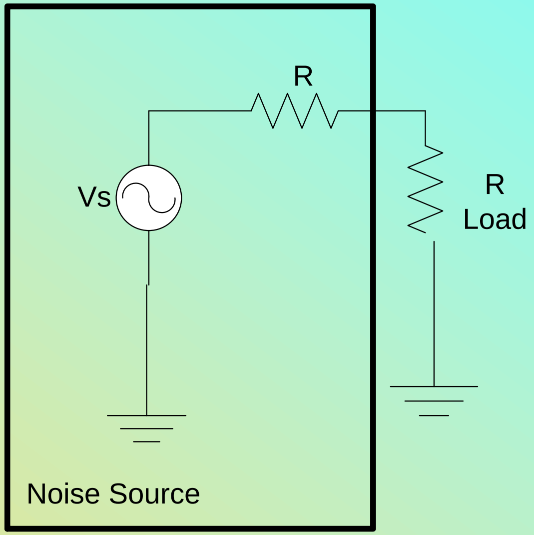


- Digital Communications
  - Causes Bit Errors
  - Not related to signal level
  - Causes timing errors
- Doppler RADAR
  - Limits the ability to identify slow moving objects
- Phase Tracking Systems
  - Causes tracking errors
- Phase Lock Loops
  - Trade off phase lock frequency tracking and noise compensation
  - Can limit phase lock loop acquisition/reacquisition

Thermal Noise Characteristics

- Thermal Noise is the random motion of electrons
  - At 0°K all motion stops - Zero Thermal Noise
  - Thermal Noise can be related to temperature
    - Excess thermal noise can be related to an increase in temperature: °K
- Thermal Noise level
  - Unknown at any instant of time
  - Statistically well behaved
    - Precisely known over a long time
    - Averaging time >>1/BW -



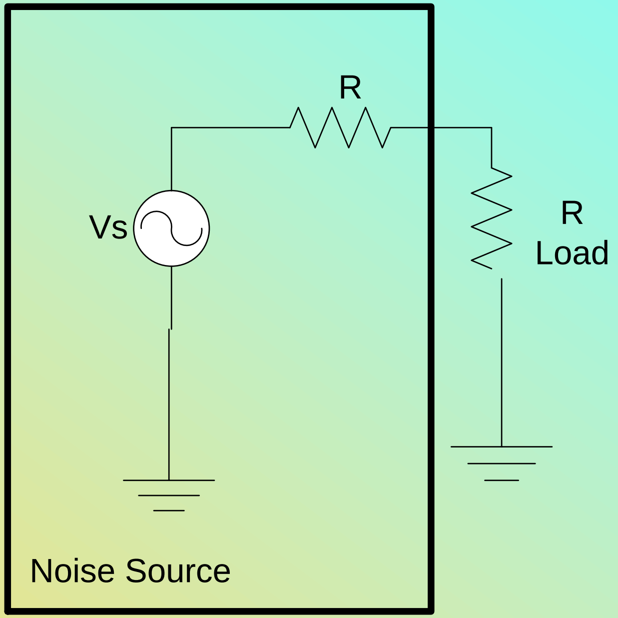


Deriving Thermal Noise

- Thermal Noise is only present in Real Elements, e.g. resistors, etc.
- Reactive elements have zero average thermal noise (L's & C's)
- Thermal noise in a real element, e.g. Resistor, is:

$$v_n = \sqrt{\frac{4h\,f\,BR}{e^{hf/kT} - 1}}$$

- h x f is momentum of a electromagnet particle
  - h = Planck's Constant: h=6.626 x 10^{-34} J*S
  - f = frequency (Hz)
- B is Band width (Hz)
- R is Resistance (Ohms)
- k is Boltzmann's constant
  - k= (-228.6 dB/ºK/Hz)
  - [Boltzman's Constant (dB)]
- T is temperature in degrees Kelvin -



Deriving Thermal Noise

$$v_n = \sqrt{\frac{4h\,f\,BR}{e^{h\,f/kT} - 1}}$$

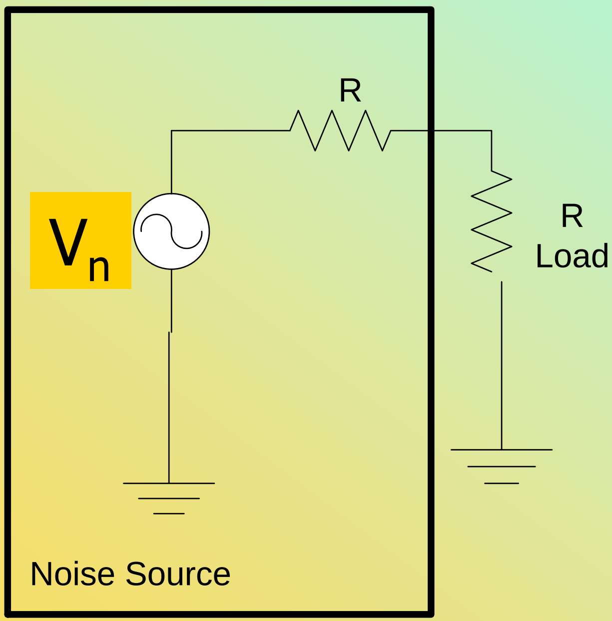
- h x f << kT

$$e^{h\,f/kT} - 1 \approx \frac{h\,f}{kT}$$

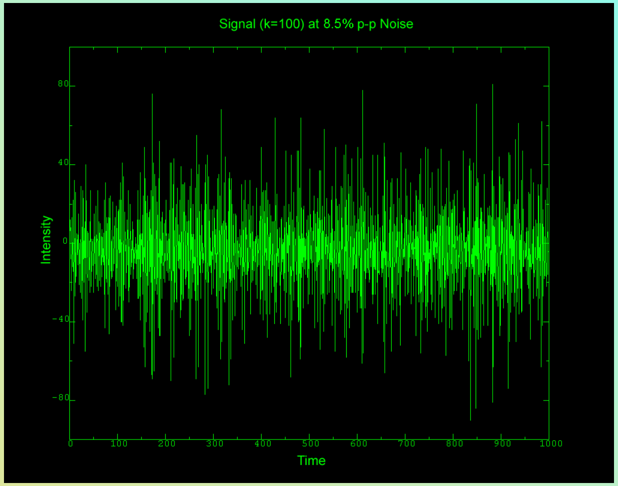
- h x f term cancels out
- Noise Voltage $V_n = \sqrt{4kTBR}$

Thermal Noise into a Load

- Noise into a matched load is: $V_n /2$



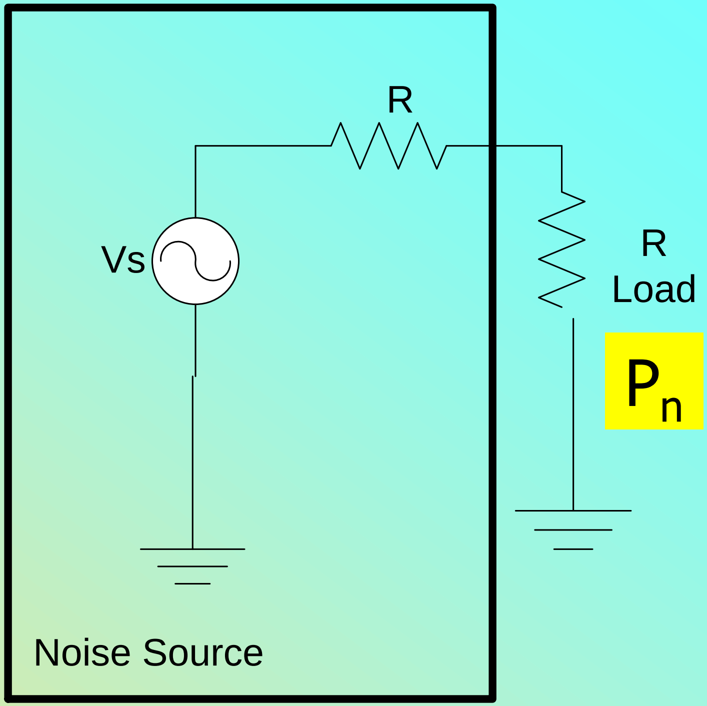
- Noise Voltage $V_n = \sqrt{4kTBR}$
- Noise Current $= I_n = V_n /(2R)$
- Noise Power $= I_n^2 R$



- Noise Power, $P_n = \left(\frac{v_n}{2R}\right)^2 R$
- $P_n = \frac{v_n^2}{4R} = kTB$

Deriving Thermal Noise

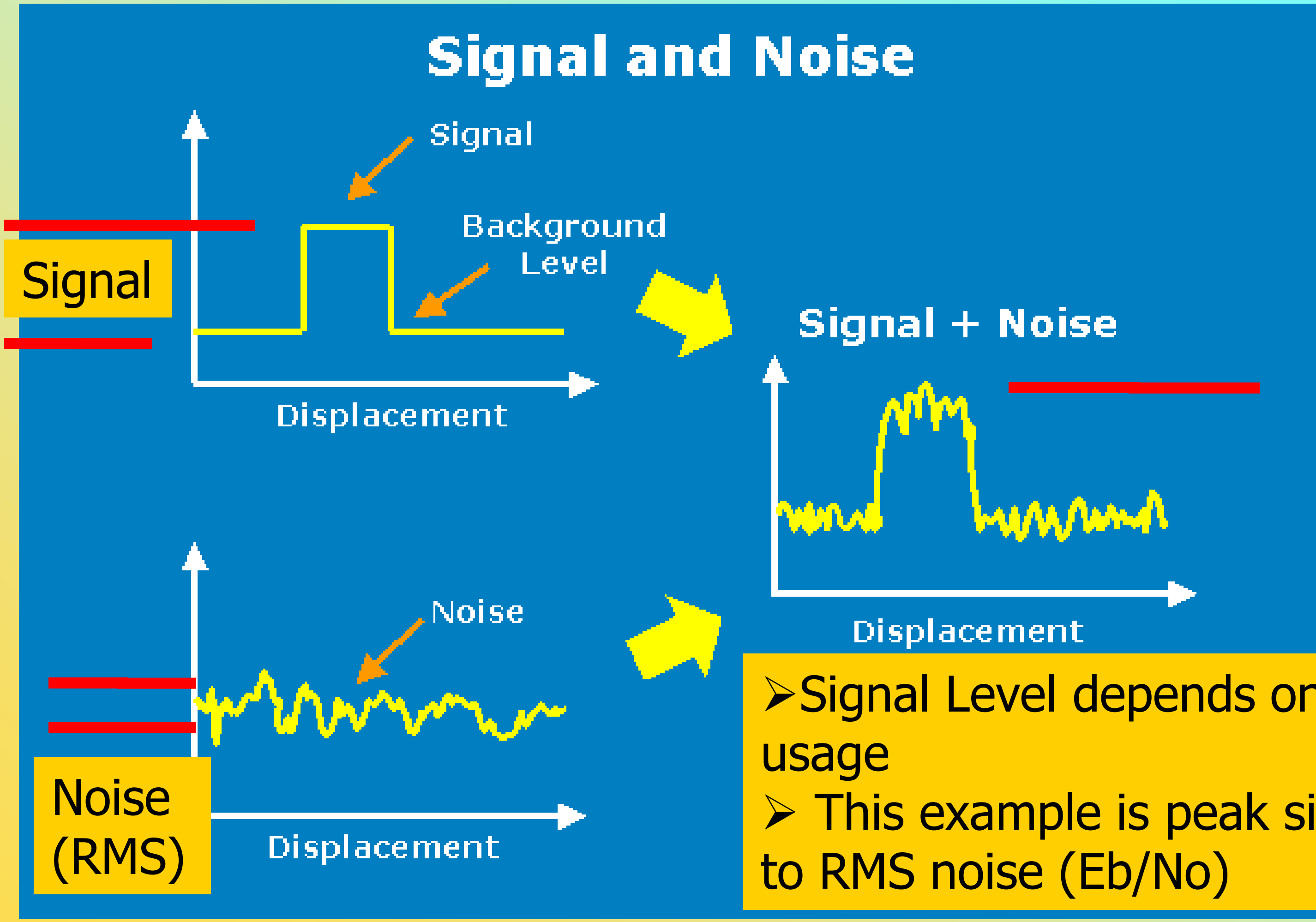
- P_n = Thermal Noise Power = kTB (Watts)
  - k = Boltzman's Constant
  - k= (-228.6 dB/ºK/Hz) [Boltzman's Constant (dB)]
  - T = Temperature in Degrees Kelvin
  - B is bandwidth in Hz
- At Room temperature T= 25 C ➔ 298 K
- kTB = 4.11 x 10^{-18} milliWatts in a 1 Hertz Bandwidth ➔ – 173.859dBm/Hz (≈-174dBm/Hz)-



k= 1.3807 × 10^{-23} joule/K

Signal to Noise Ratio

- Measure of relative signal power to noise power



- ➢Signal Level depends on usage
- ➢ This example is peak signal to RMS noise (Eb/No)

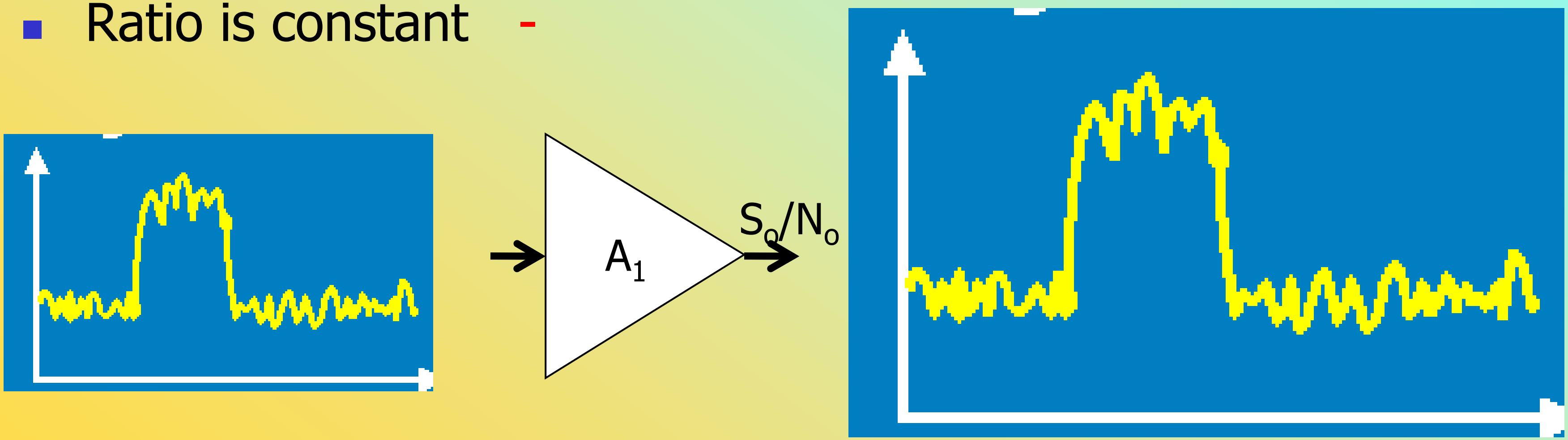
Noise Figure

- **Noise figure** is defined as a degradation in Signal to Noise Ratio

$$F = \frac{Si/Ni\ (input)}{So/No\ (output)} \geq 1$$

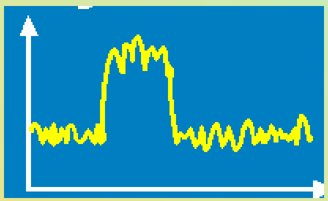
- Si/Ni is always greater than or equal to So/No
- F is the Noise Factor (Linear units)
- NF (dB) = S_{in}/N_{in} (dB) – S_o/N_o (dB)
- NF = 10 Log(F) in dB
- Amplification doesn't improve S/N
- Ratio is constant -

F >= 1
NF >= 0



Noise Figure Degradation

- Every Real Component adds Noise
- Low Noise systems
  - Amplify the input Signals & Noise
  - Minimizes the effects of other system noise generators



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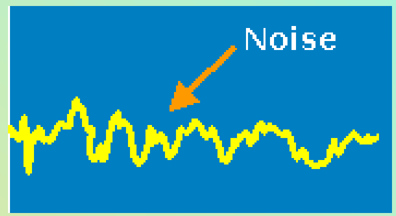
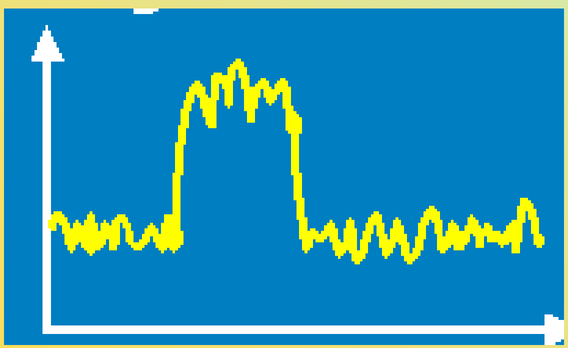


= Degrades S/N



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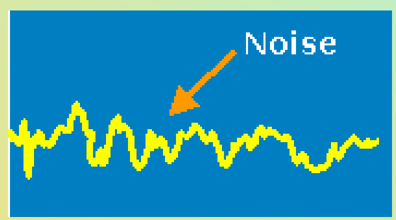
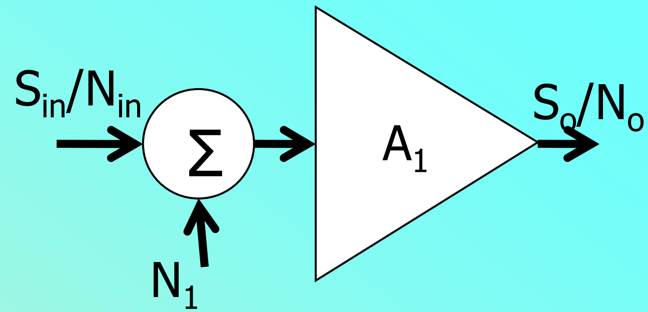


= Less degradation S/N

- S/N is degrades in every real component
  - At constant temperature and band-width -

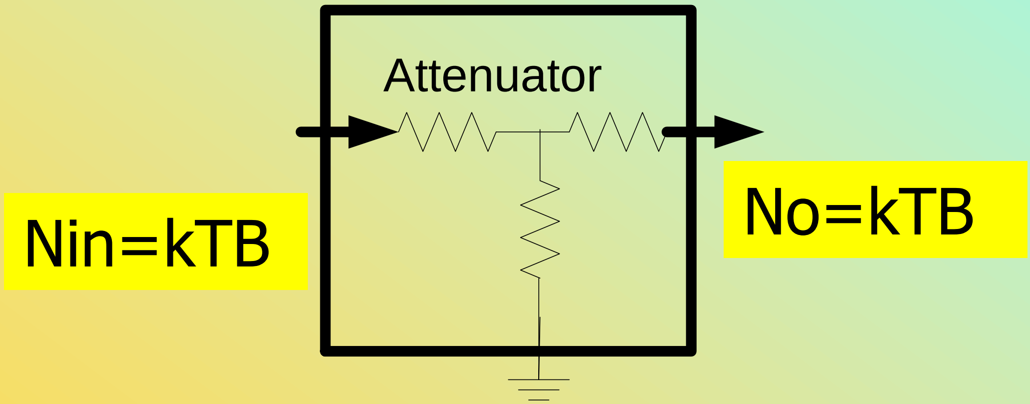
Noise Figure & Total Effective Input Noise

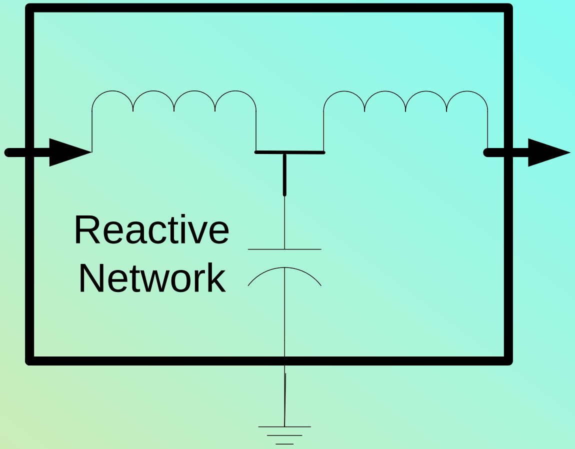


- At the input of a device
  - Signal Input (S_{in})
  - Thermal Noise (N_{in}) + Device noise (N_1)
  - All add together & get amplified
- Example of Effective Input Noise Level (N_{in}) = kTBF
  - kTB → -174dBm in a 1Hz BW
  - F → NF = 10 dB
  - B = 5MHz → 10Log(5MHz/1Hz) = 67dB
  - N_{in} = KTB(dB) + NF = -174 dBm + 10 dB + 67dB= -97 dBm in a 5 MHz Bandwidth
- Noise can be reflected to the input or output
  - Output Noise (N_o) is Input Noise times device gain (A_1) -

Noise Figure of a Passive Element

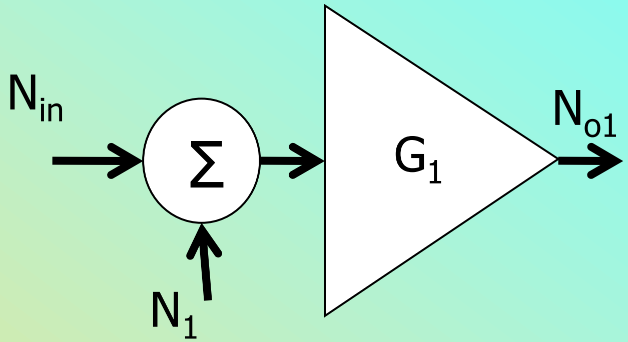
- Thermal noise does not add
  - Noise at the output of a resistor is the same as the input of a resistor
  - Signal decreases therefore S/N degrades





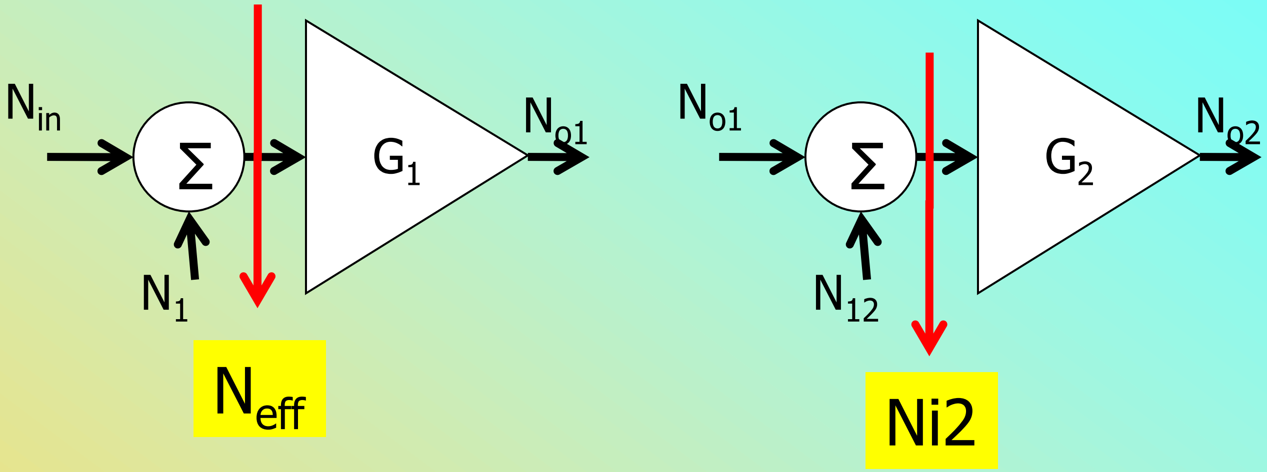
- Ideal reactive elements have no loss
  - Reactive Networks store power, don't dissipate power
  - Noise figure is 0dB if the device has no loss -

First Stage Output Noise



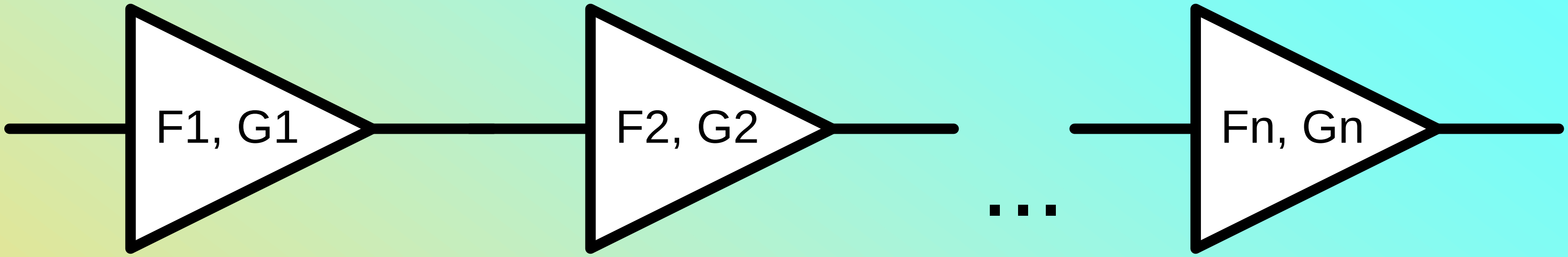
- Noise at the output of the 1st stage
  - N_{in} = kTB
  - Noise Factor = Input device noise above kTB
  - N_1 = F1*kTB –kTB = (F1-1)*kTB, F1 is the factor above kTB
  - Total input noise = kTB + (F1-1)*kTB = F1kTB
  - Total Output Noise = N_{O1} = kTB*F1*G1 -

Multistage (cascaded) System



- Noise at the input of the 2nd stage (including 2nd stage noise)
  - Ni2 = No1+ kTBF2- kTB (can't add thermal noise twice)
  - Ni2 = No1+ kTB* (F2-1)
  - Ni2= kTB*F1*G1+ kTB* (F2-1)
- Effective input noise = N_{eff}
  - N_{eff} = Ni2/G1 = kTB*F1+ kTB* (F2-1)/G1
    - Amp #2 is noiseless when you consider the input noise = N_{eff} -

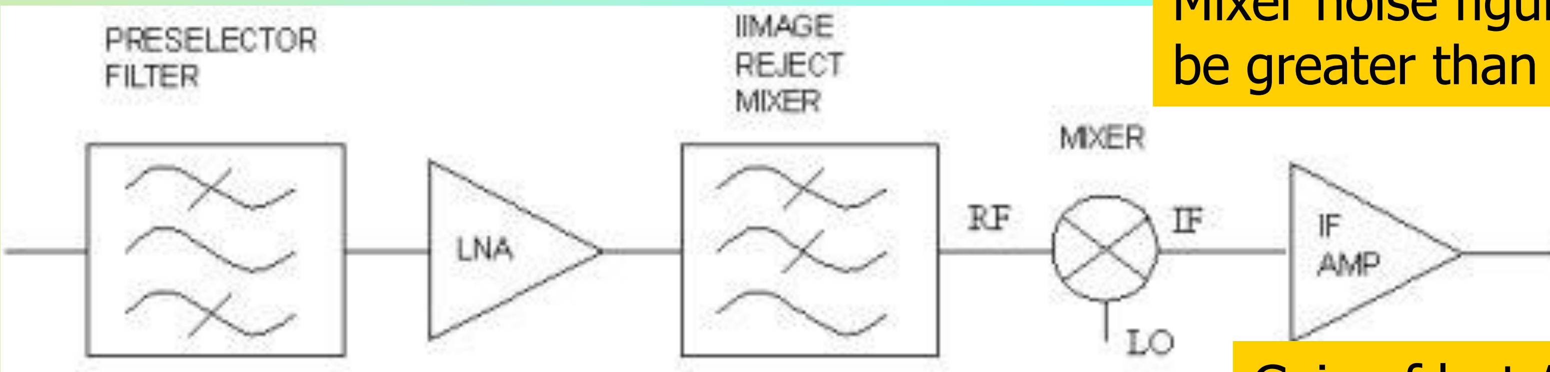
Noise Figure of a Multistage (cascaded) System



- N_{eff} = kTB(F1+[F2-1]/G1) = kTB F_{eff}
- Effective Input Noise factor F_{eff} =F1+[F2-1]/G1
- NF_{eff}=10Log(F_{eff}) ➔ Effective input Noise Figure
- Applying this formula to many stages

$$F = F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1 G_2} + \ldots + \frac{F_n - 1}{G_1 G_2 \cdots G_{n-1}}$$

Cascaded Noise Figure Example



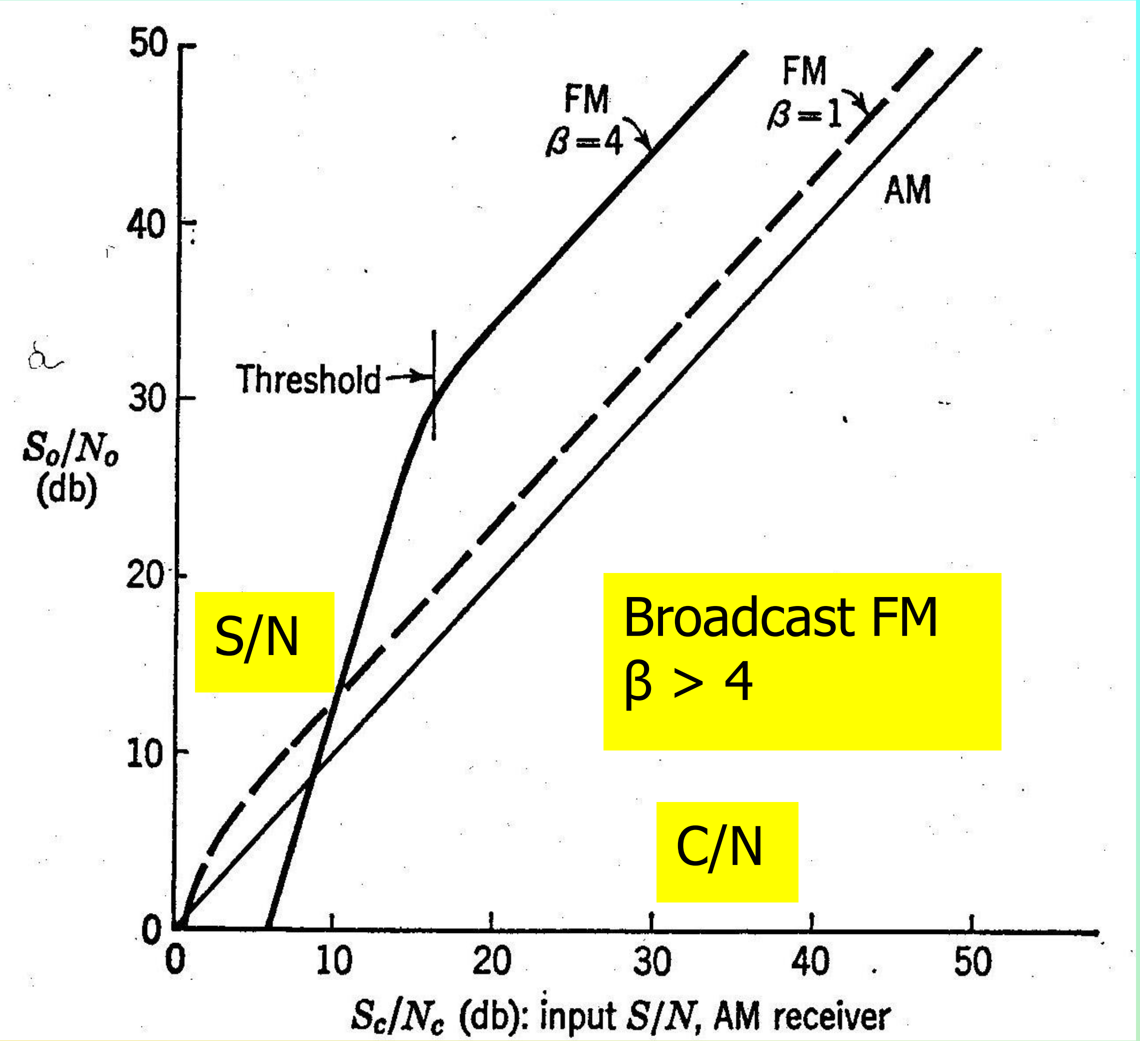
Mixer noise figures can be greater than loss

Gain of last Amp doesn't affect NF

	Preselector filter	LNA	image reject filter	mixer	IF amp
Component data (from datasheets)					
Gain (dB)	-0.5	20	-1	-6	30
NF (dB)	0.5	3	1	6	5
Cascaded gain and noise figure (convert back to dB)					
NF dB	0.5	3.5	3.505632	3.586457	3.811745
Gain (dB)	-0.5	19.5	18.5	12.5	42.5

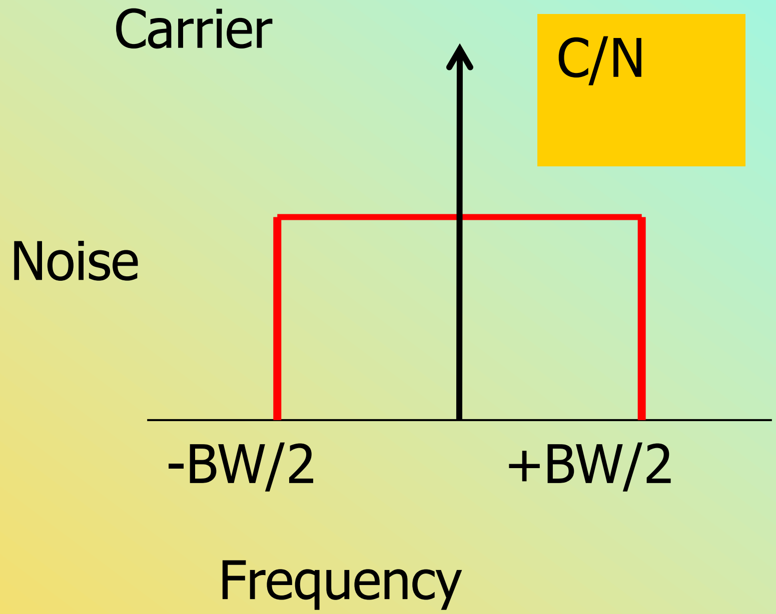
AM / FM Comparison
De-Modulated Signal to Noise

- AM & FM S/N do not have the same performance through a demodulator
- FM: S/N Improvement
  - Input S/N must be above threshold
- Phase Lock de-modulator has no Threshold effect -



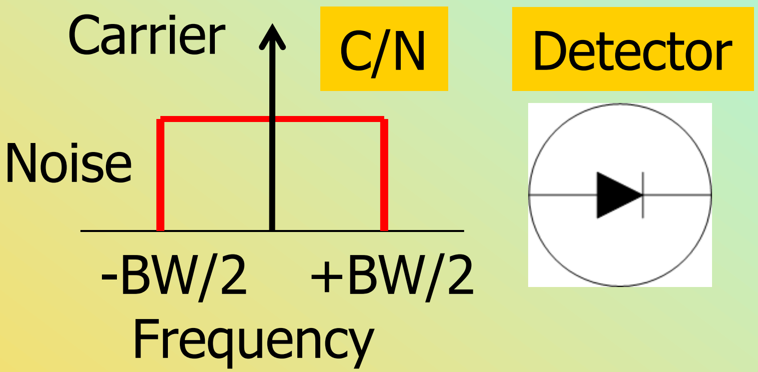
Carrier to Noise Ratio

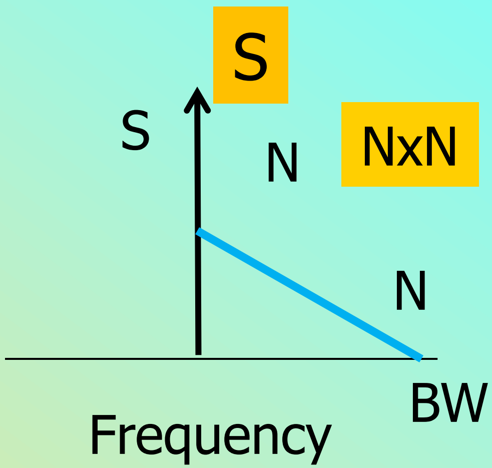
- Assume the carrier is CW with P_{ave} = C, for simplicity
- C/N = C, Carrier level divided by the noise spectral density function integrated over the spectrum –BW/2 ➔ +BW/2 -



Detected Noise

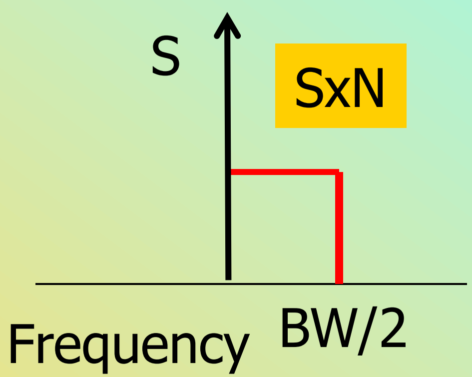
- Noise through a non-linear device (diode) produces a unique characteristics in frequency & amplitude





- Linear Slope due to convolution of a two rectangles

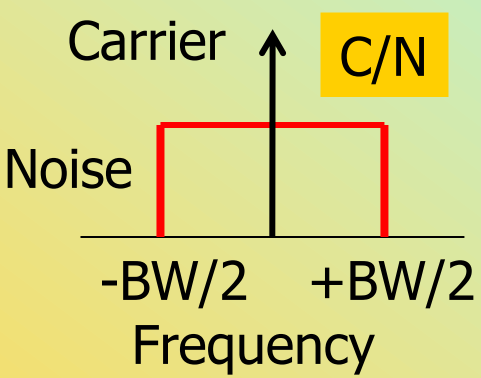
- Detected output has three components
- Noise mixing with Noise
- Noise mixing with signal
- Recovered Signal (S)

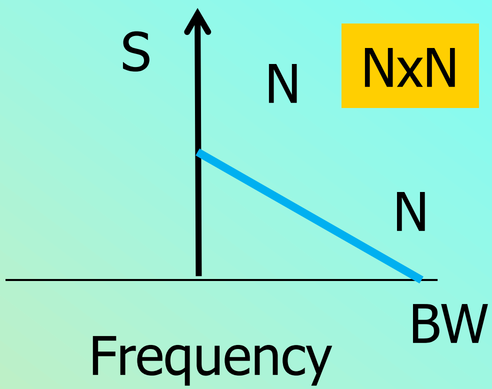


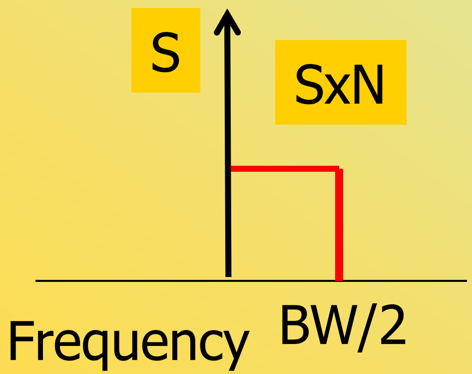
- Rectangle due to convolution of an impulse & a rectangles -

Detected Signal + Noise

- The S x N is negligible at High S/N
- At low S/N the S x N term identifies signal presence but does little in decoding the signal, usually cannot be processed
  - In Radar MDS is usually the S x N term

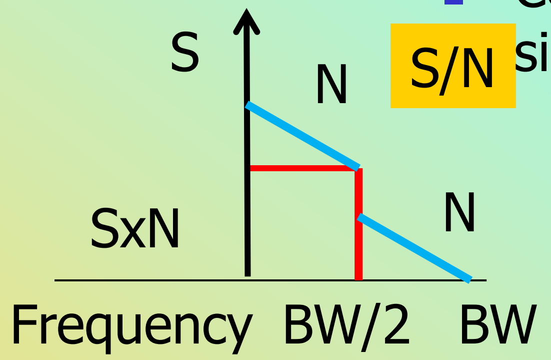






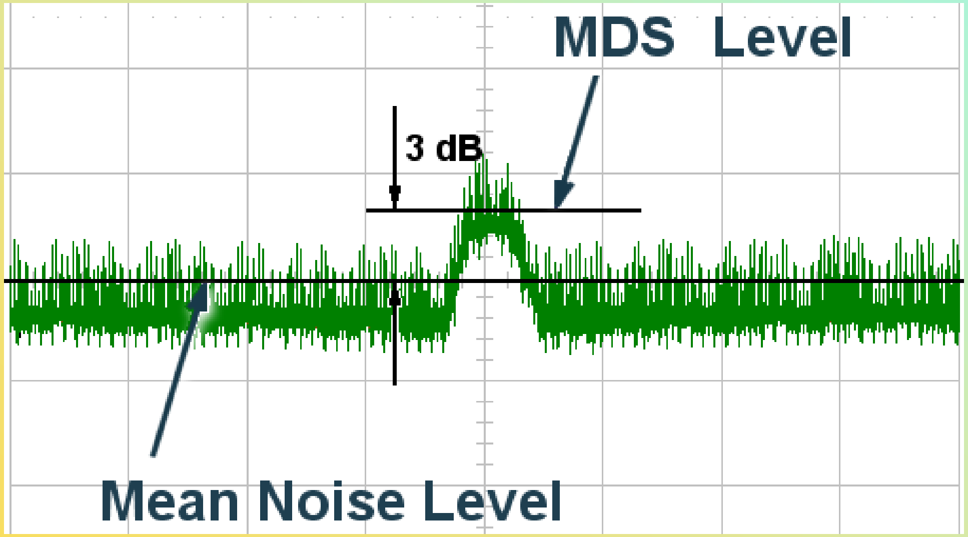
- SxN is a Rectangle due to convolution of an impulse & a noise rectangles -

- Composite signal



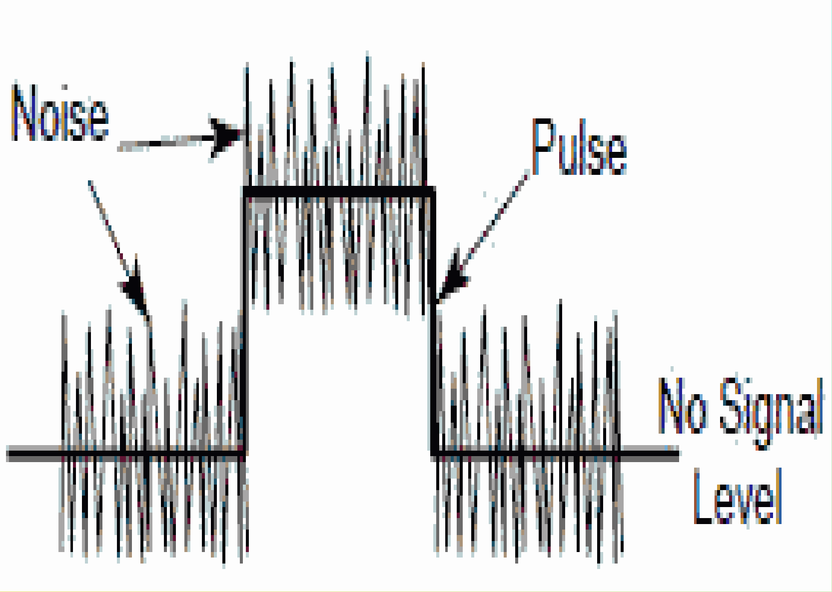
Detection at Low S/N

- Detected terms
  - S, Detected Signal
  - N x N, Noise times Noise Term
  - S x N, Signal times noise term
- The most basic Radar function is detecting signal presence
- Minimum Detectable Signal (MDS)
  - Signal presence is detected
  - Signal recovery is doubtful
  - (S + SxN)/N because SxN is only present with Signal -



Decoding Signal information

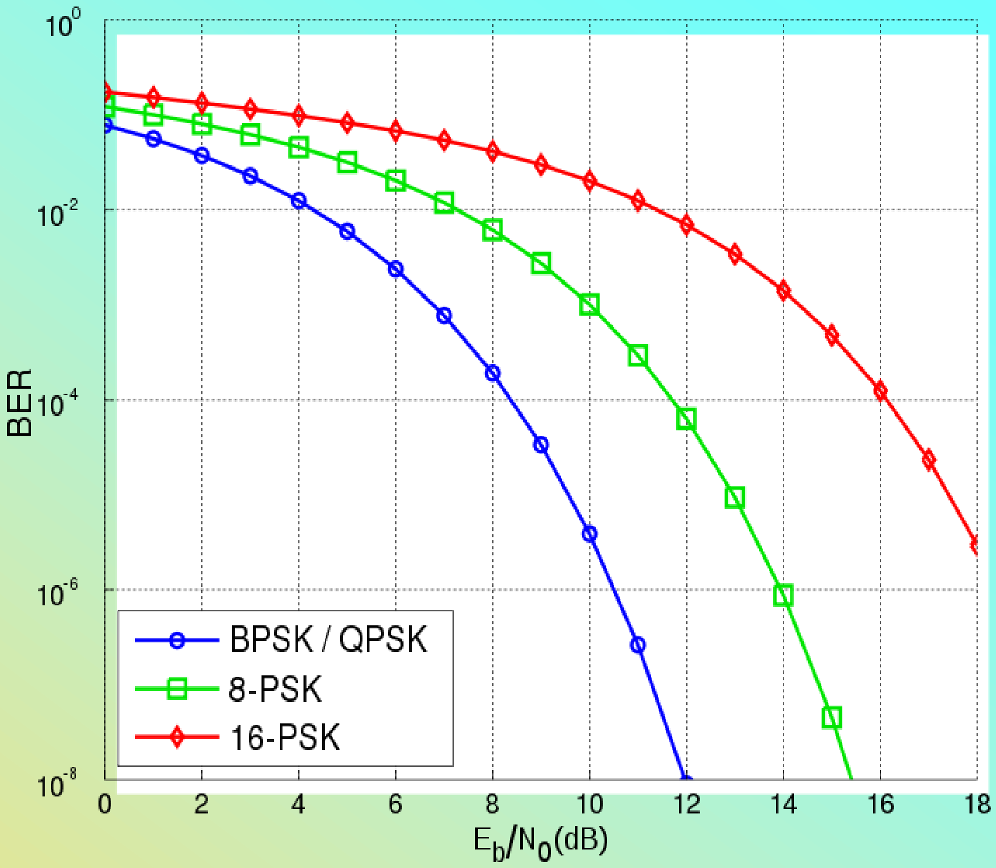
- Signal (S) must be greater than, N x N term + S x N term
  - For determination of Signal quality or Signal recovery
- Bit Error Rate (BER) the ratio S/(N+SxN) must be considered -



- Tangential Sensitivity is an ambiguous term
- 6dB<S/N<9dB

Signal vs. Noise Expressions

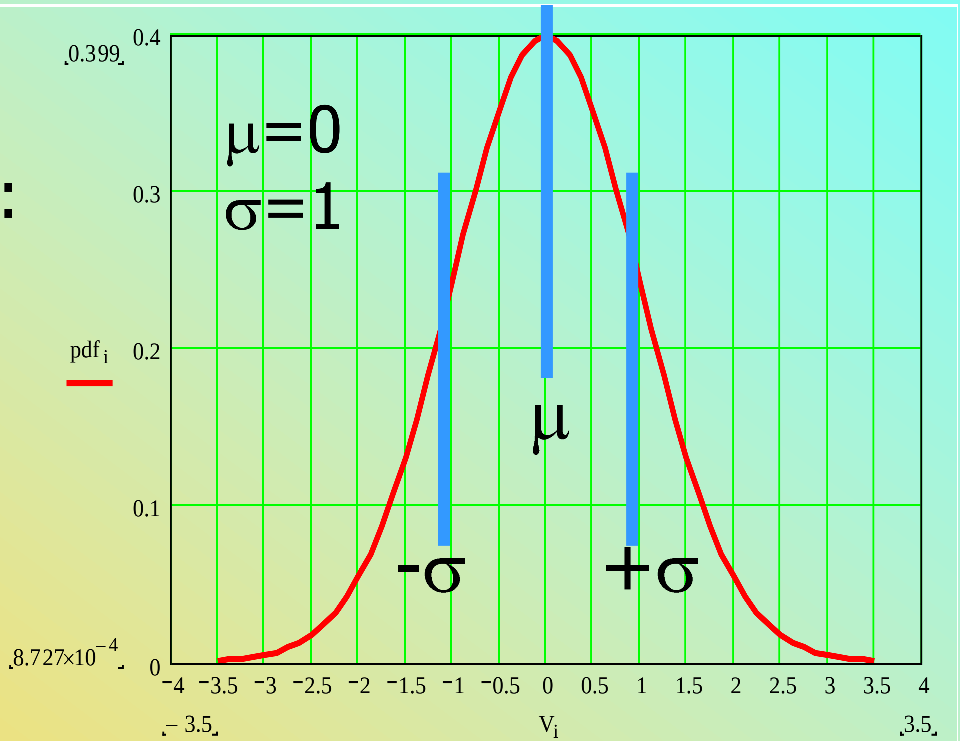
- C/N: Carrier to Noise Ratio
  - Pre-detection Signal over noise
- S/N: Signal to Noise Ratio
  - Post detection Signal over noise
- Eb/No : Bit Energy to Noise Power
  - Eb/No = So/No * (BW/Rb)
    - BW is IF bandwidth (Hz)
    - BW is related to symbol rate
    - Rb = Bit Rate (Bits/Second)
  - Assuming Signal to Noise ratio with an optimized bandwidth -



Noise as a Probability Density Function

- Noise: Gaussian Function
- Well defined amplitude probability distribution (pdf)
  - μ is Average (Mean)
    - Signal Level or Zero
  - σ=standard deviation: Relates to the function spreading
  - σ ← → RMS Noise
- Thermal Noise = kTB = -174dBm at 298 K

$$pdf := \frac{1}{\sqrt{2\cdot\pi\cdot\sigma^2}}\cdot e^{\frac{-(V-\mu)^2}{2\cdot\sigma^2}}$$

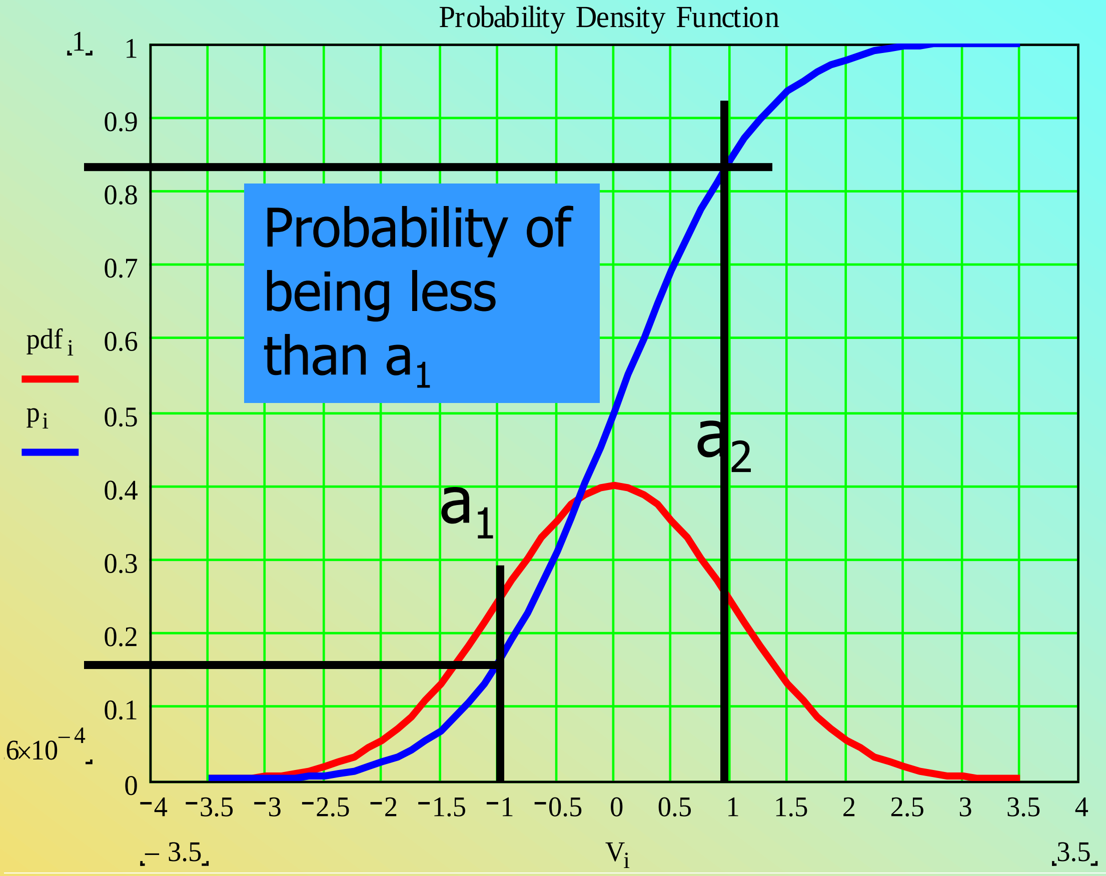


Gaussian Noise

- Total Area under the probability curve is 1
- Probability of being in any sector of the function is the area under the function

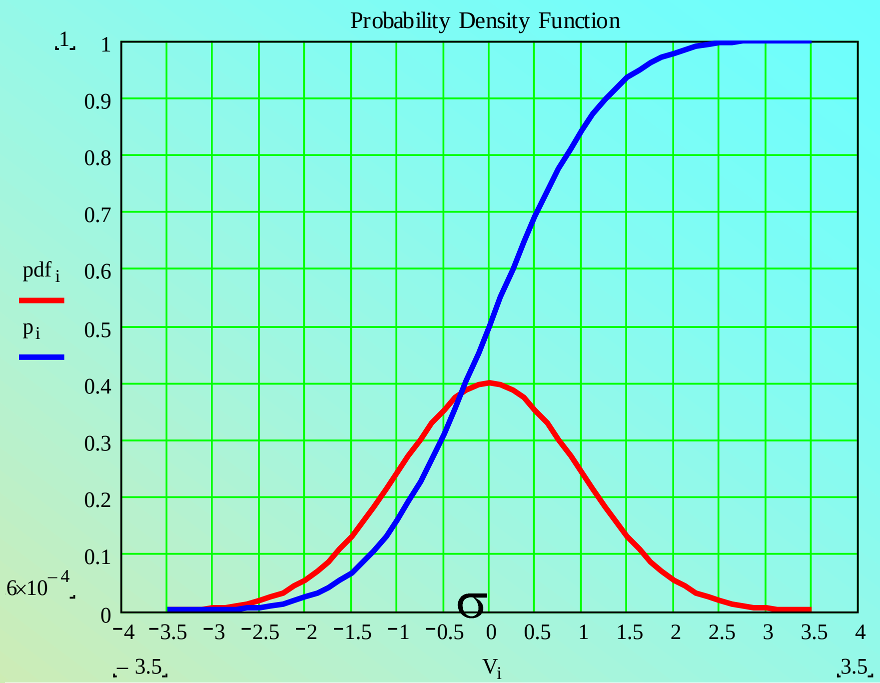
$$p_i := \frac{1}{\sqrt{2\cdot\pi\cdot\sigma^2}}\cdot\int_{-\infty}^{a_i} e^{\frac{-(V-\mu)^2}{2\cdot\sigma^2}}\,dV$$

- Integrating the Gaussian Function from - ∞ ➔ + V is a probability density function
- The probability of being from - ∞ to V is given on the Y-Axis (Blue Curve)
- The probability of being between a_1 and a_2 is the value of the pdf at a_2 minus the value at a_1
- { $P(a_2) - P(a_1)$ } -



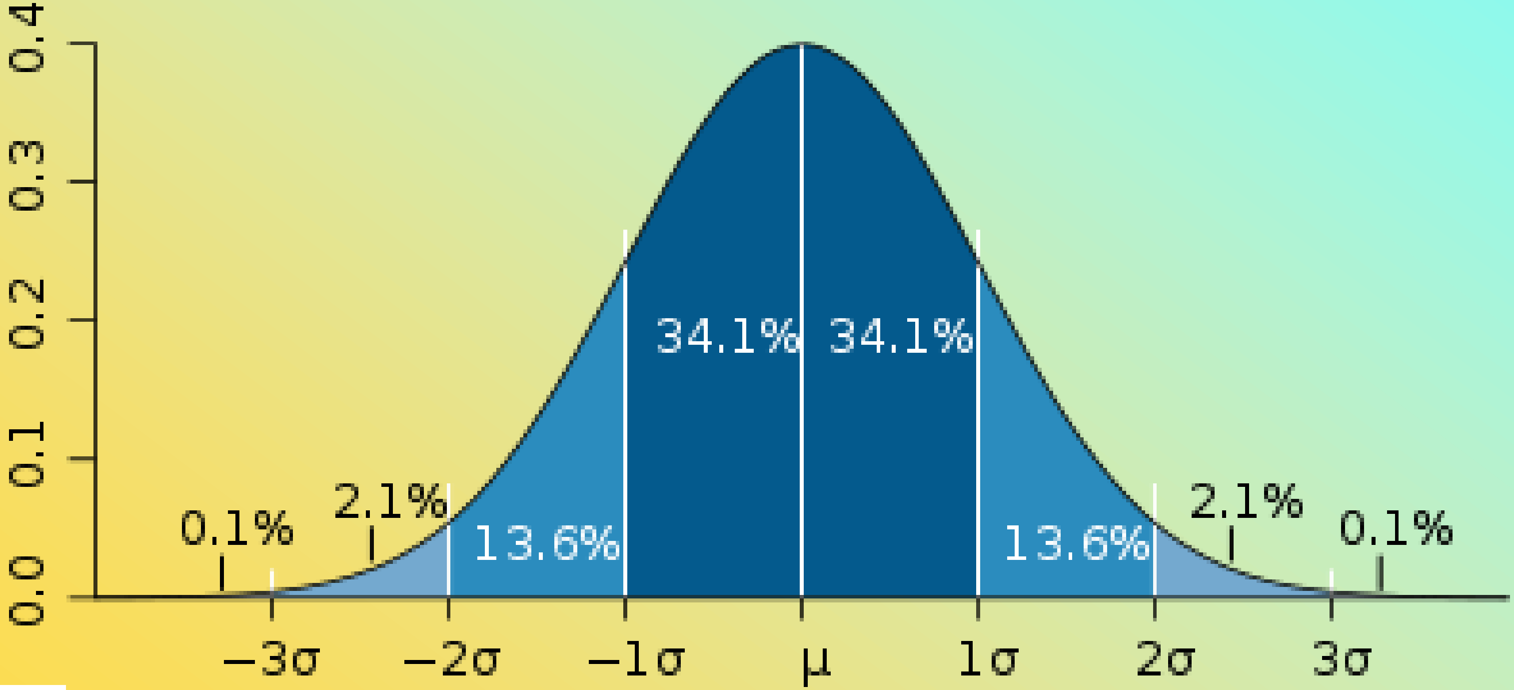
Probability, Standard Deviation & RMS Noise

- $P(V<-1\sigma)=.159$
- $P(V>1\sigma)=1-.841=.159$
- Probability of being greater 1σ (1 standard deviation)
  - $P(V<-1\sigma \& V>+1\sigma) = .318$ ➔ 31.8%
- Probability of being less than 1σ from the mean
  - $P(<|1\sigma|) = .682$ ➔ 68.2%
- $P(<|2\sigma|) = .046$ ➔ 95.4%
- $P(<|3\sigma|) = 2.7\times10^{-3}$ ➔ 99.7%
- $P(<|4\sigma|) = 6.3\times10^{-5}$ ➔ 99.994%
- $P(<|5\sigma|) = 5.7\times10^{-7}$ ➔ 99.99994%



Probabilities in a Gaussian Function

- One standard deviation from the mean (dark blue) accounts for about 68% of the set
- Two standard deviations from the mean (medium and dark blue) account for about 95%
- Three standard deviations (light, medium, and dark blue) account for about 99.7% -



Thermal Noise Effects on Threshold Performance

Signal-to-noise (S/N)

- Noise added to signal and causes a fluctuation
- S/N is the ratio of average Signal Power to average Noise Power
- Average Signal Power
- Average Noise power is RMS Noise

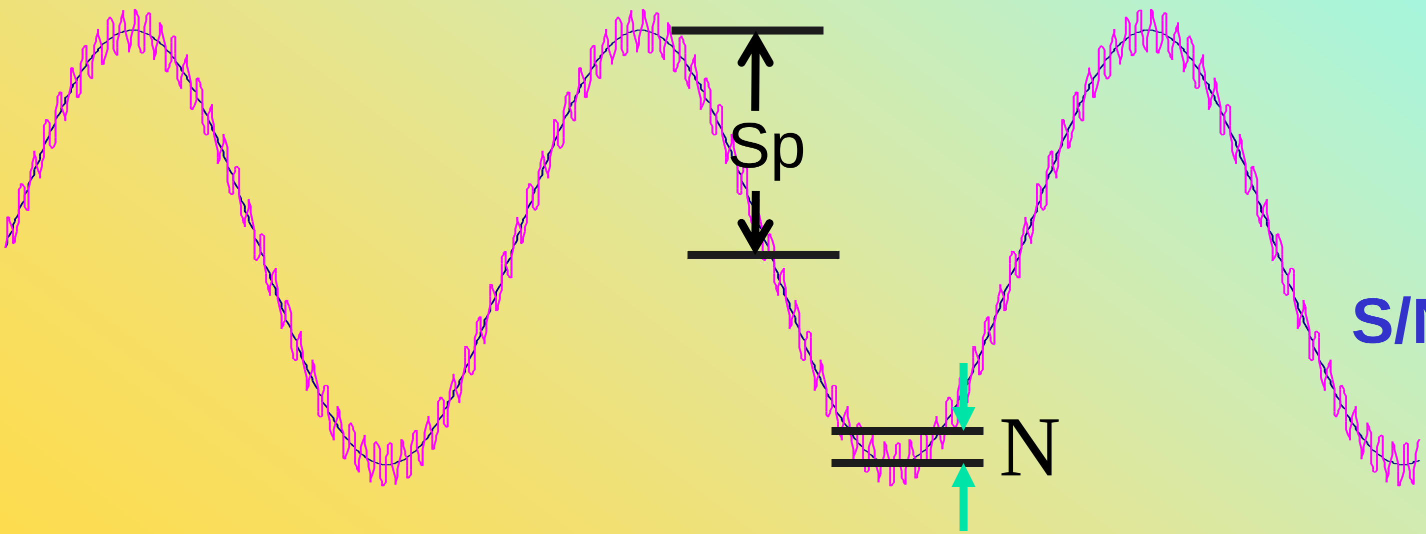
$$S = \frac{\sqrt{2}}{2} Sp$$

S = 6.15

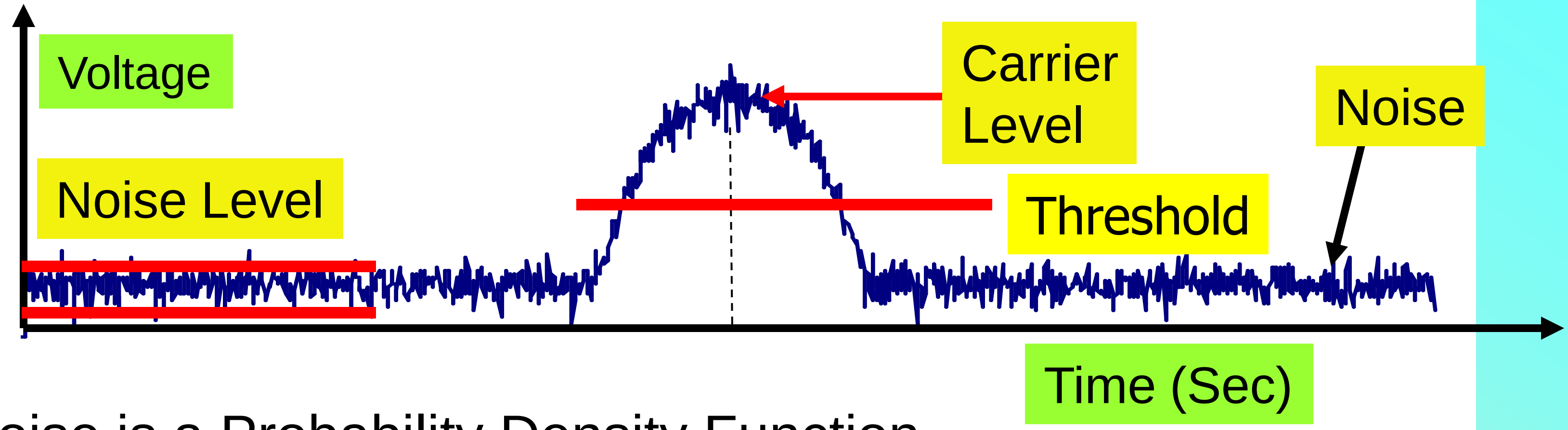
N = 0.90

S/N = 6.8

S/N_{dB} = 8.33dB



Noise Effecting Bit Error Rates (BER) in the Time Domain

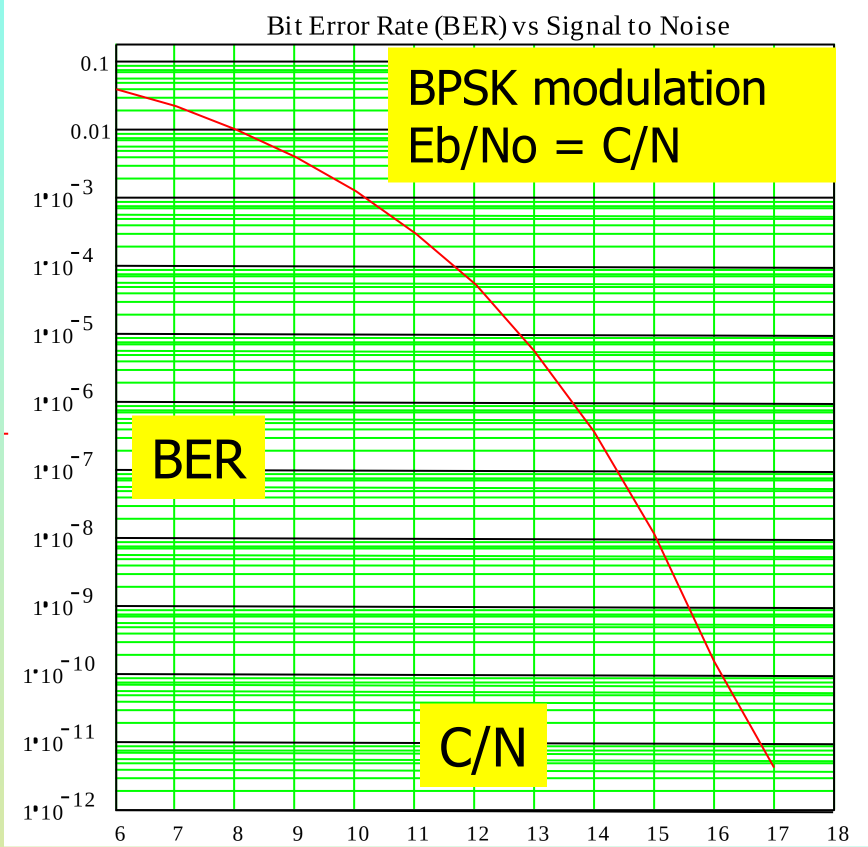


- Noise is a Probability Density Function
  - μ = Average noise level
  - σ = Standard Deviation = RMS Noise
  - RMS Noise = 1σ (Standard Deviation)
- BER is the probability of Noise exceeding the threshold
- Probability of Error is related to the number of σ's to the boundary -

- P(>|1σ|) = .318
- P(>|2σ|) = .046
- P(>|3σ|) = $2.7x10^{-3}$
- P(>|4σ|) = $6.3x10^{-5}$
- P(>|5σ|) = $5.7x10^{-7}$

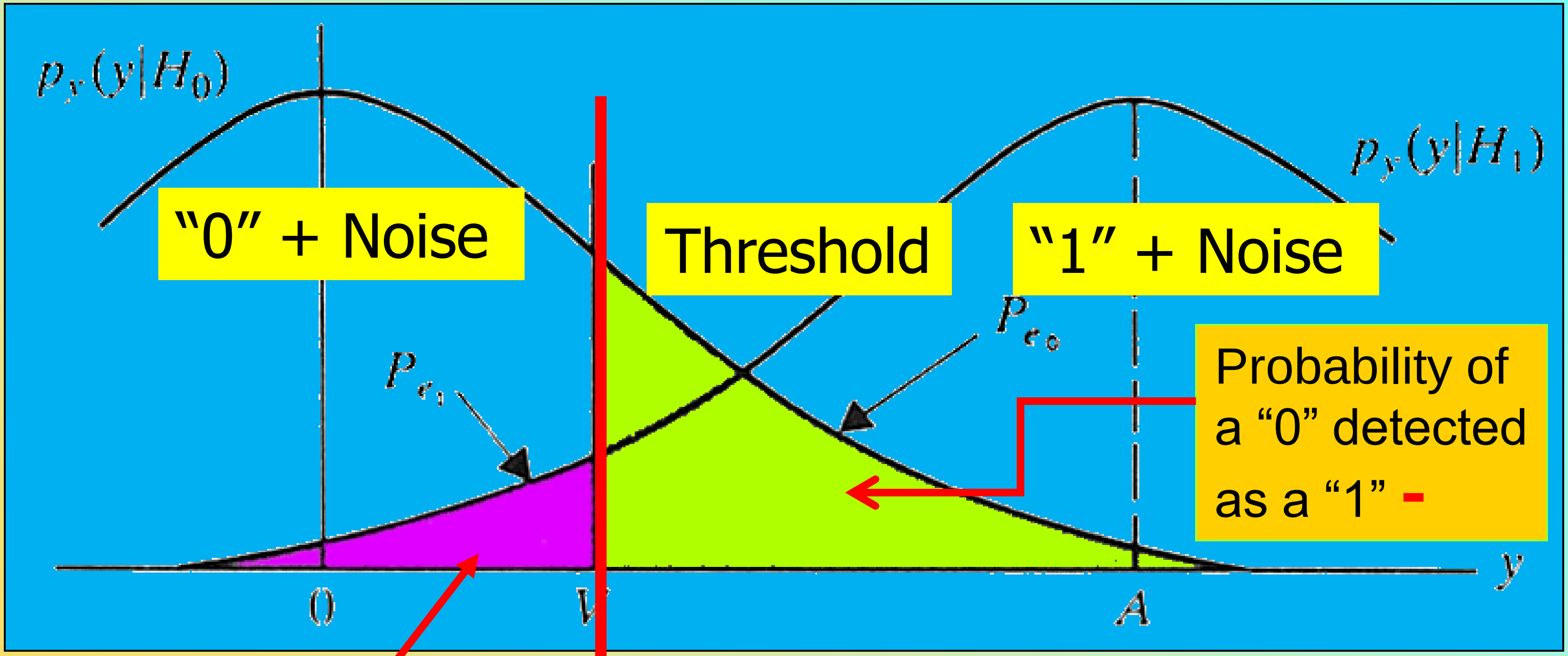
Minimum Input Signal Level – Single Signal

- System Information Example
  - C/N_{MIN} for successful signal reproduction (C/N = 10dB)
  - System Noise Figure (NF=3dB)
  - Signal Band Width (BW=10MHz)
- Minimum Signal level is S_{MIN} = -174dBm/Hz + 10Log(BW) +NF+C/N
  - Noise Level = -101 dBm
  - S_{MIN} = -91 dBm



•Digital Signals are based on a Bit Error Rate
•Analog signals are based on a visual or audio quality standard -

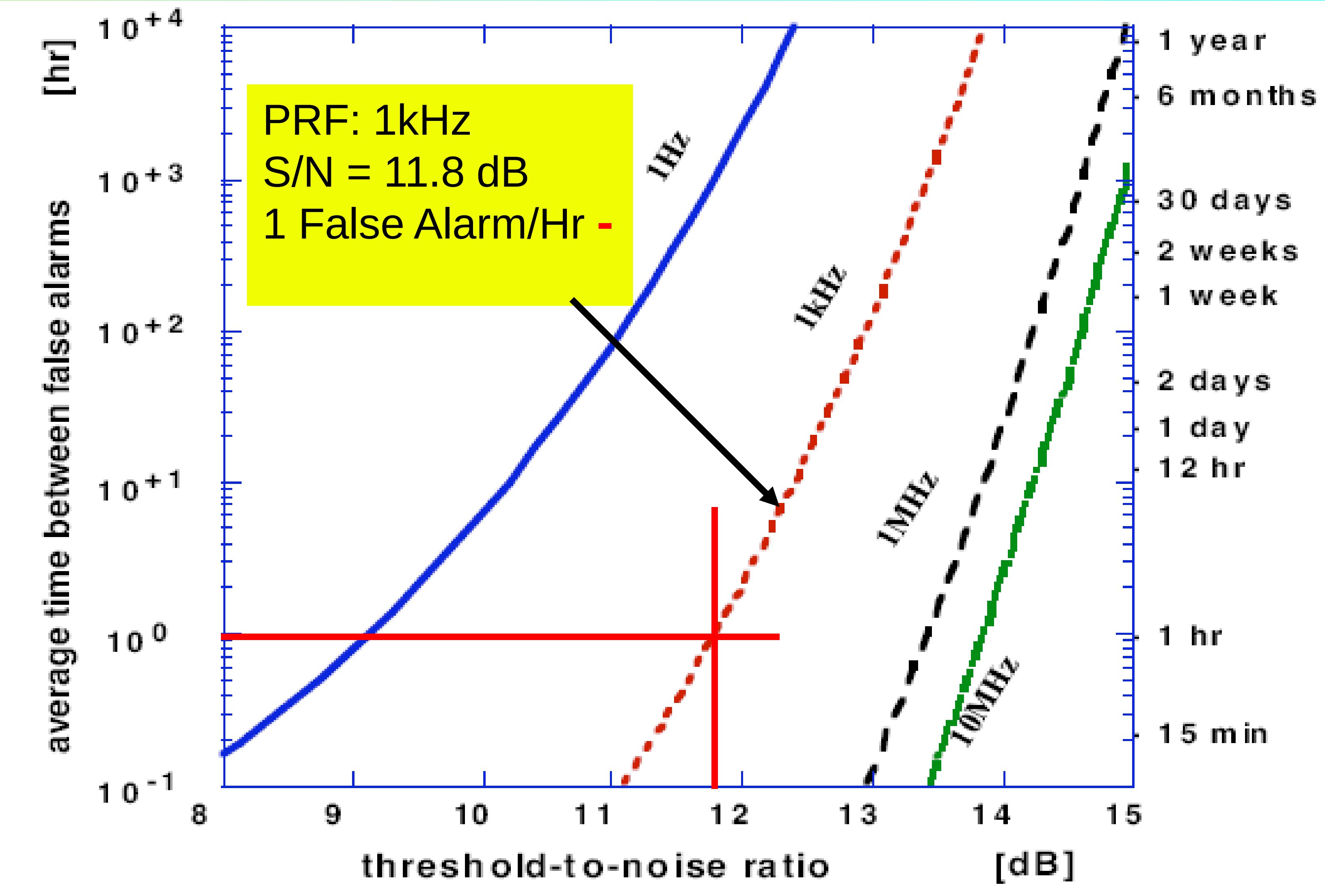
Threshold Detection Probabilities



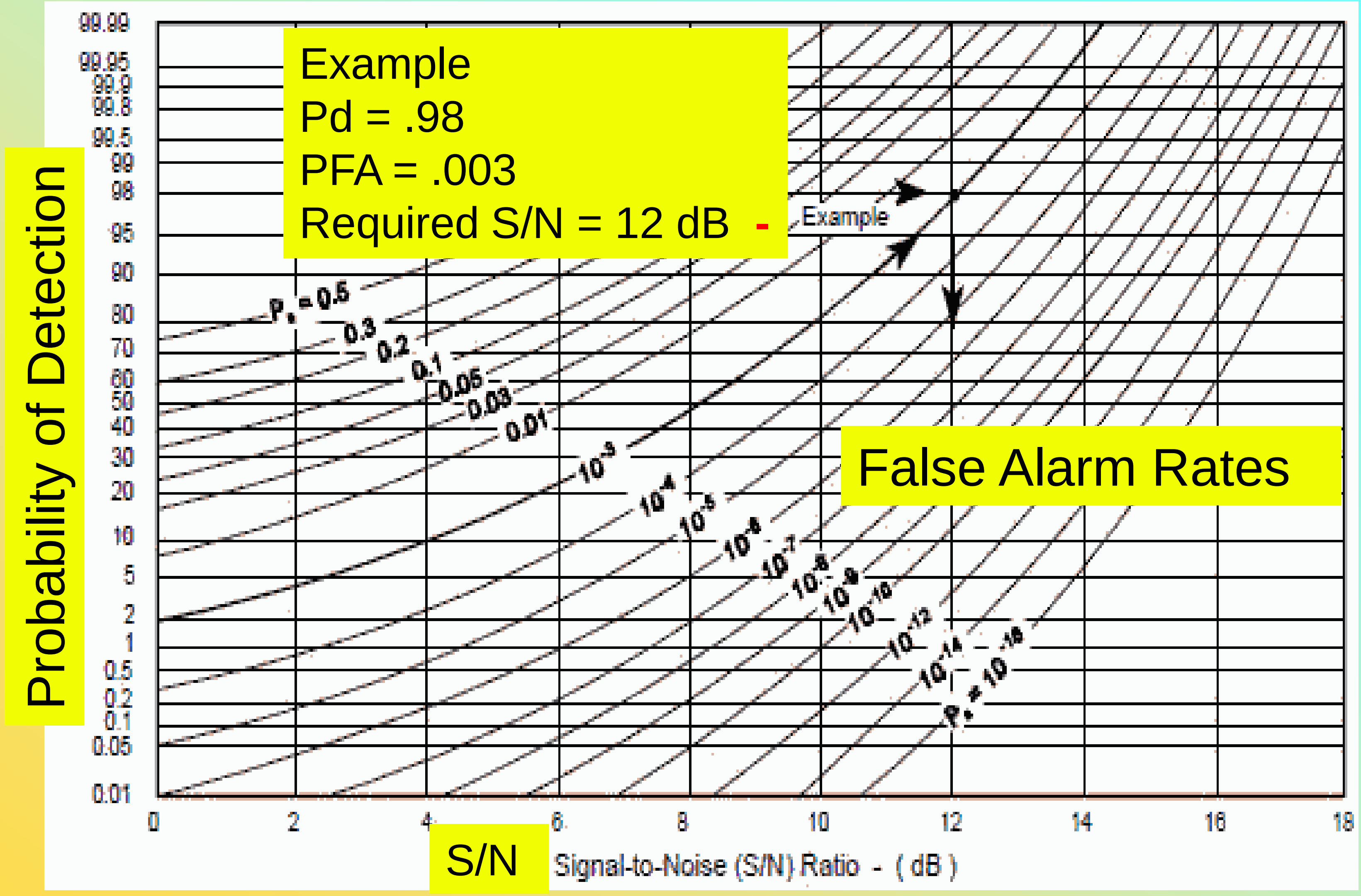
Probability of a "1" detected as a "0"

- Threshold can be varied
- Probability can be skewed

RADAR - Average False Alarm Rate vs Threshold to Noise Ratio



Detection Probability & False Alarm Rate



Trade Off is probability of detection vs. probability of false alarms

Modulation

Generalized Modulated Carrier

$$Xc(t) := \mathrm{Re}\cdot\left(Ac\cdot e^{j\cdot\theta c(t)}\right)$$

$$Xc(t) := Ac\cdot\cos\left(\theta c(t)\right)$$

$$\theta c(t) := 2\cdot\pi\cdot Fc\cdot t + \phi(t)$$

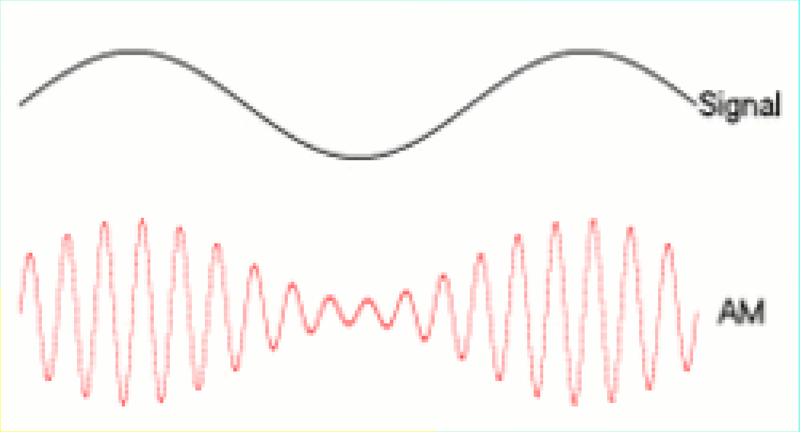
Note: No Information in Amplitude
∴ Power Amplifier can be Non-Linear -

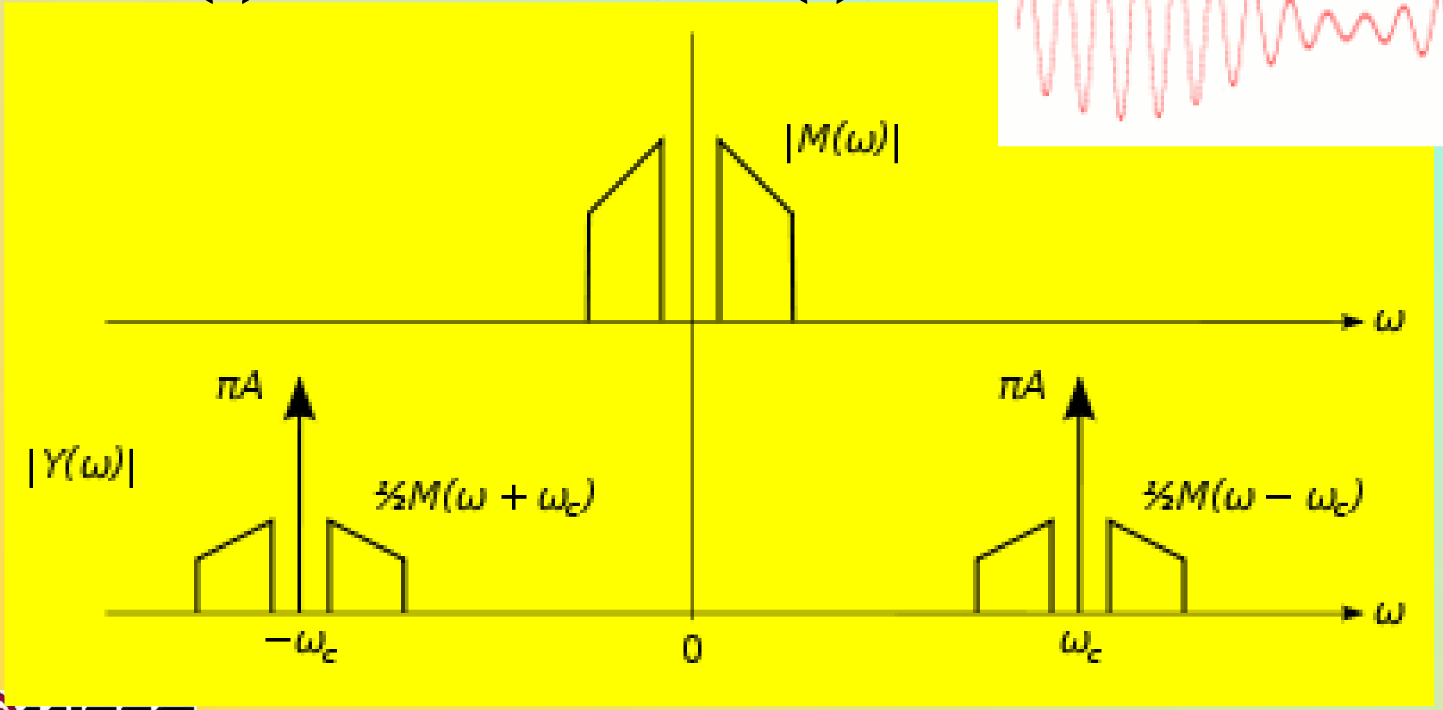
- Xc(t) = Modulated carrier
- Ac = carrier amplitude
- Θc(t) = Instantaneous phase
- Fc = average carrier frequency
- Φ(t) = instantaneous phase around the average frequency Fc
- Instantaneous Frequency = d Φ(t) / dt

AM Modulation

- Translation of Baseband spectrum to a carrier frequency
- Ac is function of time

$$Xc(t) := Ac \cdot \cos(\theta c(t))$$





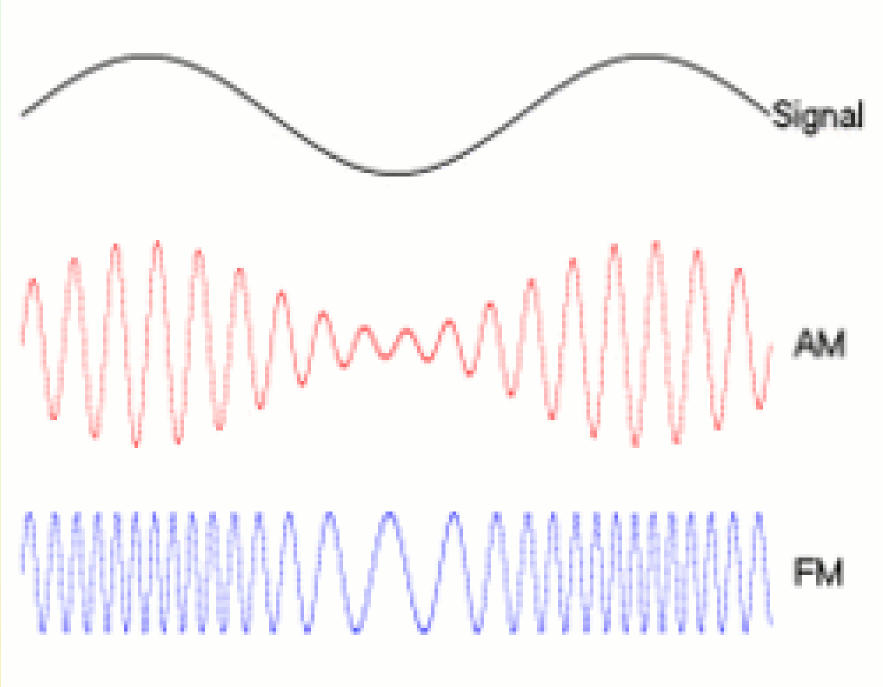
Frequency / Phase Modulation

Phase/Frequency (Exponential) Modulation

$$Xc(t) := Ac \cdot \cos(\theta c(t))$$

$$\theta c(t) := 2 \cdot \pi \cdot Fc \cdot t + \phi(t)$$

- Ac is constant
- Information is contained in φ(t)



FM Modulation Index (β)

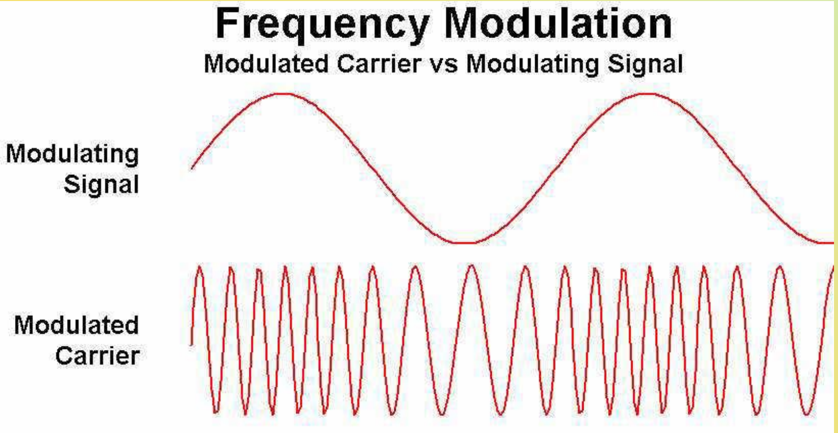
$\Phi(t)$ = Instantaneous Phase variation around carrier Fc

$$Xc(t) := Ac \cdot \cos(\theta c(t))$$

$$\theta c(t) := 2 \cdot \pi \cdot Fc \cdot t + \phi(t)$$

Xc(t) = Ac Cos [2п Fc t + φ (t)]

$$\phi(t) := 2 \cdot \pi \cdot k_f \cdot \int_{-\infty}^{t} m(\tau)\, d\tau$$



- m(t) = Information waveform
- Fi = d $\Phi(t)$ / dt = Instantaneous Frequency around carrier Fc
- Fi = Kf m(t)
- Kf = Gain Constant
  - m(t) is normalized to ±1
  - Kf = ΔF
- ΔF = Peak One sided Frequency Deviation

FM Modulation Index (β)

Xc(t) = Ac Cos [2п Fc t + φ (t)]

Φ(t) = Instantaneous Phase variation around carrier Fc

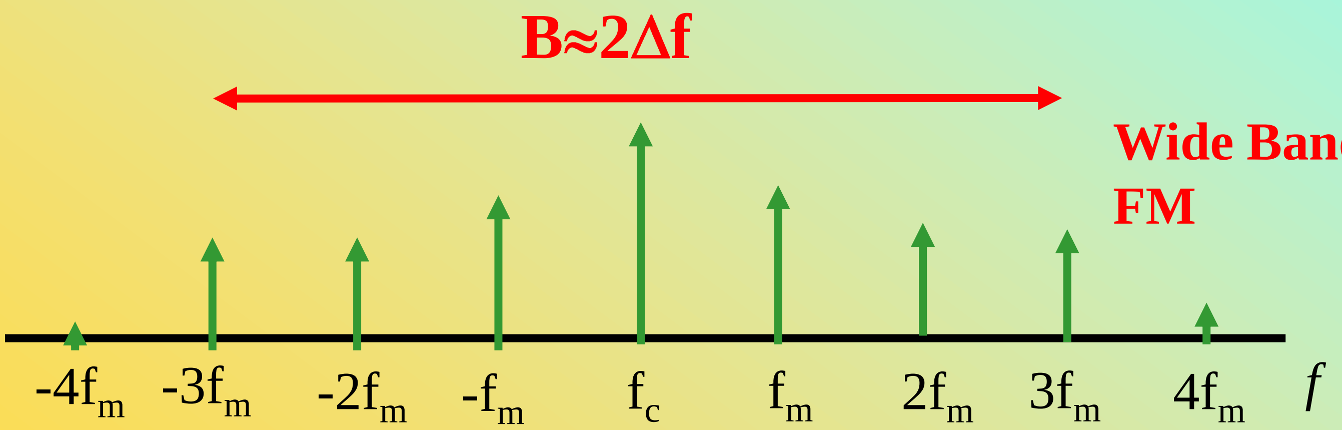
-

$$\phi(t) := 2\cdot\pi\cdot k_f \cdot \int_{-\infty}^{t} m(\tau)\, d\tau$$

- Kf = ΔF
- If m(τ) = cos(2*π*Fm *τ)   [sinusoidal modulation]
- Integrating m(t)
- Φ(t) = [(2*π*ΔF) / (2*π*Fm)] * sin (2*π*Fm *τ)
- Φ(t) = (ΔF / Fm) * sin (2*π*Fm *τ)
- β = ΔF / Fm = modulation index (Radians)
- Φ(t) = β * sin (2*π*Fm *τ)   -

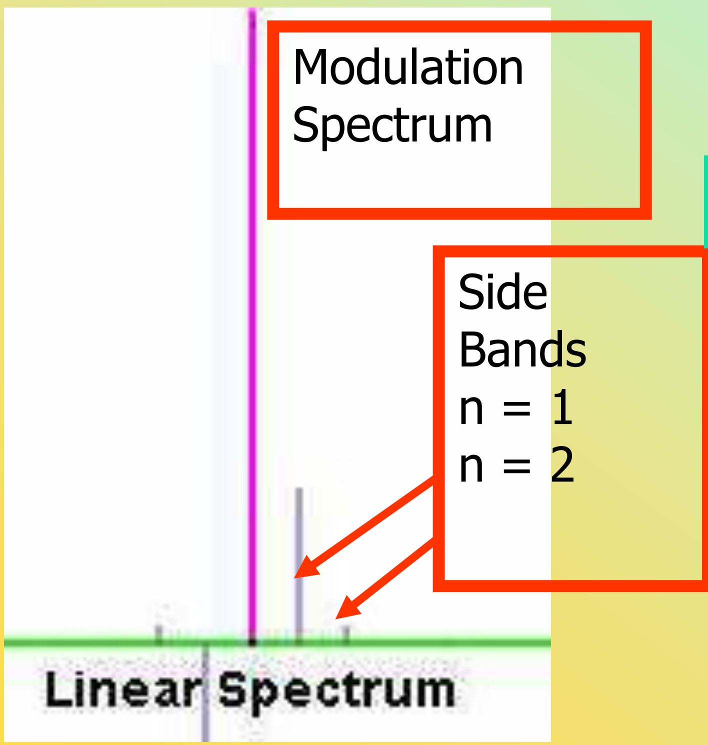
FM Spectral Analysis

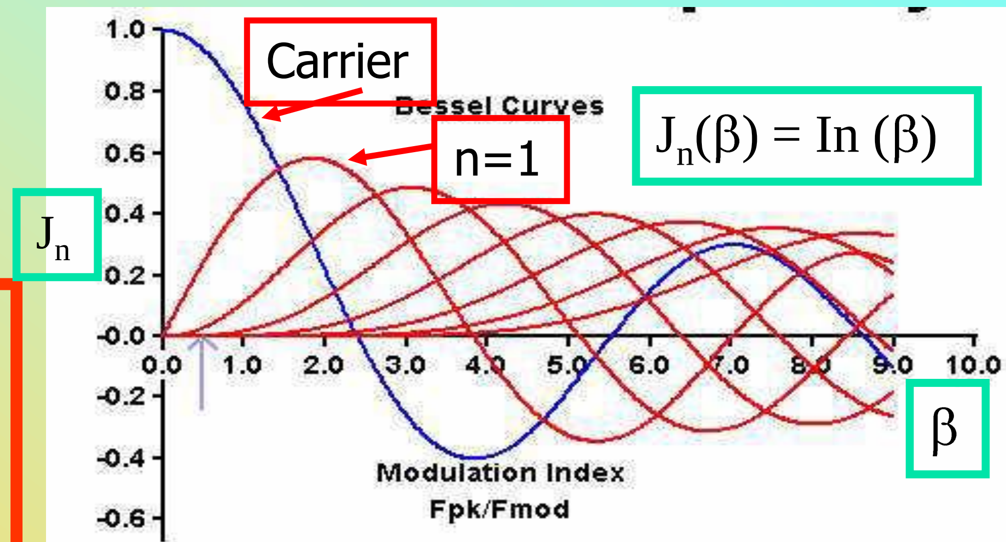
- $Xc(t)=A_c\cos(2\pi f_c t+2\pi k_f \int m(\tau)\,d\tau)$
- For sinusoidal modulation: $m(t)=\cos(2\pi f_m t)$
- Xc(f) is the Fourier Transform of Xc(t)
- Xc(f) sequence of δ functions at multiples of f_m from f_c
  - δ functions at $f_c \pm nf_m$
- Amplitudes are Bessel Coefficients of the first kind, Order n and independent variable β $[J_n(\beta)]$ -



Frequency / Phase Modulation Side Bands

- $J_n(\beta)$ = Bessel Function of the First kind, order n, Argument β
- n = side band number from carrier
- β = Modulation index in Radians
- Sideband Levels $J_n(\beta)$ (Linear units)
- Levels in dBc = $20Log_{10}\,[J_n(\beta)]$ -



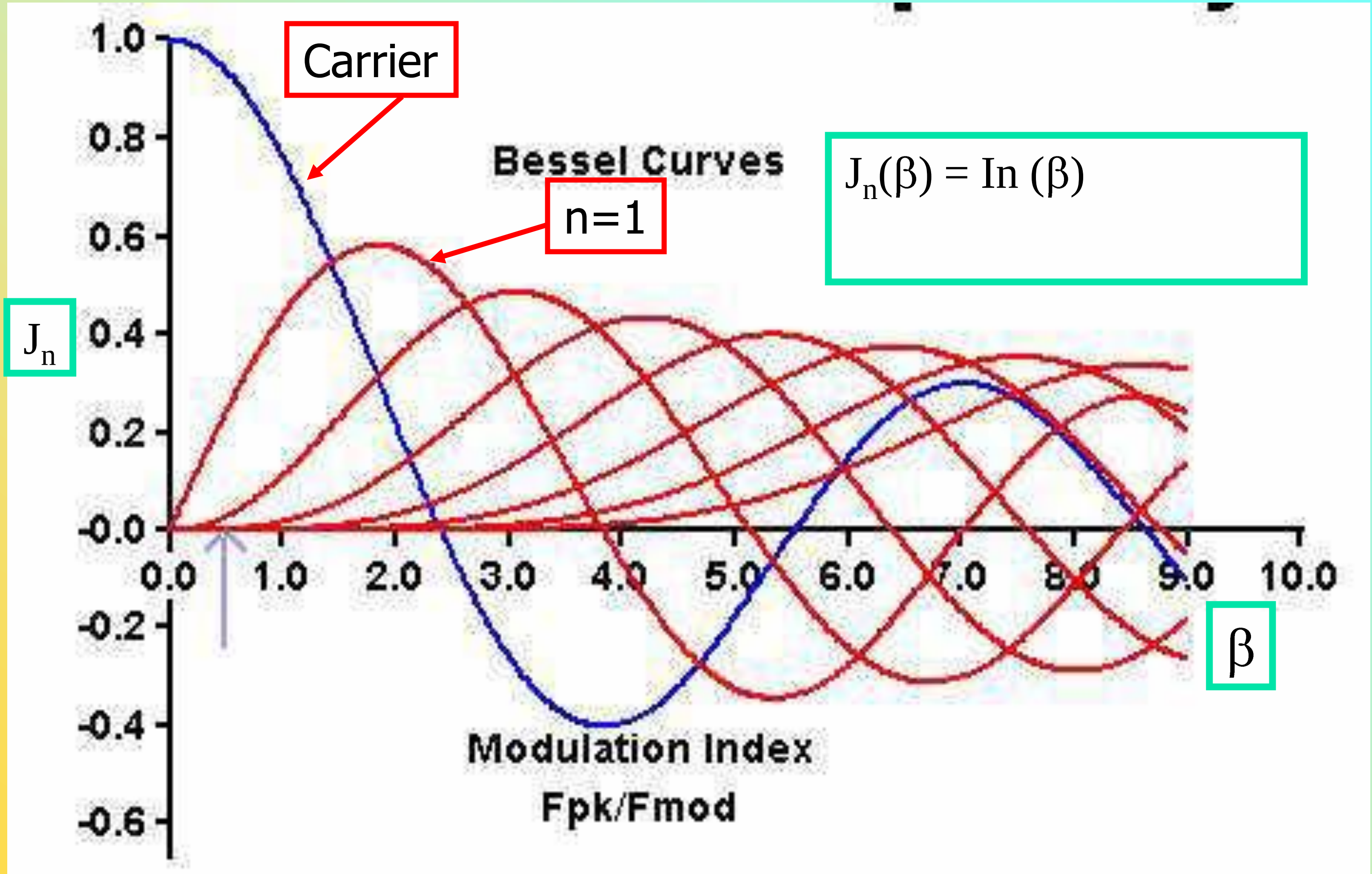


Bessel Function Solution

$$In(\beta) := \frac{1}{2\cdot\pi}\cdot\int_{-\pi}^{\pi} e^{\beta\cdot\cos(\theta)}\cdot\cos(n\cdot\theta)\,d\theta$$

Bessel Function (Side Band) Levels

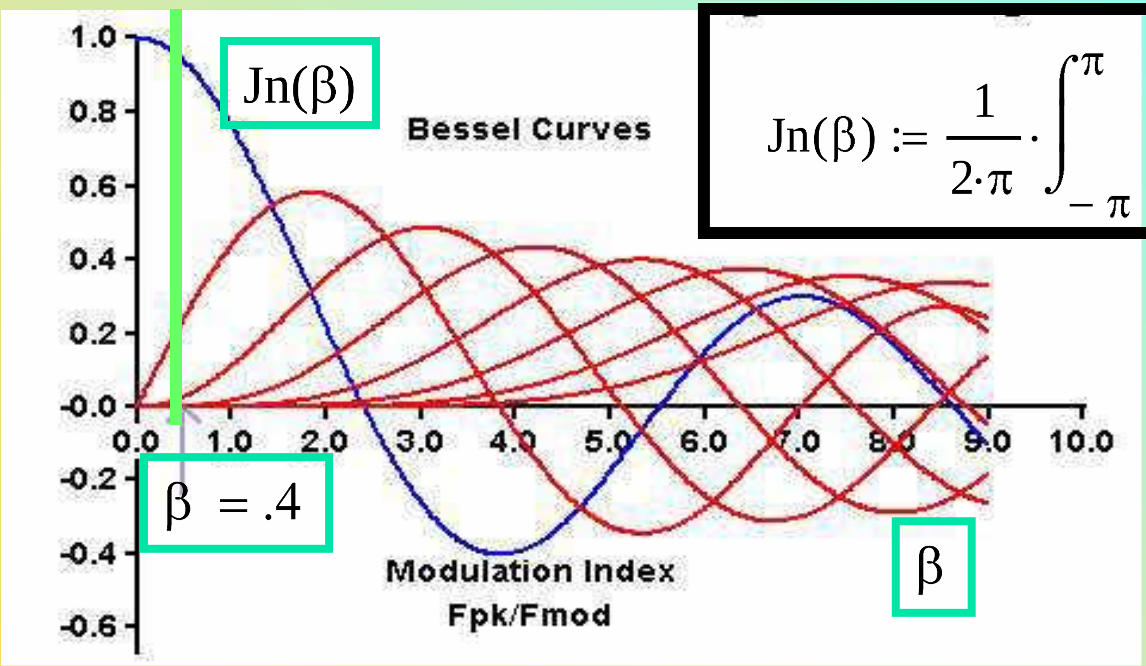
- **Note for Low Beta, Higher order sidebands are not significant** -



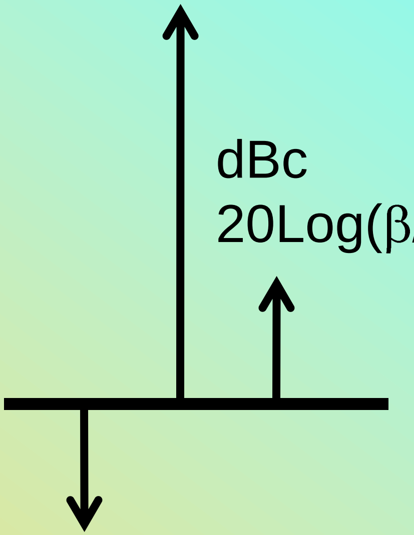
MITEQ

Frequency Modulation - Low Beta

- ➢Bessel Function of the First kind, N order, Argument β
- ➢Low Beta (β<1) has only 2 significant sidebands



$$Jn(\beta) := \frac{1}{2\cdot\pi}\cdot\int_{-\pi}^{\pi} e^{\beta\cdot\cos(\theta)}\cdot\cos(n\cdot\theta)\, d\theta$$



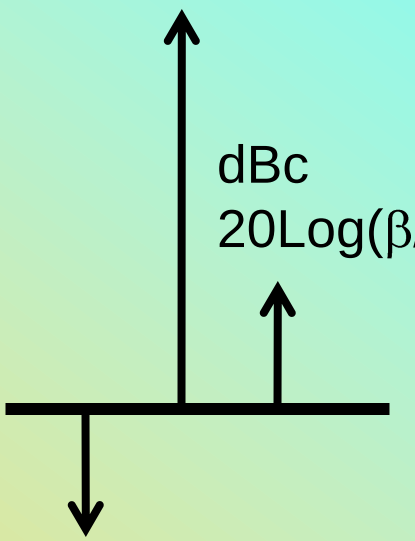
- Sideband Level = dBc=20Log(β/2)
- AM sidebands are in phase
- FM sidebands are out of phase

Phase Modulation

$$Xc(t) := Ac \cdot \cos\left(\theta c(t)\right) \qquad \theta c(t) := 2 \cdot \pi \cdot Fc \cdot t + \phi(t)$$

- Phase Modulation: $\Phi(t)$
- $\Phi(t) = \beta * m(t)$: β = peak phase deviation
  - β = Modulation Index in Radians, same as FM
  - m(t) = information normalized to 6 1
- $Xc(t) = Ac*\cos(2*\pi*Fc*t + \beta * m(t))$
- β is the same for PM or FM
- For small β
- Sideband Level = dBc=20Log(β/2) -

Line Spectrum

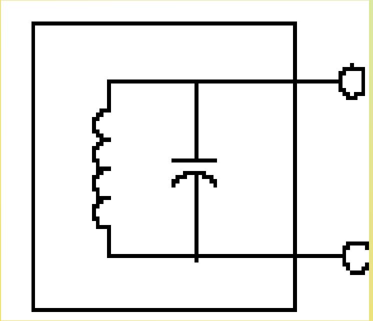


Oscillator Basics

- Negative Resistance Oscillators
- Feedback Oscillators

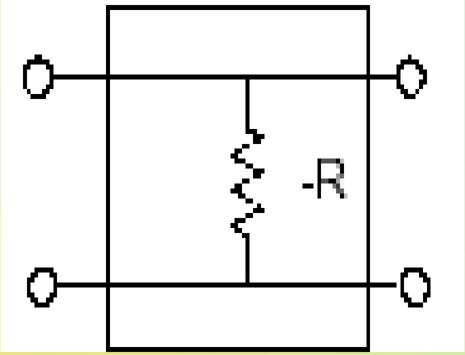
Negative Resistance Oscillators - Basic Configuration

Resonator Circuit



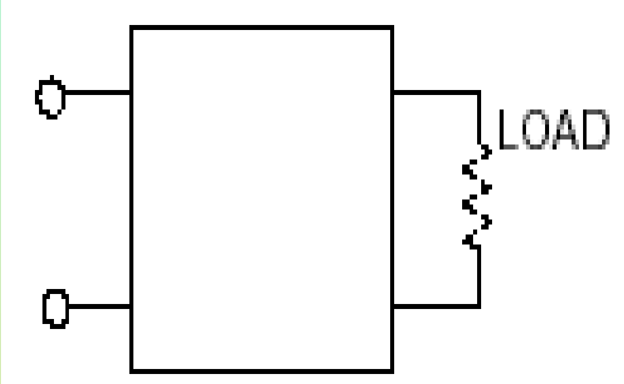
Resonator:
LC, Stub,
Varactor Tuned
Circuit, YIG, etc.

Active Circuit



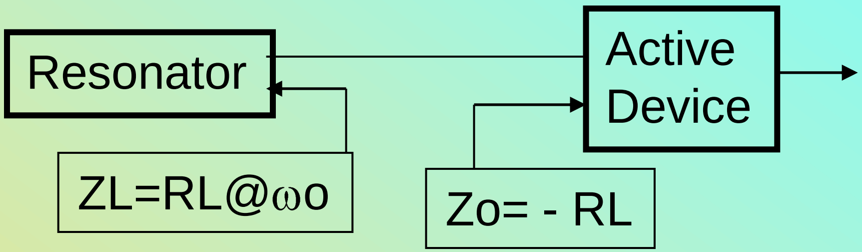
Transistor,Tunnel
Diode, Gunn
Diode, etc.

Output Network



Passive Matching
Ckt & Buffer
Amplifier -

Theory of Negative Resistance Oscillators



Reflection coefficient

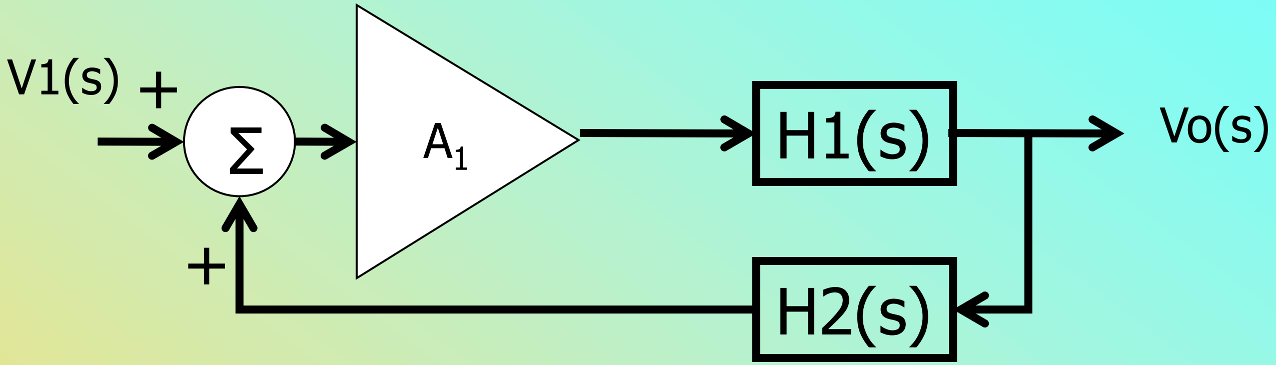
$$\rho := \frac{Vr}{Vi}$$

$$\rho := \frac{(ZL - Zo)}{ZL + Zo}$$

Resonator is a One port network

- at Resonance (Fo)
  - ZL is real only at the resonant frequency (ZL(Fo))
  - ZL(Fo) = -Zo
  - Result: Reflected voltage without an incident voltage (oscillates)
- An Emitter Follower is a classic negative resistance device
- Technique used at microwave frequencies
  - Spacing between components often precludes the establishment of a well defined feedback path. -

Feedback Oscillators (Two port networks)



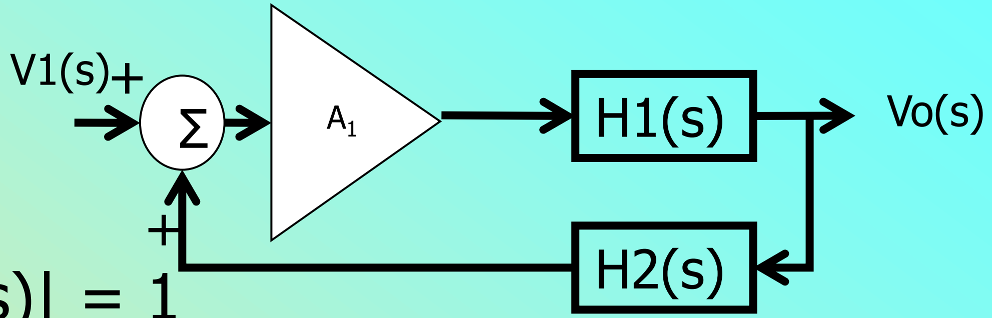
- (V1+Vo*H2)*A*H1=Vo
- V1*A*H1 = V0(1-A*H1*H2)

$$\frac{Vo}{V1} := \frac{(A \cdot H1(s))}{1 - A \cdot H1(s) \cdot H2(s)}$$

- **A*H1(s)*H2(s) = open loop gain = AL(s)** -

Barkhausen Criteria

- Barkhausen criteria for a feedback oscillator
  - open loop gain = 1
  - open loop phase = 0
- |A*H1(s)*H2(s)| = |AL(s)| = 1
- Angle (A*H1(s)*H2(s)) = 0
- s = ω_o (for sinusoidal signals)
- Re AL(ω_o) = 1
- Im AL(ω_o) = 0
- Transfer function blows up (Output with no Input) - Oscillation
  - Vo is finite when V1 = 0 -



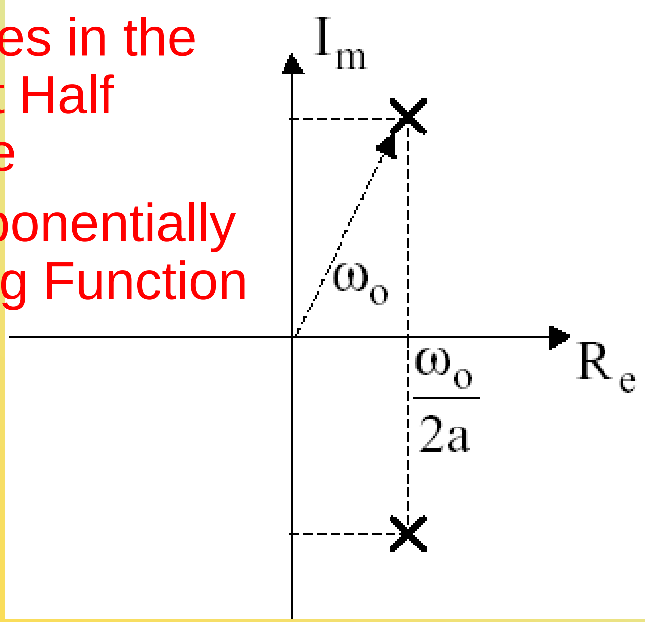
$$\frac{Vo}{V1} := \frac{(A \cdot H1(s))}{1 - A \cdot H1(s) \cdot H2(s)}$$

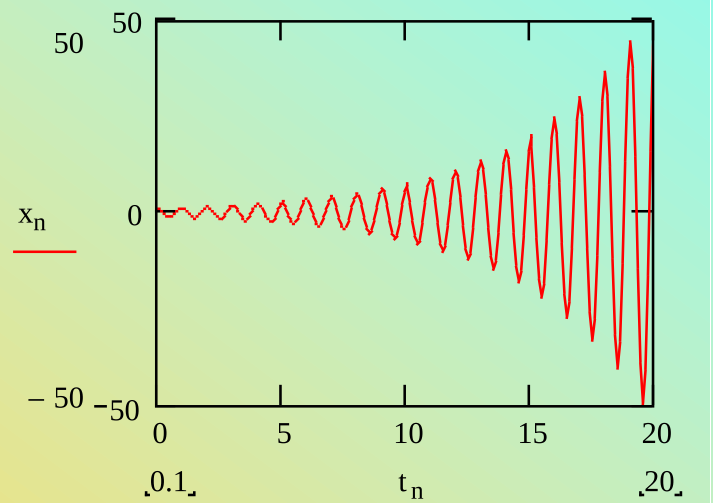
Starting an Oscillator

- To start an oscillator it must be triggered
  - Trigger mechanism: Noise or a Turn-On transient
- Open loop gain must be greater than unity
- Phase is zero degrees (exponentially rising function)

$$x_n := e^{\alpha \cdot t_n} \cdot \cos\left(2 \cdot \pi \cdot \omega \cdot t_n\right)$$

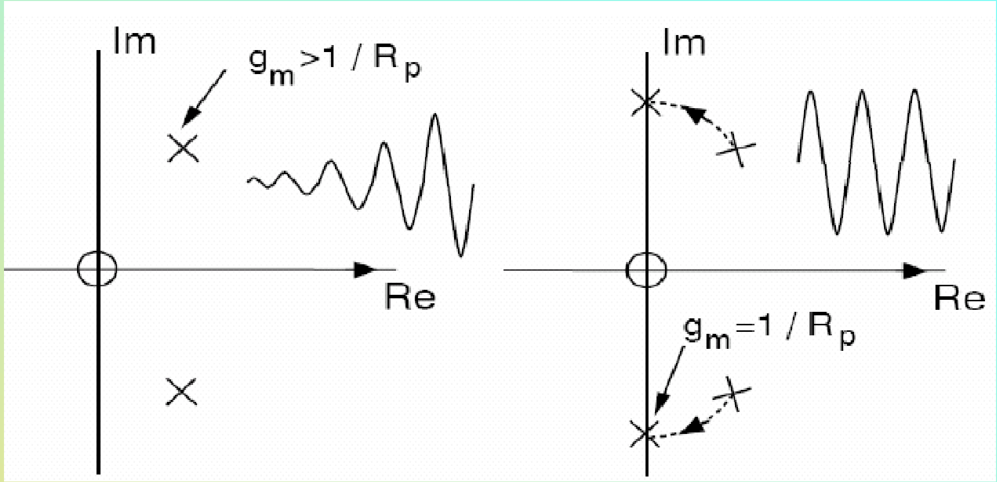
- Poles in the Right Half Plane
- Exponentially Rising Function





α = Real Part of A*H1(s)*H2(s), = >1

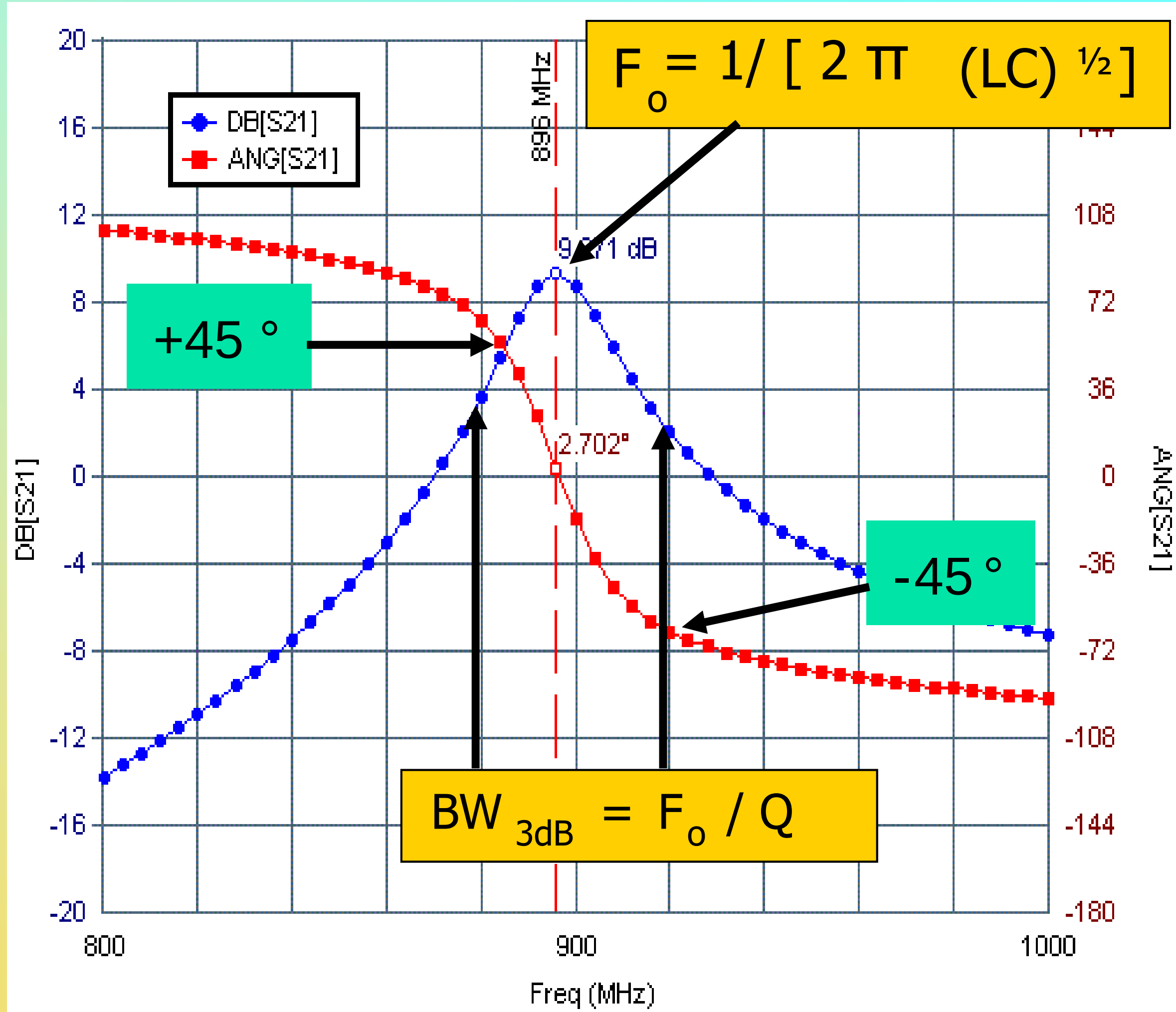
Amplitude Stabilization



- As amplitude increases Gain decreases the effective gm (transconductance gain) is reduced
- Poles move toward the Imaginary axis
- Oscillation amplitude stabilizes when the poles are on the imaginary axis
- Self correcting feedback (variable gm) maintains the poles on the axis and stabilizes the amplitude -

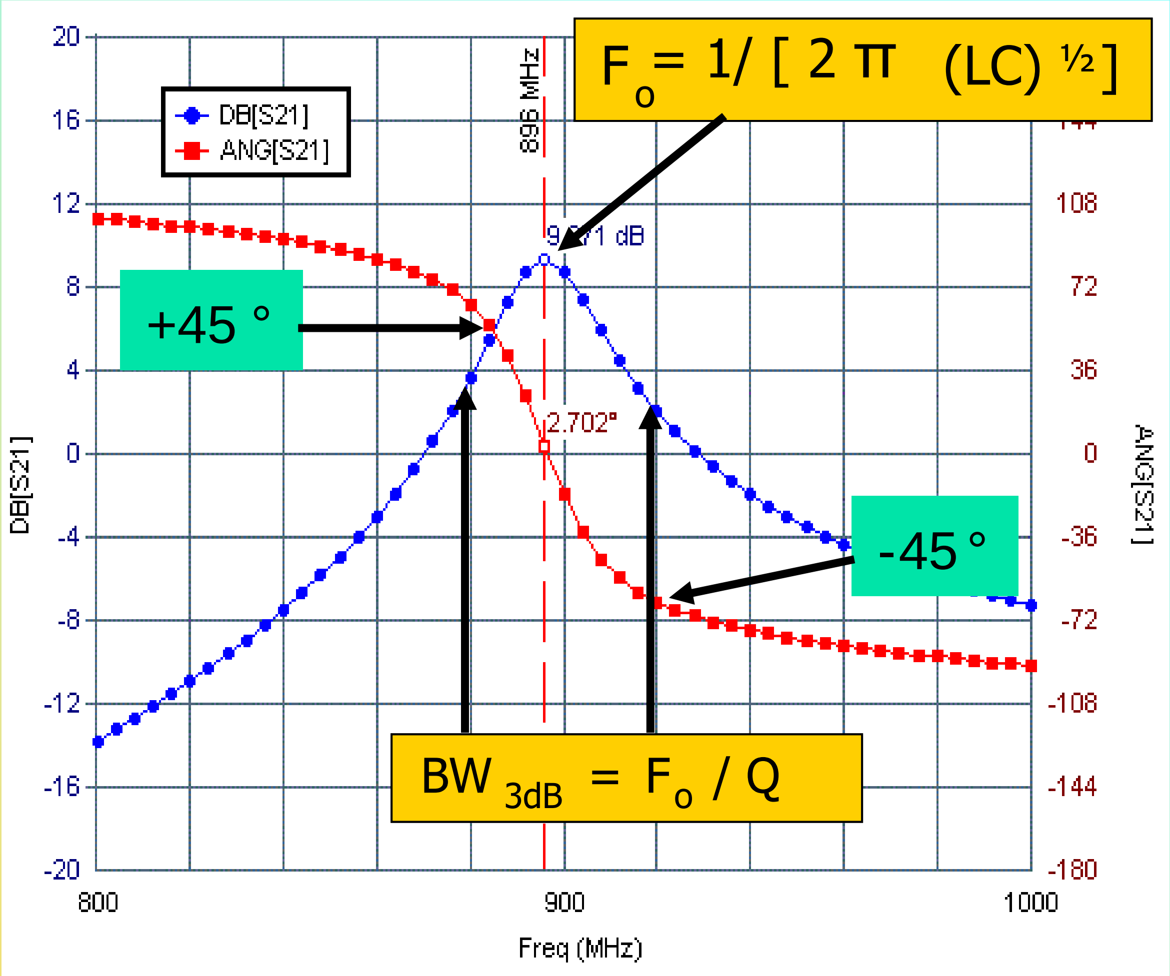
Frequency Stability Analysis

- Conditions for Oscillation
  - Sufficient gain in the 3 dB bandwidth (Open Loop Gain>1)
  - At Fo; Sum of all components around the loop are real (Resistive, Zero Phase)
- Circuit oscillates at resonance $\varpi_o = 1/(LC)^{½} = 2*\pi*F_0$



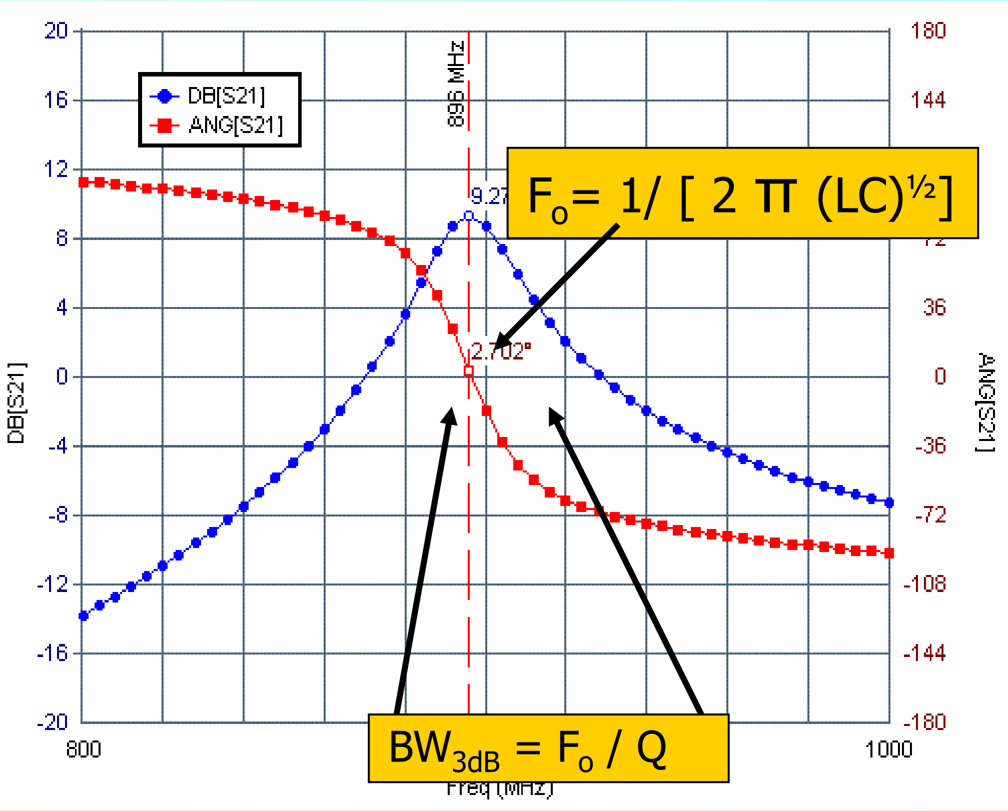
Coarse & Fine Frequency Stability

- Coarse frequency of oscillation is determined by the resonant frequency - **Amplitude**
- Fine Frequency of oscillation is determined by **PHASE**
  - Loop phase shift is automatically compensated
  - Phase changes forces frequency off of F_0
- 3 dB bandwidth provides +/-45° compensating phase



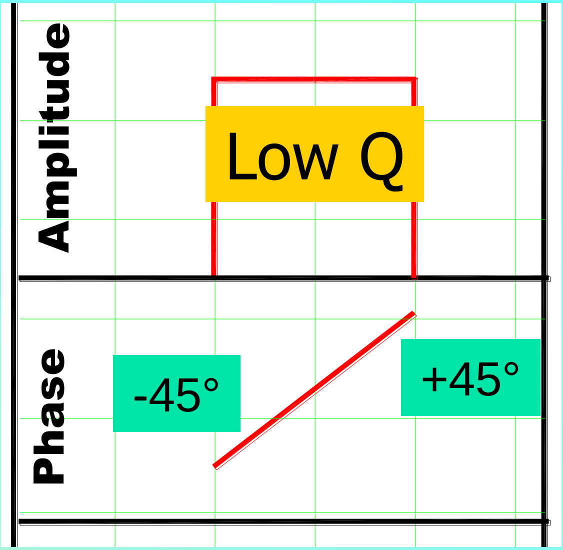
Oscillator Stability

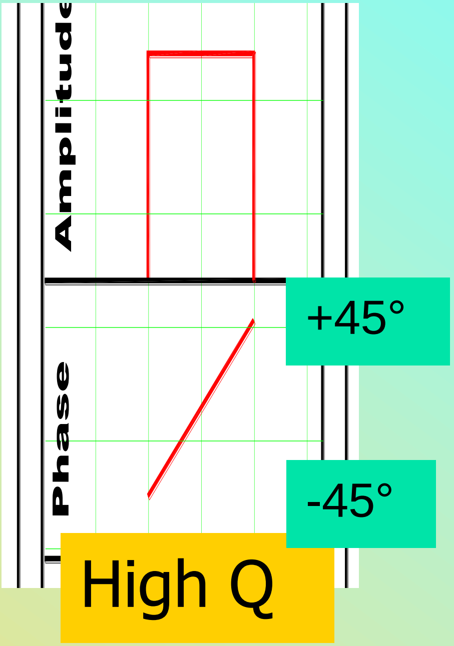
- Factors Affecting Oscillator Stability
  - Stability of the Resonator
  - Q of the resonator
- Causes of Oscillator Frequency Drift
  - Change in resonant frequency
  - Change of Open Loop Phase
  - Amplitude Changes
    - Oscillators operate in a non-linear mode
    - Changes in Amplitude changes phase -



Parasitic Phase Shifts vs Frequency Stability

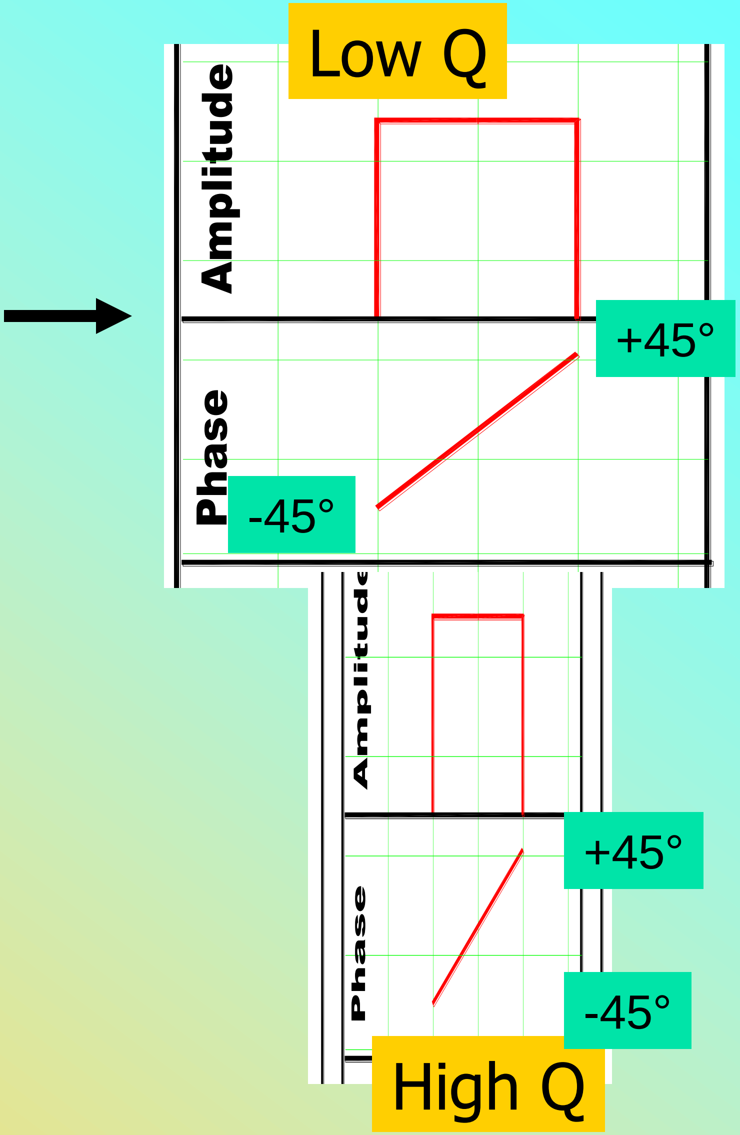
- $Q = F_0/BW_{3dB}$ ➔ $BW_{3dB} = F_0/ Q$
- 1 Pole Resonant Circuit
  - 3 dB bandwidth shifts +/- 45°
- Phase change
  - If maximum $\Delta F_0 = BW_{3dB}$
  - $(\Delta F_0 / \Delta\varphi)$ sensitivity of the frequency to phase changes
  - $(\Delta F_0 / \Delta\varphi) \approx BW_{3dB} / 90°$
  - $\Delta F_0 = BW_{3dB} = F_0/ Q$
  - $\Delta F_0 / \Delta\varphi \approx =[F_0/Q]/90°$ (Hz/Deg)
  - **$\Delta F_0 / \Delta\varphi \approx F_0/(Q*90°)$ (Hz/Deg)**
  - Higher Q Smaller $\Delta F_0 / \Delta\varphi$ (phase)





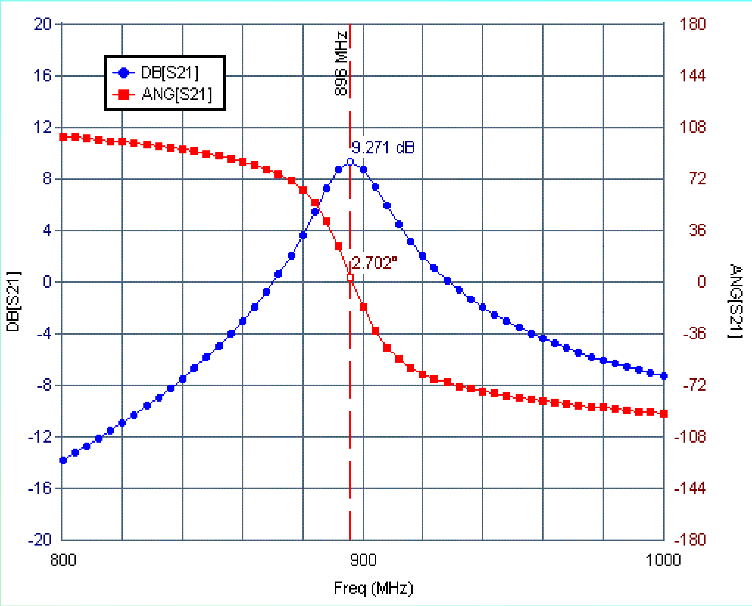
Parasitic Phase Shifts vs Frequency Stability

- Frequency stability vs Phase is proportional to Q
  - Phase changes around the loop
  - Loop Self Corrects Phase Variations
- Parasitic Phase shifts have less effect on frequency in Higher Q circuits -



Frequency Stability – Resonator Dependent

- Center Frequency Resonator (Fo)
- Q of the Resonator
  - Phase Stability (A function of $Q=Fo/BW_{3dB}$)
- **$\Delta F_0 / \Delta\varphi$ (Hz/Deg) ≈ $F_0 / [\,90°\,Q]$**



	Q	Q	Stability
	Min	Max	PPM/C
LC Resonators:	50	150	100
Cavity resonators	500	1000	10
Dielectric resonators:	2,000	10000	1
SAW devices:	300	10000	0.1
Crystals	50000	1000000	0.01

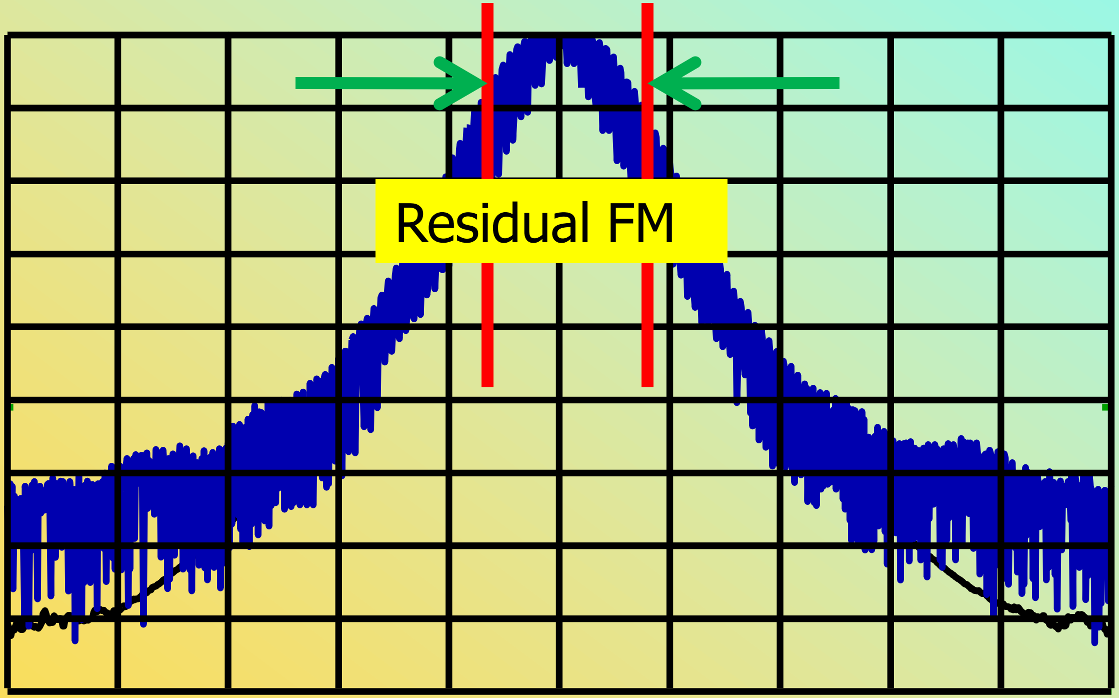
-

Oscillator Stability

- Long Term Frequency Stability
  - Usually a function of the Resonator Center Frequency Stability
  - Change in Frequency (ΔF) with respect to center frequency (Fo)
  - Stated as ΔF/Fo in Parts Per Million (PPM)
  - Time frame: Typically hours to years
  - Stability over Temperature
- Short Term Frequency Stability
  - Usually a function of noise perturbations
  - Residual FM
  - Allen Variance
  - Phase Noise -

Residual FM
Slow Moving Frequency Variations

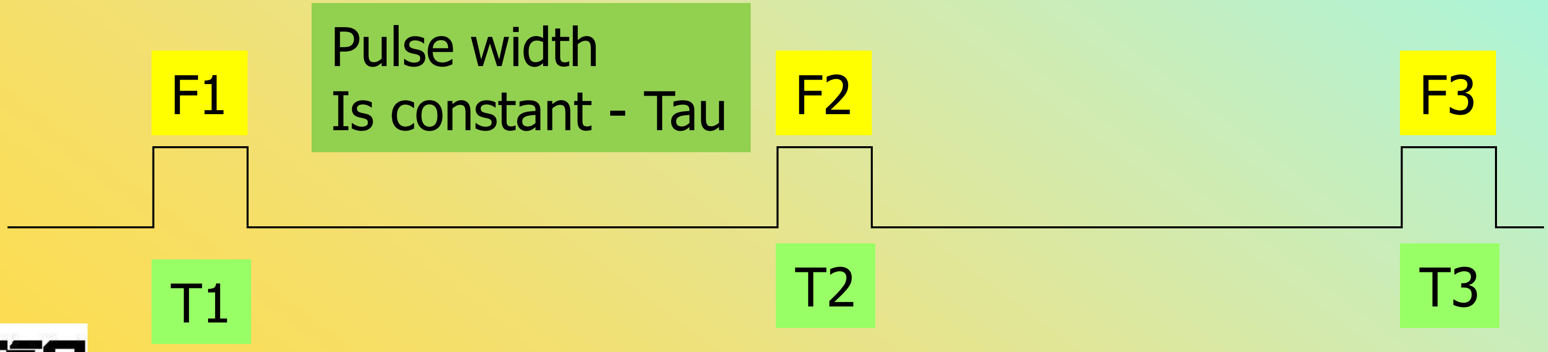
- Change in frequency ΔF is much greater than the rate of frequency change, fm ($\Delta F/fm = \beta >> 1$)
- Spectrum has a flat top
  - Peak to Peak change in frequency is the Residual FM
- Typically measured 6dB down from the peak -



Allen Variance

Phase / Frequency Noise Variations >1 Second

- Defines accuracy of clocks
- One half of the time average over the sum of the squares of the differences between successive readings of the frequency deviation sampled over the sampling period.
- Allen variance is function of the time period used between samples1
- Measure frequency at time interval T2-T1
- (F2-F1) / F1 is the fractional change in frequency over time interval T2-T1 -

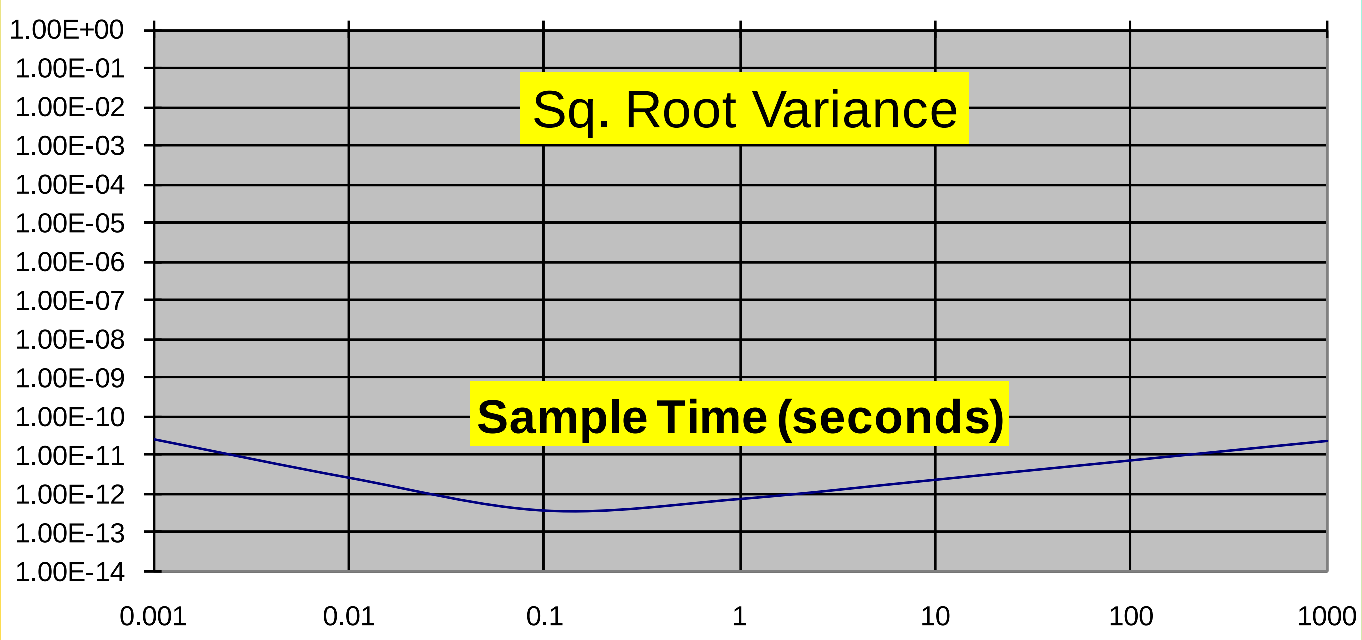


Two-point Allen variance - $\sigma_y(\tau)$

- Time domain measure of oscillator instability.
- It can be directly measured using a frequency counter
  - Repetitively measure the oscillator frequency over a time period τau.
- Allen variance is the expected value of the RMS change in frequency with each sample normalized by the oscillator frequency.
- Data is in Parts per Million or Parts per Billion
  -

Allen Variance Computation

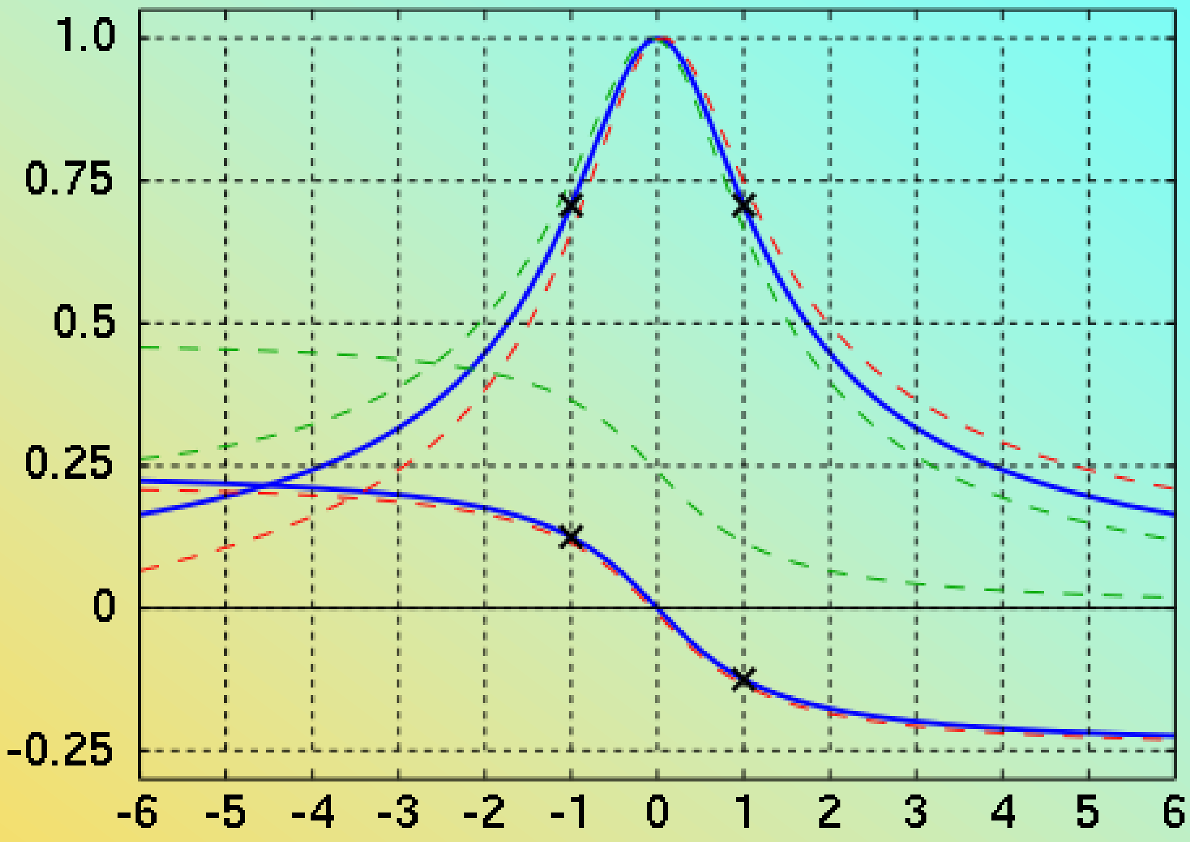
- Typical specification might be frequency variation in 100 seconds
- Take two sample of frequency a 100 seconds apart
- Repeat the measurement
- Allen variance is the ½ the square root of the sum of the squares of all the samples taken -



- Inverse of time between samples time is carrier offset
- 0.001 Hz to 1kHz

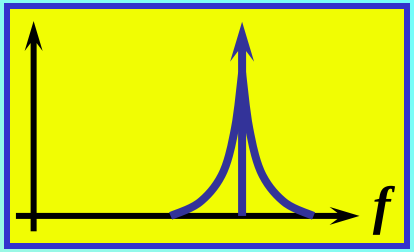
Frequency Stability and its Effect on Phase Noise

- Resonators Stability – Does Not Effect Phase Noise
- Phase Sensitivity Effects Phase Noise -



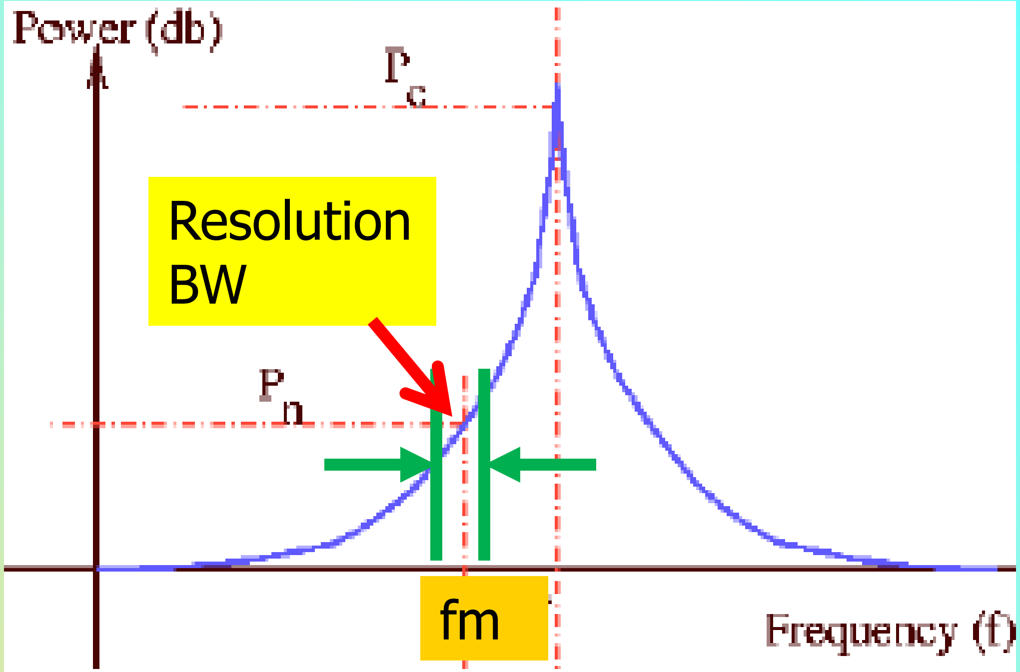
Phase Noise - Short Term Stability

- Measures oscillator Stability over short periods of time
  - Typically 0.1 Seconds to 0.1 microseconds`
- Noise varies the oscillator phase/frequency
  - **Not amplitude related**
- Noise level increases close to the carrier
  - Typical offset frequencies of interest: 10Hz to 10MHz
  - Stability closer to the carrier is measured using Allen Variance
  - Noise further from the carrier is usually masked by AM thermal noise
- Phase Noise cannot be eliminated or affected by filtering
- Phase & Frequency are related:
  - Frequency is the change in phase with respect to time
  - $\Delta\varphi / \Delta t \rightarrow d\varphi/dt$ as $t \rightarrow 0$ -



Short Time Phase / Frequency Noise (<1 Second)

- Specified and measured as a spectral density function typically in a 1 Hz bandwidth
- Normalized to dBc/Hz at a given offset from the carrier
- Level relates phase noise in degrees



  - Modulation index (β) of noise in a 1 Hz bandwidth
  - Level in dB = 20 Log (β/2) where β is in radians -

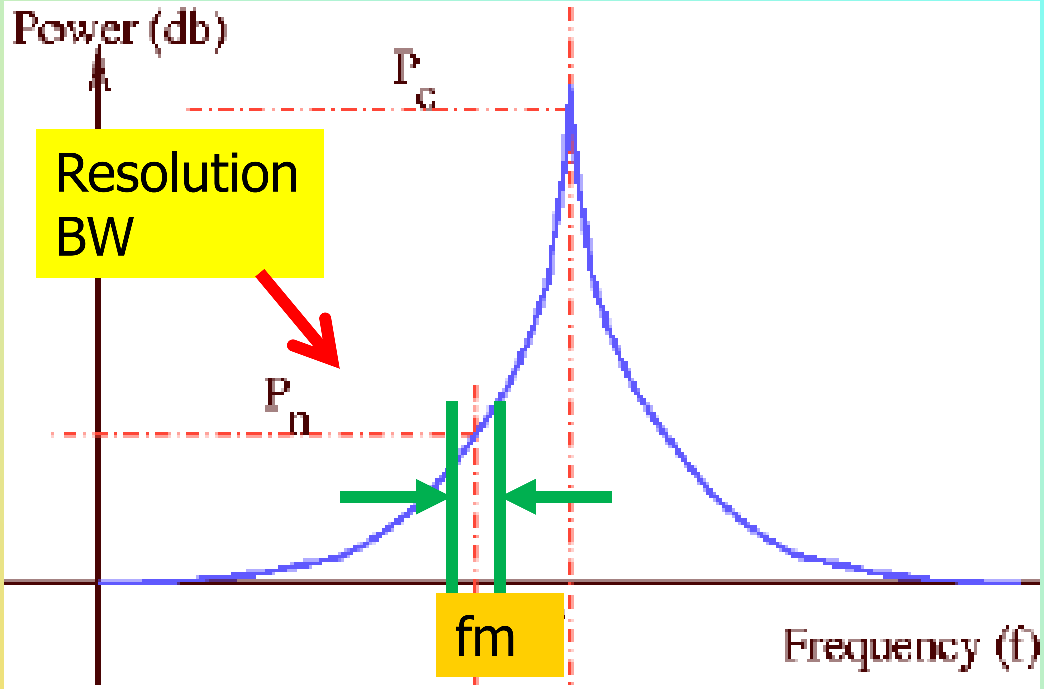
Phase Noise Measurement

- Measurement at a frequency offset from the carrier (fm) is the time interval of phase variation
  - 1 kHz offset is phase variation in 1 millisecond
  - Resolution Bandwidth is the dwell time of the measurement

➢ 1Hz resolution bandwidth is a 1 second measurement time

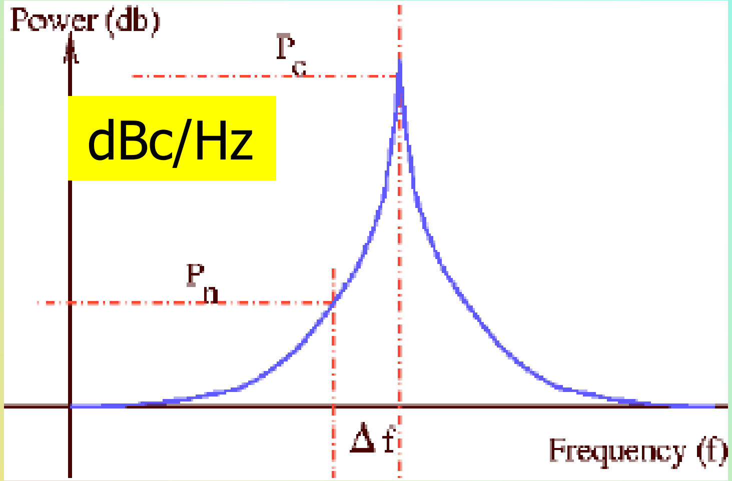
➢ a 1Hz resolution bandwidth at 1kHz from the carrier

   ➢ Measuring phase variation in 1 millisecond averaged 1000 times (1Hz) -



Measurement Data

- Data is normalized to a 1Hz resolution bandwidth
- Data is actually taken at much faster rates
- In automated test equipment
  - Rates are shortened as the analyzer gets further from the carrier
  - Accurate measurement don't require averaging 1000 times   -



Conclusion

- Thermal Noise can be thought of as a vector with a Gaussian amplitude at any phase
- This vector add to the desired signal and creates an uncertainty in the signal characteristic
- If thermal noise changes the phase characteristic of the device it has to be evaluated as phase modulation
- This phase modulation has a Gaussian phase distribution which adds to the phase characteristic of the desired signal
- Phase Noise is dominant close to the carrier (greater than thermal noise)
- Demodulation close to the carrier must consider Phase Noise levels as well as amplitude related thermal noise levels
- Part 2 will focus:
  - Phase Noise Generation
  - Phase Noise Models
  - Effects on Digital Modulation -