Topic 4

Practical Magnetic Design: Inductors and Coupled Inductors

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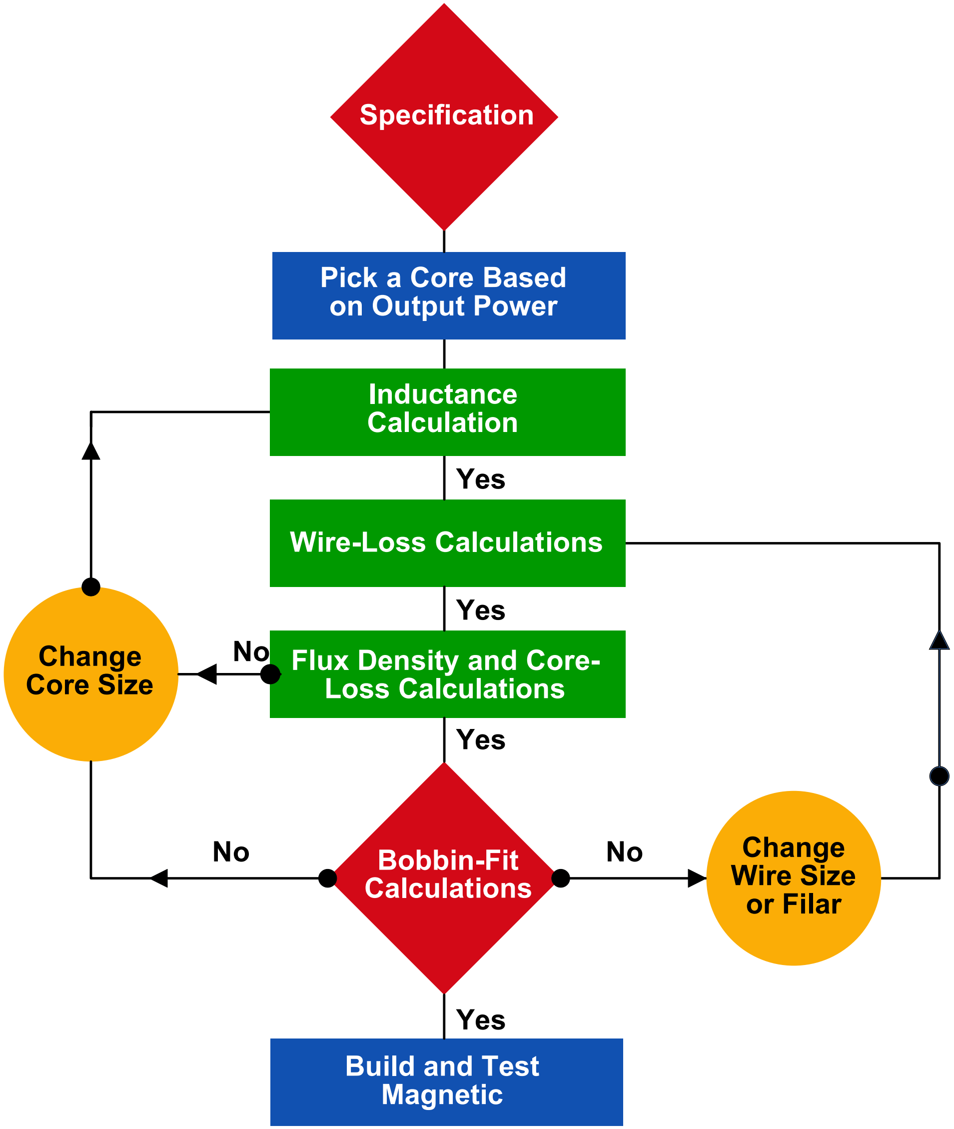
Agenda

- **Theory of operation and design equations**
  - Design flow diagram discussion
  - Inductance calculations
  - Ampere’s law for magnetizing force
  - Faraday’s law for flux density
  - BH curves and magnetic saturation
  - Core loss
  - AC and DC wire losses
  - Effective permeability and inductance rolloff
  - Coupled-inductor winding layers and considerations
- **Example – flyback-coupled inductor design using EFD core**

Design Flow

Specifications required:

1. Inductance
2. Turns
3. Peak current
4. RMS current
5. Output power
6. Frequency



Magnetic Parameters and Conversion Factors

		SI	CGS	CGS to SI
Flux Density	B	Tesla	Gauss	10^{-4}
Field Intensity	H	A-T/m	Oersted	$1,000/4\pi$
Permeability	μ	$4\pi \times 10^{-7}$	1	$4\pi \times 10^{-7}$
Area	Ae	m^2	cm^2	10^{-4}
Length	lg, le	m	cm	10^{-2}

Inductance

The inductance, L, of a wound core can be calculated from the core geometry using the following equation:

$$L = \frac{0.4 \times \pi \times N^2 \times Ae \times 10^{-8}}{lg + \frac{le}{\mu}}$$

$\mu =$ Core permeablity

$N =$ Number of turns

$Ae =$ Core cross-section (mm^2)

$le =$ Core magnetic path length (mm)

$lg =$ Gap (mm)

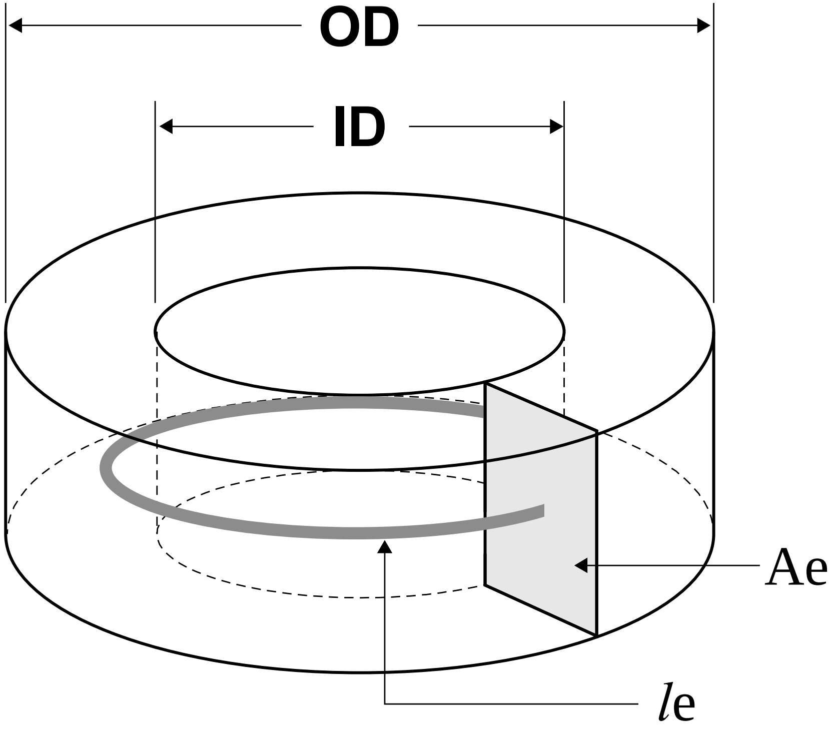
Manufacturers list inductance for a given core and gap as inductance per turn squared, referred to as Al.

$$L = Al \times N^2$$

For Al in nH/per turn^2

Core Geometry

Mean magnetic path length (le) and effective area (Ae) are given in most data sheets, but le can also be calculated using the following equation:



$$le = \frac{\pi(OD-ID)}{\ln\left(\frac{OD}{ID}\right)}$$

Ampere’s Law

Ampere’s law states that the total magnetic force along a closed path is proportional to the ampere-turns in the winding that the path passes through.

SI: $\mathrm{F} = \oint \mathrm{H} \times \mathrm{d}l = \mathrm{N} \times \mathrm{I} \approx \mathrm{H} \times l\mathrm{e}$

$$\mathrm{H} \approx \frac{\mathrm{N} \times \mathrm{I}}{l\mathrm{e}} \text{ampere-turns/meter(A-T/m)}$$

CGS: $\mathrm{F} = \oint \mathrm{H} \times \mathrm{d}l = 0.4 \times \pi \times \mathrm{N} \times \mathrm{I} \approx \mathrm{H} \times l\mathrm{e}$

$$\mathrm{H} \approx \frac{0.4 \times \pi \times \mathrm{N} \times \mathrm{I}}{l\mathrm{e}} \text{(Oersteds)}$$

H = Magnetizing force

le = Core magnetic path length (m for SI, cm for CGS)

N = Number of turns

I = Peak magnetizing current (amperes)

Faraday's Law

The total magnetic flux Φ passing through a surface of area Ae is related to the flux density β.

SI:

$$\Delta\Phi = \frac{1}{N}\int E \times dt$$

$$E = N \times \frac{d\Phi}{dt} \approx N \times Ae \times \frac{d\beta}{dt} \Rightarrow \beta = \frac{\int E \times dt}{N \times Ae}$$

β = Flux density (Tesla)

CGS:

$$\Delta\Phi = \frac{10^8}{N}\int E \times dt$$

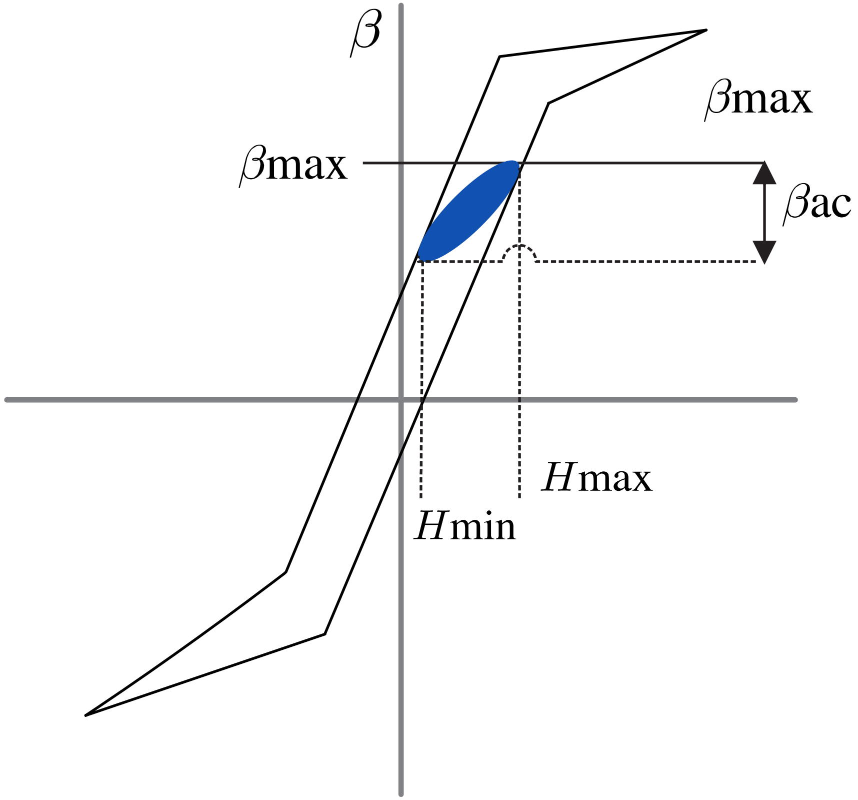
$$E = \frac{N}{10^8} \times \frac{d\Phi}{dt} \approx \frac{N \times Ae \times d\beta}{10^8 \times dt} \Rightarrow \beta = \frac{\int E \times dt \times 10^8}{N \times Ae}$$

β = Flux density (Gauss)

E = Voltage across coil (V)

Ae = Effective core area (m^2)

BH Curves



Bac is the AC flux density and Bmax is the peak flux density.

$$\beta ac = \frac{Vin \times ton}{Ae \times Np} \quad \text{Tesla}$$

$$\beta max = \frac{Lp \times Ip}{Ae \times Np} \quad \text{Tesla}$$

β = Flux density (Tesla)

Np = Number of primary turns

ton = On time (s)

Ae = Effective core area (m^2)

Vin = Voltage (V)

Lp = Primary inductance (H)

Ip = Peak primary current (A)

Core Loss

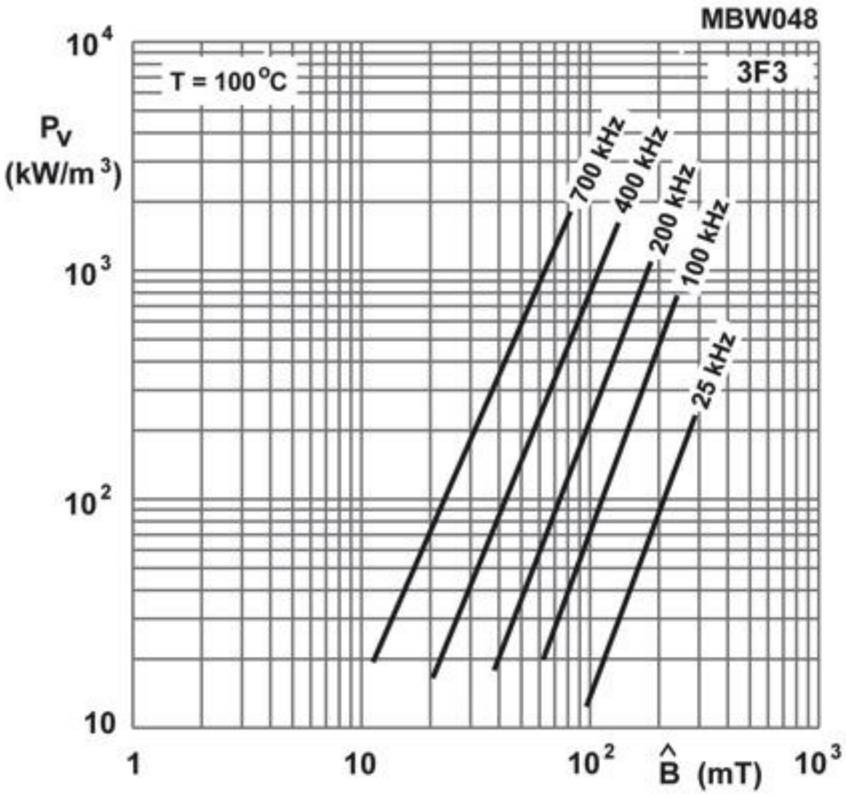
For a Ferroxcube core:

$$\beta acpeak = \frac{\beta ac}{2}$$

Convert Ve from mm^3 to m^3

$$Ve(m^3) = \frac{Ve}{1{,}000^3}$$

$$Pcoreloss = Pv(KW/m^3) \times Ve(m^3)$$



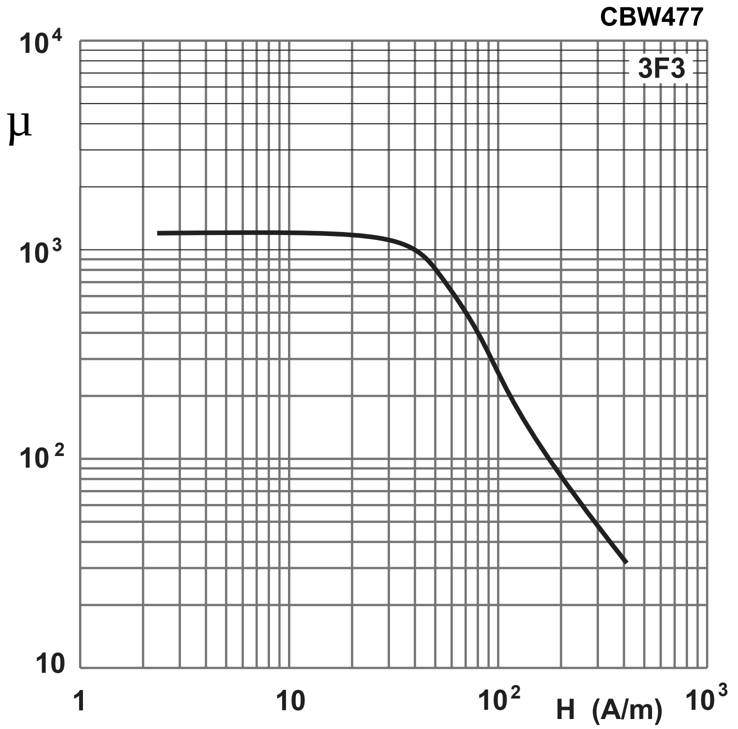
Specific power loss as a function of one-half of peak-to-peak flux density with frequency as a parameter.

Courtesy of Ferroxcube

Permeability and Inductance Rolloff

As current is increased in a magnetic and the flux density gets closer to core saturation, the permeability of the core starts to roll off.

$$L = \frac{0.4 \times \pi \times N^2 \times Ae \times 10^{-8}}{lg + \frac{le}{\mu}}$$

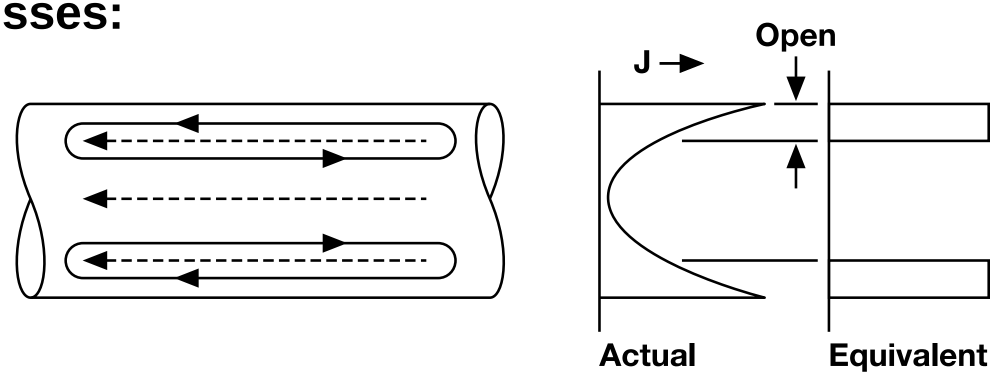


Permeability as a function of magnetizing force (H) (courtesy of Ferroxcube).

Courtesy of Ferroxcube

AC–DC Wire Loss

Determine wire area and AC losses:



$$\text{Wire area required} = \text{Irms} / \text{J}$$

$$\text{J} = 400\ \text{A/cm}^2$$

$$\delta(\text{skin depth cm}) = \sqrt{\frac{\rho}{\pi \times \mu \times \text{f}}} \approx \frac{7.6}{\sqrt{\text{f}}}\text{at } 100°\text{C}$$

$$\text{Number of wires required} = \frac{\dfrac{\text{Wire area required}}{\text{Area of wire chosen}}}{\text{Ratio of AC to DC losses}}$$

$$\text{Total winding resistance} = \text{AC–DC wire resistance of selected wire} \times \text{Number of turns} \times \frac{\text{Length per turn}}{\text{Number of wires used}}$$

AC–DC Wire Loss

Using look-up tables from the “Magnetics Design Handbook (MAG100A)” by Llyod H. Dixon:

Determine the wire area in cm^2 and the radius in cm

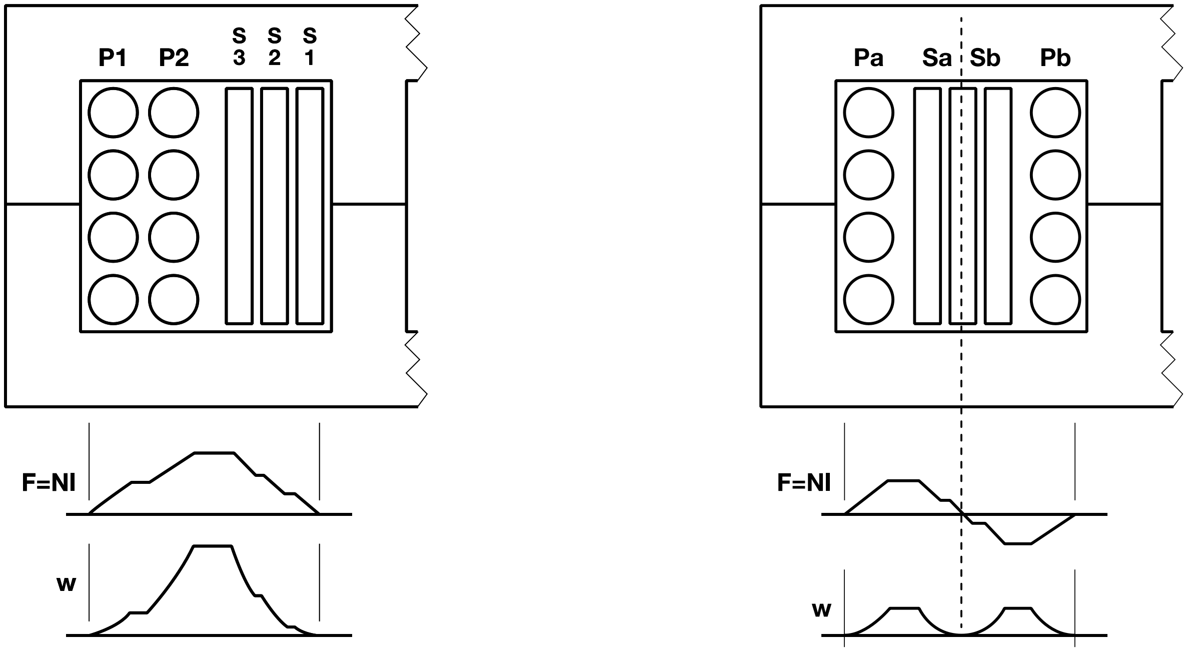
$$\text{Annular inner ring (cm}^2) = \pi \times \left\{ \text{radius of wire}^2 - [\text{wire radius(cm)} - \text{skin depth(cm)}]^2 \right\}$$

$$\text{Ratio Rac to Rdc (Rskin)} = \frac{\text{Area of wire in (cm}^2)}{\text{Annular inner ring (cm}^2)}$$

$$\text{AC–DC wire resistance} = \text{DC resistance of chosen wire} \times \text{Ratio Rac to Rdc (Rskin)}$$

Proximity Effects

AC current in a conductor induces eddy currents in adjacent conductors by a process called the proximity effect.



Power loss Pm in layer m is:

$$Pm = I^2\left[(m-1)^2 + m^2\right]\left(\frac{wire\_dia}{d} \times Rdc\right)$$

Winding Factor and Bobbin Fit Calculations

Winding data and area product for EFD20/10/7 coil former with 10-solder pads

Number of Sections	Winding Area (mm²)	Minimum Winding Width (mm)	Average Length of Turn (mm)	Area Product Ae x Aw (mm⁴)	Type Number
1	27.7	13.5	34.1	859	CPHS-EFD20-1S-10P

Courtesy of Ferroxcube

Winding factor for a bobbin should be in the .3 to .7 range depending on isolation requirements

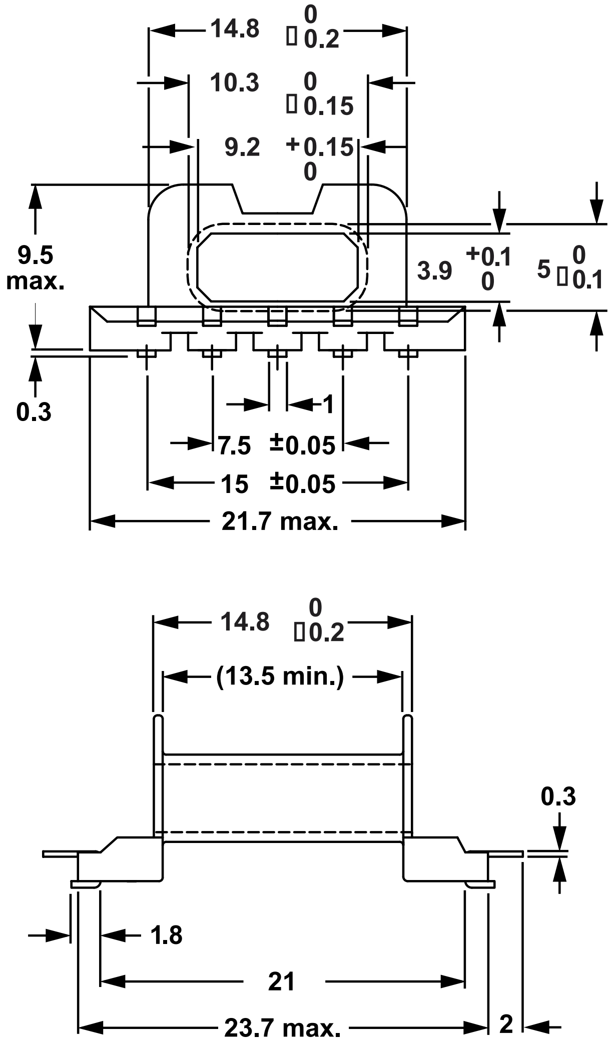
$$\text{Turns per layer} = \frac{\text{Bobbin width (mm)}}{\text{Diameter of wire in (mm)}} \text{-2}$$

$$\text{Build up} = \frac{\text{Winding area in (mm)}}{\text{Winding width in (mm)}}$$

$$\text{Number of avaliable layers} = \frac{\text{Buildup}}{\text{Diameter of chosen wire}}$$

$$\text{Total bobbin turns} = \text{Turns per layer} \times \text{Number of layers}$$

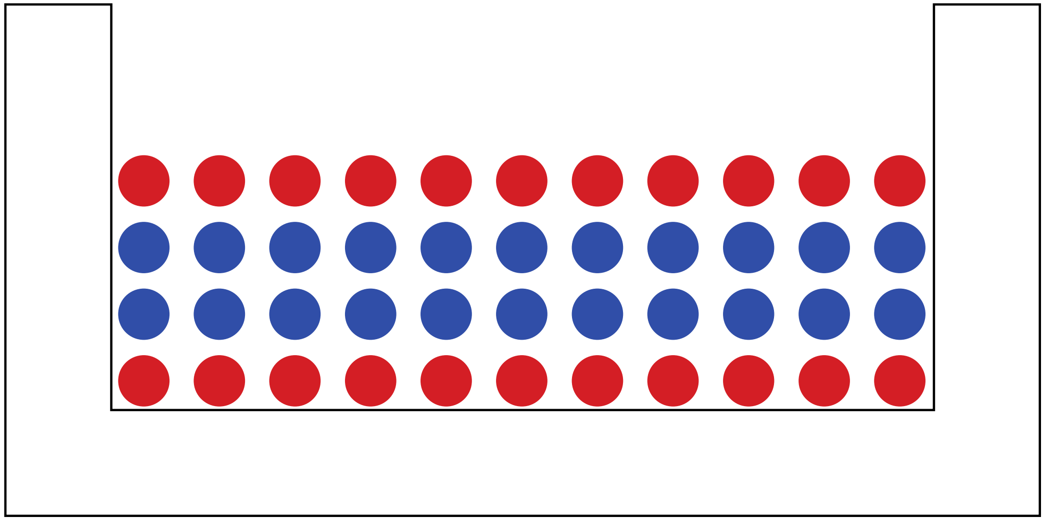
$$\text{Winding fill factor} = \frac{\text{Turns needed}}{\text{Total bobbin turns avaliable}}$$



Dimmensions in mm

Transformer Layers

Transformer layout is very important because it affects primary to secondary coupling and leakage inductance.



Fourth layer: 32awg single filar half of primary.

Third layer: 32awg trifilar.

Second layer: 32awg trifilar.

First layer: 32awg single filar half of primary.

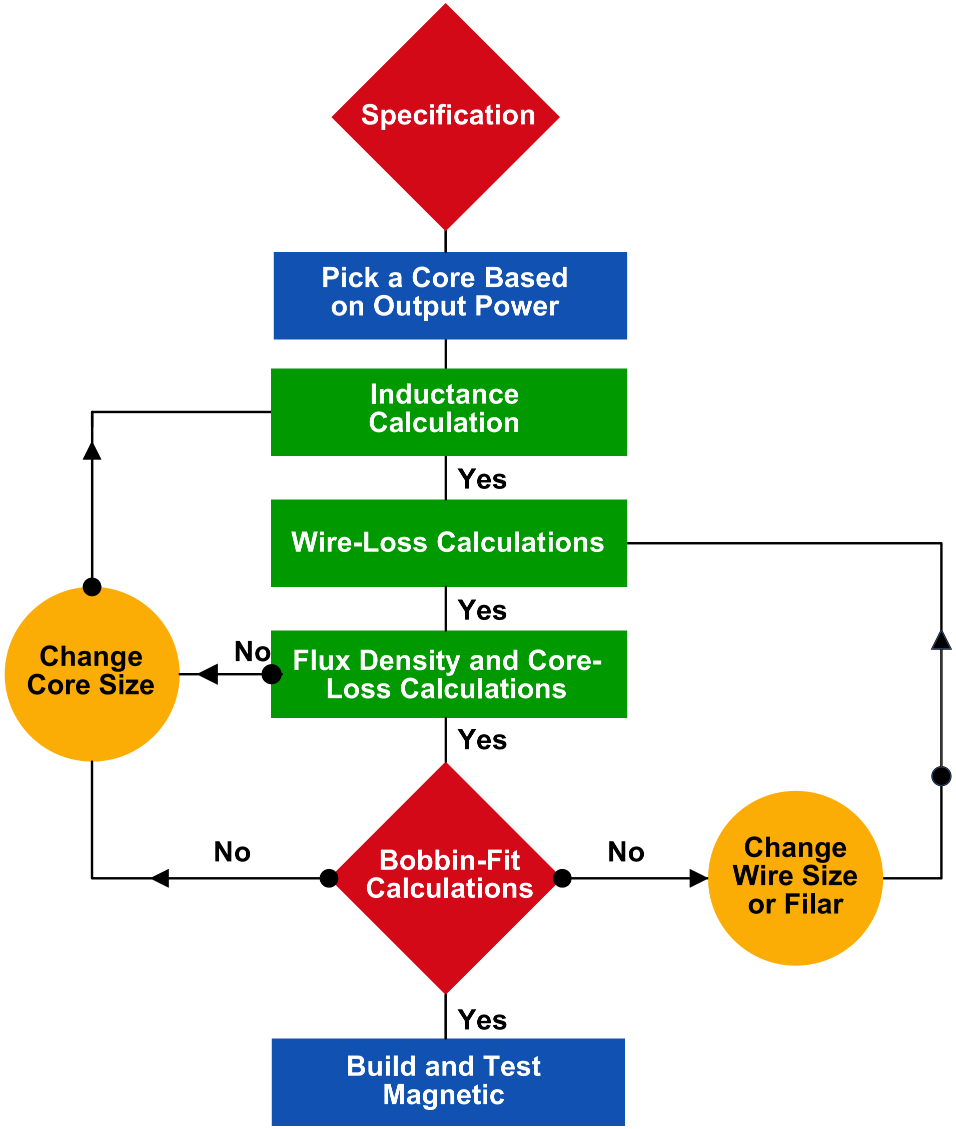
Design Flow Summary

- Complete specification
- Pick a core (see section 4, “Power Transformer Design” by Lloyd H. Dixon)
- Calculate inductance and turns-based Al
- Calculate copper loss
- Calculate flux density and core loss
- Calculate temperature rise (see “Constructing Your Power Supply – Layout Considerations” by Robert Kollman)
- Iterate as required to get the turns to layer nicely on the core for good coupling and low leakage inductance

Flyback-Coupled Inductor Example

Specifications required:

1. Inductance
2. Turns
3. Peak current
4. RMS current
5. Output power
6. Frequency



Flyback-Coupled Inductor Example

Specifications required:

General	
Topology	Quasi-Resonant Flyback
Main Output Power	10 W
Maximum Switching Frequency at Full Load	140 kHz
Input	
Minimum Input Voltage	85 VAC, 76 VDC
Maximum Input Voltage	265 VAC, 375 VDC
Primary Peak Current	1.155 A
Primary RMS Current	0.425 A
Outputs	
Secondary Output Voltage	5 V
Secondary Peak Current	13.861 A
Secondary RMS Current	5.382 A
Bias Voltage	16 V
Bias Current	50 mA
Inductance and Turns Ratio	
Primary Inductance	190.918 µH
Leakage Inductance	3.818 µH
Primary-to-Secondary Turns Ratio	12

Flyback-Coupled Inductor Example

- Core selection is based on
  - Core size
  - Core material
  - Core gap

See section 5 in “Magnetics Design Handbook”
by Lloyd H. Dixon (MAG100A)

- An EFD20 core with 3F3 material was chosen
  - Al was set to 82 nH
  - Key core and bobbin data

Winding Area (mm^2)	Min. Winding Width (mm)	Length per Turn (mm)	Area Product (mm^4)	Buildup (mm)	Effective Volume (mm^3)
27.7	13.5	34.1	859	2.052	1460

Flyback-Coupled Inductor Example

- Calculate primary turns, Np, based on required inductance and *Al* value

$$\mathrm{Np} = \left(\frac{\mathrm{Lp}}{Al}\right)^{1/2}$$

$$\mathrm{Lp} = 190.918\ \mu\mathrm{H}$$

$$Al = 82\ \mathrm{nH}$$

$$\mathrm{Np} = 48$$

- Calculate secondary turns-based turns ratio

$$\mathrm{Ns} = \frac{\mathrm{Np}}{\text{Turns ratio}}$$

$$\text{Turns ratio} = 12$$

$$\mathrm{Ns} = 4$$

Flyback-Coupled Inductor Example

- Determine required primary wire area based on current density, J

$$\text{Primary wire area} = \frac{\text{Ipri\_max}}{\text{J}}$$

$$\text{J} = 400\ \text{A/cm}^2$$

$$\text{Ipri\_max} = 0.425\ \text{A}$$

$$\text{Primary wire area} = 1.0625 \times 10^{-3}\ \text{cm}^2$$

- Determine skin depth at 100 C

$$\delta(\text{skin depth cm}) \approx \frac{7.6}{\sqrt{\text{f}}}$$

$$\text{f} = 140\ \text{kHz}$$

$$\text{Skin depth} = 0.0203\ \text{cm}$$

Flyback-Coupled Inductor Example

- Calculate annular ring area

$$\text{Annular inner ring}\left(\text{cm}^2\right) = \pi \times \left\{\text{radius of wire}^2 - \left[\text{wire radius (cm)} - \text{skin depth (cm)}\right]^2\right\}$$

  - If skin depth is greater than wire radius annular ring area = wire area

- Calculate AC to DC resistance, Rac/Rdc ratio

$$\text{Ratio Rac to Rdc (Rskin)} = \frac{\text{Area of wire in }(\text{cm}^2)}{\text{Annular inner ring }(\text{cm}^2)}$$

AWG	Cu Radius (cm)	Cu Area (cm^2)	Annular Ring Area (cm^2)	Rac/Rdc Rskin
24	0.0255	0.002047	0.001958	1.0453
26	0.02	0.001287	0.001287	1.0000
28	0.016	0.00081	0.000810	1.0000
30	0.0125	0.000509	0.000509	1.0000

Flyback-Coupled Inductor Example

- Calculate number of wires required based on wire area, current density and Rac/Rdc ratio

$$\text{Numbers of wires required} = \frac{\dfrac{\text{Wire area required}}{\text{Area of wire chosen}}}{\text{Ratio of AC to DC losses}}$$

AWG	Wire Area Required (cm^2)	Cu Area (cm^2)	Rac/Rdc Rskin	Number of Wires Required
24	0.0010625	0.002047	1.0453	1
26	0.0010625	0.001287	1.0000	1
28	0.0010625	0.00081	1.0000	2
30	0.0010625	0.000509	1.0000	3
32	0.0010625	0.00032	1.0000	4

Flyback-Coupled Inductor Example

- Look at how the windings might fit in the bobbin window
  - Initially estimate that half the area is used by the primary

$$\text{Turns per layer} = \frac{\text{Bobbin width (mm)}}{\text{Diameter of wire (mm)}}$$

$$\text{Number of available layers} = \frac{\text{Buildup}}{\text{Diameter of chosen wire}} - 2$$

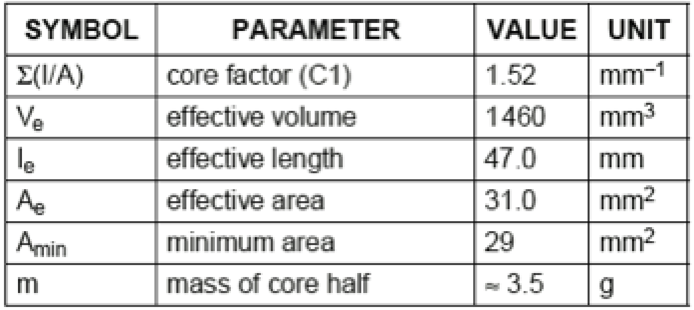
$$\text{Buildup} = \frac{\text{Winding area in (mm)}}{\text{Winding width in (mm)}}$$

AWG	Insulated Diameter (cm)	Min. Winding Width (mm)	Turns per Layer	Available Layers	Number of Wires Required
24	0.057	13.5	21	3	1
26	0.046	13.5	27	4	1
28	0.037	13.5	34	5	2
30	0.03	13.5	43	6	3
32	0.024	13.5	54	8	4

Flyback-Coupled Inductor Example

Estimate primary winding copper losses

- One wire of wire gauge 26 was chosen



SYMBOL	PARAMETER	VALUE	UNIT
$\Sigma(l/A)$	core factor (C1)	1.52	mm^{-1}
V_e	effective volume	1460	mm^3
l_e	effective length	47.0	mm
A_e	effective area	31.0	mm^2
A_{min}	minimum area	29	mm^2
m	mass of core half	≈ 3.5	g

Effective core parameters (courtesy of Ferroxcube).

$$\text{Length\_per\_turn\_cm} = \frac{34.1\ \text{mm}}{10} \qquad \text{Length\_per\_turn\_cm} = 3.41\ \text{cm}$$

$$\text{Rcu\_pri\_26awg} = \text{Rcu26\_100°C} \times \text{Np} \times \frac{\text{Length per turn}}{\text{Number of primary wires}}$$

$$\text{Rcu\_pri\_26awg} = 0.293\ \Omega$$

$$\text{Pcu\_pri\_loss\_26awg} = \text{Ipri\_rms}^2 \times \text{Rcu\_pri\_26awg}$$

$$\text{Pcu\_pri\_loss\_26awg} = 0.053\ \text{W}$$

Flyback-Coupled Inductor Example

Determine secondary wire AWG and losses: $\mathrm{Ns} = 4$

$$\text{Secondary wire area} = \frac{\text{Isecondary}}{\text{J}} = 14 \times 10^{-3}\,\text{cm}^{2} \qquad \text{where J} = 400\ \text{A/cm}^{2}$$

$$\text{Number\_of\_wires\_sec\_28awg} = \frac{\text{Area\_req\_sec\_cm}^{2}}{\dfrac{\text{Area\_28awg\_cm}^{2}}{\text{Rskin\_28awg\_100°C}}} \qquad \text{Number\_of\_wires\_sec\_28awg} = 16.667$$

$$\text{Number\_of\_wires\_sec\_28awg\_used} = 5$$

$$\text{Rcu\_sec\_28awg} = \text{Rcu28\_100°C} \times \text{Ns} \times \frac{\text{Length per turn}}{\text{Number of secondary wires}}$$

$$\text{Rcu\_sec\_28awg} = 7.761 \times 10^{-3}$$

$$\text{Pcu\_sec\_loss\_28awg} = \text{Isec\_rms}^{2} \times \text{Rcu\_sec\_28awg}$$

$$\text{Pcu\_sec\_loss\_28awg} = 0.226\ \text{W}$$

Flyback-Coupled Inductor Example

Determine bias wire AWG and losses:

$$\text{Number of bias wires} = 1 \qquad \text{Nbs} = 13$$

$$\text{Rcu\_bias\_32awg} = \text{Rcu32\_100°C} \times \text{Nsb} \times \frac{\text{Length per turn}}{\text{Number of bias wires}}$$

$$\text{Rcu\_bias\_32awg} = 225 \times 10^{-3}$$

$$\text{Pcu\_bias\_loss\_32awg} = \text{Ibias\_rms}^{2} \times \text{Rcu\_bias\_32awg}$$

$$\text{Pcu\_bias\_loss\_32awg} = 4.197 \times 10^{-4}\ \text{W}$$

Flyback-Coupled Inductor Example

Determine core flux density:

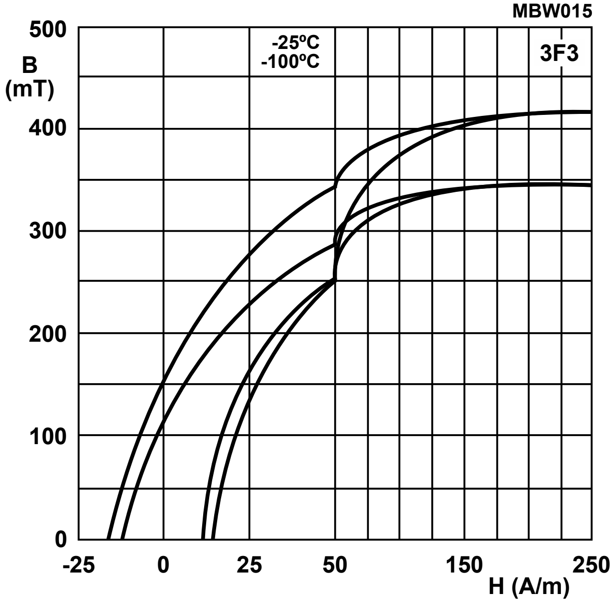
$$\text{Ae} = 31\ \text{mm}^2 \quad \text{Np} = 48 \quad \text{Ipri\_p} = 1.155\ \text{A} \quad \text{tonmax} = 2.9\ \mu\text{s}$$

$$\text{Bac} = \frac{\text{Vinmin} \times \text{tonmax}}{\text{Ae} \times \text{Np}}$$

$$\text{Bac} = 148.17\ \text{mT}$$

$$\text{Bmax} = \frac{\text{Lp} \times \text{Ipri\_p}}{\text{Ae} \times \text{Np}}$$

$$\text{Bmax} = 148.19\ \text{mT}$$



Courtesy of Ferroxcube

Flyback-Coupled Inductor Example

Determine core loss:

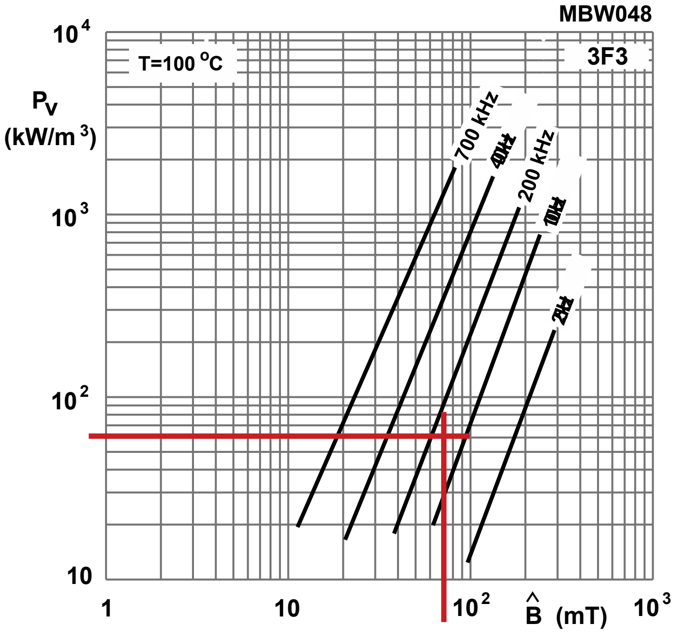
$$\text{Bunipolar} = \frac{\text{Bac}}{2}$$

$$\text{Bunipolar\_mt} = 74.1\ \text{mT}$$

$$\text{From graph Pcore} = 60\ \text{kW/m}^3$$

$$\text{Ve} = 1460\ \text{mm}^3 \qquad \text{Ve\_per\_m}^3 = \frac{\text{Ve}}{1{,}000^3}$$

$$\text{Pcore\_loss} = \text{Pcore} \times \text{Ve\_per\_m}^3 \qquad \text{Pcore\_loss} = 0.088\ \text{W}$$



Specific power loss as a function of Peak flux density with frequency as a parameter.

Courtesy of Ferroxcube

Flyback-Coupled Inductor Example

Magnetic power dissipation:

$$\text{Pcu\_loss\_tot} = \text{Pcu\_pri\_loss} + \text{Pcu\_sec\_loss} + \text{Pcu\_bias\_loss}$$

$$\text{Ptot\_mag} = \text{Pcu\_loss\_tot} + \text{Pcore\_loss} \qquad \text{Ptot\_mag} = 0.367\ \text{W}$$

Determine bobbin fit factor:

AWG	Copper Diameter in cm with Insulation	Copper Area in cm^2 with Insulation
26	0.046	0.001671
28	0.037	0.001083
30	0.03	0.000704
32	0.024	0.000459

$$\text{bobbin\_width\_mm} = 13.5\ \text{mm} \qquad \text{bobbin\_wdth\_cm} = \text{bobbin\_width\_mm} \times .1$$

$$\text{bobbin\_width\_cm} = 1.35\ \text{cm}$$

Flyback-Coupled Inductor Example

Determine bobbin fit factor continued:

$$\text{Turns\_per\_layer26} = \frac{\text{bobbin\_width\_cm}}{\text{dia\_26awg\_Inso\_cm}} - 2 \qquad \text{Turns\_per\_layer26} \approx 27$$

$$\text{Turns\_per\_layer28} \approx 34 \qquad \text{Turns\_per\_layer30} \approx 43$$

$$\text{Winding\_area\_mm} = 27.7\ \text{mm}^2 \qquad \text{Winding\_area\_cm}^2 = .277$$

$$\text{Build\_up\_cm} = \frac{\text{winding\_area\_cm}^2}{\text{bobbin\_width\_cm}} \qquad \text{Build\_up} = .205\ \text{cm}$$

$$\text{Layers} = \frac{\text{Build\_up\_cm}}{\text{dia\_26awg\_Inso\_cm}} \qquad \text{Layers} \approx 4$$

$$\text{Total\_bobbin\_turns\_26awg} = \text{Turns\_per\_layer} \times \text{Layers} \qquad \text{Total\_bobbin\_turns\_26awg} = 108$$

$$\text{Turns\_needed} = \text{Np} \times \text{number\_of\_wires\_pri} + \text{Ns} \times \text{number\_of\_wires\_sec} +$$
$$\text{Nsb} \times \text{number\_of\_wires\_bias} \qquad \text{Turns\_needed} = 81$$

$$\text{Winding\_factor} = \frac{\text{Turns\_needed}}{\text{Total\_bobbin\_turns\_26awg}} \qquad \text{Winding\_factor} = .75$$

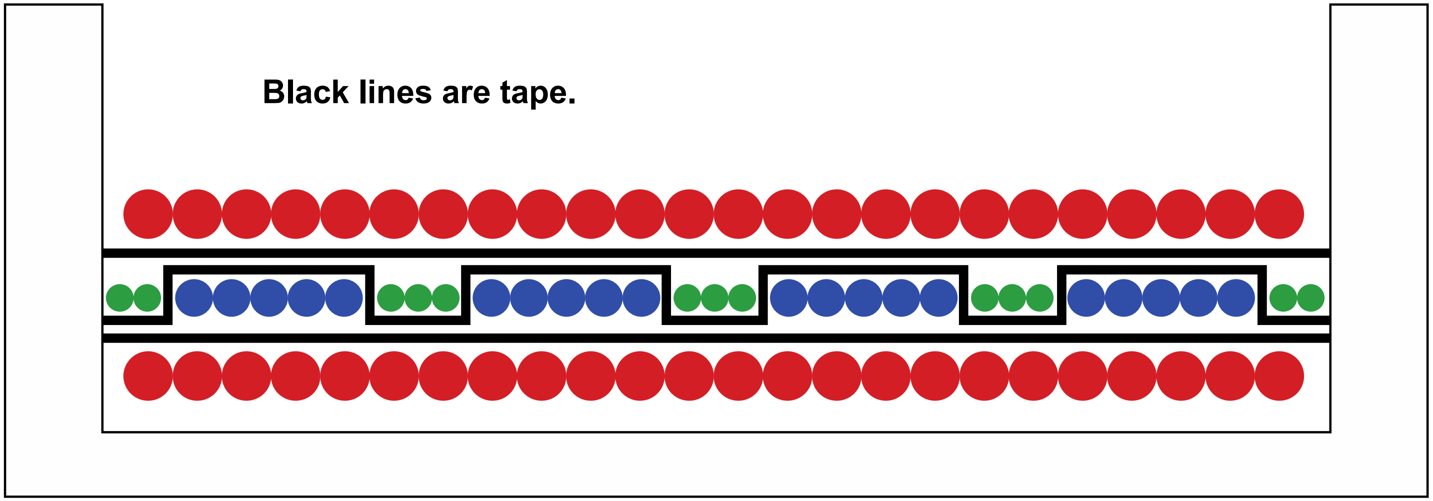
Flyback-Coupled Inductor Example

Coupled inductor layout:

First layer: 24 turns of 26awg single filar half of primary

Second layer: 20 turns of 28awg five filar secondary plus 13 turns of 30awg bias winding

Third layer: 24 turns of 26awg single filar half of primary



Black lines are tape.

Summary

- Discussed theory of operation and design flow
- Determined inductance, flux density, core loss, AC and DC wire losses
- Discussed BH curves and magnetic saturation, effective permeability and inductance rolloff
- Discussed transformer winding layers and how to interleave the winding for good coupling and low leakage
- Went through an example of a flyback-coupled inductor design using EFD core