Resonant Phenomena in HV Converters

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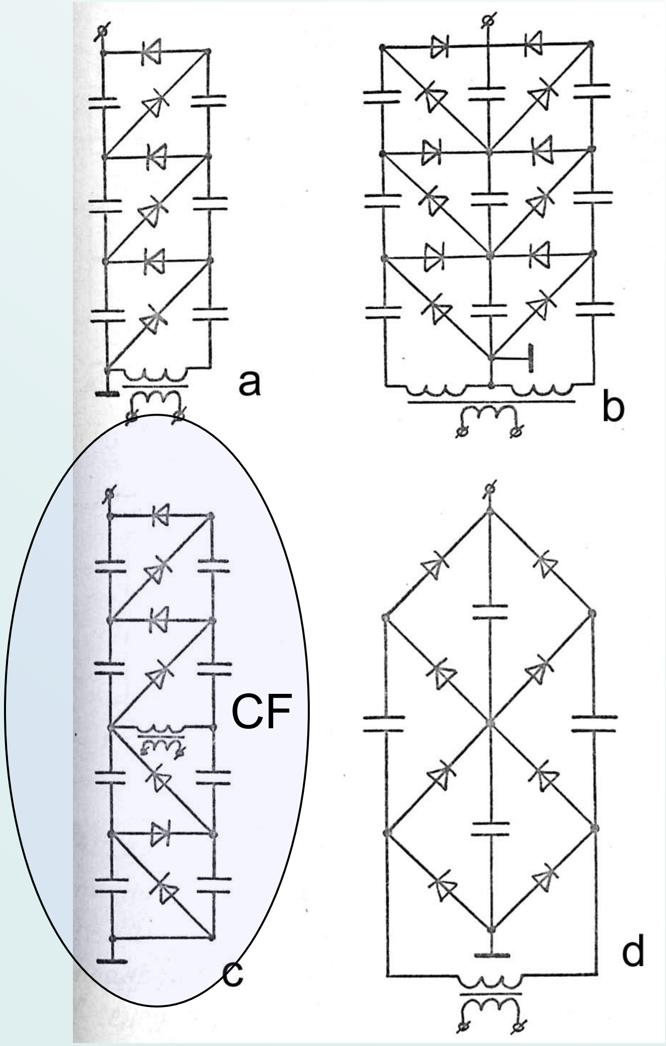
Outline

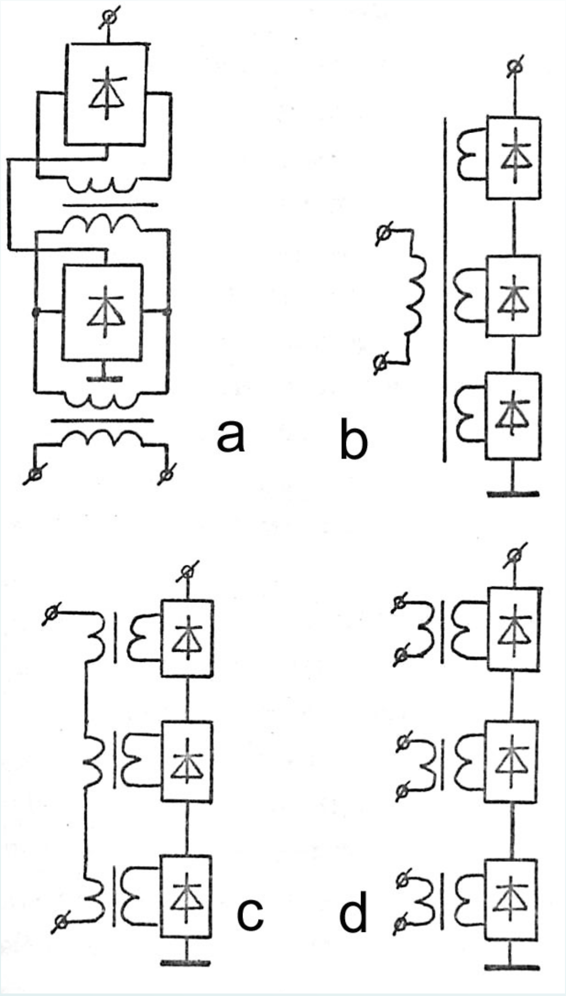
- Introduction
  - General remarks on HV converters, voltage multiplier (VM) topologies, and range of applicable parameters
  - Equivalent circuits of HV transformers and converters, including VM
- No-load operation at PWM square-wave excitation
- Transformer parasitic parameters calculation and measurement
- Analytical estimates of VM voltage drop, voltage compression along stages, *K*, and added equivalent parasitic capacitance, C_{add}
- VM-transformer circuit simulation
- Integration of parasitics
- Backlash from secondary winding
- Conclusions

General remarks on HV DC-to-DC converters relevant to this presentation

- *V* from tens to hundreds kV, *P*=0.01 ÷200kW, switching frequency f_s to ~100kHz
- Essence of HV converter design is combining high dielectric strength with reliable operation of semiconductor switches:
  - large distance between low and HV (or high potential) elements--large leakage inductance
  - large parasitic capacitances, especially for potted designs
  - Bizarre current and voltage waveforms: increased switches losses
- reactive currents strongly affect electromagnetic processes that, at no-load conditions, are *dominated* by parasitics
- Both *HV side* parasitics and VM construction components impact on current waveforms of HV transformer and inverter power switches is termed here as “resonant” behavior

Selected topologies of voltage multiplication



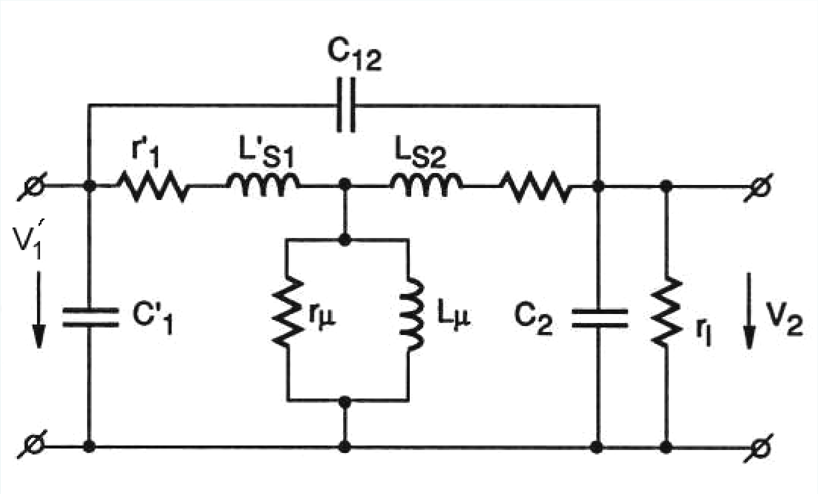


applicable both to line and high frequency

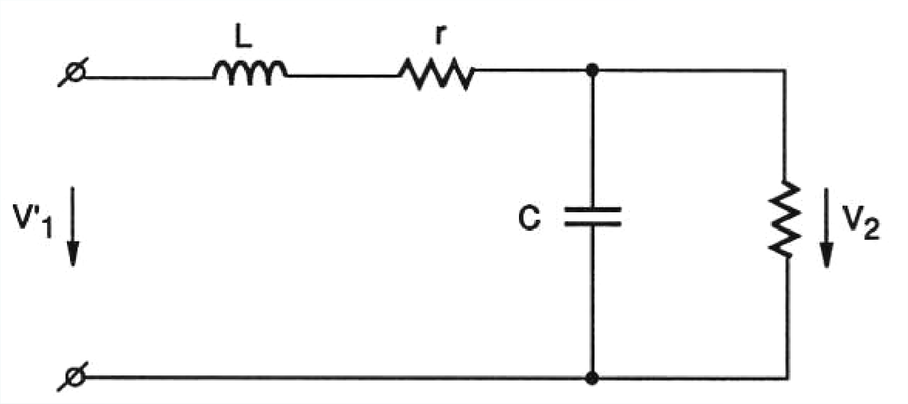
From A. Pokryvailo, "Study and Development of High Voltage Switch-Mode DC Power Supplies for X-Ray Analytical Apparatus" Dissertation, Leningrad, 1987. In Russian

HV TRANSFORMER
equivalent circuits with lumped parameters:

may *not* be applicable to analysis of fast transient phenomena



Equivalent circuit of high voltage transformer. All values are reflected to secondary. L_s - leakage inductances, C – parasitic capacitances, resistors – losses in windings, HV insulation and core.



Simplified equivalent circuit of high voltage transformer. Magnetizing inductance L_μ>>L_s; L≈L_{s2}; C≈C_2. Works well for closed magnetic systems (U, E-cores, etc.). Inapplicable for open cores (e.g., rods).

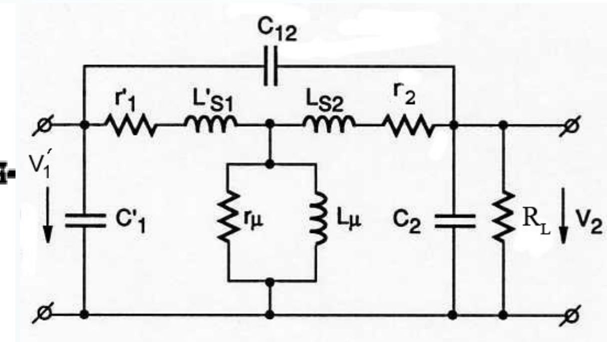
Extensive experimental justification for transformers with U-cores in: A. Pokryvailo, Dissertation, Leningrad, 1987.

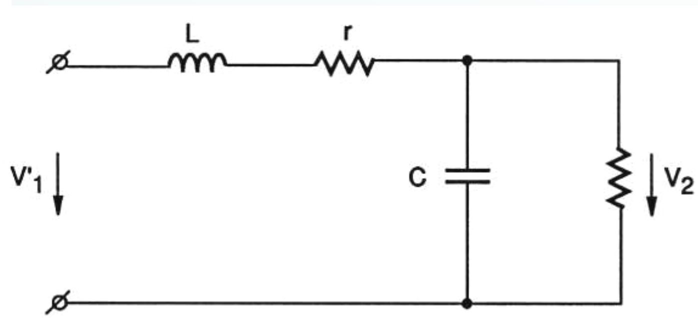
Frequency response of HV transformer

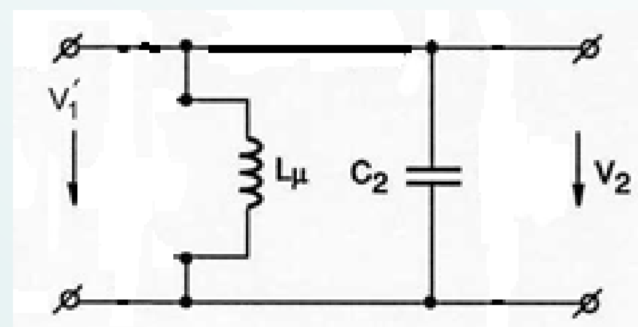
Очевидно, что экстремумы обусловлены резонансами в структуре трансформатора, причем минимум тока, согласно эквивалентной схеме, следует отнести за счет резонанса в параллельном контуре, образованном L_μ и цепочкой L_{s_2}, C_2, а максимум вызван резонансом в последовательном колебательном контуре L'_{s_1}; L_{s_2}, C_2. Действительно, пренебрегая влиянием r'_1, C'_1, C_{12}, r_μ, r_2, можно записать выражение для входной проводимости трансформатора

$$y = \frac{\omega(L_{s_2}+L_\mu) - 1/\omega C_2}{\omega^2\left[L'_{s_1}(L_{s_2}+L_\mu)+L_\mu L_{s_2}\right] - \frac{L'_{s_1}+L_\mu}{C_2}},$$

откуда следует, что $y = 0$ при $\omega_1 = 1/\sqrt{(L_\mu+L_{s_2})C_2}$ и, если принять $L_\mu \gg L'_{s_1}, L_\mu \gg L_{s_2}$, $y = \infty$ при $\omega_0 = 1/\sqrt{L\,C_2}$, где $L = L'_{s_1}+L_{s_2}$. Тогда $\omega_1 \approx 1/\sqrt{L_\mu C_2}$.



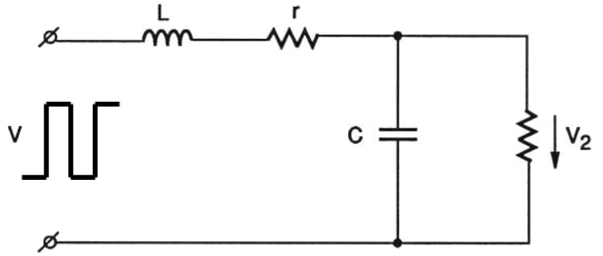




y is transformer input conductance

No-Load at Symmetrical Square Wave Excitation

Fourier analysis



Let $\bar{t} = t/T$ be non-dimensional time, $T = 2\pi/\omega$ - period of exciting voltage, $\overline{\omega} = \omega/\omega_0$ - dimensionless angular frequency, $\omega_0 = 1/\sqrt{LC}$ - resonant angular frequency of transformer, and $\rho = \sqrt{LC}$ - transformer wave impedance. When primary winding is excited by symmetrical square wave voltage (duty cycle D=0.5) with amplitude E, voltage across secondary is given by expression

$$v_2 = \sum_{k=1}^{\infty} V_{mk}^* \sin(2\pi k\bar{t} - \Psi_k^*) \quad (1)$$

Here V_{mk}^*, Ψ_k^* are the amplitude and the phase of the k^{th} harmonic, respectively:

$$V_{mk}^* = \frac{4E}{k\pi} \frac{1}{\sqrt{\left(1 + r/R_L - k^2\overline{\omega}^2\right)^2 + \left(kd\overline{\omega}\right)^2}}, \quad (1a)$$

$$\Psi_k^* = a\tan\frac{kd\overline{\omega}}{1 - \overline{\omega}r/R_L - k^2\overline{\omega}^2}. \quad (1b)$$

Parameter $d = \rho/R_L + r/\rho$ characterizes the damping of the oscillating system. Overvoltage coefficient (OVC) K is defined as ratio of secondary voltage amplitude to E:

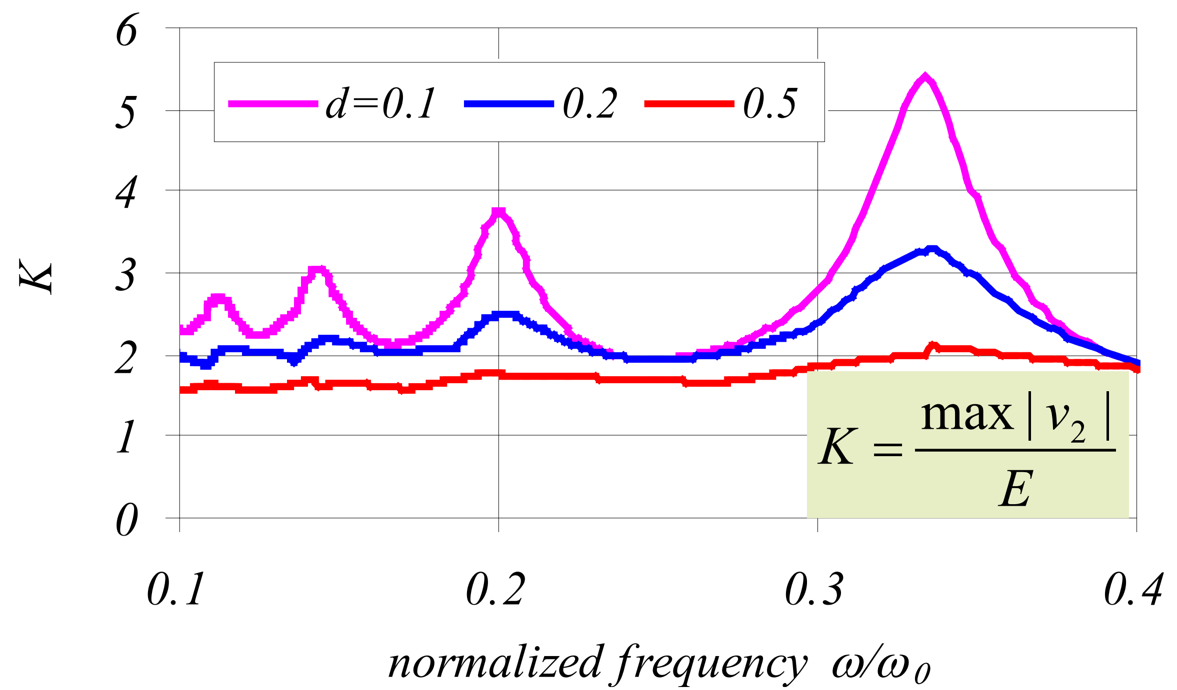
$$K = \frac{\max|v_2|}{E}$$

For PWM with duty cycle *D* (unpublished)

$$V_{mk}^* = \frac{4E}{k\pi} \frac{\cos\left[k\frac{\pi}{2}(1-D)\right]}{\sqrt{\left(1 + r/R_L - k^2\overline{\omega}^2\right)^2 + \left(kd\overline{\omega}\right)^2}}$$

A. Pokryvailo and V. Pokryvailo, "Some Features of Operation of High Voltage Switch-Mode Power Supplies for X-Ray Apparatus", *Instrumentation and Methods of X-Ray Analysis,* Vol. 35, 1986.

No-load at Symmetrical Square Wave Excitation cont.



$$K = \frac{\max|v_2|}{E}$$

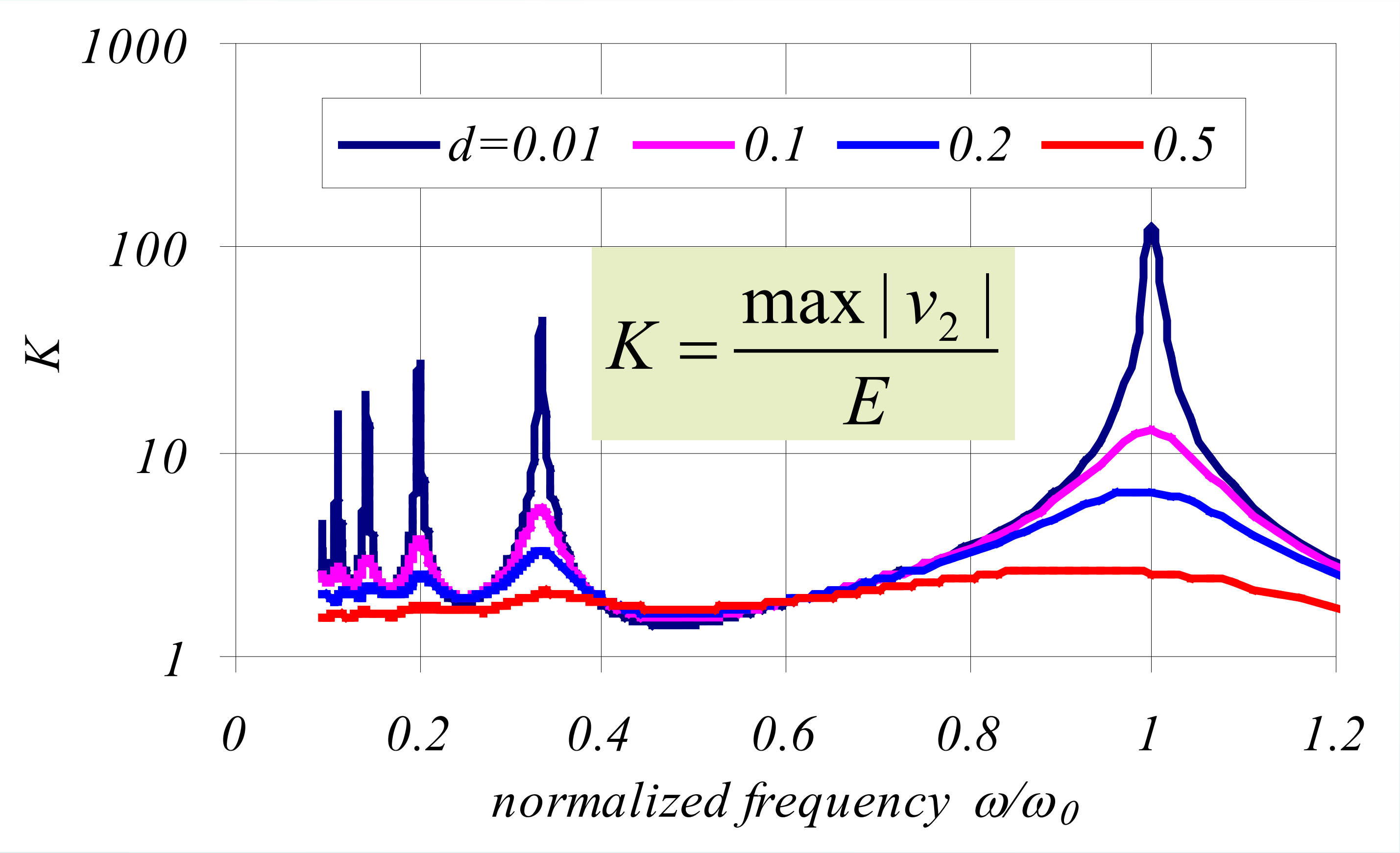
K=1 - no overvoltage (OV), or the output is equal to the input. Already at this point, we can draw an important conclusion: at no-load the converter output voltage is **at least double** of that calculated according to the transformation ratio.

A. Pokryvailo and V. Pokryvailo, "Some Features of Operation of High Voltage Switch-Mode Power Supplies for X-Ray Apparatus", *Instrumentation and Methods of X-Ray Analysis,* Vol. 35, 1986.

A. Pokryvailo, "On Electromagnetic Processes in HV Transformers of Switching-Mode Power Supplies at No-Load Conditions", Proc. 27th Int. Power Modulators Symp., Washington DC, 2006

No-load at Symmetrical Square Wave Excitation cont.

Resonance on fundamental frequency; low damping



No-load at quasi-symmetrical square-wave excitation – Experimental

Note peaks on even harmonics

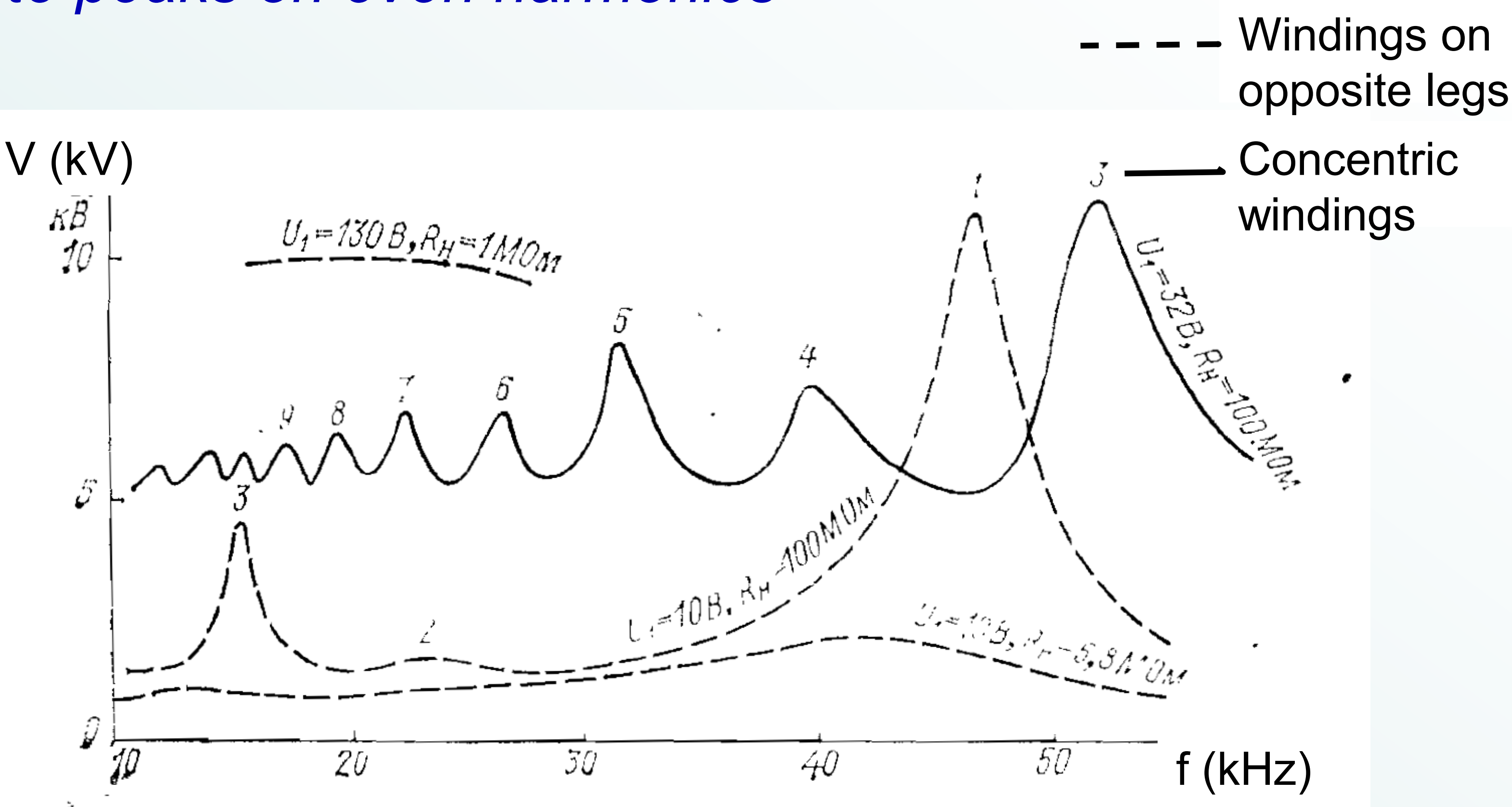
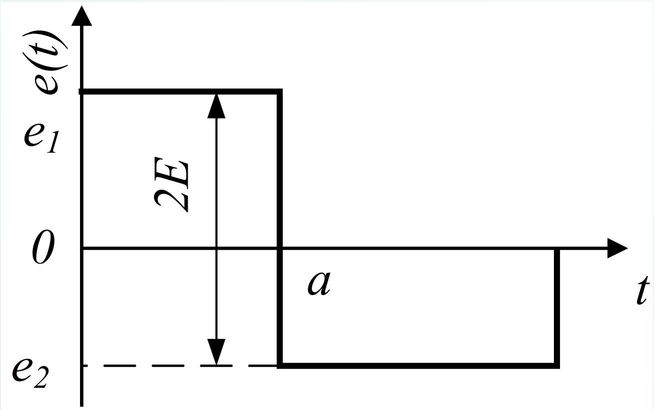


Рис. 4. АЧХ высоковольтного генератора: (1—9) — номера гармоник в максимумах АЧХ; (———) — обмотки расположены на одном керне, (– – –) — на разных кернах

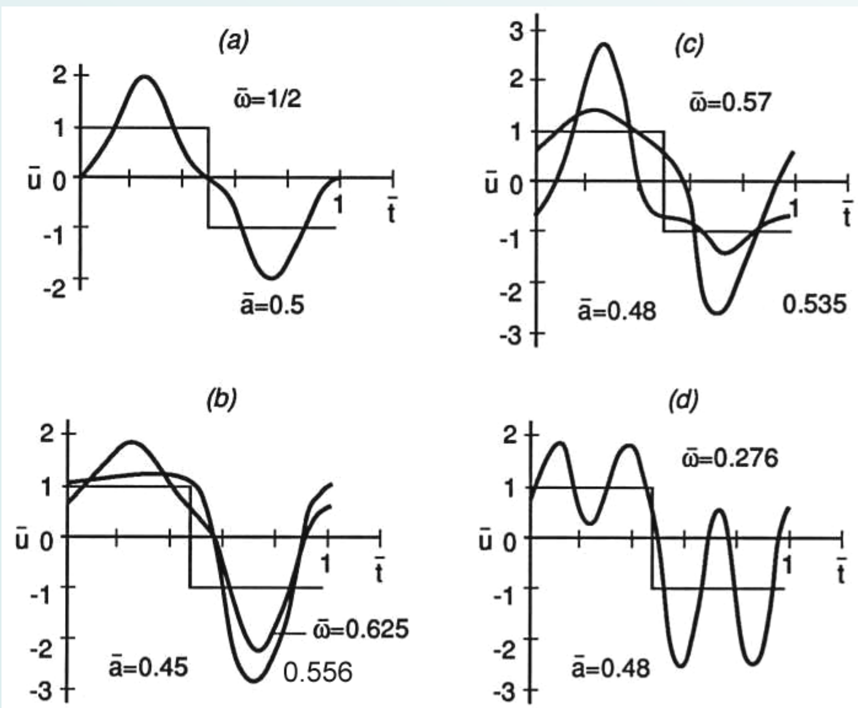
A. Pokryvailo and V. Pokryvailo, "Some Features of Operation of High Voltage Switch-Mode Power Supplies for X-Ray Apparatus", *Instrumentation and Methods of X-Ray Analysis,* Vol. 35, 1986.

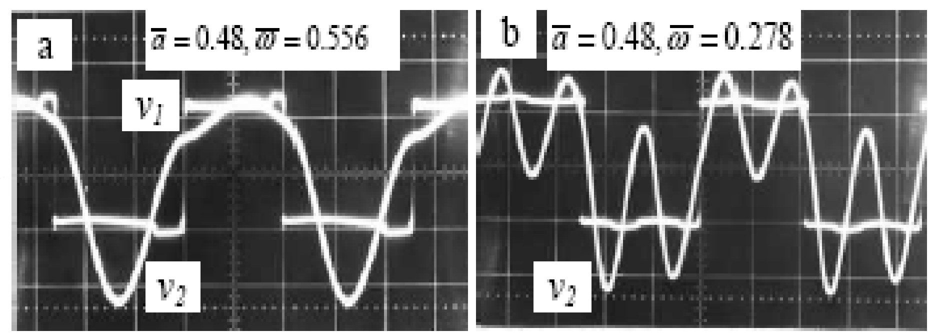
No-load at *asymmetrical* square-wave excitation



$$\overline{v} = 2\left[\frac{\sin\beta\frac{\overline{\alpha}-1}{2}}{\sin\beta/2}\cos\beta\left(\overline{t}-\overline{a}/2\right)+1-\overline{a}\right]$$

where $\overline{\beta} = 2\pi/\overline{\omega}$, $\overline{i} = i/I^*$, $\overline{v} = v/E$
$I^* = E/\rho$, $\overline{a} = a/T$, and $2E = e_1 - e$

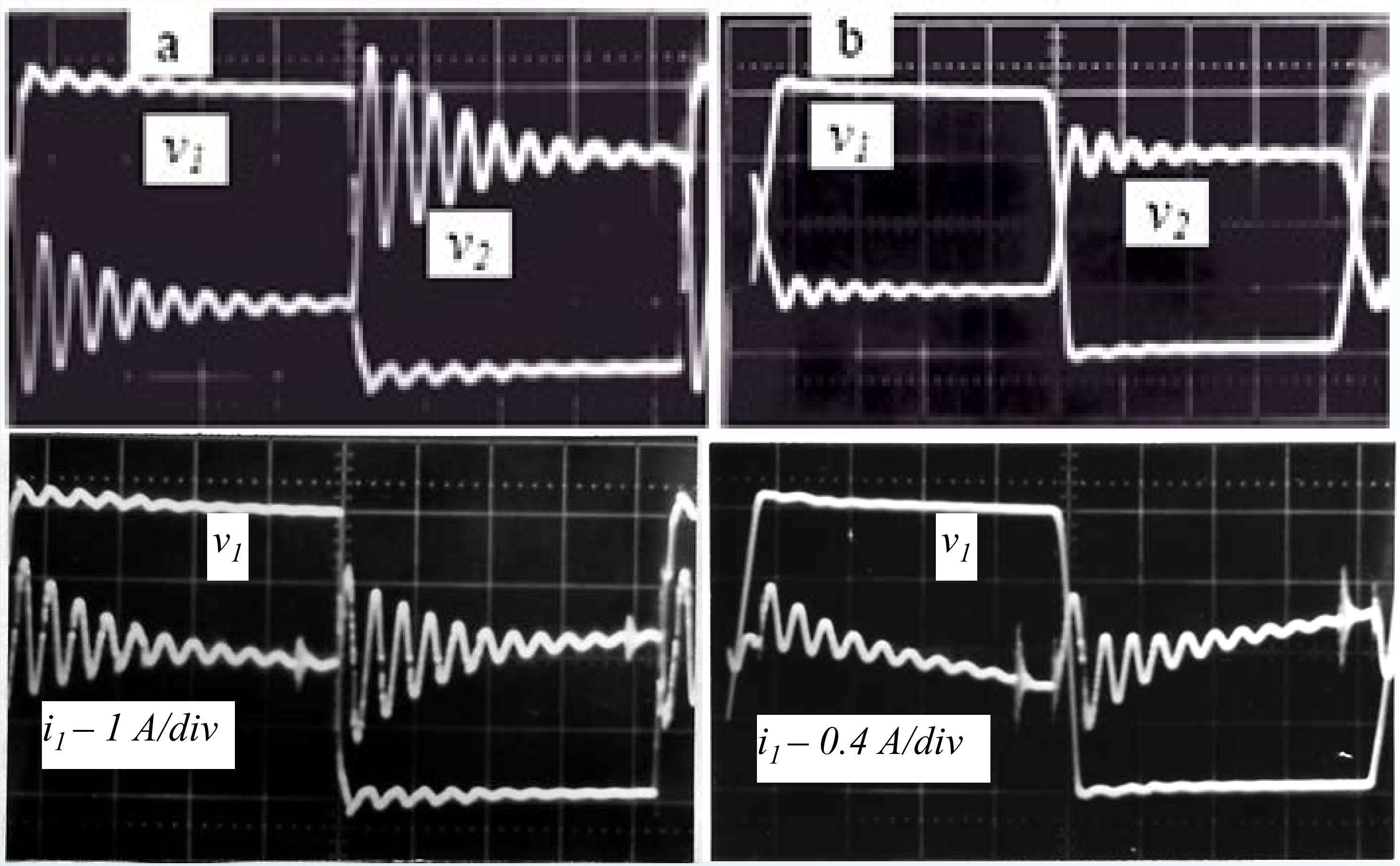




Voltage waveforms of high voltage transformer corresponding to (b), (d) mode of operation. a - v_1 - 5 V/div, v_2 - 200 V/div. Horizontal – 10 µs/div. b - v_1 - 20 V/div, v_2 - 1 kV/div. Horizontal – 20 µs/div.

OVC (K) dependence on risetime

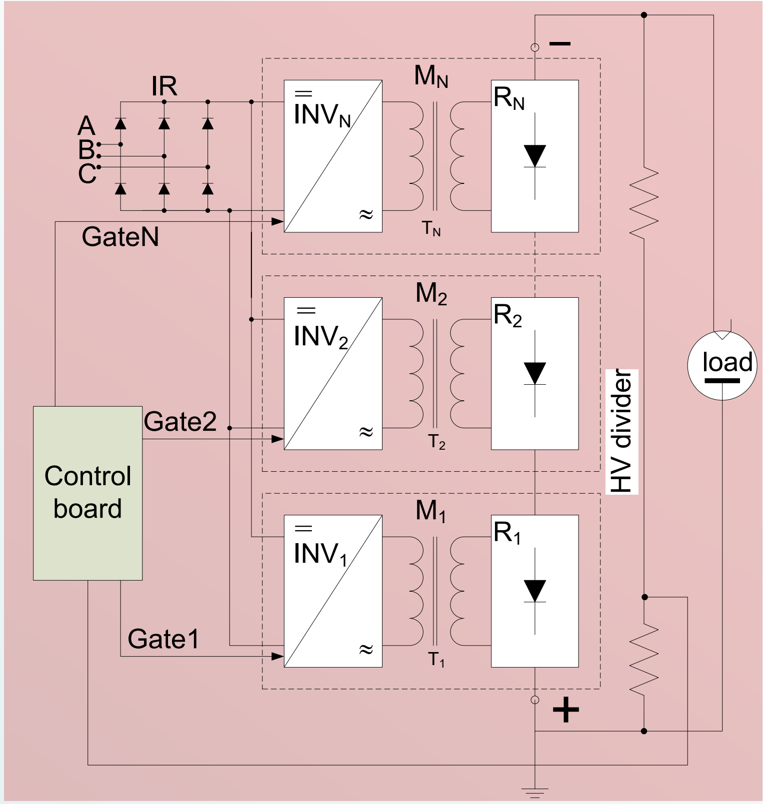
$K \rightarrow 3$ at steep risetime



Voltage waveforms of high voltage transformer for steep (a) and slow risetime (b). Turns ratio 1000:30. v_1 - 20 V/div, v_2 - 1 kV/div. Horizontal – 10 μs/div.

HV converter with square-wave inverter voltage at light or no-load – some takeouts

1. Minimum OVC≡K=2, realized on even harmonics
2. Operation in the vicinity of fundamental frequency enables frequency regulation based entirely on the transformer/VM parasitics. This approach was popular for low-power.
3. Multicell converters: uniformity of cells' output voltages is a charged issue



HV Converter Parasitic Parameters

- Leakage Inductance
  - Has very large influence on switching performance
  - May be easily tuned to be integrated into tank circuit in resonant converters
- Parasitic capacitance of transformer and multiplier
  - Poorly controlled; tank circuit optimization is usually relegated to (leakage) inductance
  - Strongly influences resonant behavior at no-load and under load, especially with large number of stages
- Up-front knowledge allows accurate simulation

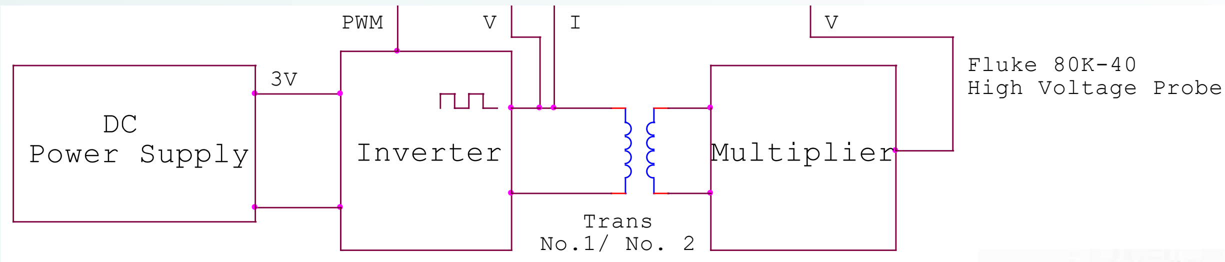
Measuring selected transformer parameters

- Ubiquitous method of measuring L_s is with LC-meter: connecting it to one of the windings and shorting the other. This implies using *simplified* equivalent circuit ($L_\mu \to \infty$).
- If L_s is known, C_p can be found by measuring *series-resonance* frequency and using *simplified* circuit Figure. *Parallel resonance* can be used if L_μ is known.
- For weak coupling, mutual inductance *M* knowledge is desirable. *M* can be found from relation

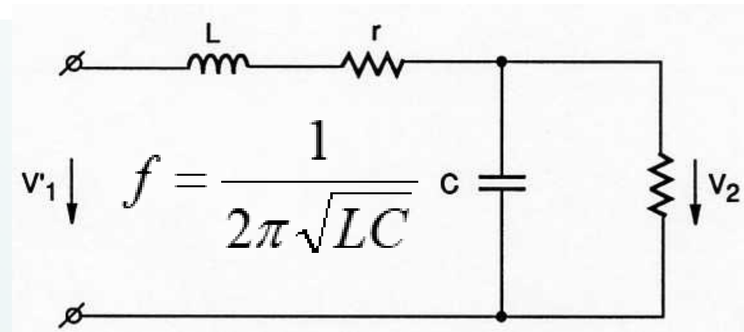
$$|M| = \frac{L_{eq1} - L_{eq2}}{4} \quad (3)$$

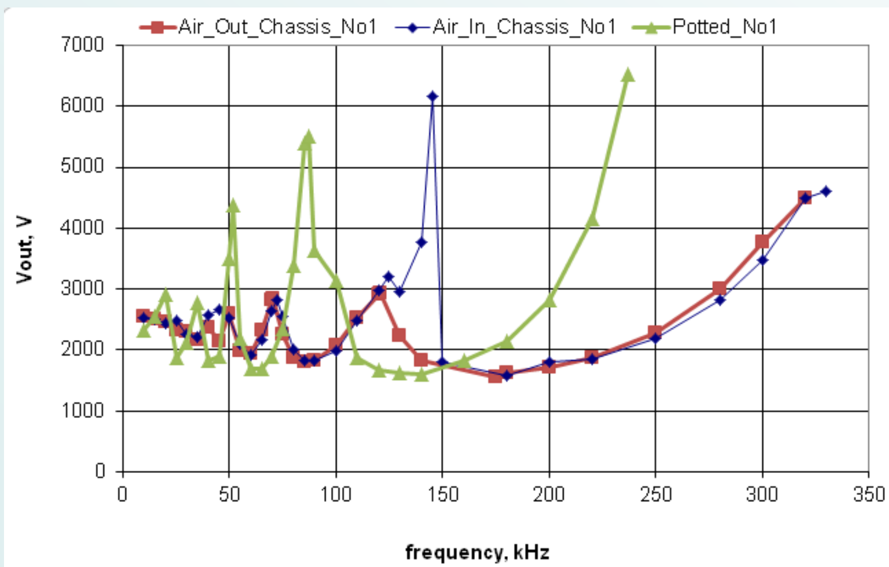
where L_{eq1}, L_{eq2}, are inductances of primary and secondary coils in series connection, in matched and opposite order, respectively (in their actual position!). Note that a) *L1, L2* knowledge is not necessary; b) for numeric field analysis, (3) requires setting simulation accordingly

How to measure C_p with "primitive" means?



Driving transformer/rectifier with variable-frequency square-wave source (inverter)



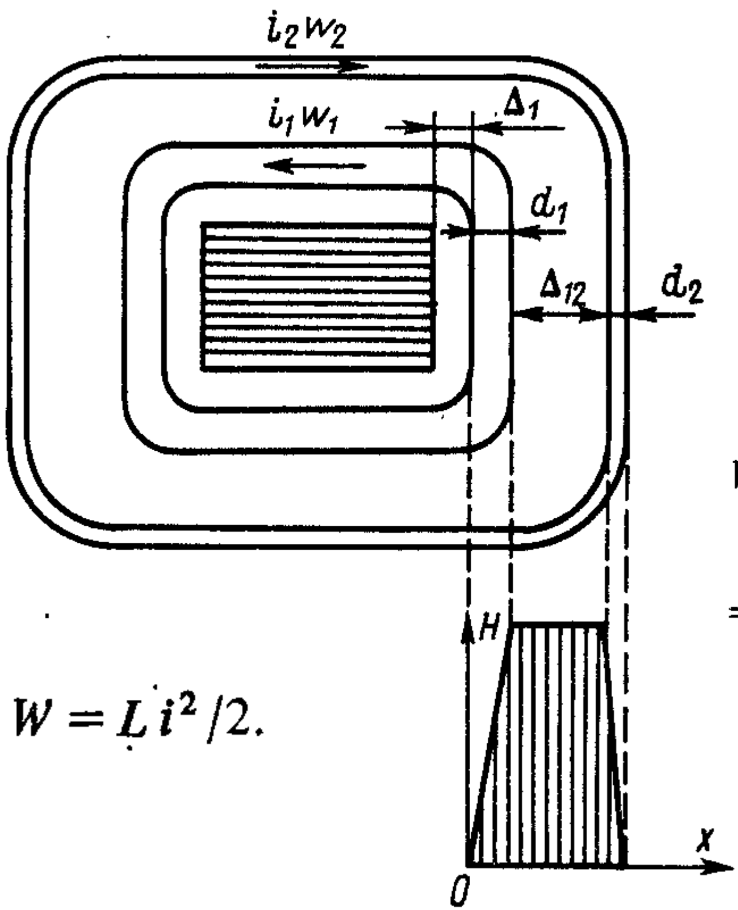


	Trans No.1		
	Air (out of chassis)	Air (in chassis)	Potted
f_5th (Hz):	7.07E+04	7.23E+04	5.16E+04
f_3rd(Hz):	1.21E+05	1.25E+05	8.70E+04
f_0 (Hz):	358250	367650	259500
Ls (H):	2.60E-03	2.60E-03	2.60E-03
Cp (F):	7.60E-11	7.22E-11	1.45E-10

$$f_0 = \frac{5 \times f_5 + 3 \times f_3}{2}$$

Frequency response of transformer No. 1 for three scenarios; multiple resonances on odd harmonics; L_s=const, C_p changes

Calculating Leakage inductance



$$W = L i^2 / 2.$$

$$H_{1x} = \frac{i_1 w_1}{h} \frac{x}{d_1} = \frac{i_2 w_2}{h} \frac{x}{d_1};$$

$$H_{12} = i_1 w_1 / h = i_2 w_2 / h;$$

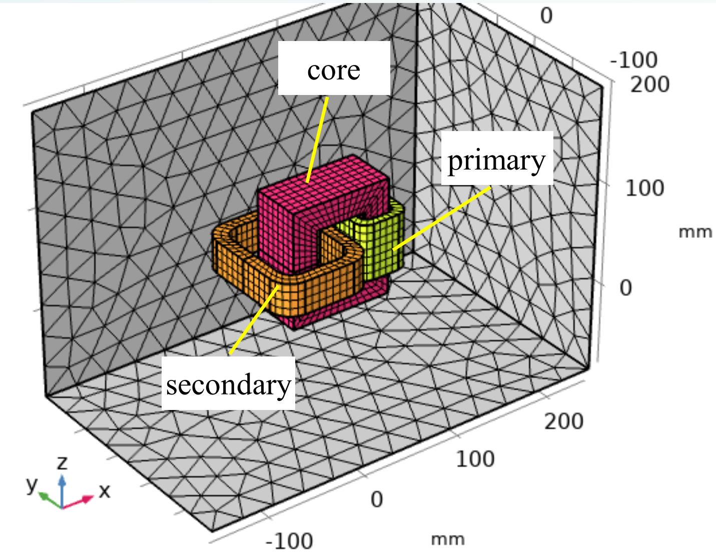
$$H_{2x} = \frac{i_2 w_2}{h} \left(1 - \frac{x}{d_2}\right)$$

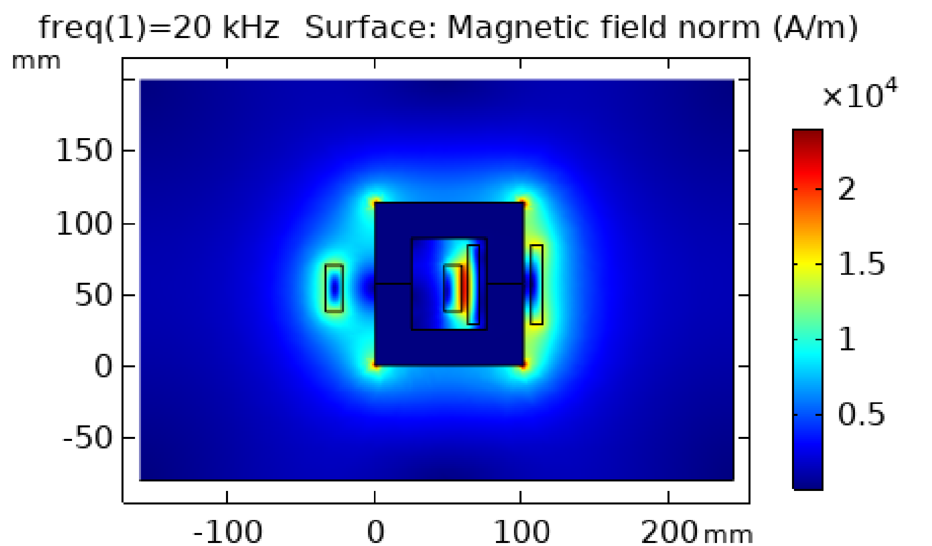
$$W = \frac{\mu_0 h p}{2} \left[\int_0^{d_1} H_{1x}^2 dx + H_{12}^2 \Delta_{12} + \int_{d_1 + \Delta_{12}}^{d_1 + \Delta_{12} + d_2} H_{2x}^2 dx \right] = \frac{\mu_0 i_2^2 w_2^2 p}{2h} \left(\Delta_{12} + \frac{d_1 + d_2}{3} \right).$$

$$L_{s\,(2)} = \frac{\mu_0 w_2^2 p}{h} \left(\Delta_{12} + \frac{d_1 + d_2}{3} \right);$$

1. Still, A., “Principles of Transformer Design”, NY, Wiley, 1919, 216pp.
2. G. N. Glasoe et al., “Pulse Generators”, Dover, NY, 1948, 741pp.
3. Vdovin, S.: "Design of Pulse Transformers". Energia, Moscow, 1971, 148pp. (In Russian).

Calculating leakage inductance *numerically*





Windings on separate legs. Core 2xU100/57/25. w1=10, w2=320. Ls2=33mH. Primary driven by current, secondary shorted (MF-EC interfaces used).
Leakage inductance ~four times larger compared to case of concentric windings

- Analytical estimates are difficult for the case of *not overlapping* windings. Leakage field is truly 3D, and simulations become especially useful.
- To allow field to “leak”, bounding box must be “large”. Its size has major impact on results.

Pokryvailo, A., “Use of COMSOL Multiphysics in High Voltage Electronic Transformers Design”, *Proc. COMSOL Conference 2019,* Boston MA, 2019

Air-core transformer (*weak magnetic coupling)*

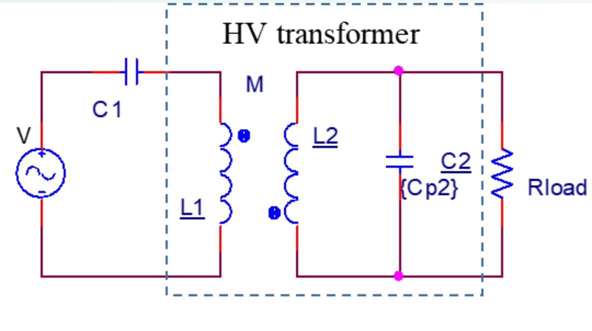
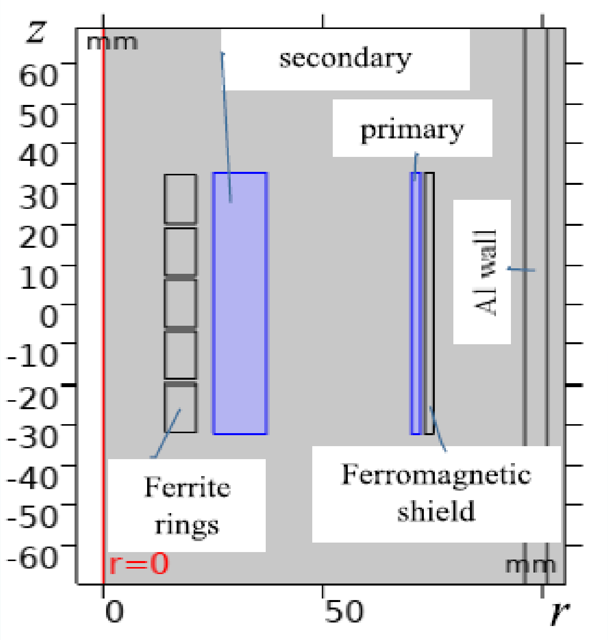


Fig. 2. Two inductively-coupled loops. C2 is parasitic capacitance of transformer/load.



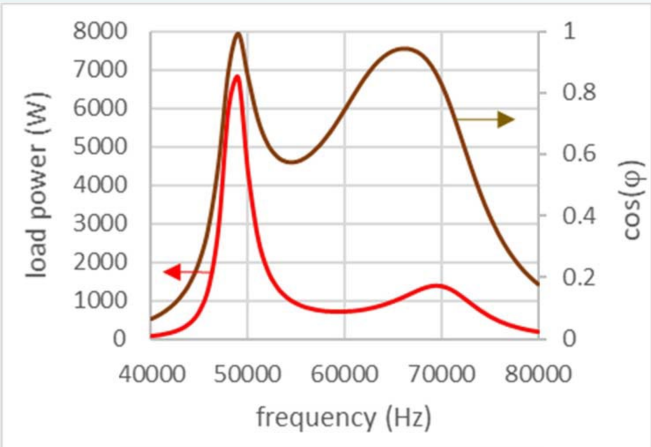
1-kW, 20-kV transformer prototype. Outer cylinder carries primary $w1$ = 50T, Litz 420/38. It is covered by a ferromagnetic shield. Inside cylinder carries secondary (12 sections, number of turns varied from 2000 to 3500). Secondary can be lined with ferrite toroids. Cross section shows homogenized secondary (COMSOL axisymmetric model).

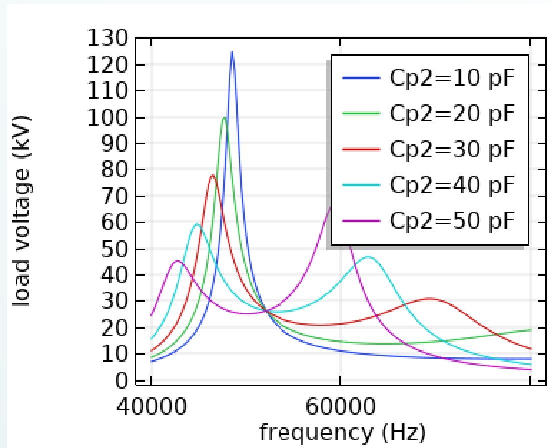
Coupled MF-circuit simulation synergistically analyses both the geometry and the coils' interaction with external circuitry

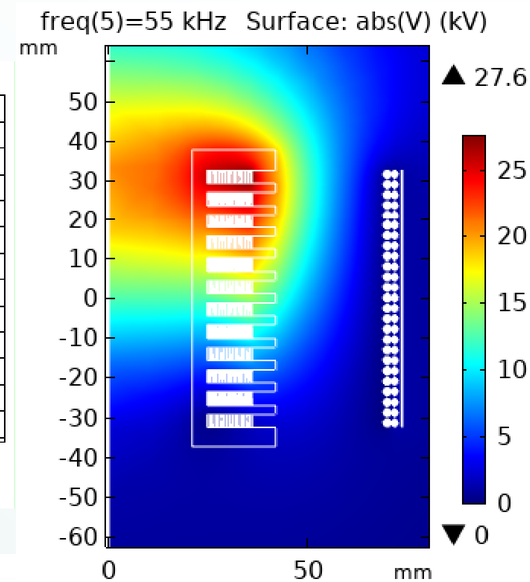
A. Pokryvailo and H. Dave, "Coupled Magnetic Field-Circuit Analysis of Inductive Power Transfer in High-Potential Transformers", IEEE Transactions on Plasma Science, vol. 48, no. 10, pp. 3279-3288, Oct. 2020.

Coupled field and circuit simulation

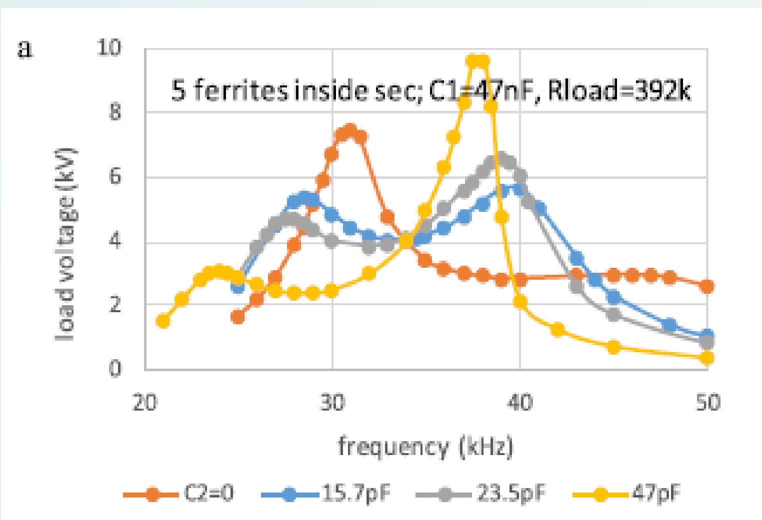
Inductive power transfer

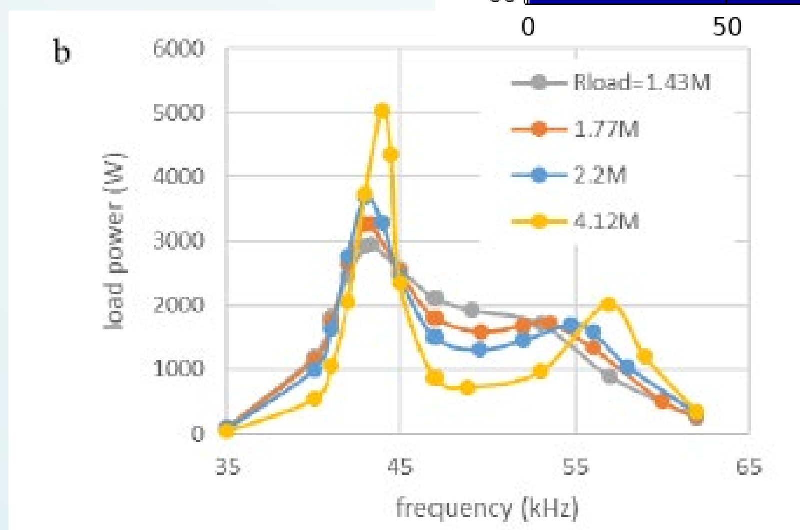






Simulated





Experimental

Note resonances and resonant frequency split

Transformer *dynamic* capacitance C_T

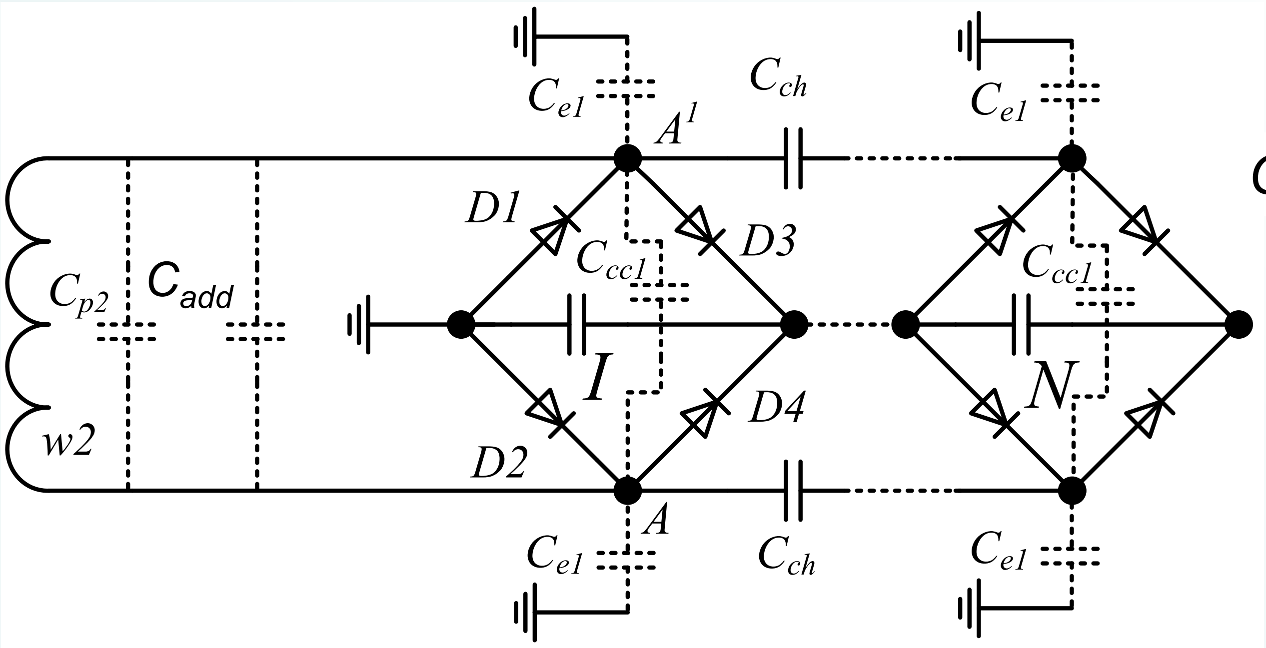
- May be calculated as (E is energy at voltage V(t)):

$$C_T=f(t)=2E(t)/V(t)^2 \qquad (1)$$

- Voltage distribution in windings may be not linear during leading and trailing edges
- Calculation is simplified if voltage distribution in windings assumed to be linear
- $C_T=C_1+C_{12}+C_2$ (reflected to a chosen winding), all of them possible calculate by (1)
- For HV transformers (large transformation ratio), $C_T \approx C_2$

1. G. N. Glasoe et al., “Pulse Generators”, Dover, NY, 1948, p. 518-520.
2. S. Vdovin, "Design of Pulse Transformers". Energia, Moscow, 1971, 148pp. (In Russian). 2nd Ed. 1991
3. L. Dalessandro et al., "Self-Capacitance of High-Voltage Transformers," TPEL, vol. 22, 2007.

VM Parasitic Capacitance



$C_{add} = C_{cc} + C_e/2$

N-stage bridge VM with its parasitic capacitances; C_{p2} is parasitic capacitance of transformer

- In simulations, C_{e1}, C_{cc1} *usually* can be replaced by $C_{m2} \equiv C_{add}$; asymmetry may have bizarre effects
- In first approximation, C_{m2} is proportional to the number of stages and remains invariant

Analytical estimates (no-load, following [1], [8])

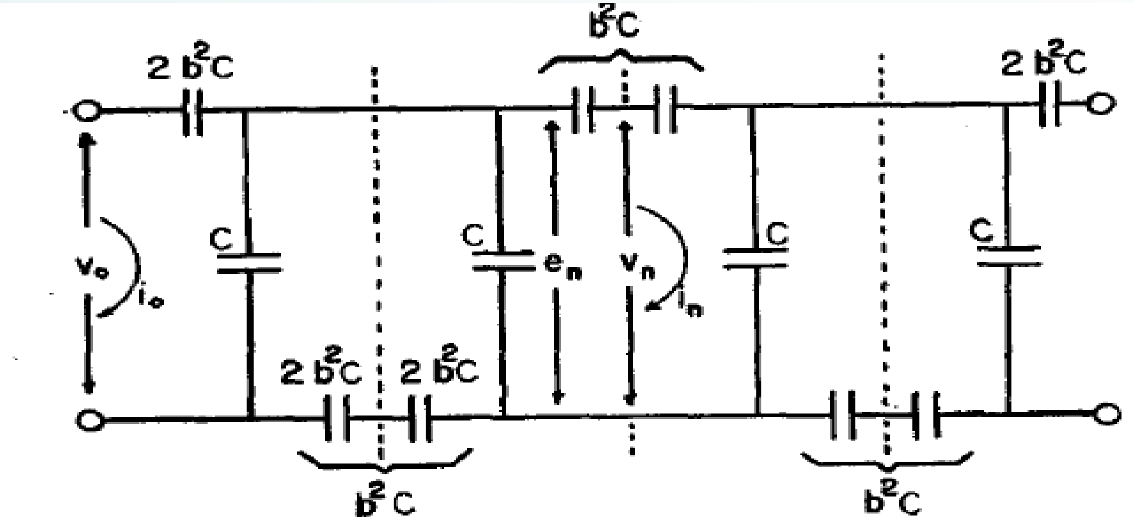


FIG. 2. Transmission line equivalent to a conventional four-stage Cockcroft-Walton circuit under no load.

[1] E. Everhart and P. Lorrain, "The Cockcroft-Walton Voltage Multiplying Circuit", Review of Scientific Instruments 24, 221 (1953).

[8] B. I. Albertinski *et. al.*, "Cascade Generators", Moscow, 1980. In Russian.

$$vn(n,b) := v_0 \frac{\cosh\left[2\cdot\frac{(N-n)}{b}\right]}{\cosh\left(\frac{2N}{b}\right)}$$

$$comp(n,b) := \frac{1}{vn(n,b)}$$

K≡ *comp* –compression between stages

N - the total number of stages

n - the stage number along the column

b - relates C_m to *C*

Input voltage v_0 is multiplied, ideally, to $2Nv_0$.

A. Pokryvailo, "Analysis of Resonant Behavior of Voltage Multiplier," 2022 IEEE IPMHVC, Knoxville, TN, USA, 2022, pp. 21-24.

Compression at no-load

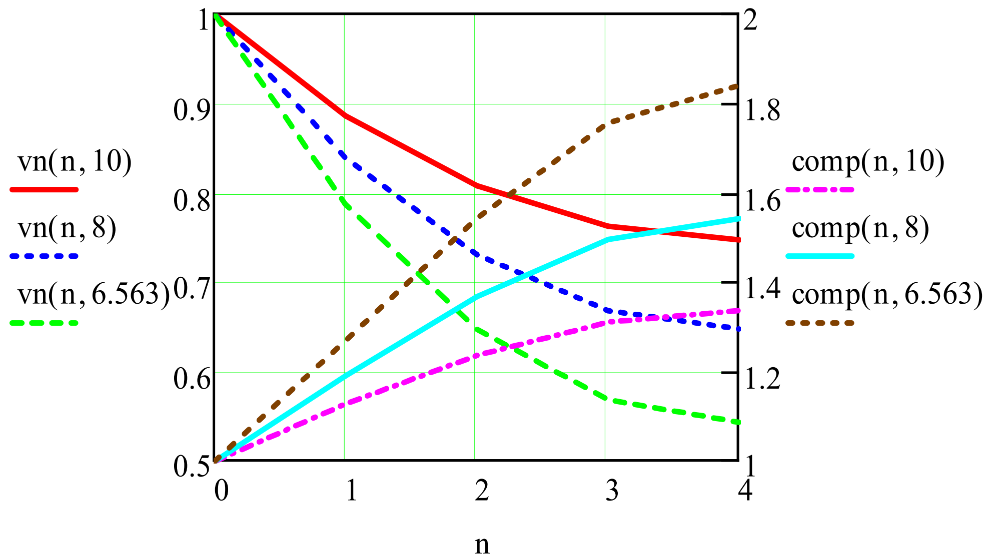
Lumped parasitic C transferred to transformer terminals

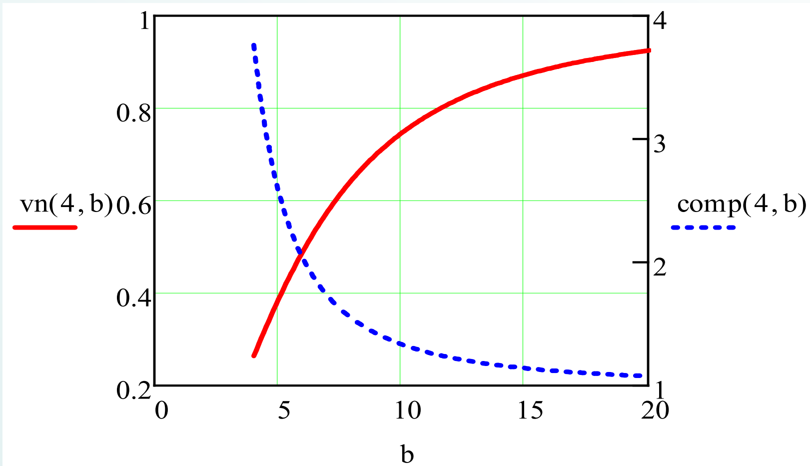
$$C_{add} = bC\tanh\left(\frac{2N}{b}\right) \quad (3).$$

If 2N/b<<1 (negligible parasitics),

$$C_{add} = 2NC \quad (4).$$

If VM is center fed, the values obtained by (3), (4) should be doubled. For the above values and the 8-stage multiplier (four up, four down), C_{add}=36pF calculated by (3), and C_{add}=52pF calculated by (4)





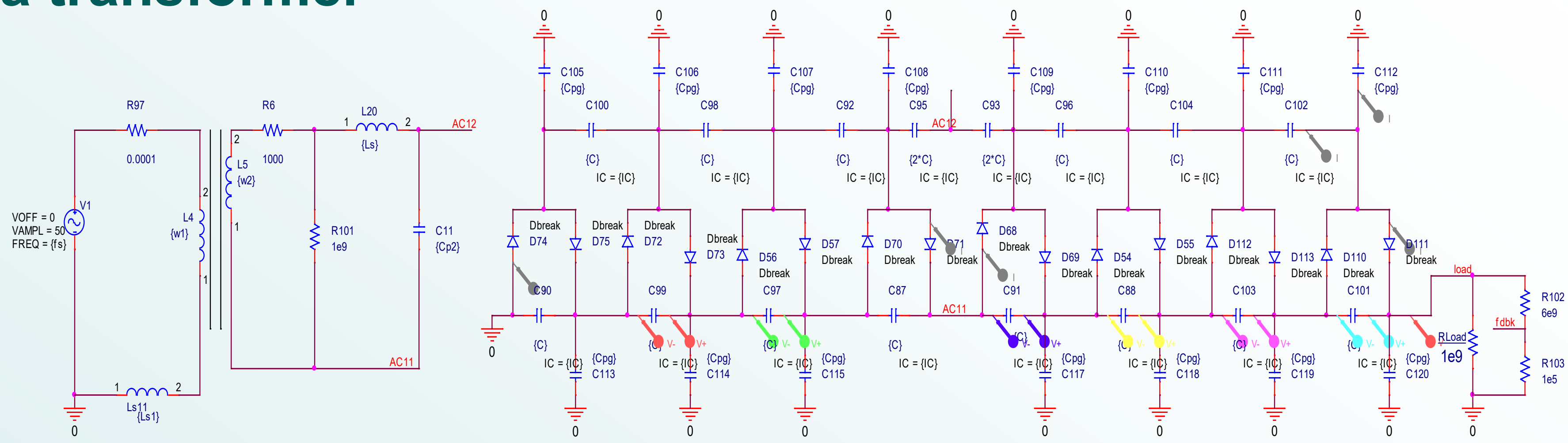
CIRCUIT SIMULATIONS

- VM directly fed by a sine wave source (w/o transformer)
  - Diode junction capacitance/inter-column capacitances modeled
  - Column capacitances to ground modeled
- **VM fed by a sine wave source via a ”real-world” transformer**

A. Pokryvailo, "Analysis of Resonant Behavior of Voltage Multiplier," 2022 IEEE IPMHVC, Knoxville, TN, USA, 2022, pp. 21-24.

Spellman High Voltage Electronics Corporation

VM center-fed by a sine wave source via a transformer



VM fed via transformer: C=0.14nF C_{pg}=3.25pF C_{p2}=12pF R_{load}=100MΩ				
Stage	V_{stage}, kV	K	top, kV	bottom, kV
bot1	9.24	1.68	33.1	28.8
bot2	7.66			
bot3	6.42			
bot4	5.50			
top1	9.50	1.29		
top2	8.44			
top3	7.74			
top4	7.38			

Compression *K* asymmetrical for bottom and top halves. C=0.14nF

VM fed by a sine wave source via a transformer - resonances

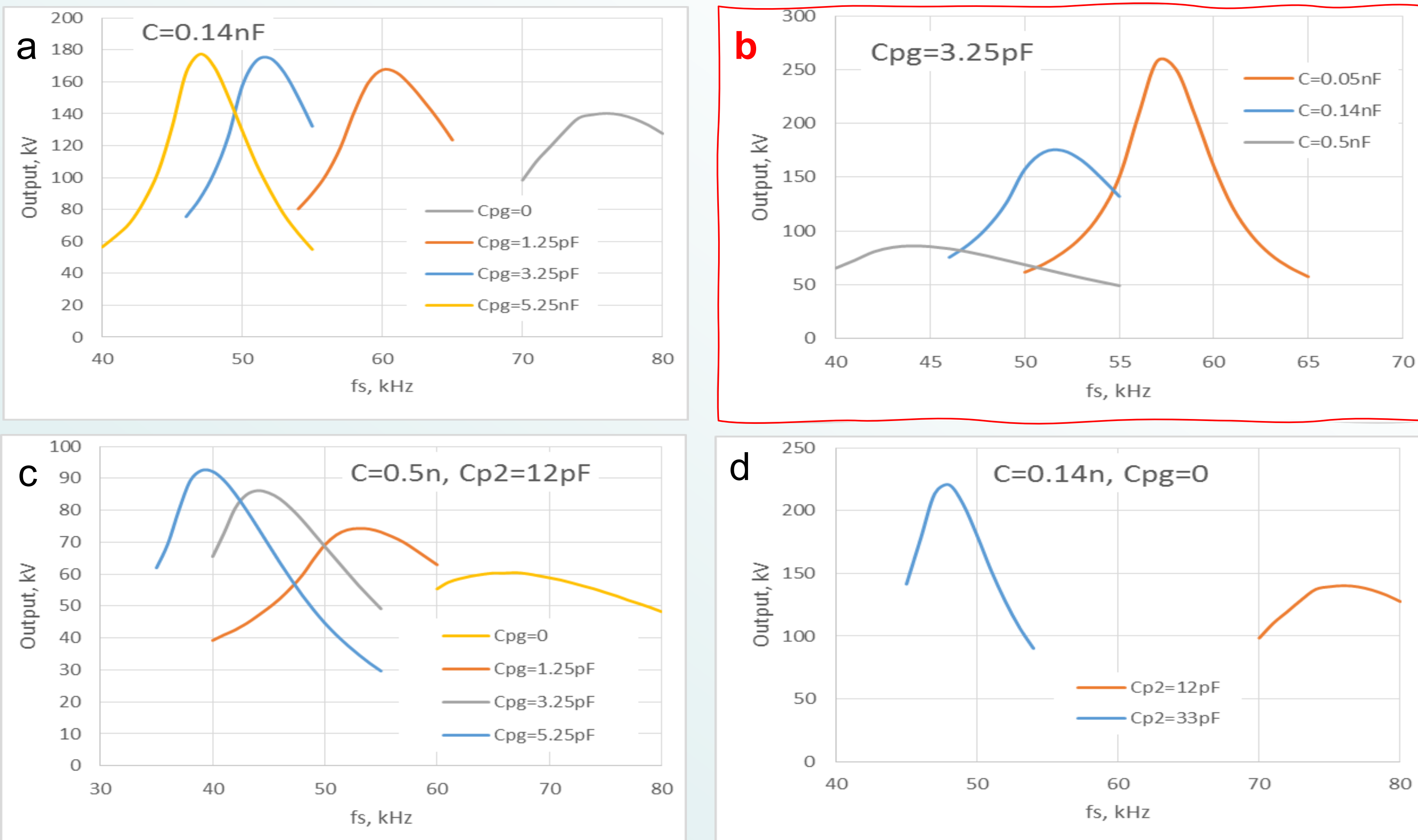


Figure 7**b** shows that VM becomes more resonant with smaller *C's*, and resonant frequency shifts to higher values. This is more noticeable with larger number of stages N.

Multiplier with small capacitances *C* is more resonant!

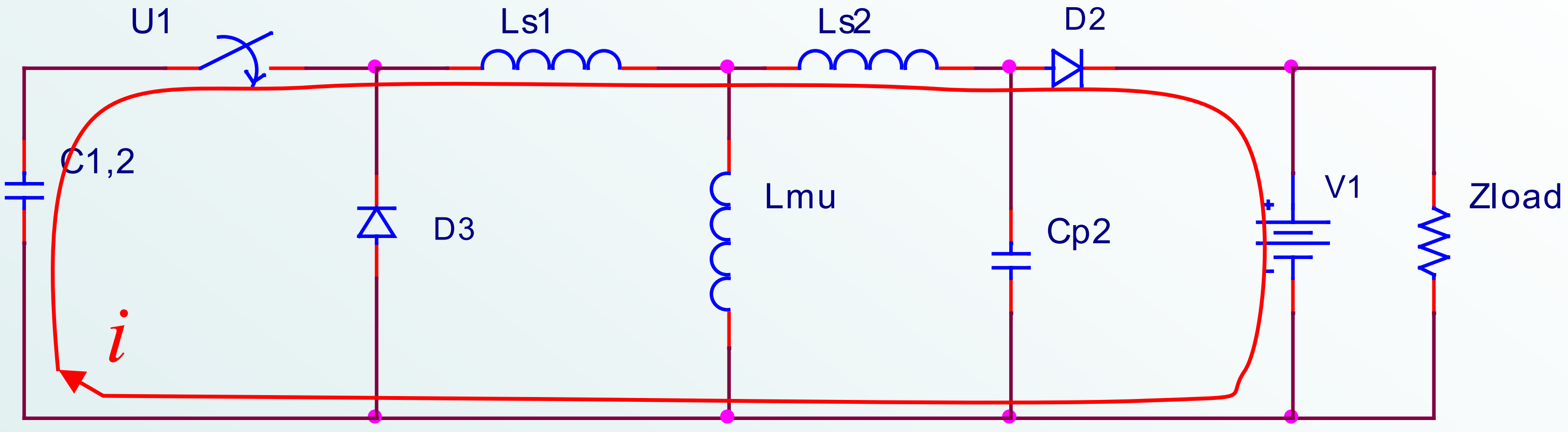
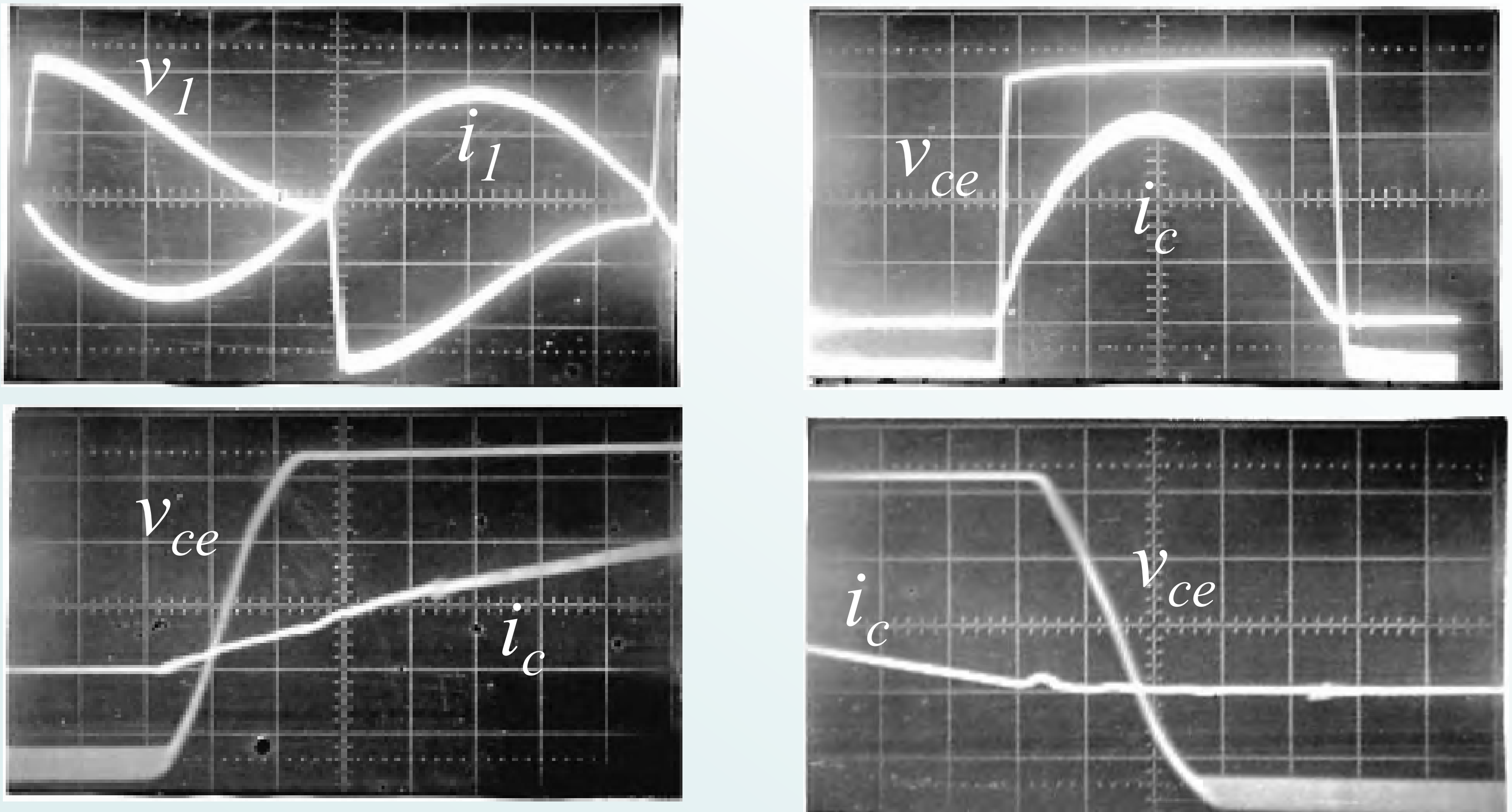


Figure 4. Equivalent circuit for interval of multiplier caps' C charge. V1 replaces C's if C>>C1,2; V1, hence C's, are in series with C1,2. (There are more subtle phenomena yet…)

A. Pokryvailo "Analysis of half-bridge converter with energy dosing", unpublished, Nov. 2006

Integration of parasitics:

almost always possible to avoid addition of inductors and/or capacitors to tank circuit, also with hard switching ☺

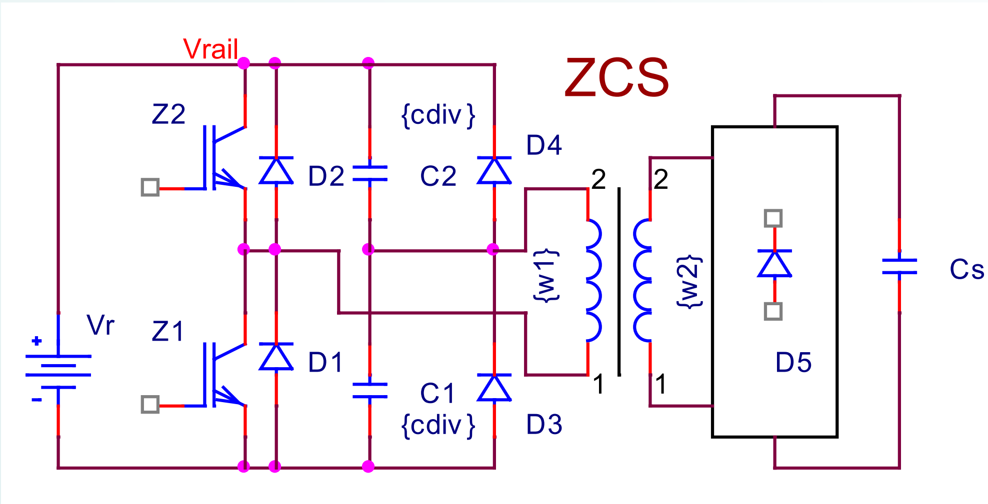


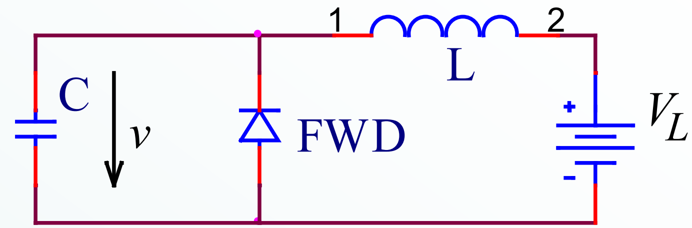
Series resonance. v_1, v_{ce} *- 200V/div;* i_1 *– 2A/div,* i_c *– 1A/div;* f_s*=20.4kHz.*

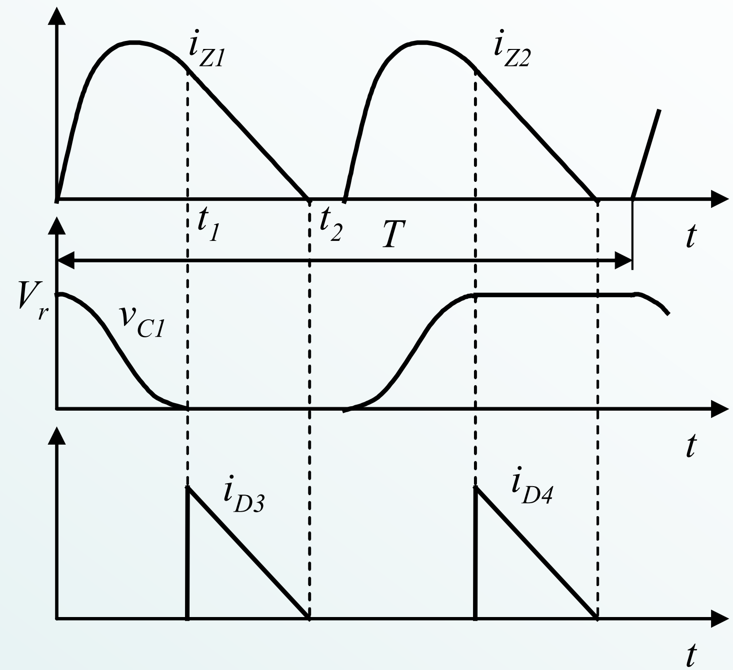
A. Pokryvailo, Dissertation, Leningrad, 1987.

Integration of parasitics cont.

Energy-dosing inverter (also called clamped diode inverter)







- Tight control of energy transfer
- Low RMS currents
- Inherent limitation of short circuit current and voltages
- leakage inductance of HV transformers incorporated into resonant tank - no external inductors

1. B.D. Bedford and R.G. Hoft, 'Principles of Inverter Circuits", Wiley, NY, 1964.
2. B. Kurchik, A. Pokryvailo and A. Schwarz, "HV Converter for Capacitor Charging", Pribory i Tekhnika Experimenta, 1990, Translation to English Plenum Publishing Corp.

Integration of parasitics (cont.)

Energy-dosing inverter

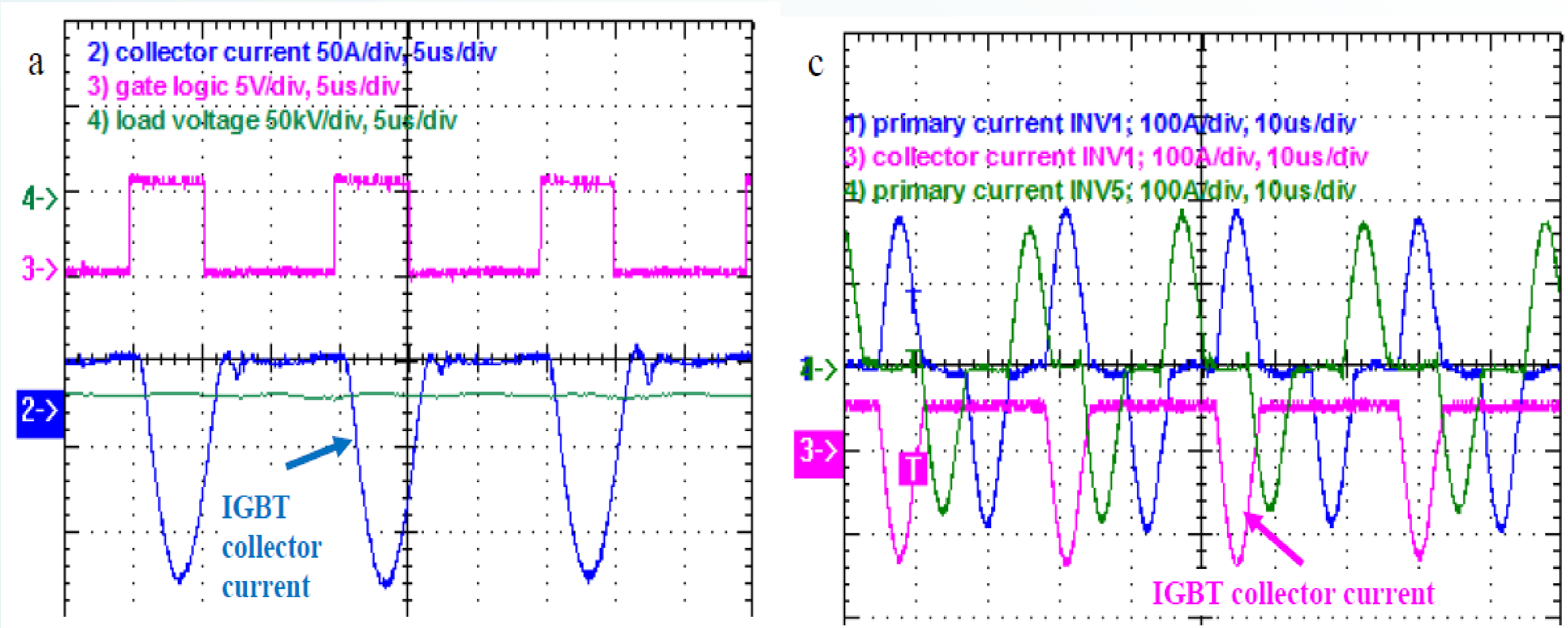
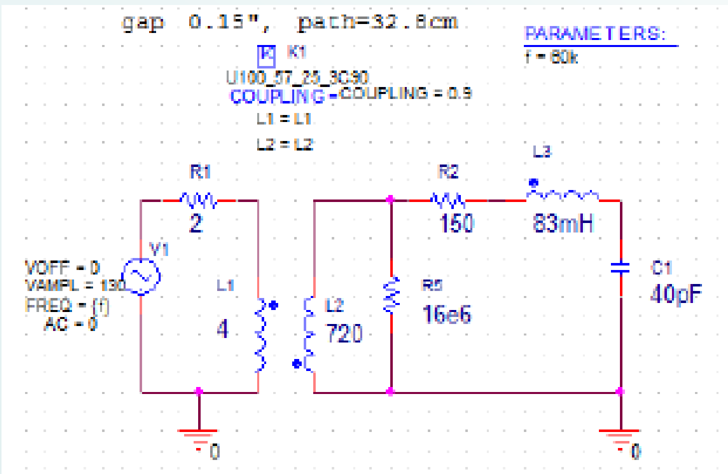


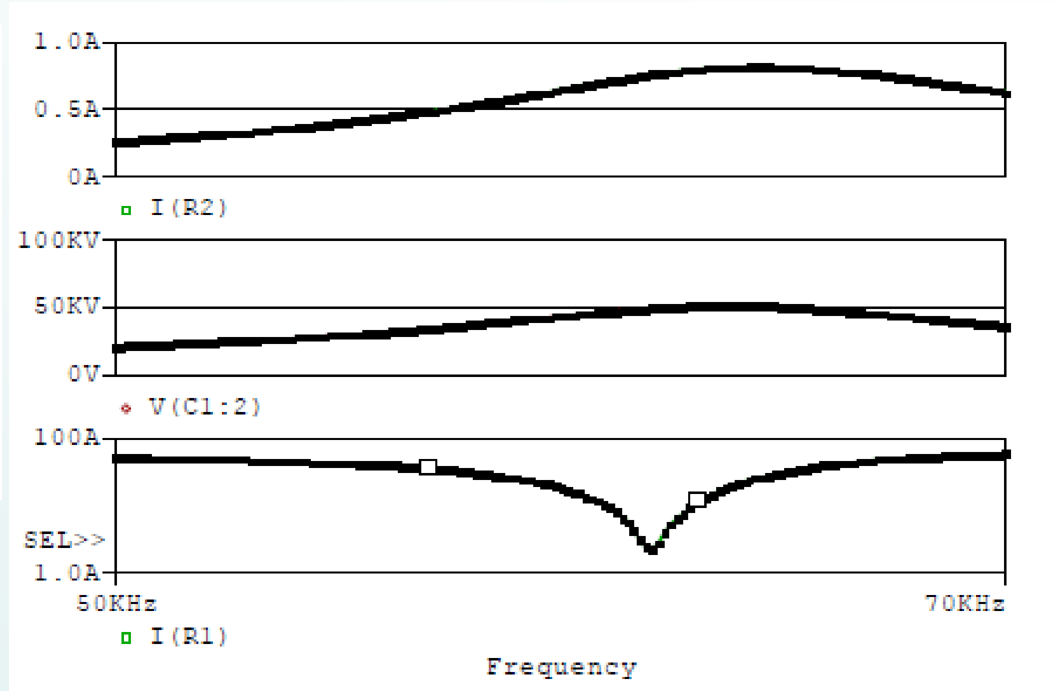
Fig. 12. Series resonance. 120 kV@102 kW. a - low line, f_s=67 kHz; c – high line, fs=45 kHz.

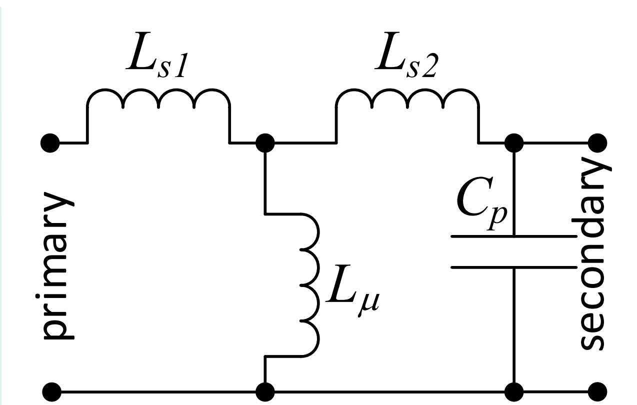
A. Pokryvailo et al., "A 100 kW High Voltage Power Supply for Dual Energy Computer Tomography Applications," IEEE TDEI., vol. 22, no. 4, pp. 1945–1953, Aug. 2015

Integration of parasitics (cont.)

parallel resonance to minimize consumption at no-load



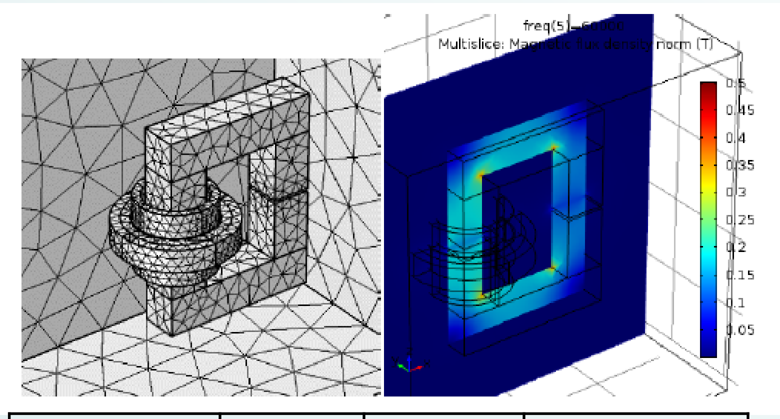




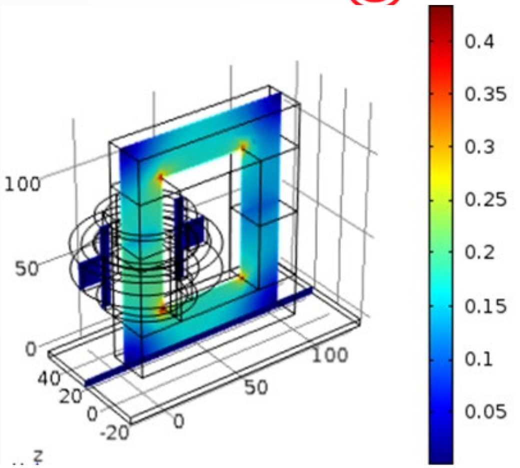
Can be analyzed both in Circuit and Field formulations. **Note gapped core (gap=0.15”).**

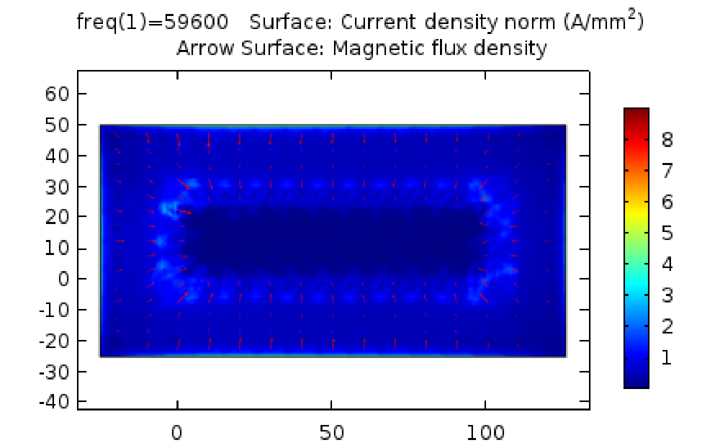
Integration of parasitics

parallel resonance (cont.)



freq (Hz)	I1 (A)	I2 (A)	Vload (V)
58000	11.342	0.53153	36464
58500	9.44	0.53776	36576
59000	7.557	0.54397	36685
59500	5.7071	0.55017	36791
60000	3.9304	0.55636	36895
60500	2.3831	0.56253	36996
61000	**1.7899**	**0.56868**	**37094**
61500	2.8209	0.57482	37189
62000	4.4582	0.58094	37282



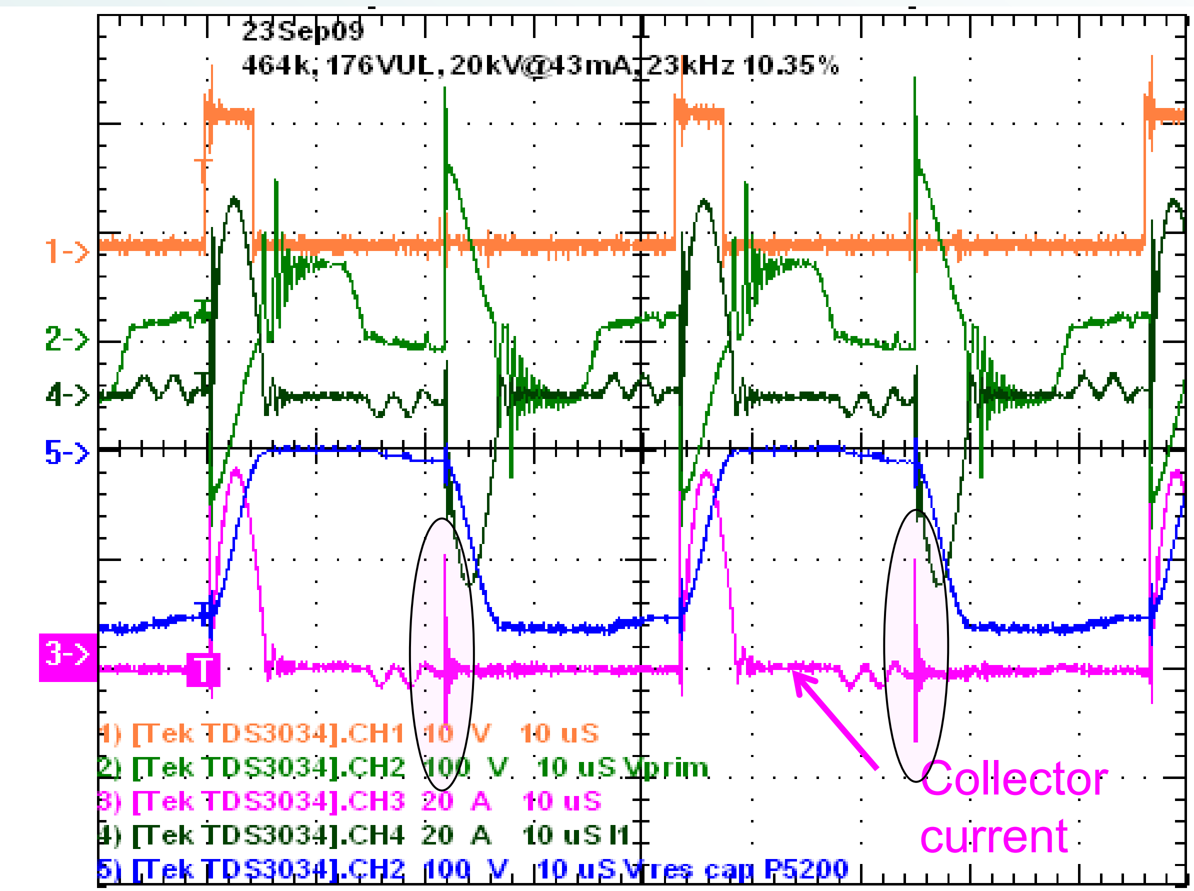


Construction elements: aluminum plate under core increases resonant frequency (slightly)

Field simulation. **gap=0.15”**. C1=40pF. Parallel resonance at 61kHz (close to experiment).

An “exotic” case – backlash from secondary:

shoot-through currents induced by parasitic capacitances during dead time



70kV@50kW peak power energy-dosing converter.

Shoot-thru currents at light load.

The problem arises with transformer parasitics. The secondary and VM parasitic capacitances stay charged during dead time, and then may discharge thru secondary. The processes can be tracked with Spice simulations.

CONCLUSIONS

- Performance of HV converters, to a great extent, is determined by parasitics. Their rigorous representation is necessary for operation understanding.
- Multiple resonances may be generated on high harmonics of primary voltage. They generate overvoltages at light load.
- Multiplier C_p may dominate over parasitic capacitance of transformer. In first approximation, C_{add} is proportional to number of stages.
- In conjunction with the HV transformer, parasitics modify the VM-transformer resonant characteristics by shifting resonant frequency to lower values at higher C_{pg}. A corollary: for very high voltages, housing VM in gas decreases compression by reducing C_{pg}'s. This was one of many reasons of using compressed gases in HV electronics.
- Almost always, parasitics can be integrated into tank circuit, also with hard switching☺

Many cited references are available at www.researchgate.net