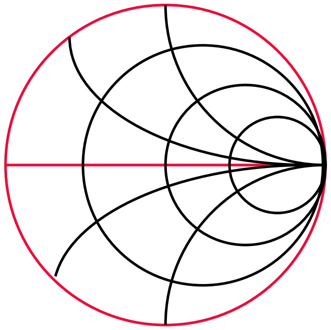
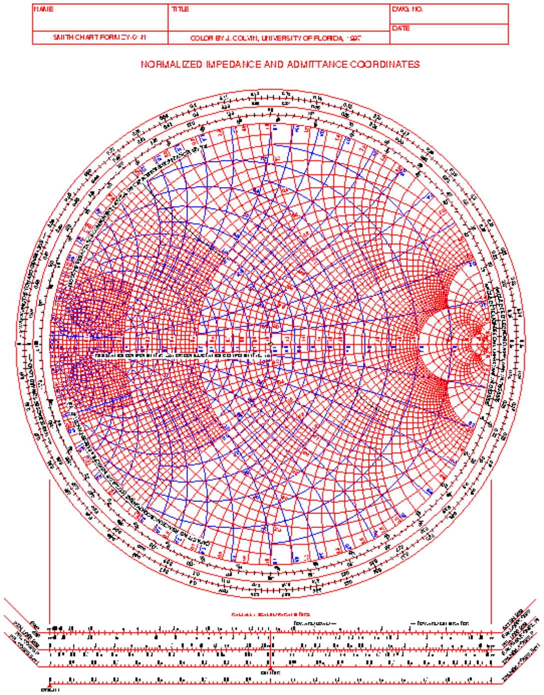
# Smith Charts



Frank A. Lynch

***W4FAL***

1905 - 1987



# Smith Chart History

- Invented by Phillip H. Smith in 1939
- Used to solve a variety of transmission line and waveguide problems

## Basic Uses

For evaluating the rectangular components, or the magnitude and phase of an input impedance or admittance, voltage, current, and related transmission functions at all points along a transmission line, including:

- Complex voltage and current reflections coefficients
- Complex voltage and current transmission coefficents
- Power reflection and transmission coefficients
- Reflection Loss
- Return Loss
- Standing Wave Loss Factor
- Maximum and minimum of voltage and current, and SWR
- Shape, position, and phase distribution along voltage and current standing waves

# Basic Uses (continued)

For evaluating the effects of line attenuation on each of the previously mentioned parameters and on related transmission line functions at all positions along the line.

For evaluating input-output transfer functions.

# Specific Uses

- Evaluating input reactance or susceptance of open and shorted stubs.
- Evaluating effects of shunt and series impedances on the impedance of a transmission line.
- For displaying and evaluating the input impedance characteristics of resonant and anti-resonant stubs including the bandwidth and Q.
- Designing impedance matching networks using single or multiple open or shorted stubs.
- Designing impedance matching networks using quarter wave line sections.
- Designing impedance matching networks using lumped L-C components.
- For displaying complex impedances verses frequency.
- For displaying s-parameters of a network verses frequency.

# The Formula!

$$Z_x = Z_0 \frac{Z_L + Z_0 \tanh \gamma x}{Z_0 + Z_L \tanh \gamma x}$$

$$\gamma = \alpha + j\beta$$

This is why the chart was developed.. Before calculators and computers the above formula's were impossible! They're still not easy today.

# The Smith Chart

$Z = R +/- j X$

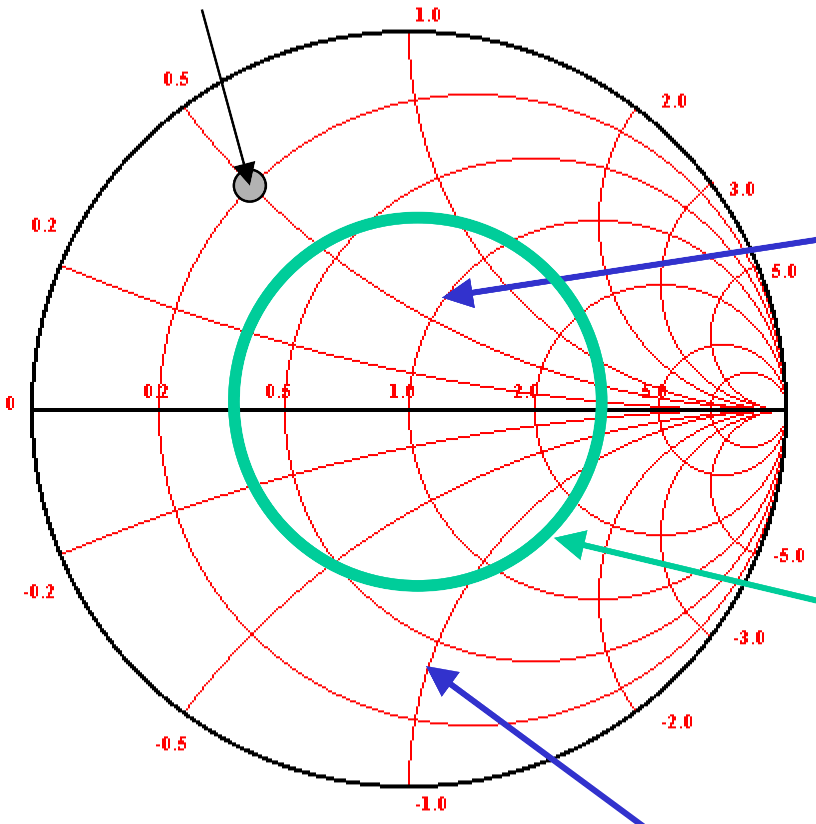
**Impedance**

The chart is normalized (Zn) so that any characteristic impedance (Zo) can be used.

$$Zn = \frac{R}{Zo} +/- j \frac{X}{Zo}$$

$Y = G +/- j B$

**Admittance**



$Zn = 0.2 +j0.5$

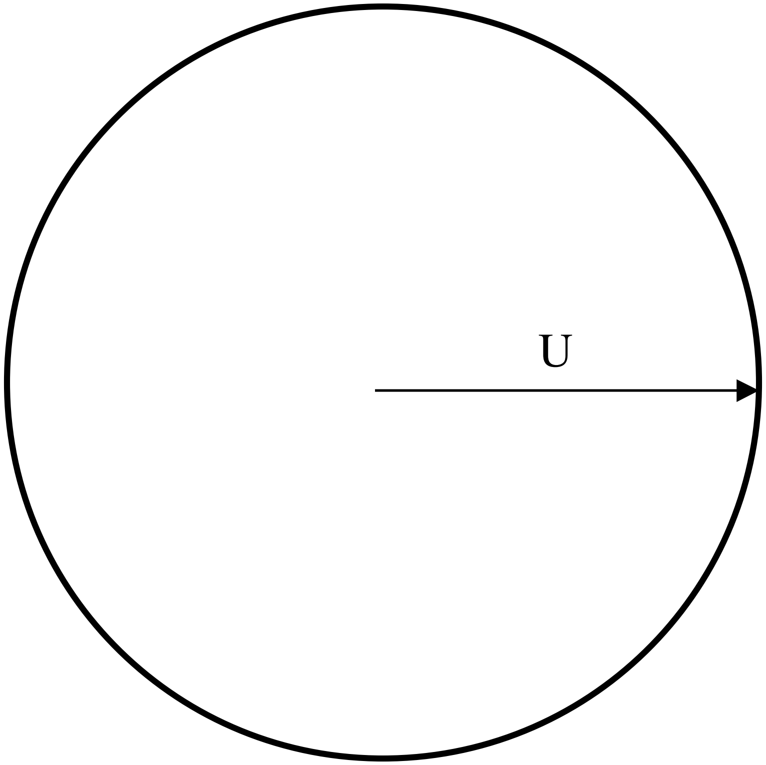
Constant R circles

Constant VSWR circles

Constant X circles

# Building the Smith Chart

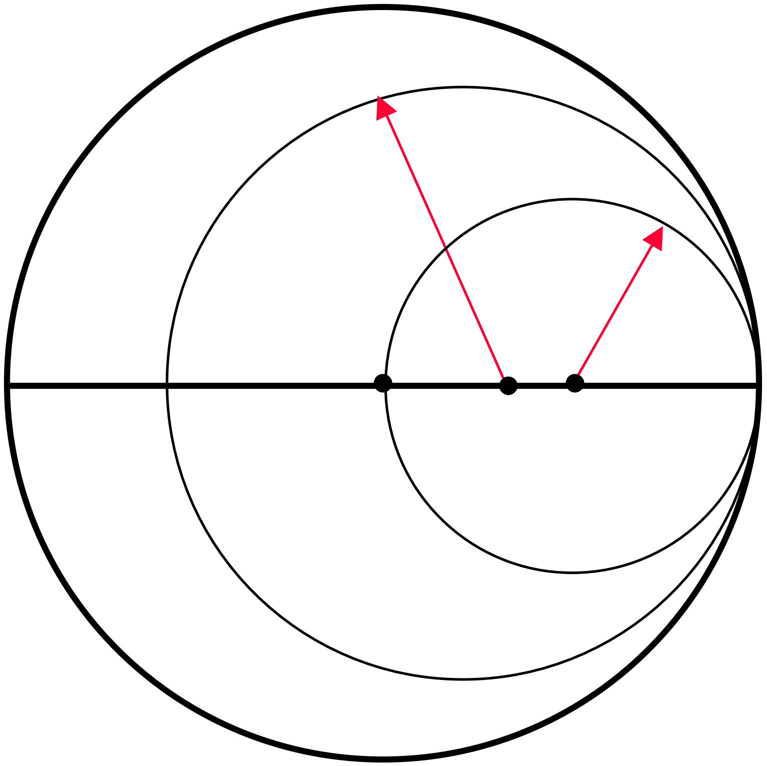
Start with circle of radius U.



This forms the outer boundary of the Smith Chart. It is in fact the R = 0 constant resistance circle.

# Building the Smith Chart

Now add the X = 0 axis.



Then start adding constant resistance circles. All circles pass through the R = infinity point and have a radius of:

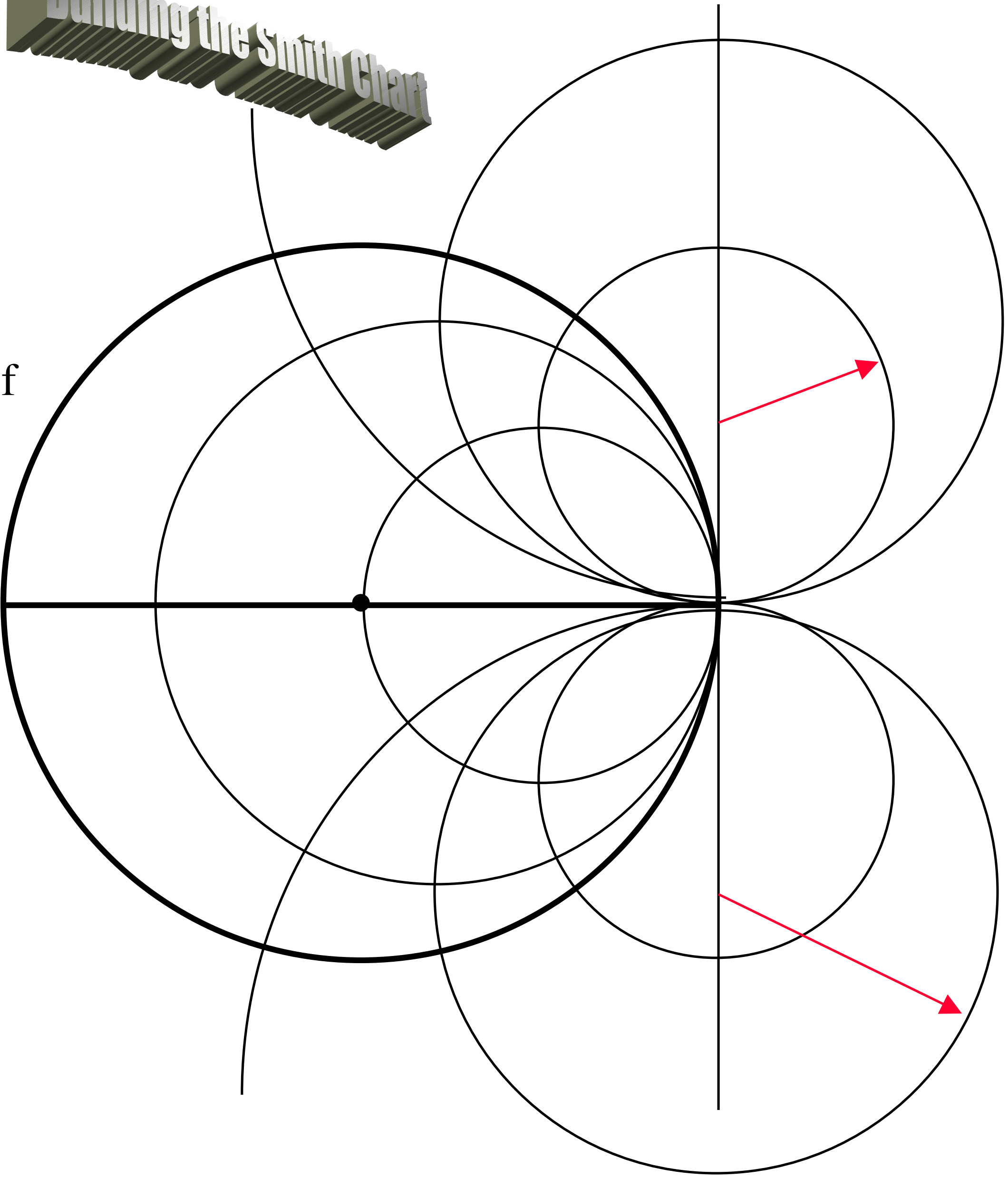
$$\frac{1}{1 + \frac{R}{Zo}}$$

# Building the Smith Chart

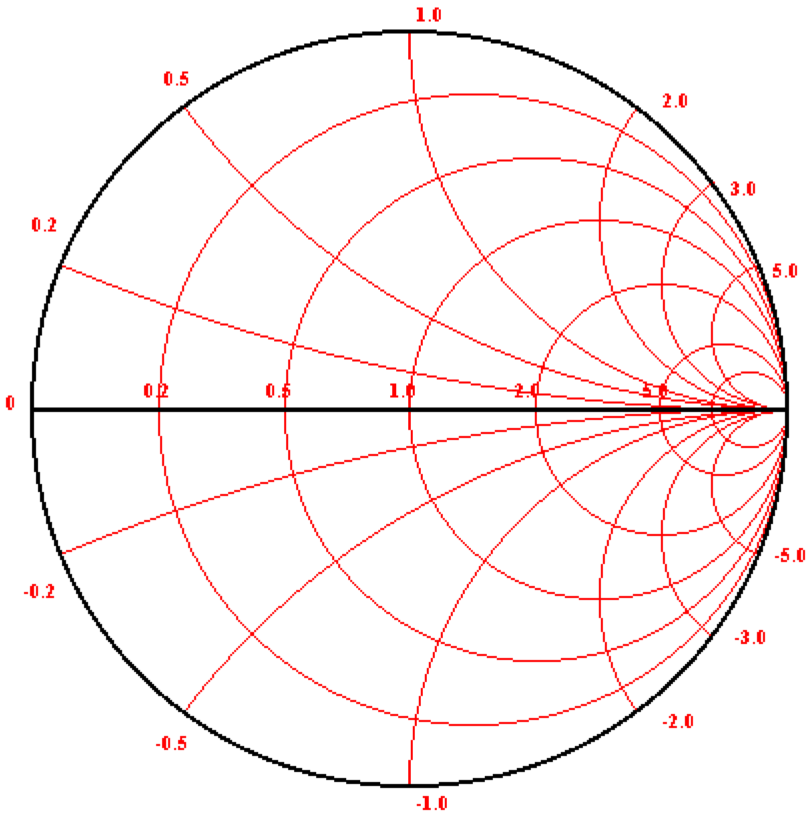
Now add constant reactance circles

The radius of each of these circles is:

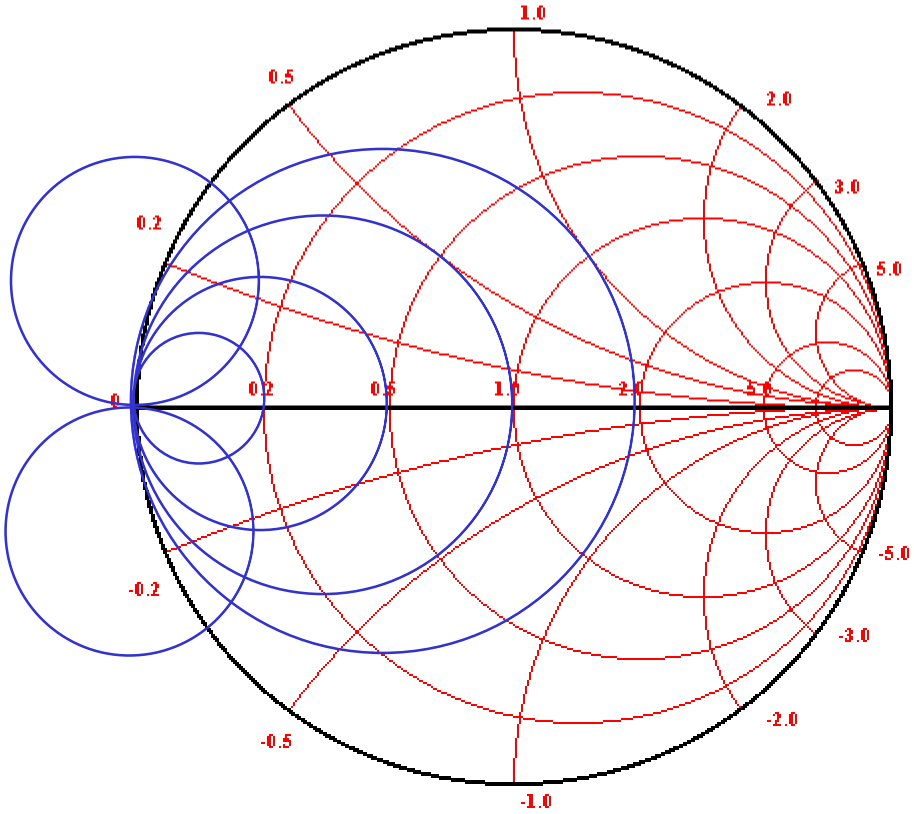
$$\frac{1}{\frac{X}{Z_o}}$$



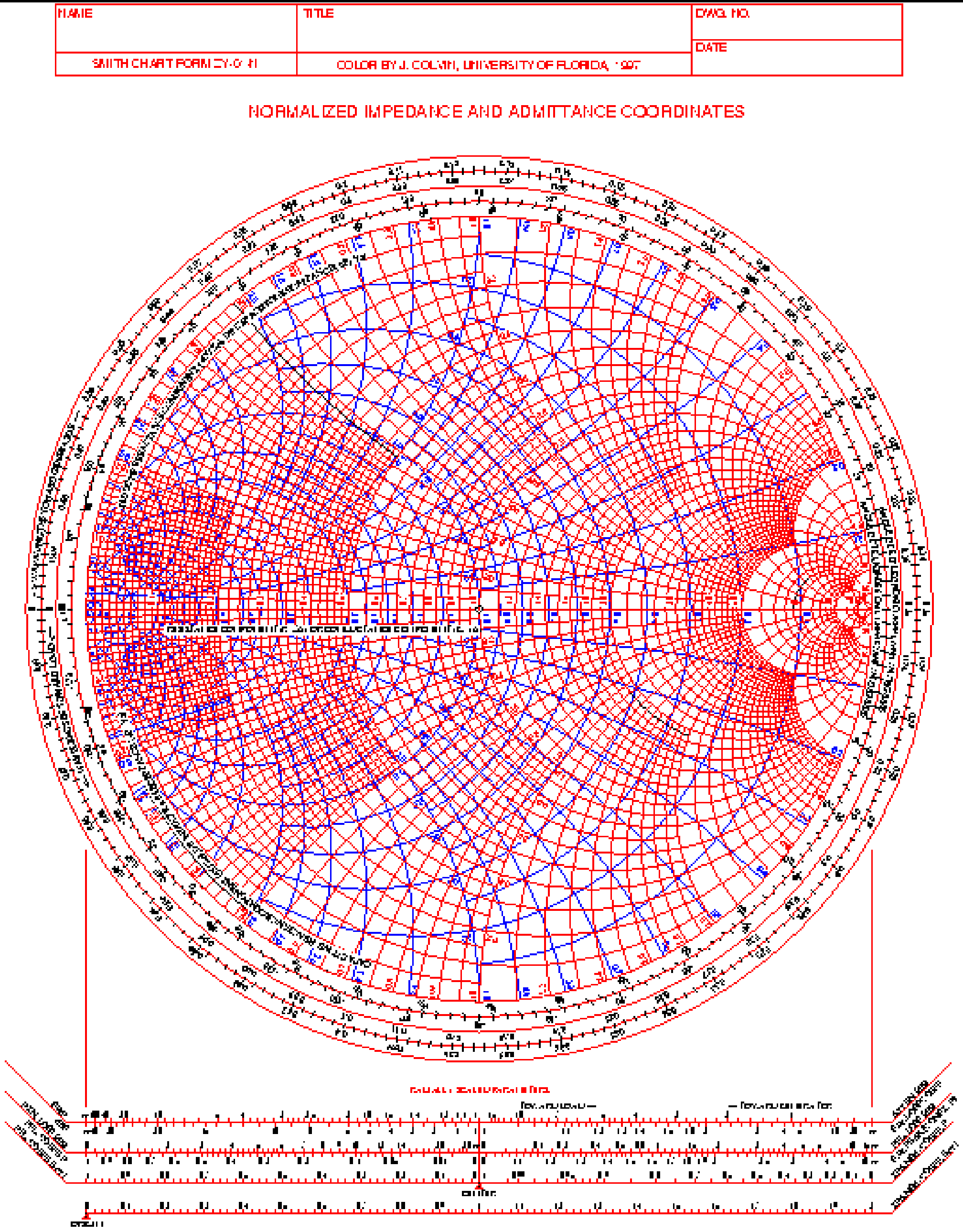
When it’s complete we have something that looks like this



There's also a mirror image of the chart that instead of having constant resistance circles, and constant reactance curves, has instead constant conductance circles and constant susceptance curves.



# The offical version!



| | | |
|---|---|---|
| NAME | TITLE | DWG. NO. |
| SMITH CHART FORM ZY-01-N | COLOR BY J. COLVIN, UNIVERSITY OF FLORIDA, 1997 | DATE |

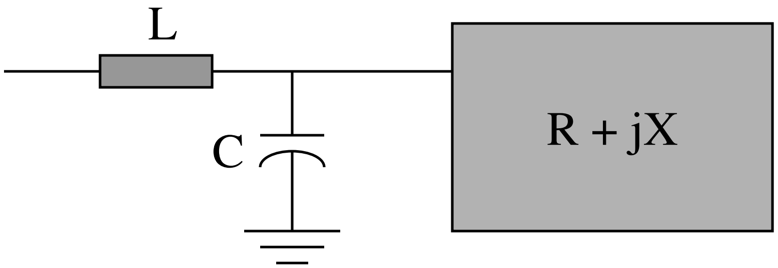
NORMALIZED IMPEDANCE AND ADMITTANCE COORDINATES

## Using the Smith Chart to solve impedance matching problems

The Smith Chart can be used to design a lumped (L-C) matching network to match one impedance to another.

Each LC element behaves in a certain way on the chart

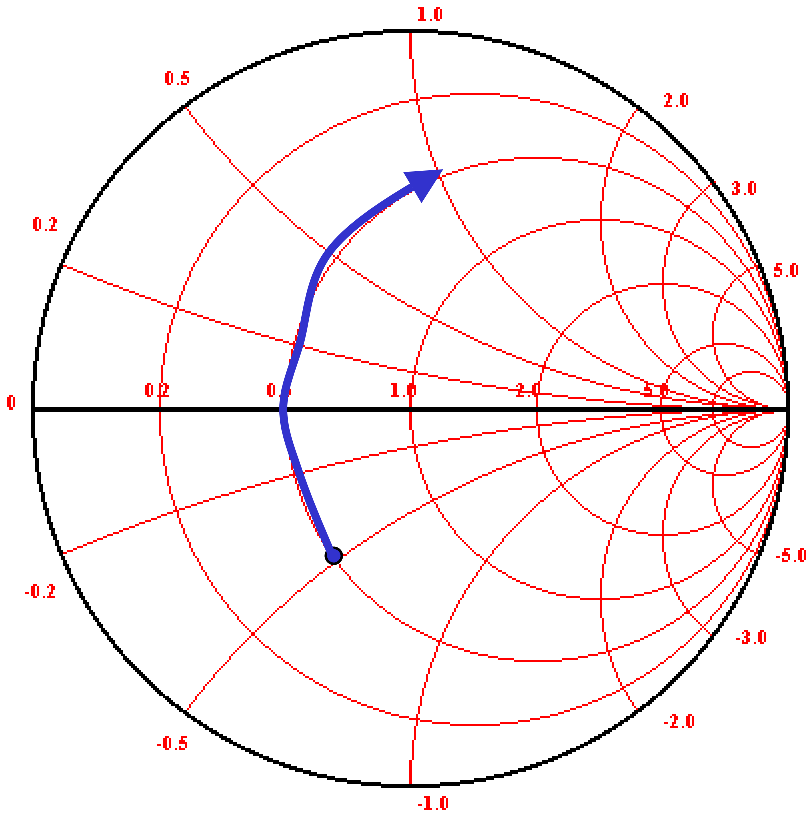
Typical LC elements include:



Series Inductors
Series Capacitors
Shunt Inductors
Shunt Capacitors

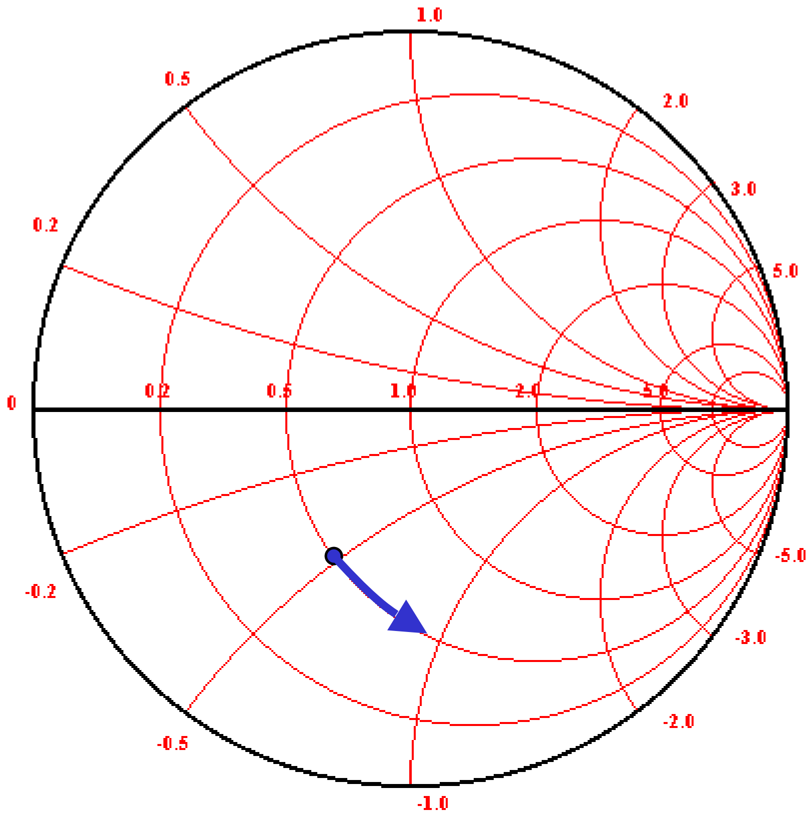
# Series Inductors

Moves clockwise along circles of constant resistance



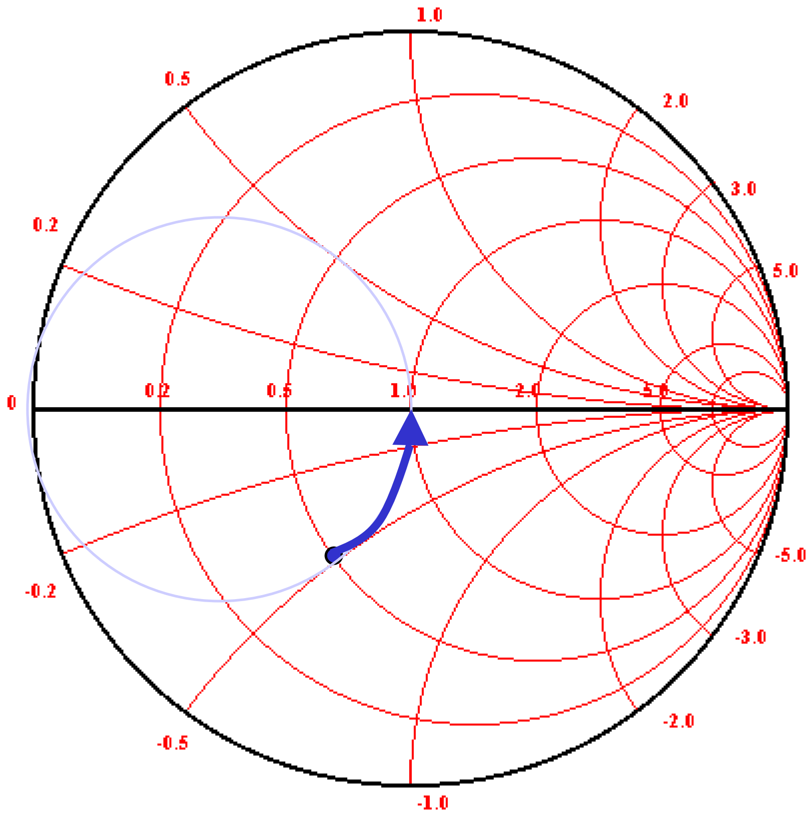
# Series Capacitors

Moves counter-clockwise along circles of constant resistance



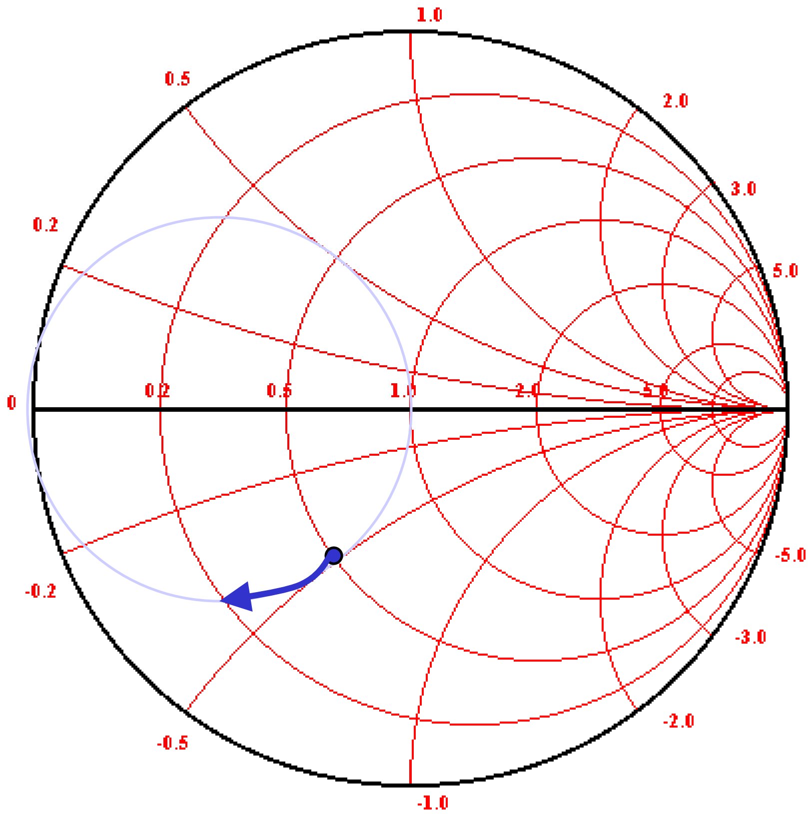
# Shunt Inductors

Moves counter-clockwise along circles of constant conductance



# Shunt Capacitors

Moves clockwise along circles of constant conductance



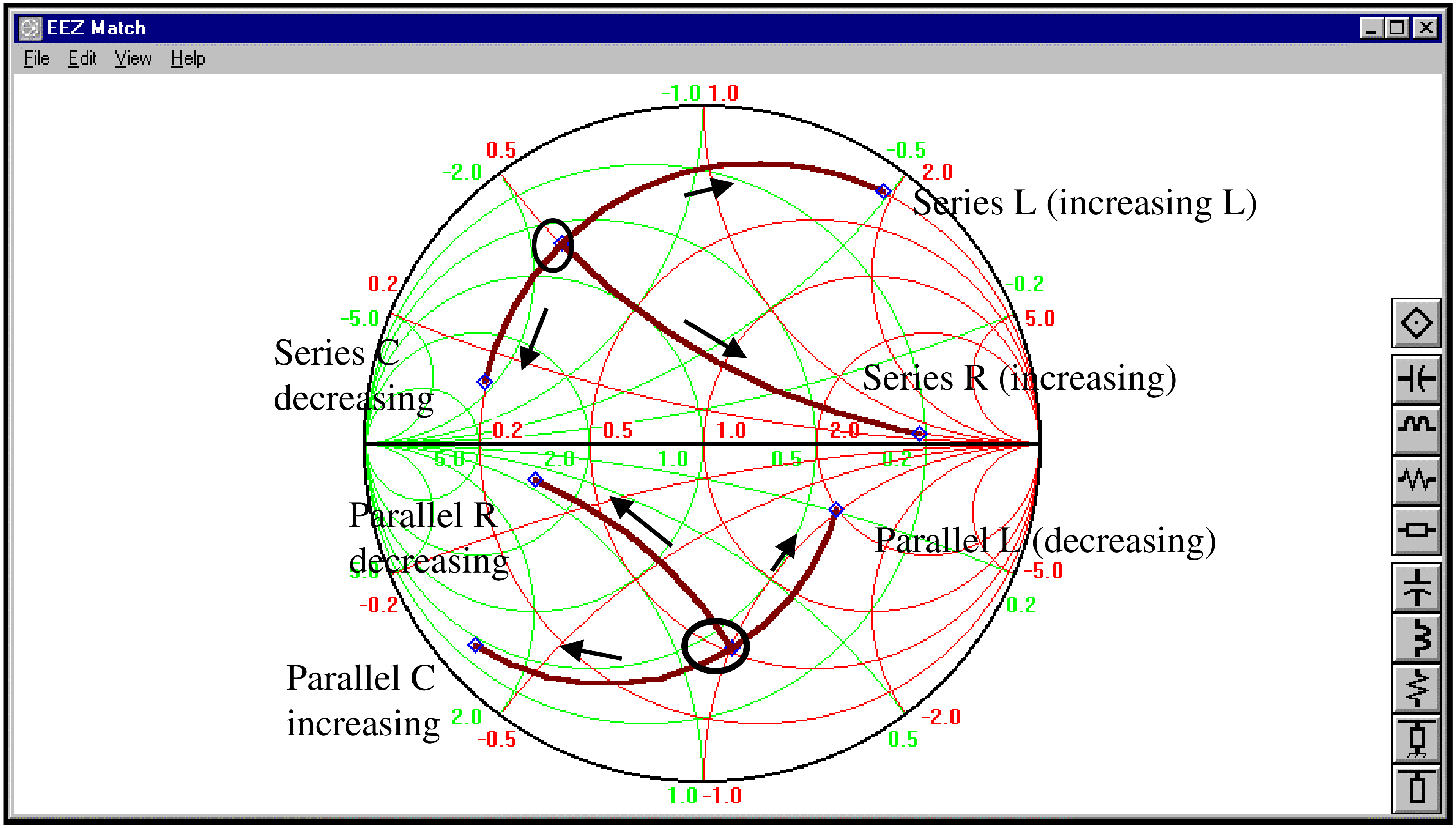
# How do I calculate physical values of Inductance and Capacitance from normalized values on the chart

Series L $+jX_{norm} = (2\pi f L)/Z_o$
Series C $-jX_{norm} = 1/ 2\pi f C Z_o$
Series R $R_{norm} = R/Z_o$

Parallel L $-jB_{norm} = Z_o/(2\pi f L)$
Parallel C $+jB_{norm} = 2\pi f C Z_o$
Parallel R $G_{norm} = Z_o/R$

Since I know what Zo is,
and I know what the frequency is,
I can solve for real values of R, C,
and L.

# What Components do on the Smith Chart….



Series L (increasing L)

Series C decreasing

Series R (increasing)

Parallel R decreasing

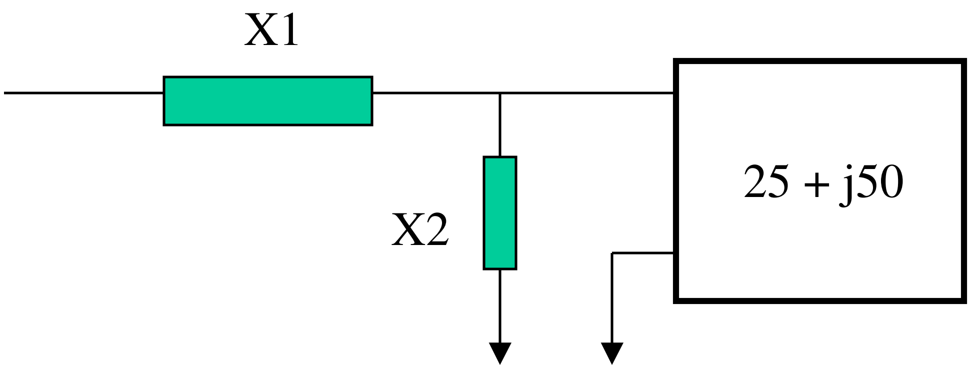
Parallel L (decreasing)

Parallel C increasing

Remember that all values on the chart are NORMALIZED to Zo!
Component values on the chart will be Reactances or Susceptances,
the actual values must be computed based on the Freq and Zo...

Example: Find X1 and X2 to match to 50 ohms

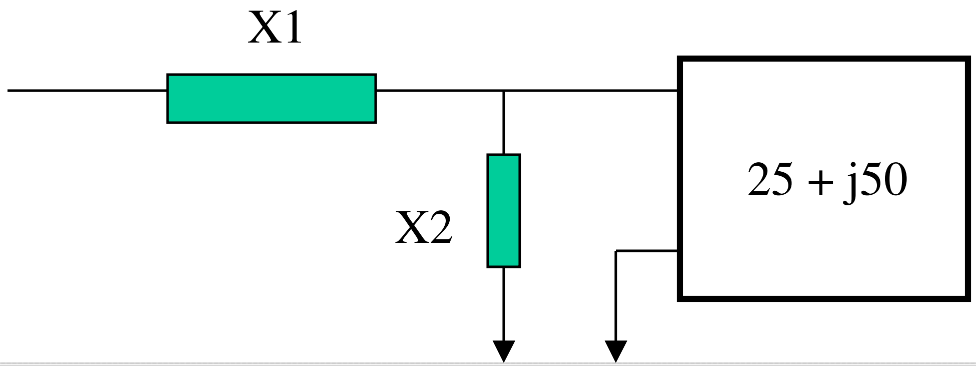
Freq = 28 MHz



# Let’s do some matching with L’s and C’s

Pick Zo (50 Ohms sounds OK…)

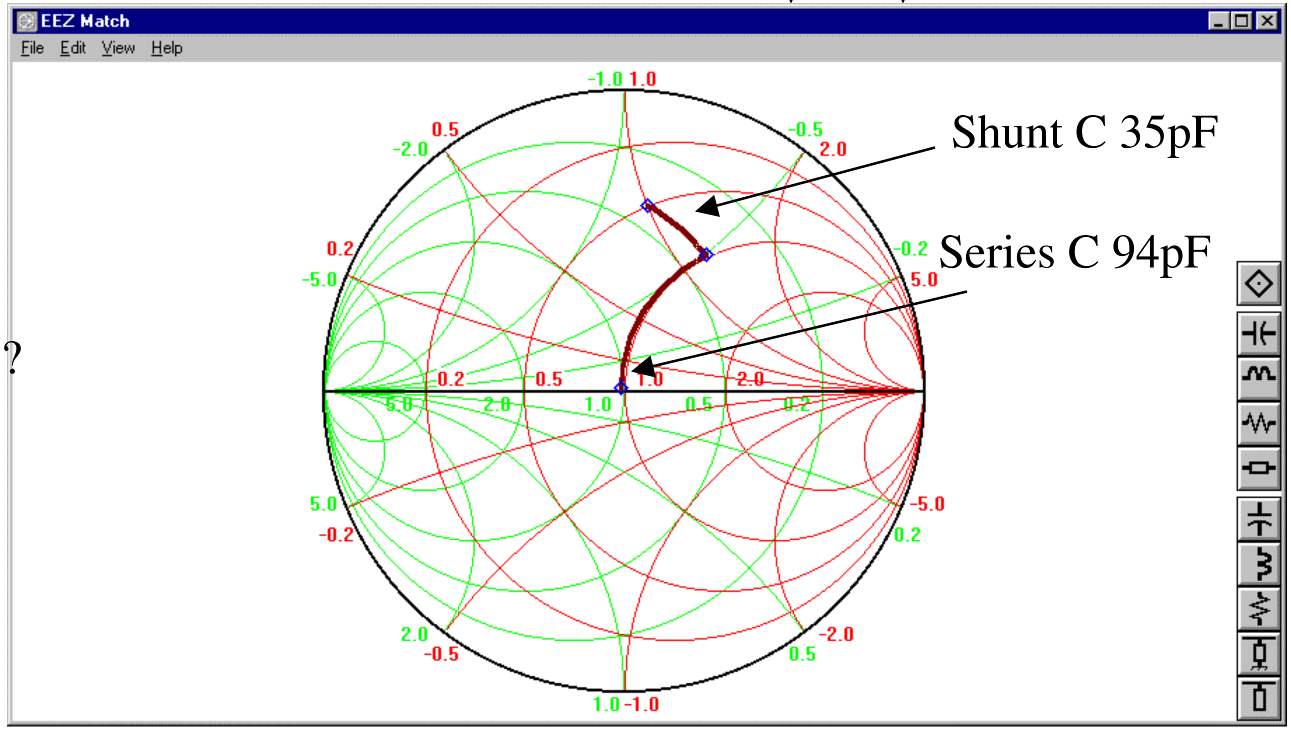
Frequency is 28 MHz



The shunt Cap, transforms the R part of the Z to 1 (on the unit R circle..) from there a simple series C will take out the inductive reactance..

Easy as π

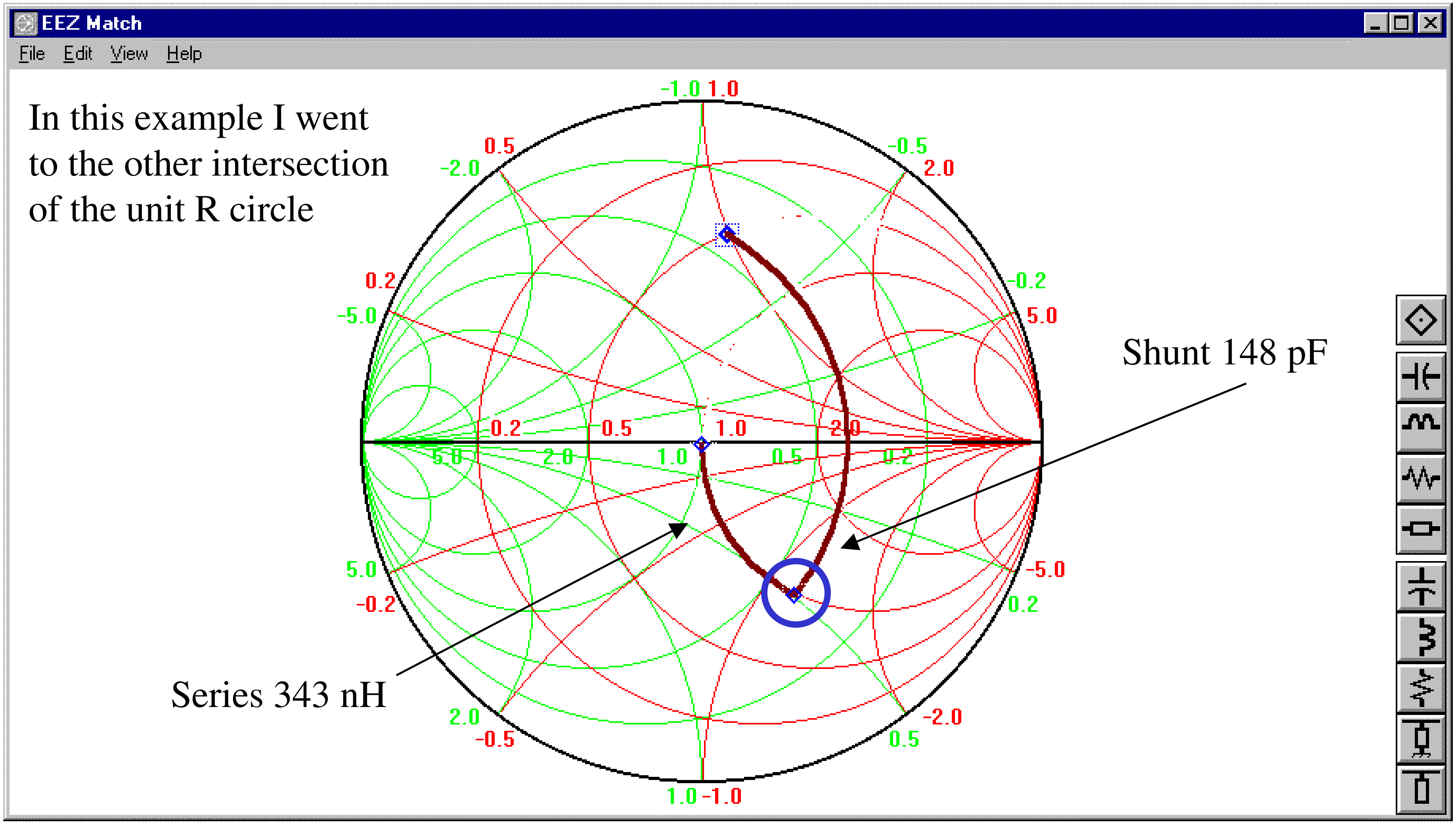
Are there other ways to match this same impedance??



Shunt C 35pF

Series C 94pF

# More than one way to skin a cat!



In this example I went to the other intersection of the unit R circle

Shunt 148 pF

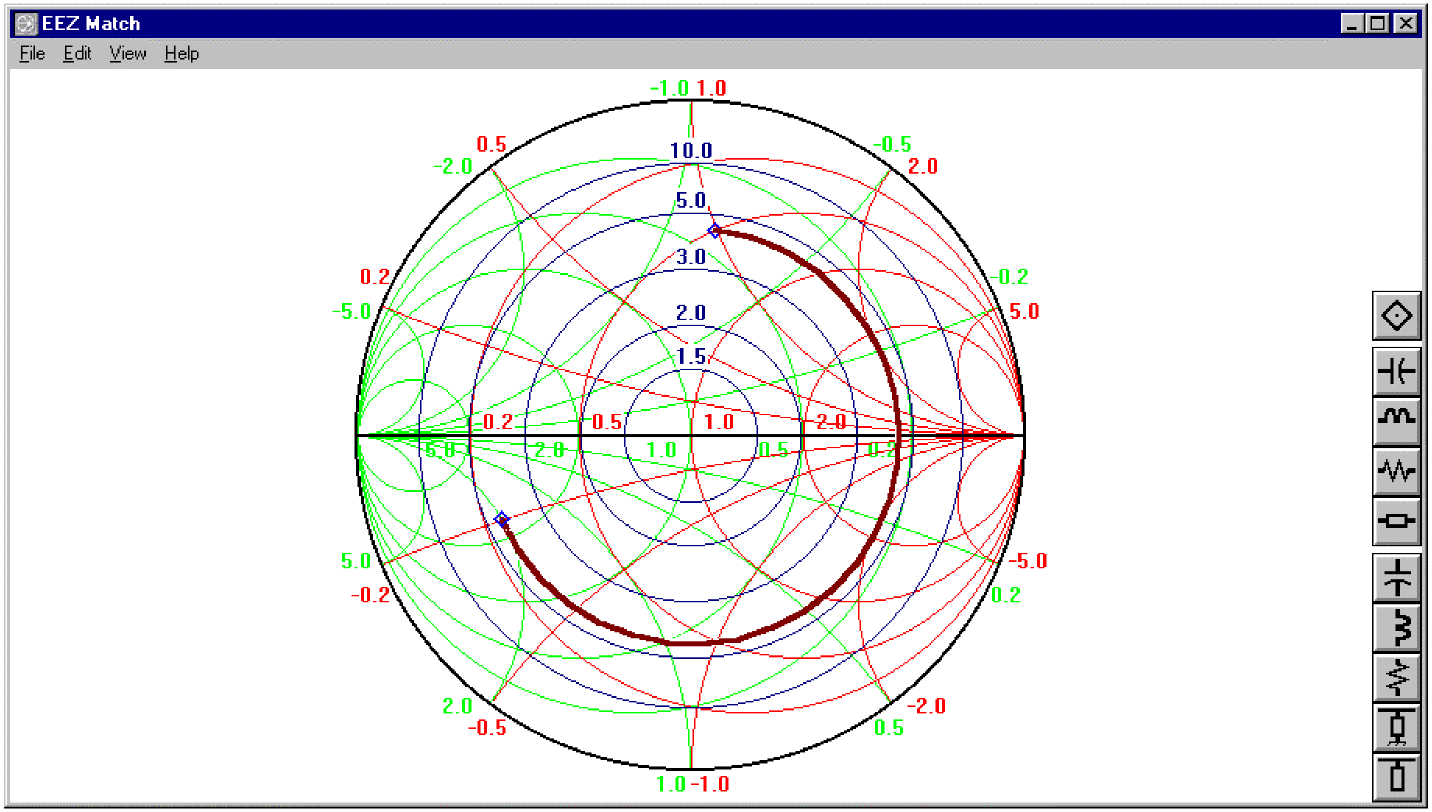
Series 343 nH

# Distributed Elements

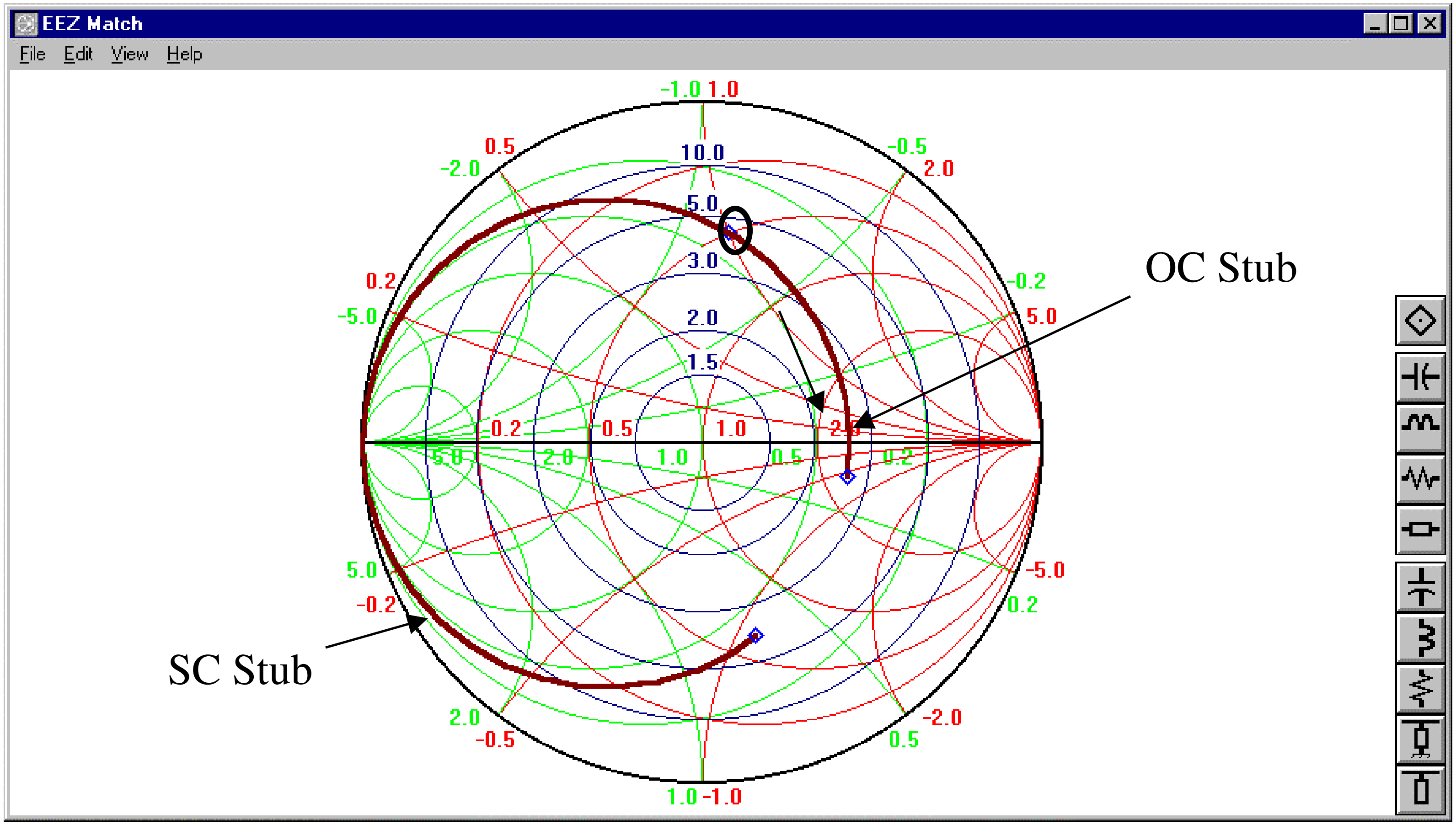
- Last but NOT least, transmission lines, microstrips, striplines and waveguides can all be made to perform impedance transformation and matching functions just like their lumped element cousins…
- There are some important differences that are best seen on the Smith Chart.
- Observe how series transmission lines of various Zo's and physical lengths transform the impedance…
- Also notice the effect of shunt sections of transmission lines (these are called stubs). Stubs can be either open circuited or short circuited...

Here we have a transmission line in series with the same load. Note that the behavior is different.. The impedance is transformed along constant VSWR circles (something new!). In this example the impedance was transformed to a completely new value by a 120 degree (about 3.6m at this freq), piece of 50 ohm coax cable.

What do stubs do?

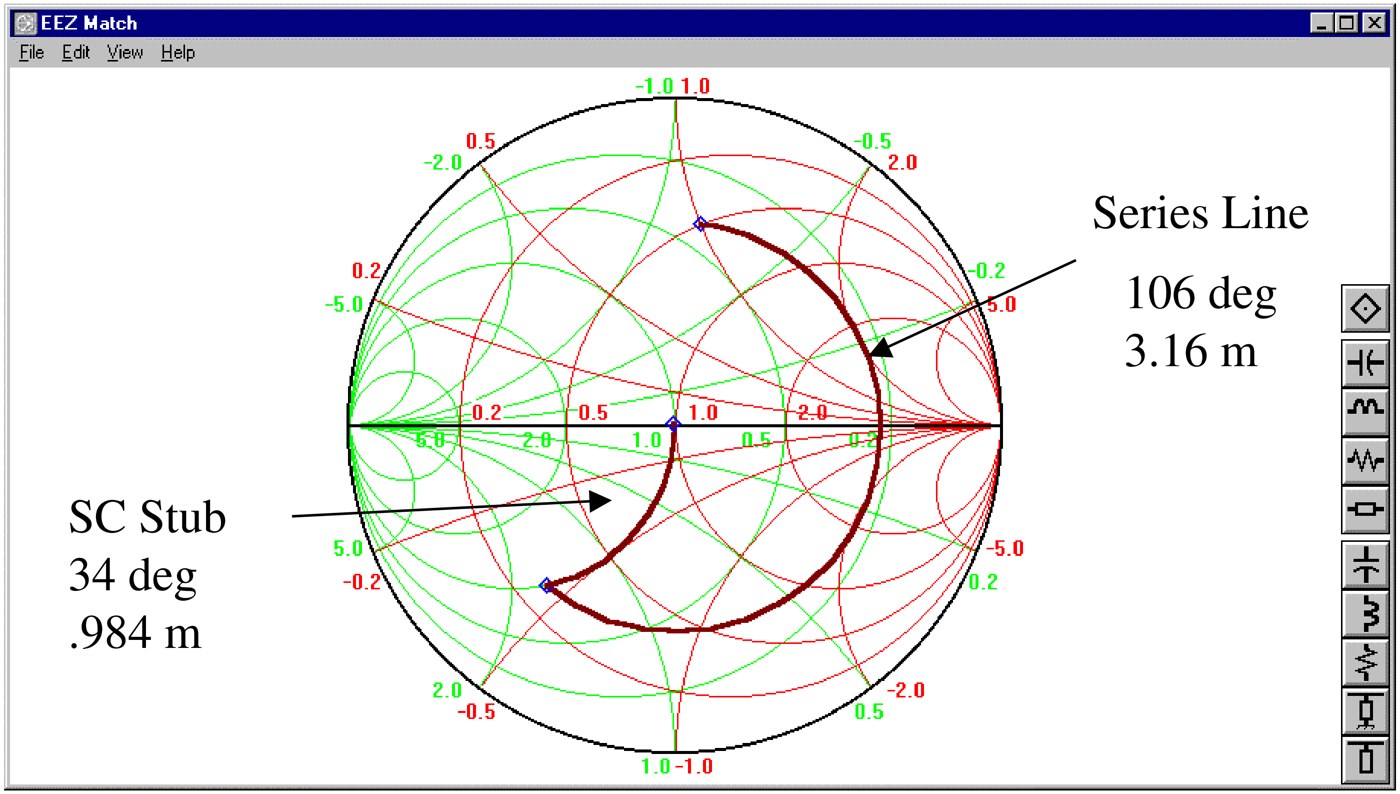


Stubs can rotate all the way around the chart (unlike shunt L's and C's), but along circles of constant conductance (Like L's and C's).



## Matching can be done entirely with Transmission Line Sections

50 Ohm Line is used



Series Line

106 deg

3.16 m

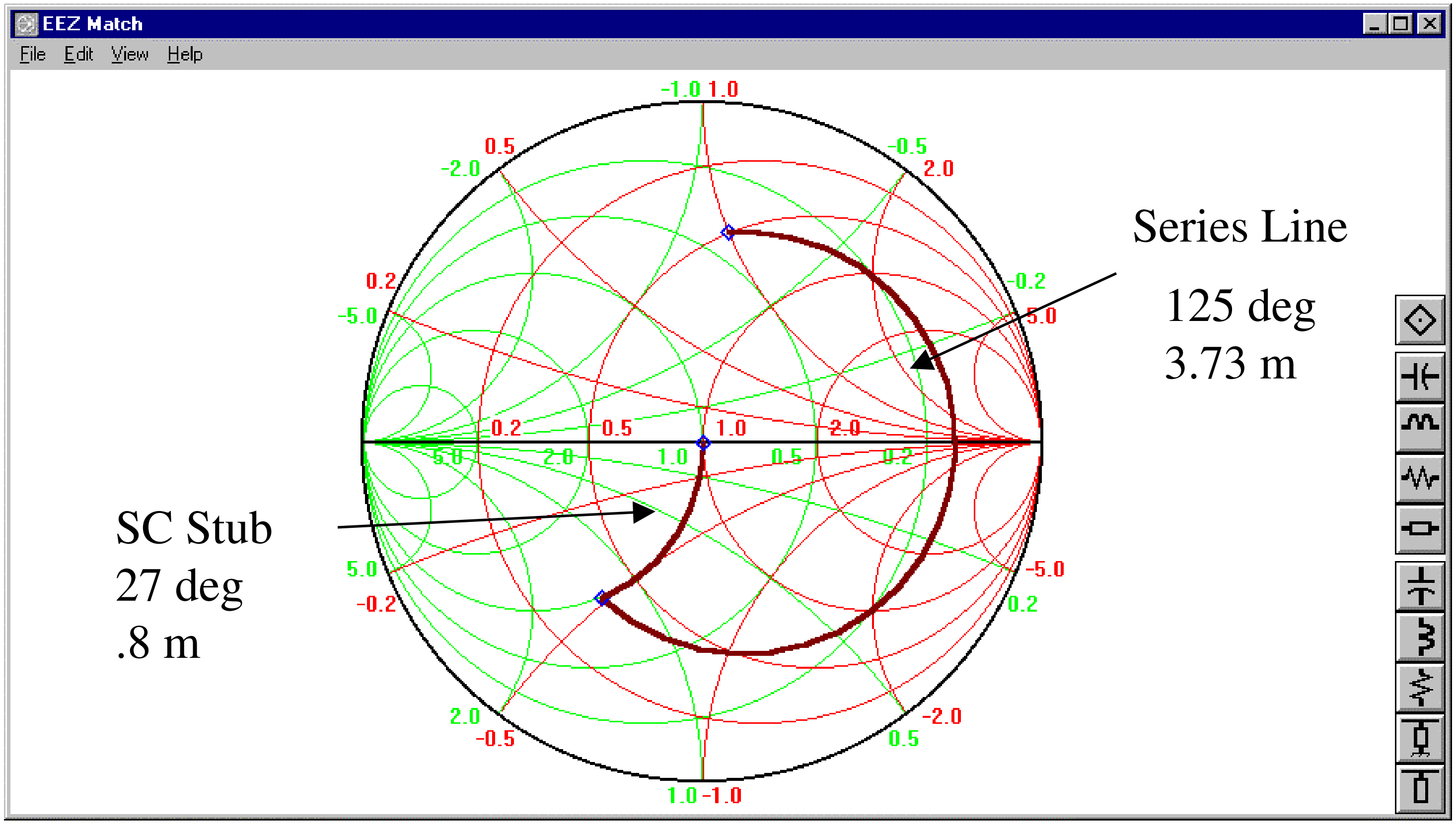
SC Stub

34 deg

.984 m

# Matching can be done entirely with Transmission Line Sections

This time with 75 Ohm

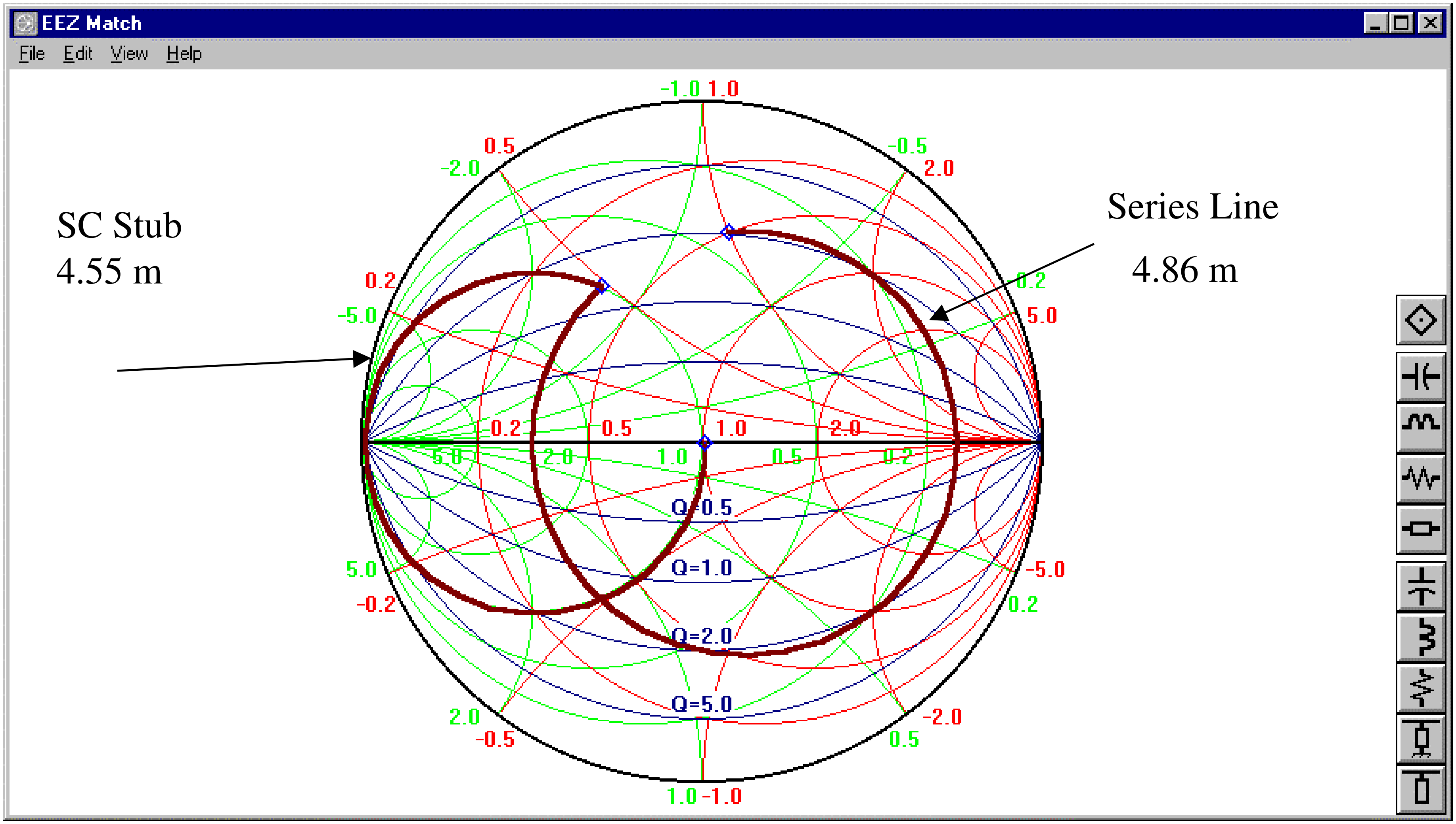


Series Line

125 deg
3.73 m

SC Stub
27 deg
.8 m

And if you want to waste coax…. There are almost always more than one way to achieve a match!

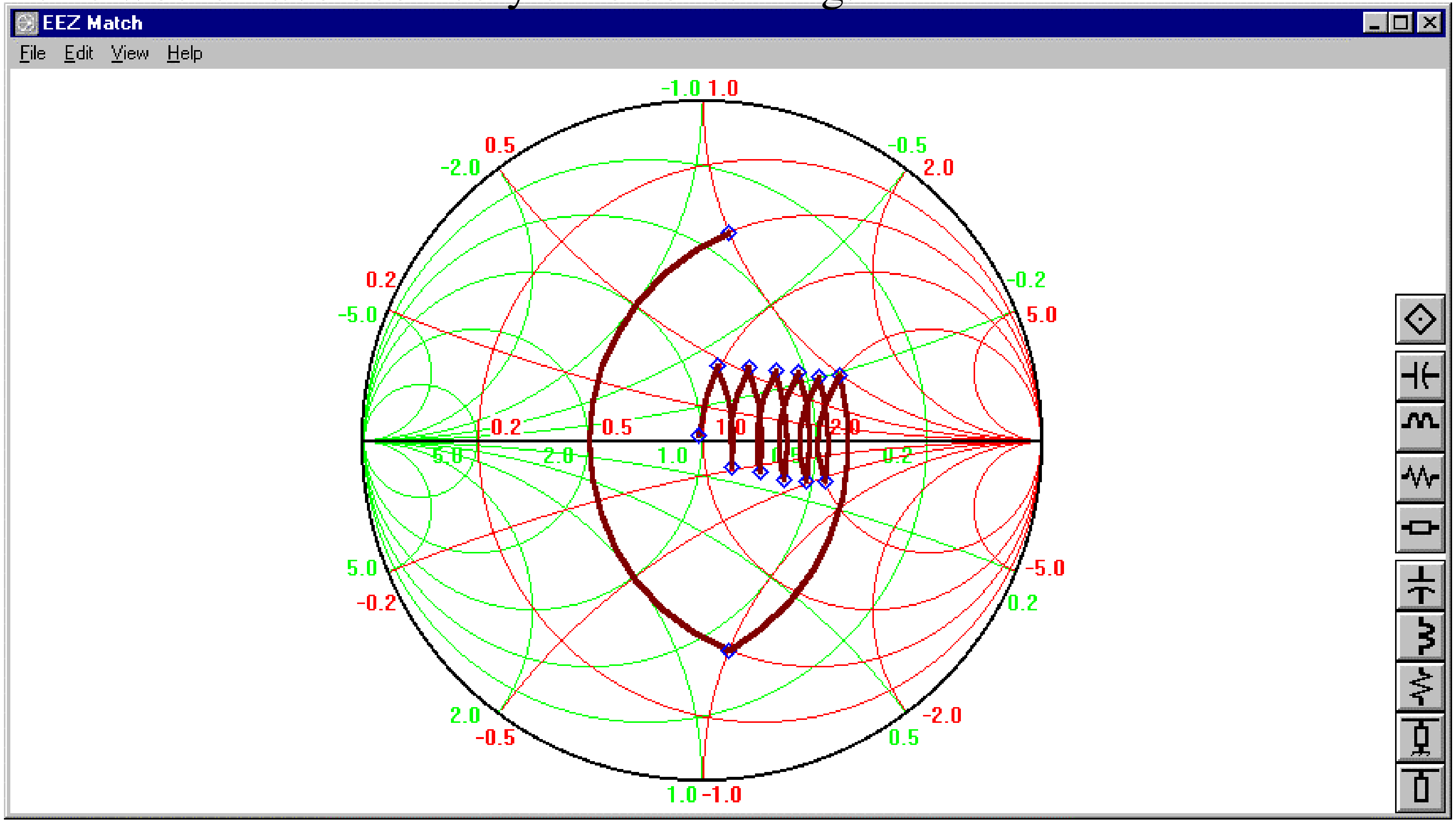


SC Stub
4.55 m

Series Line
4.86 m

# Using Many Lumped Elements

Although the graph below was done on a software program, this complex (5L's, 5C's) matching network could have easily been done on a paper smith chart. The same calculations to do this would have been very time consuming.



# Plotting vector data VS frequency

Vector Data : Data that has both real and imaginary parts (R +/- jX) or Magnitude and Phase (like S- Parameters)

The Smith Chart makes an excellent backdrop for showing how this data varies with frequency. (Bonus.. What are Rieke Diagrams??)