

Ambertec, P.E., P.C.

Using Recursive
Differential Equations

c. 2009 Ambertec, P.E., P.C..





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Just two differential equations

$$\mathbf{v = L * di / dt}$$

$$\mathbf{i = C * dv / dt}$$

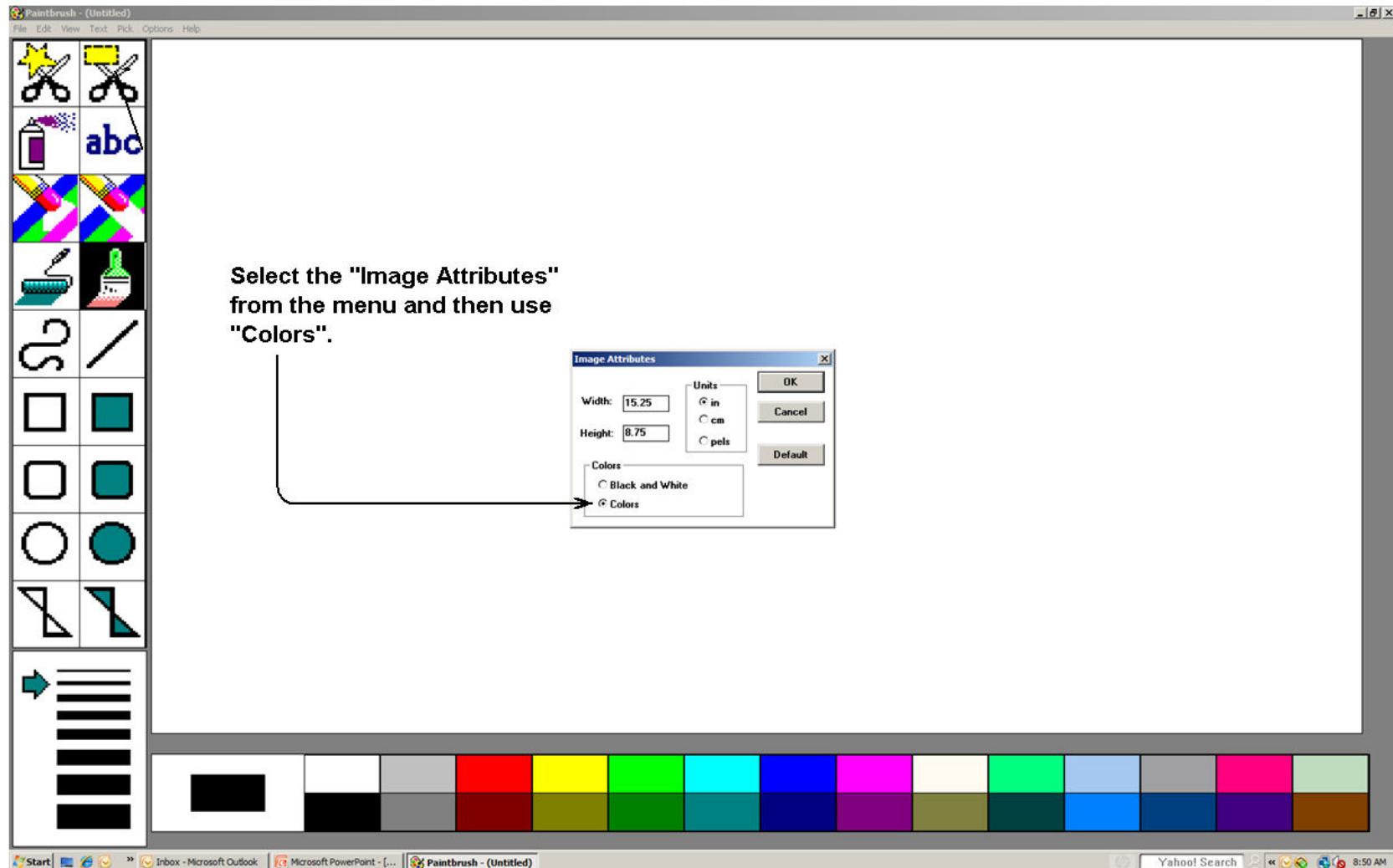
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Name	Size	Type	Modified
 Gwbasic	78KB	Application	7/7/86 1:00 PM
 Pbrush	180KB	Application	12/31/93 2:11 AM
 Pbrush	40KB	Help File	12/31/93 2:11 AM
 pbrush.dll	7KB	Application Extension	12/31/93 2:11 AM

These four files are old, but while tallying up to only 305 KB, they can be used for analyses that are as varied and versatile as that kind of thing can be.

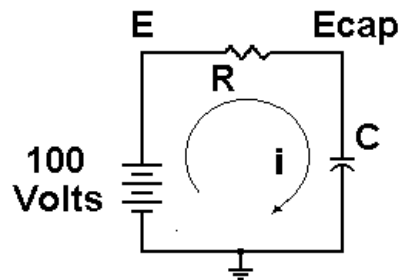
There are even some things you can do with them that some modern software can't do.

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This is a simple RC network charging from a DC source.
The first order differential equation is put into a loop which simulates
in software, the actual process under consideration.



Iterative loop:

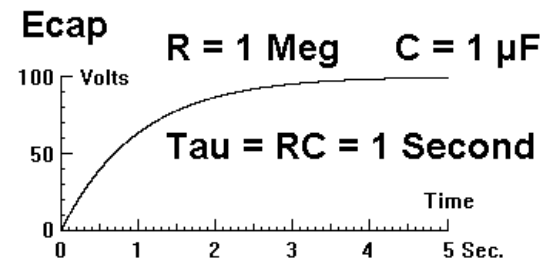
$$i = (E - E_{cap}) / R$$

$$dE_{cap} = i \, dt / C$$

$$E_{cap} = E_{cap} + dE_{cap}$$

$$t = t + dt$$

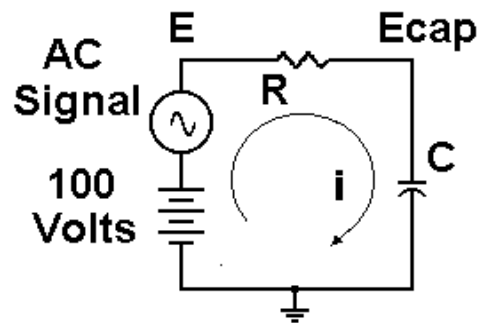
A capacitor charging from a starting voltage
of zero from a 100 volt DC source.



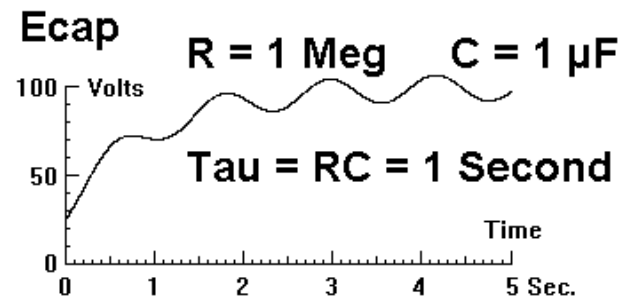
```
150 ECAP=0
160 READ EDC,R,C,DT:DATA 100,1e6,1e-6,.001
170 FOR T=0 TO 5 STEP DT:E=EDC
180 I=(E-ECAP)/R:DV=I*DT/C:ECAP=ECAP+DV:GOSUB 110:NEXT T
```

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When we add an AC source in series with the DC source, the output waveform reflects the effects of both of them.



A capacitor charging from a starting voltage of 25 volts from a 100 volt DC source



Iterative loop:

$$i = (E - E_{cap}) / R$$

$$dE_{cap} = i \, dt / C$$

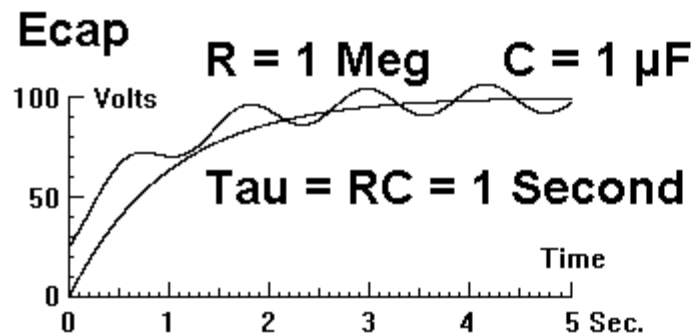
$$E_{cap} = E_{cap} + dE_{cap}$$

$$t = t + dt$$

```
150 ECAP=25
160 READ EDC,R,C,DT:DATA 100,1e6,1e-6,.001
170 FOR T=0 TO 5 STEP DT:E=EDC+40*SIN(300*T*PI/180)
180 I=(E-ECAP)/R:DU=I*DT/C:ECAP=ECAP+DU:GOSUB 110:NEXT T
```

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Looking at an overlay:

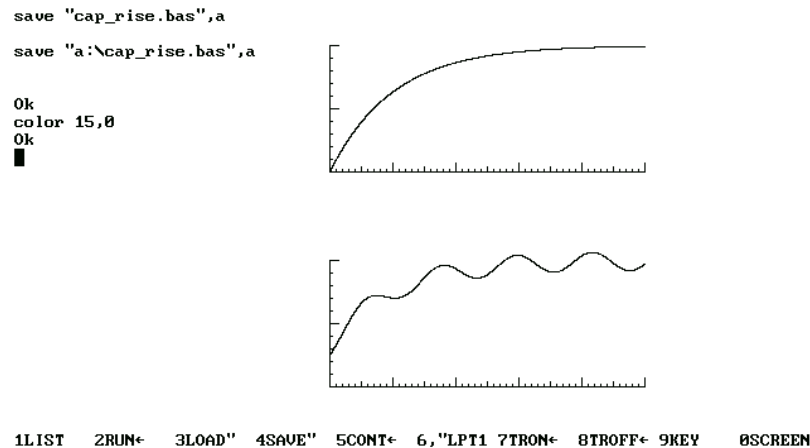


When we overlay the two results, we see that their E_{cap} values are starting from two different initial values (zero and 25 volts), but are heading to the same average value of 100 volts.

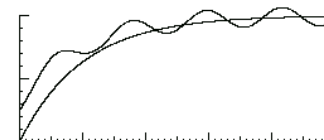
cap_rise.bas

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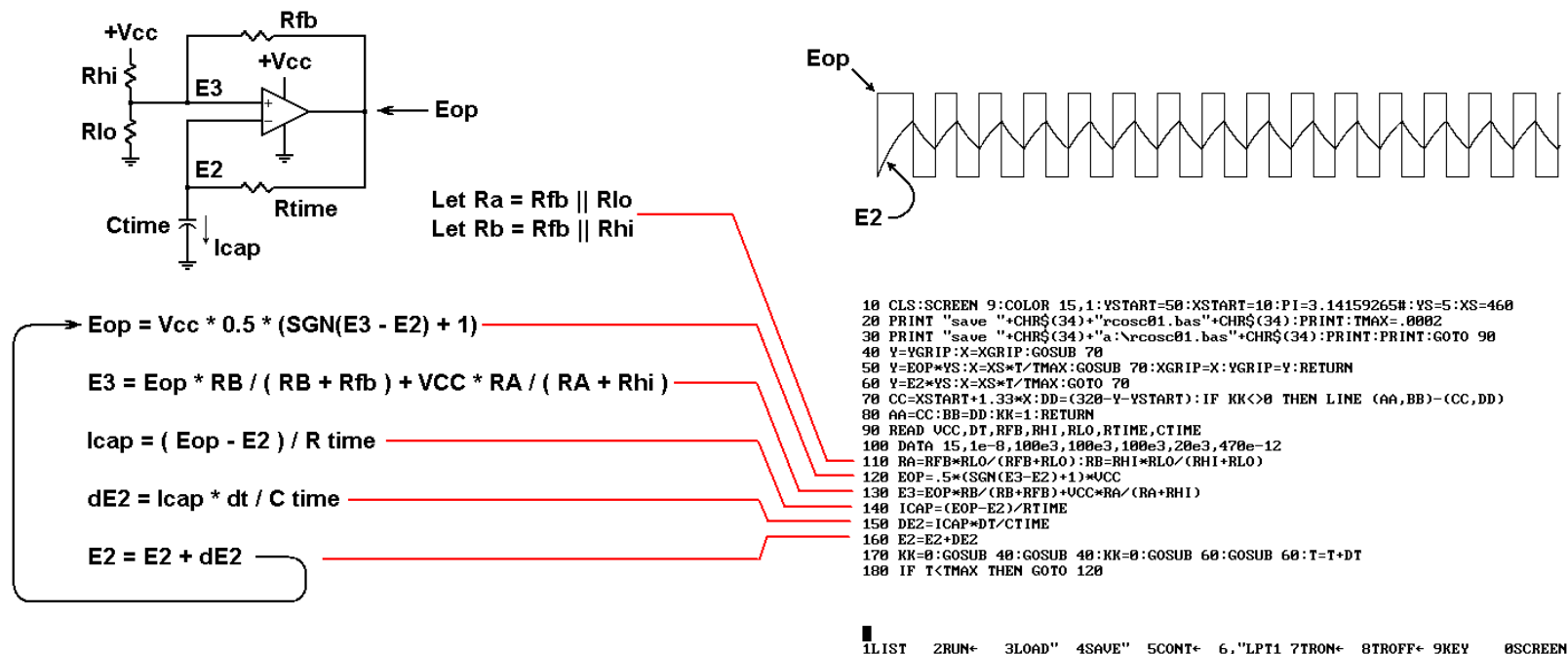
Putting a picture together.



Now move things around
and annotate any way you
want.



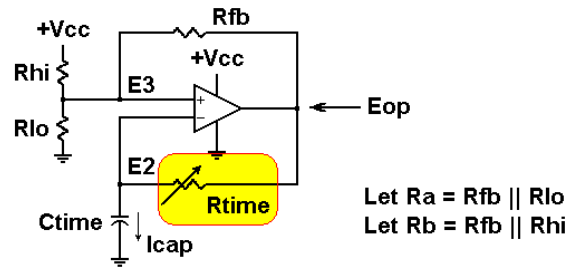
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This oscillator simulation uses fixed values of its resistors and capacitors.

rcosc01.bas

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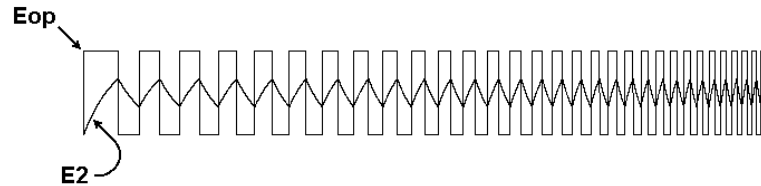
$$Eop = Vcc * 0.5 * (SGN(E3 - E2) + 1)$$

$$E3 = Eop * RB / (RB + Rfb) + VCC * RA / (RA + Rhi)$$

$$Icap = (Eop - E2) / Rtime$$

$$dE2 = Icap * dt / Ctime$$

$$E2 = E2 + dE2$$



```

10 CLS:SCREEN 9:COLOR 15,1:VSTART=50:XSTART=10:PI=3.14159265#:VS=5:XS=460
20 PRINT "save "+CHR$(34)+"rcosc02.bas"+CHR$(34):PRINT:TMAX=.0002
30 PRINT "save "+CHR$(34)+"a:\rcosc02.bas"+CHR$(34):PRINT:PRINT:GOTO 90
40 V=VGRIP:X=XGRIP:GOSUB 70
50 V=EOP*VS:X=XS*T/TMAX:GOSUB 70:XGRIP=X:VGRIP=V:RETURN
60 V=E2*VS:X=XS*T/TMAX:GOTO 70
70 CC=XSTART+1.33*X:DD=(320-V-VSTART):IF KK<>0 THEN LINE (AA,BB)-(CC,DD)
80 AA=CC:BB=DD:KK=1:RETURN
90 READ VCC,DT,RFB,RHI,RLO,RT,CTIME
100 DATA 15,1e-8,100e3,100e3,100e3,20e3,470e-12
110 RA=RFB*RLO/(RFB+RLO):RB=RHI*RLO/(RHI+RLO)
120 EOP=.5*(SGN(E3-E2)+1)*VCC:E3=EOP*RB/(RB+RFB)+VCC*RA/(RA+RHI)
130 REM
140 RTIME=RT*(1-.8*T/TMAX):REM Here is where the value of Rtime is varied.
150 REM
160 ICAP=(EOP-E2)/RTIME:DE2=ICAP*DT/CTIME:E2=E2+DE2
170 KK=0:GOSUB 40:GOSUB 40:KK=0:GOSUB 60:GOSUB 60:T=T+DT
180 IF T<TMAX THEN GOTO 120

```

1LIST 2RUN+ 3LOAD+ 4SAVE+ 5CONT+ 6,"LPT1 7TRON+ 8TROFF+ 9KEY 0SCREEN

In this simulation, the value of the timing resistance is arbitrarily varied during the time interval under examination.

rcosc02.bas

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Drawing pictures in GWBASIC

```
10 CLS:SCREEN 9:COLOR 15,1:YSTART=150:XSTART=100:PI=3.14159254#
20 PRINT "save "+CHR$(34)+"how2draw.bas"+CHR$(34):PRINT
30 PRINT "save "+CHR$(34)+"a:how2draw.bas"+CHR$(34):PRINT:PRINT:GOTO 60
40 CC=XSTART+X:DD=(320-Y-YSTART):IF KK<>0 THEN LINE (AA,BB)-(CC,DD)
50 AA=CC:BB=DD:KK=1:RETURN
60 REM
```

This short program is the basis of drawing pictures. If the variable called "KK" is non-zero, a line will be drawn between two points set by the values of "AA", the previous x-axis coordinate, "CC", the new x-axis coordinate, "BB", the previous y-axis coordinate and "DD", the new y-axis coordinate. (Yes, that is a mouthful.)

If the value of "KK" zero, then no line is drawn, but new x-axis and y-axis coordinates are memorized as the starting point for drawing the next line.

how2draw.bas

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Horizontal axis issue:

The basic (yes, pun intended) GWBASIC subroutine for doing pictures:

```
10 CLS:SCREEN 9:COLOR 15,1:YSTART=150:XSTART=100:PI=3.14159254#
20 PRINT "save "+CHR$(34)+"hou2draw.bas"+CHR$(34):PRINT
30 PRINT "save "+CHR$(34)+"a:hou2draw.bas"+CHR$(34):PRINT:PRINT:GOTO 60
40 CC=XSTART+X:DD=(320-Y-YSTART):IF KK<>0 THEN LINE (AA,BB)-(CC,DD)
50 AA=CC:BB=DD:KK=1:RETURN
60 REM
```

Two lines of code for drawing a spiral.

```
10 CLS:SCREEN 9:COLOR 15,1:YSTART=60:XSTART=100:PI=3.14159254#
20 PRINT "save "+CHR$(34)+"hou2dra2.bas"+CHR$(34):PRINT
30 PRINT "save "+CHR$(34)+"a:hou2dra2.bas"+CHR$(34):PRINT:PRINT:GOTO 60
40 CC=XSTART+X:DD=(320-Y-YSTART):IF KK<>0 THEN LINE (AA,BB)-(CC,DD)
50 AA=CC:BB=DD:KK=1:RETURN
60 RAD=10:FOR THETA=0 TO 360*9:X=XSTART+RAD*(THETA/360)*COS(THETA*PI/180)
70 Y=YSTART+RAD*(THETA/360)*SIN(THETA*PI/180):GOSUB 40:NEXT THETA
```

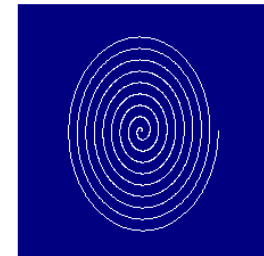
The subroutine with an added coefficient of 1.33 for the x-axis.

```
10 CLS:SCREEN 9:COLOR 15,1:YSTART=150:XSTART=100:PI=3.14159254#
20 PRINT "save "+CHR$(34)+"hou2draw.bas"+CHR$(34):PRINT
30 PRINT "save "+CHR$(34)+"a:hou2draw.bas"+CHR$(34):PRINT:PRINT:GOTO 60
40 CC=XSTART+X*1.33:DD=(320-Y-YSTART):IF KK<>0 THEN LINE (AA,BB)-(CC,DD)
50 AA=CC:BB=DD:KK=1:RETURN
60 REM
```

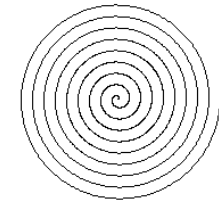
Two lines of code for drawing a spiral.

```
10 CLS:SCREEN 9:COLOR 15,1:YSTART=60:XSTART=100:PI=3.14159254#
20 PRINT "save "+CHR$(34)+"hou2dra2.bas"+CHR$(34):PRINT
30 PRINT "save "+CHR$(34)+"a:hou2dra2.bas"+CHR$(34):PRINT:PRINT:GOTO 60
40 CC=XSTART+X*1.33:DD=(320-Y-YSTART):IF KK<>0 THEN LINE (AA,BB)-(CC,DD)
50 AA=CC:BB=DD:KK=1:RETURN
60 RAD=10:FOR THETA=0 TO 360*9:X=XSTART+RAD*(THETA/360)*COS(THETA*PI/180)
70 Y=YSTART+RAD*(THETA/360)*SIN(THETA*PI/180):GOSUB 40:NEXT THETA
```

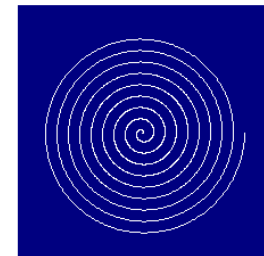
The on-screen view in GWBASIC looks like it's crunched in from side to side:



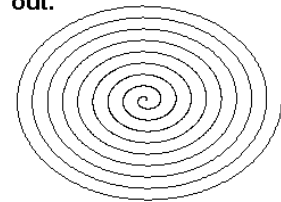
When "Print Screen" takes it to Paintbrush, the vertical to horizontal proportions are okay.



The on-screen view in GWBASIC looks like it's okay:

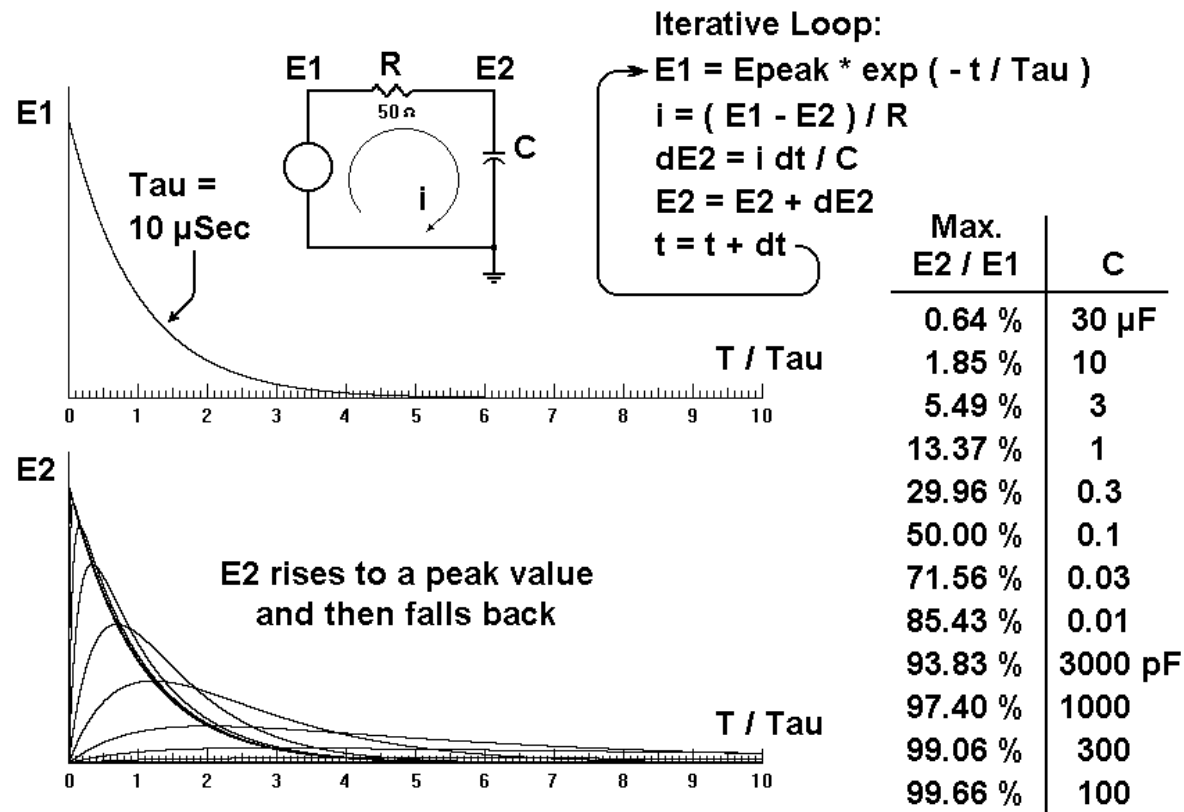


When "Print Screen" takes this image to Paintbrush, the result is horizontally spread out.



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We apply an exponentially decaying waveform to the input of a single-pole lowpass filter and calculate the output.

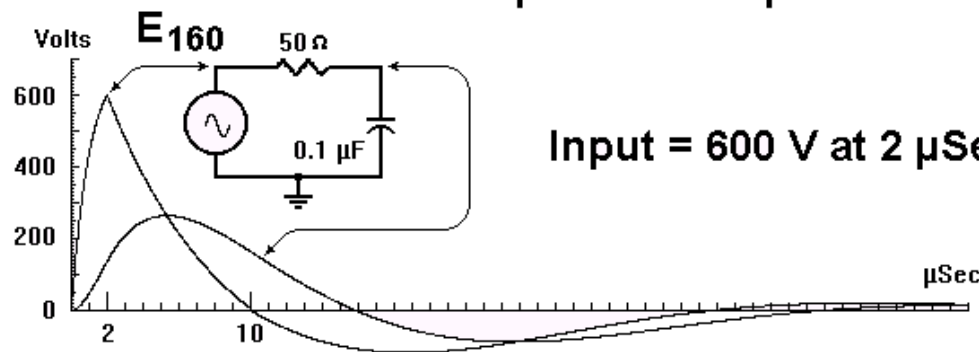


mult_rc.bas

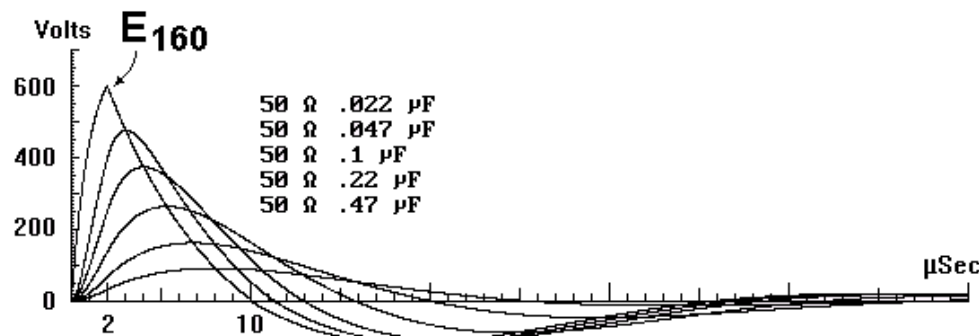
Ambertec, P.E., P.C.

We can apply agency-specified waveforms to circuit inputs:

**DO-160C Category A voltage spike input to
a 1-pole RC lowpass filter.**



Input = 600 V at 2 μSec and zero at 10 μSec.



Iterative Loop:

$$E1 = E_{160} * e^{(-t / \text{Tau})}$$

$$i = (E1 - E2) / R$$

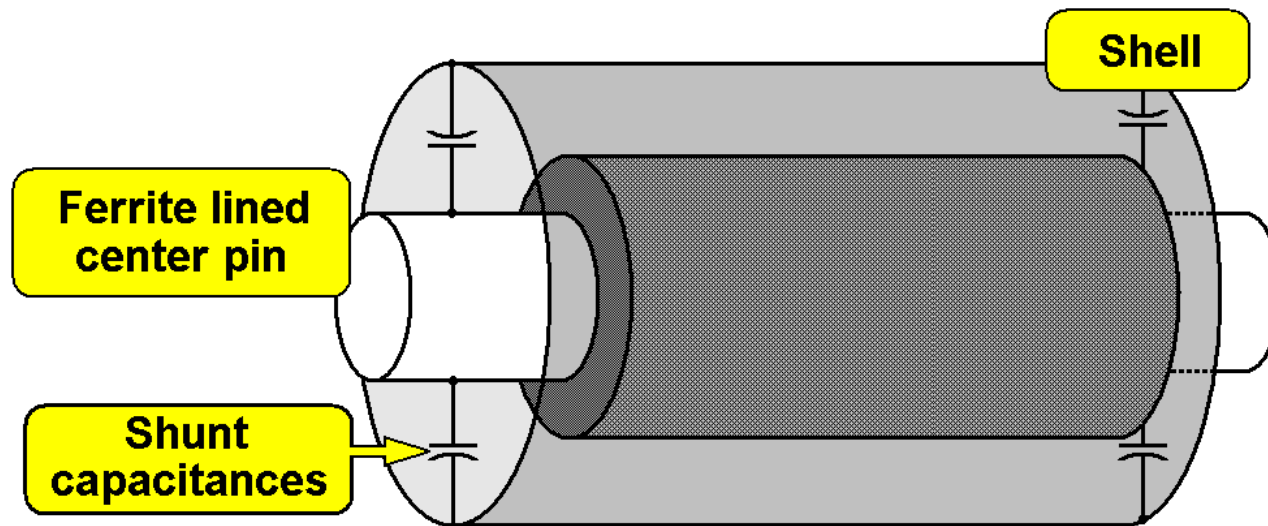
$$dE2 = i \, dt / C$$

$$E2 = E2 + dE2$$

$$t = t + dt$$

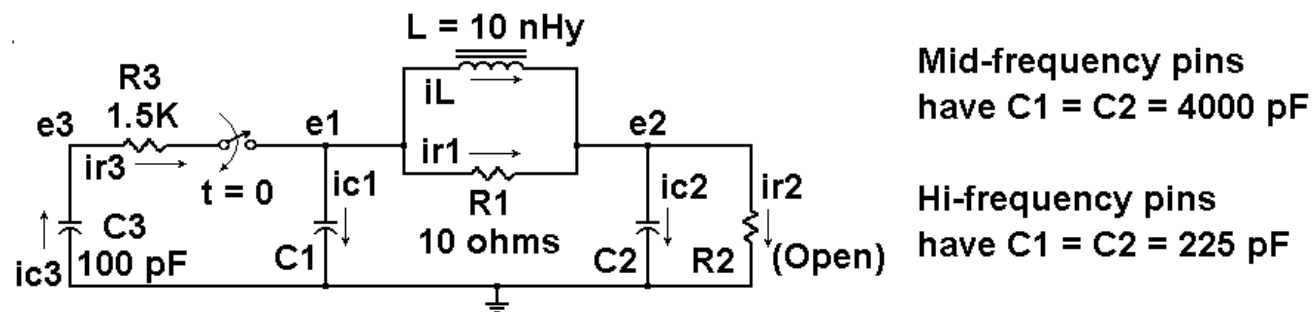
Ambertec, P.E., P.C.

Structure of EMI Filter Pin



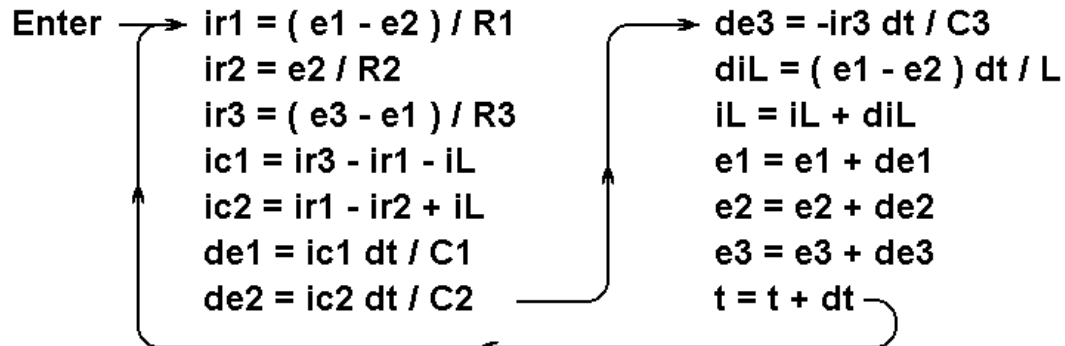
Ambertec, P.E., P.C.

The human body model of an electrostatic discharge (ESD) event is applied to a connector's filter-pin.



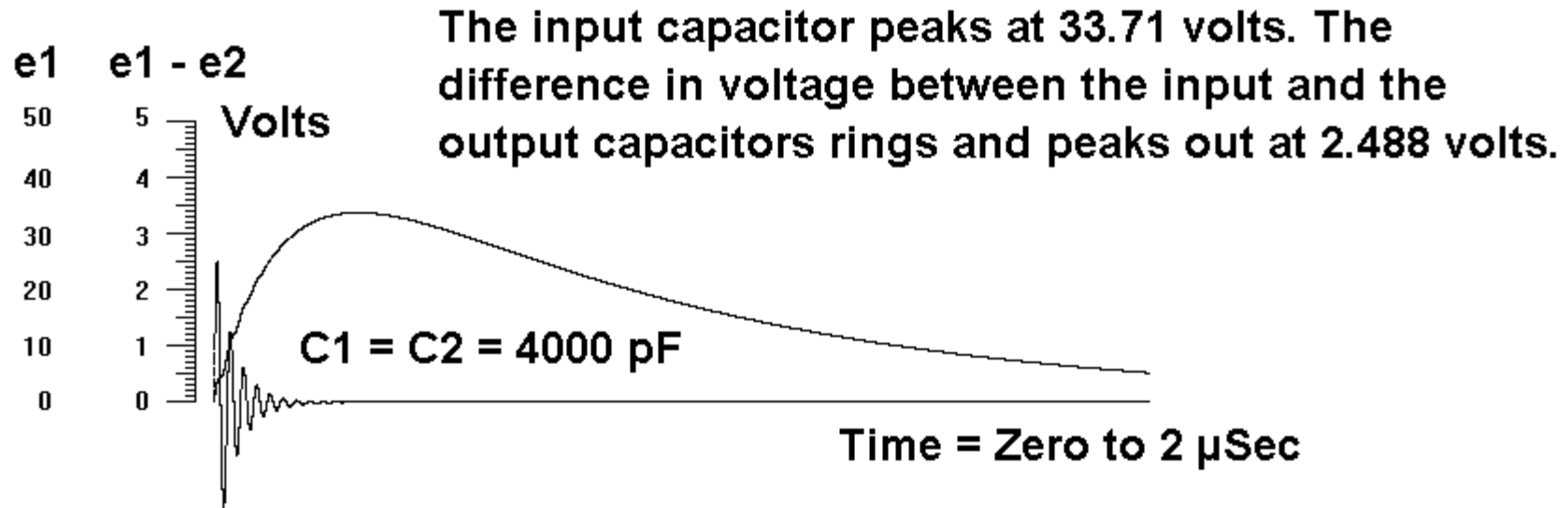
Initial conditions are $e3 = 4000 \text{ volts}$,
 $e1 = \text{zero}$, $e2 = \text{zero}$ and $iL = 0$

Iteration loop:



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The human body model of an electrostatic discharge (ESD) event is applied to a connector's filter-pin.



esdbody.bas

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See: <http://www.advansi.com/data/EffectsofSimulElectroMpulseonCA.pdf>

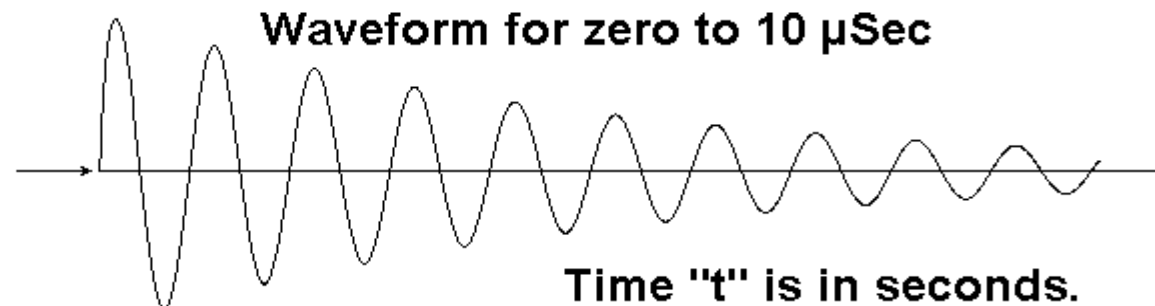
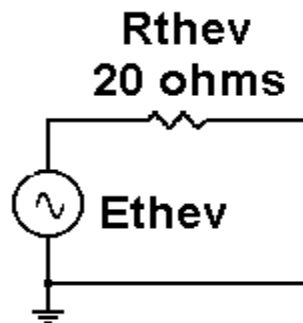
**This paper describes a variety of lightning simulations
including the Douglas Aircraft BXU7026.**

- [7] "Requirements for protection of electrical electronics equipment from lightning induced electrical transients," 88277 BXU7026, Douglas Aircraft Co., Long Beach, CA, Apr. 24, 1979.

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Lightning strike equivalent circuit of Douglas specification BXU7026.

See: <http://www.advansi.com/data/EffectsofSimulElectroMpulseonCA.pdf>



$$V_0 * (\exp (- a t) - \exp (- b t)) * \cos ((g t - d) * \pi)$$

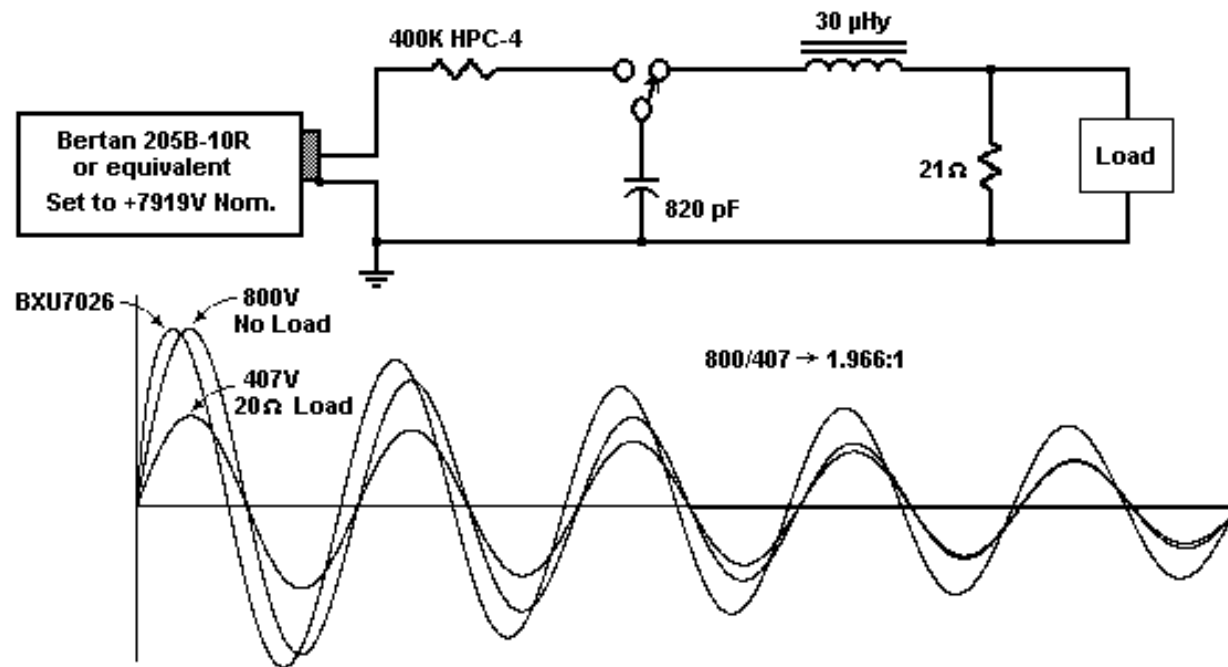
$$V_0 = 831 \quad a = 200,000 \quad b = 32,000,000$$

$$g = 2,000,000 \quad d = 0.32 \quad \pi = 3.14159....$$

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Lightning strike equivalent circuit per Boeing specification BXU7026.

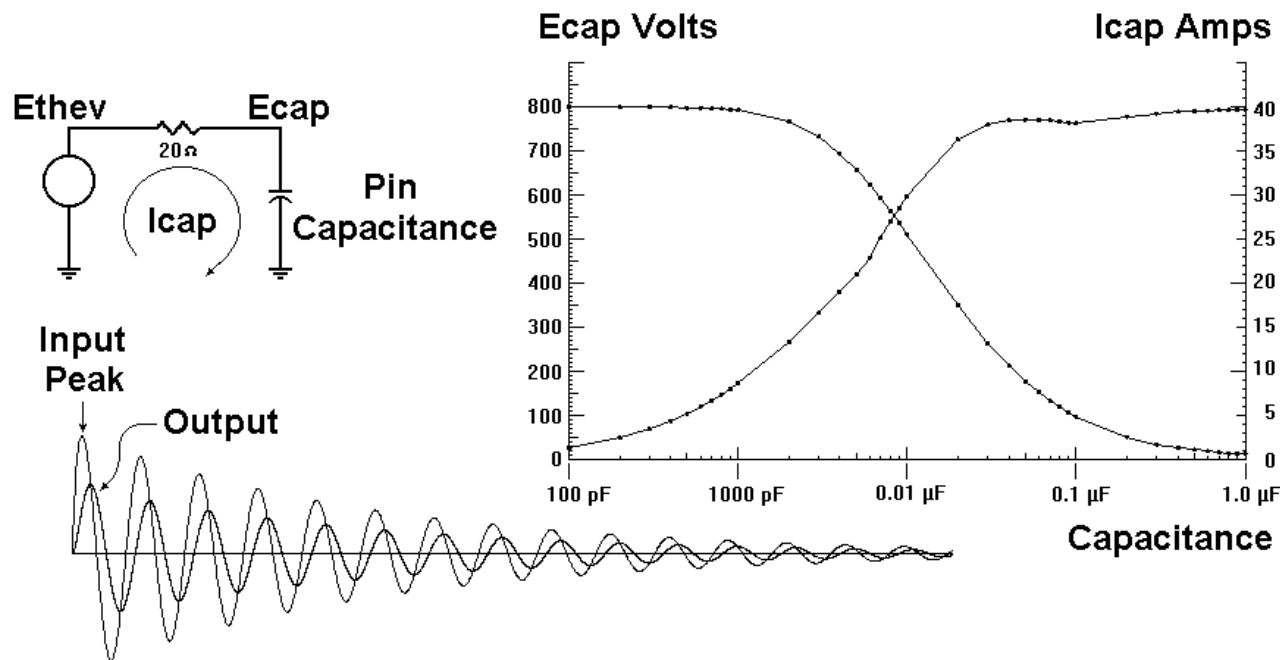
**Boeing BXU7026 Waveform Generator for
Lighting Simulation (maybe)**



bxu_sim1.bas

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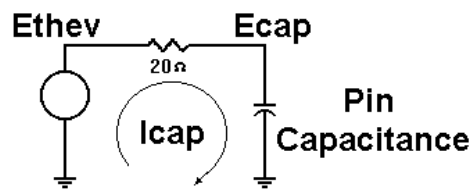
Lightning strike equivalent circuit per Boeing specification BXU7026.



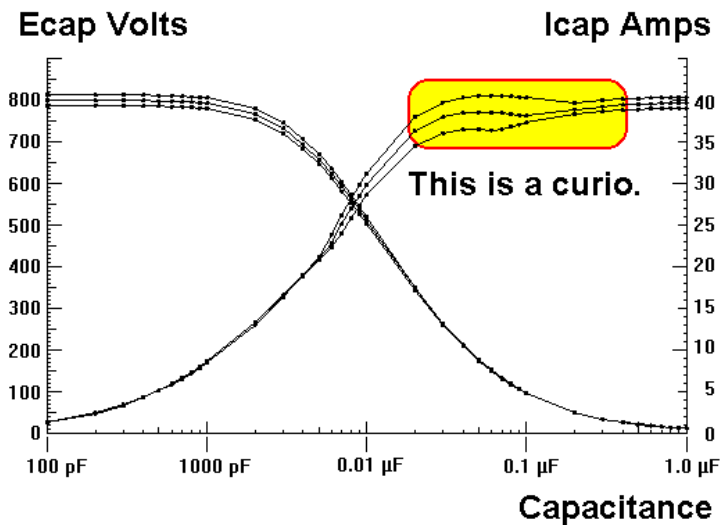
bxu7026a.bas

Ambertec, P.E., P.C.

Examining A Curio



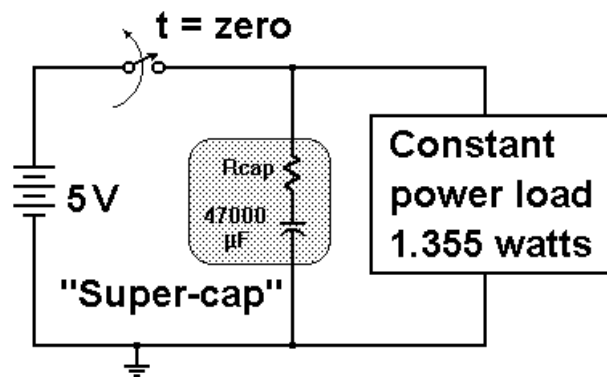
When we alter the E_{thev} slightly off of BXU7026 coefficients, the output changes a little, but the basic result is still the same.



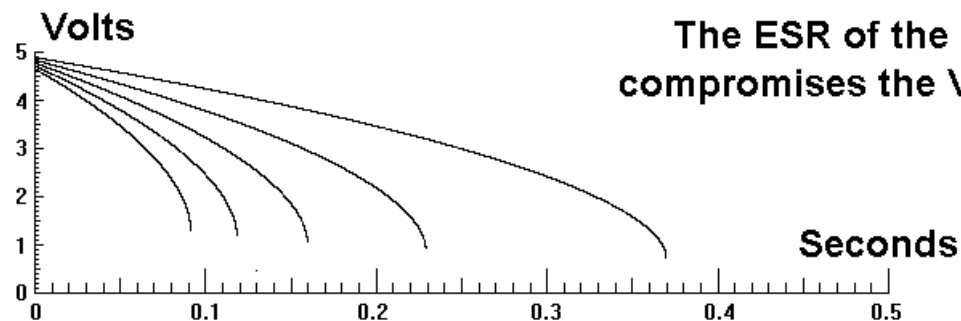
bxu7026b.bas

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Constant power decay of the voltage of a charged super-capacitor.



R_{cap}	Start	Time to 4.5 volts
0.4 ohms	4.889 volts	60.3 mSec
0.6	4.832	50.2
0.8	4.773	40.2
1.0	4.712	30.3
1.2	4.650	20.6



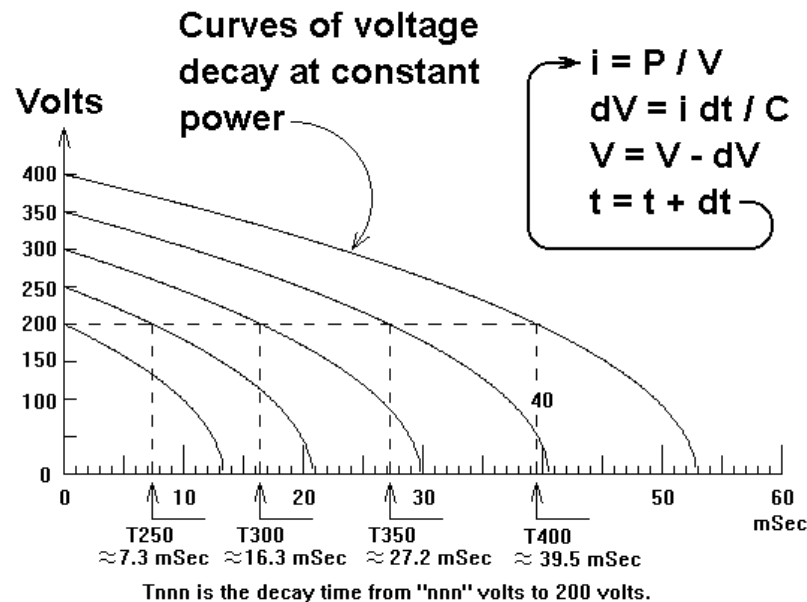
The ESR of the super-cap severely compromises the V_{CC} hold-up capability.

supercap.bas

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Constant power decay of the voltage of a charged capacitor.

Constant power decay of 470 μF capacitor for 714 watt load where the requirement is to sustain the load at 200 volts or more for a specified time.



$$i = C dV/dT$$

$$i = P/V$$

$$P/V = C dV/dT$$

$$dT * \{ P/V \} = C dV$$

$$\int dV = \{ 1/VC \} \int P dT$$

$$\int V dV = \{ 1/C \} \int P dT$$

$$0.5 * V^2 \Big|_{V_f}^{V_s} = \{ P/C \} \int dT = \{ P/C \} T \Big|_{T_f}^{T_s}$$

$$\{ P/C \} * \Delta T = 0.5 * V_s^2 - 0.5 * V_f^2$$

$$C = 2 * P * \Delta T / \{ V_s^2 - V_f^2 \}$$

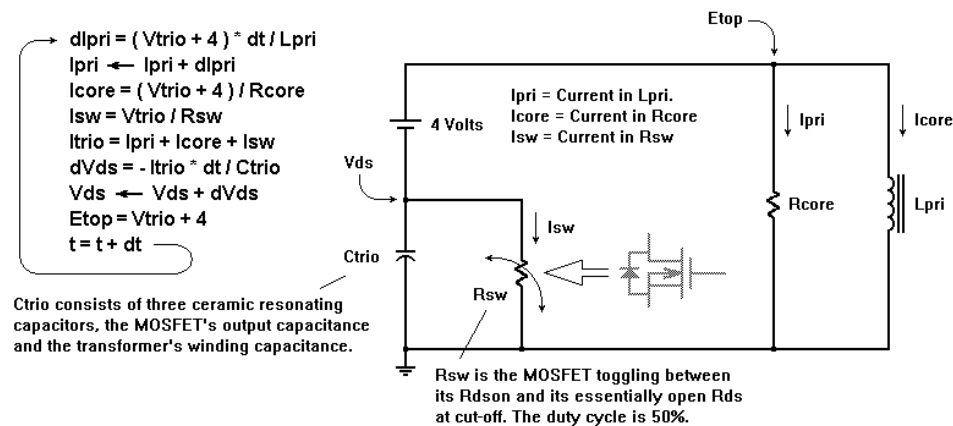
C is that capacitance which will hold a voltage starting at V_s above a final or finishing voltage V_f under a constant power load P for a time interval of ΔT .

Note that the curve for any particular starting voltage can be slid to the right where it will exactly match a portion of another curve representing a start from a higher initial voltage.

cpwrdcay.bas

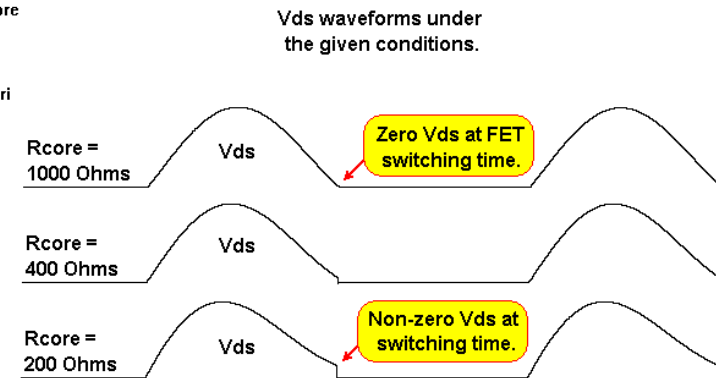
Ambertec, P.E., P.C.

Quasi-resonant power inverter



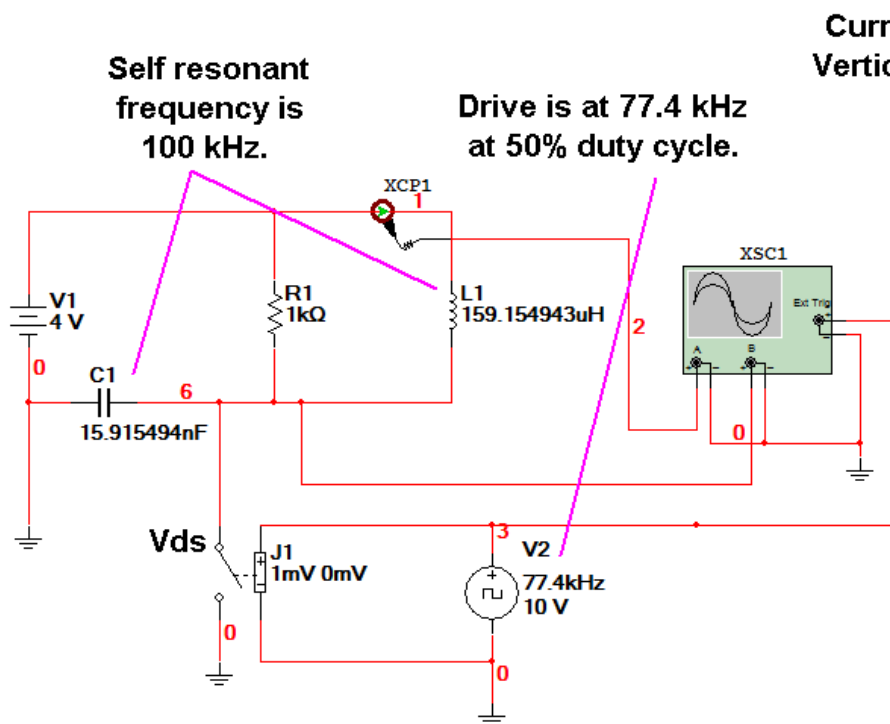
The volts RMS across R_{core} and L_{pri} vary with changes in the values of those two items as follows:

	$R_{core} = 1000 \text{ Ohms}$	$R_{core} = 400 \text{ Ohms}$	$R_{core} = 200 \text{ Ohms}$	
$L_{pri} = 200 \mu\text{Hy}$	4.959 Vrms	4.843 Vrms	4.634 Vrms	These values of Vrms are found when the drive frequency at which the R_{sw} is toggled is made 77.4% of the nominal self resonance of L_{pri} and C_{trio} (15289 pF).
175 μHy	4.963 Vrms	4.855 Vrms	4.657 Vrms	
150 μHy	4.968 Vrms	4.870 Vrms	4.685 Vrms	
100 μHy	4.978 Vrms	4.900 Vrms	4.748 Vrms	
85 μHy	4.978 Vrms	4.907 Vrms	4.768 Vrms	



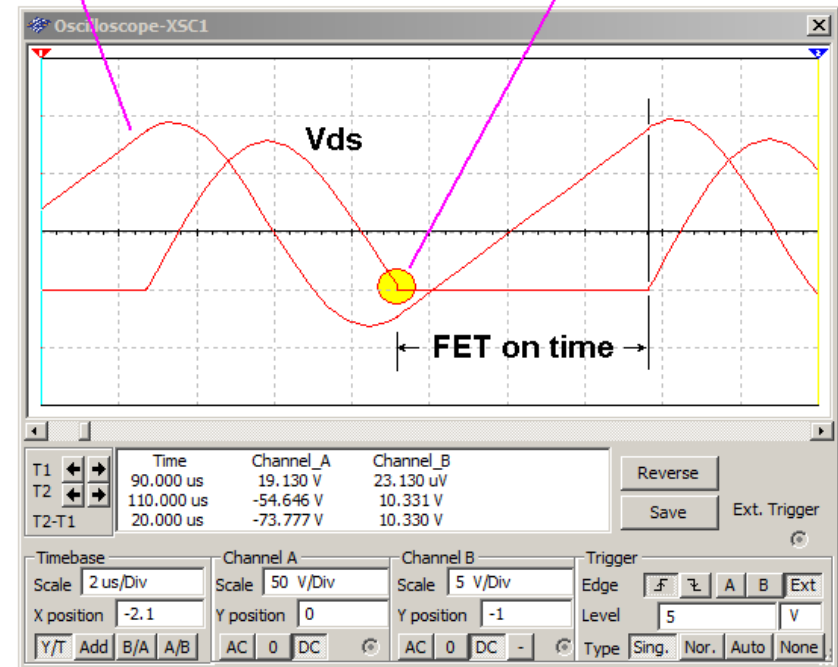
Ambertec, P.E., P.C.

SPICE Simulation



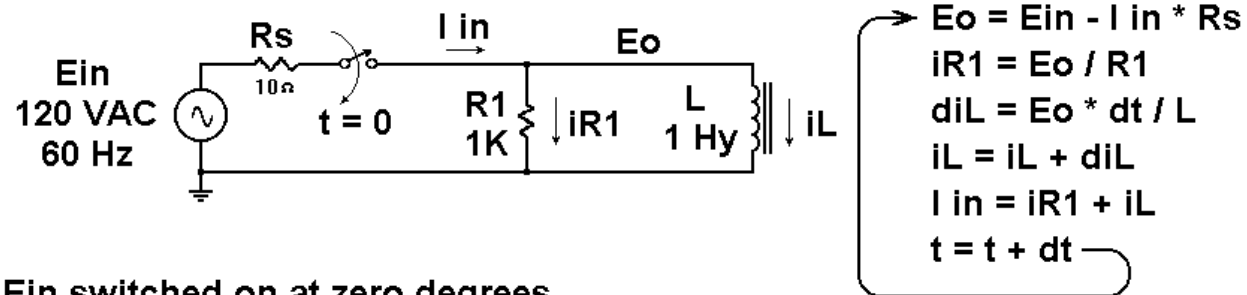
Current probe is 1V per mA.
Vertical scale = 50 mA per div.

Switch closure is almost at the zero crossover.

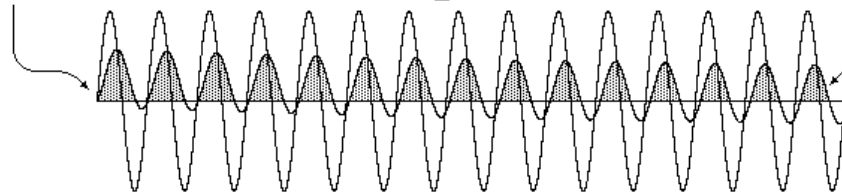


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Parallel RL response to AC input switching. (Think “AC power transformer.”)

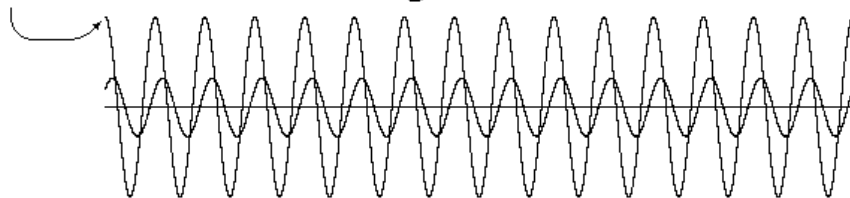


E_{in} switched on at zero degrees



The input current has a lo-o-o-o-ng duration decaying transient.

E_{in} switched on at 90 degrees



The input current is transient-free.

transpri.bas

Ambertec, P.E., P.C.

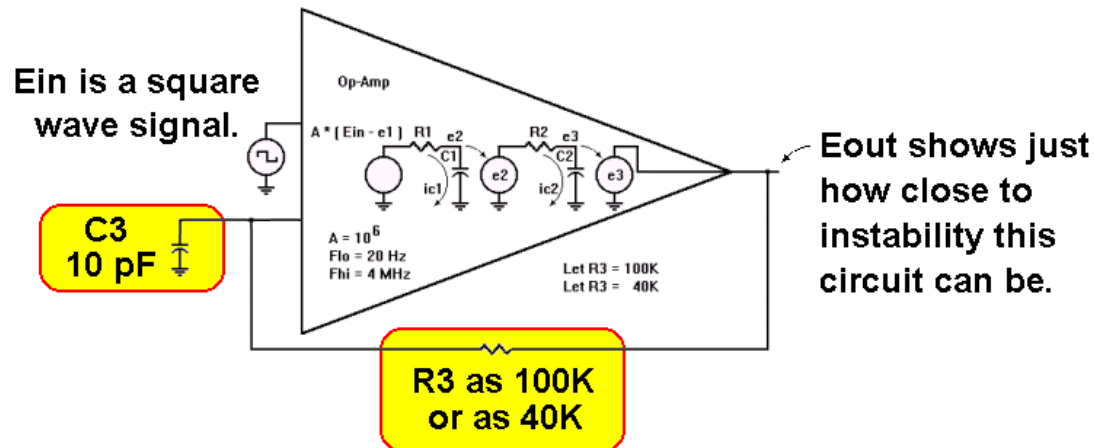
Beware of Loop Gain!



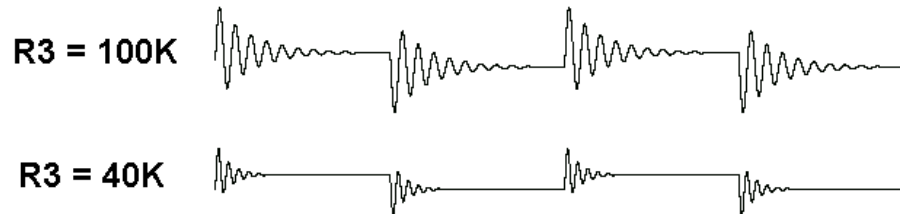
© Evan Dunn 2006

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Op-Amp Stability versus Input Capacitance (This capacitance is spelled "b-a-d")



For time = zero to 160 μSec :



You ignore this issue at your own peril!

opamposc.bas

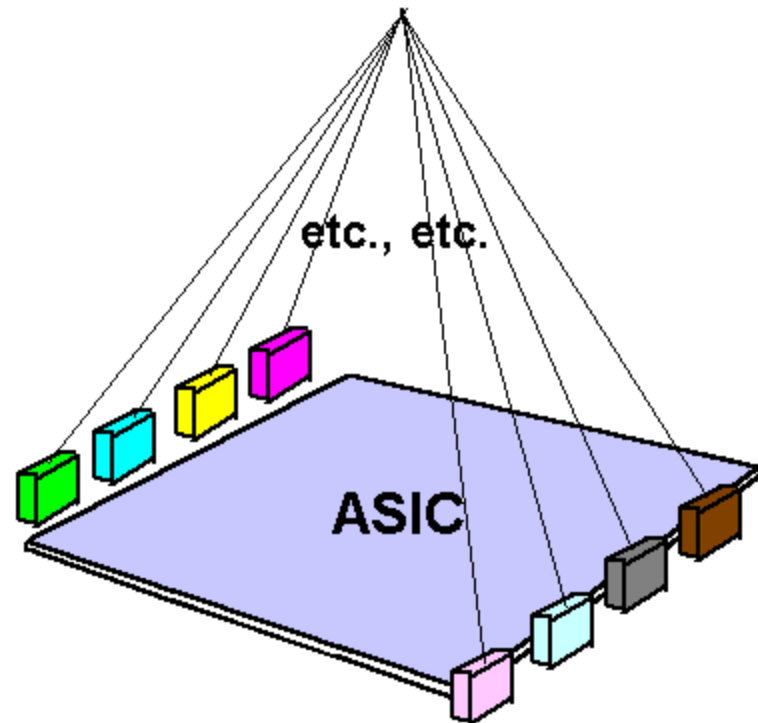


Pieter van Laer
(Dutch, Haarlem 1592/5-1642 (?)
Haarlem)

His work is on display at The
Metropolitan Museum of Art

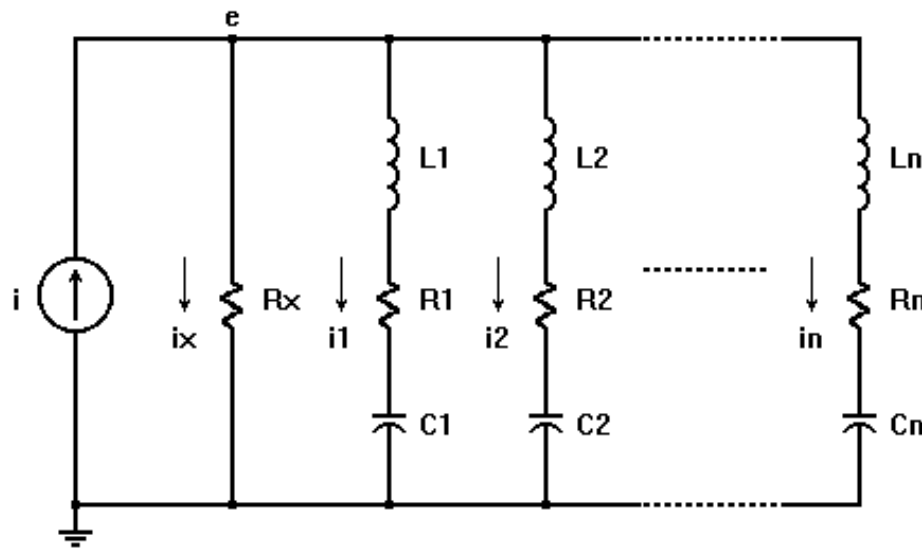
A Common Situation

Lots of rail voltage
bypass capacitors



Ambertec, P.E., P.C.

Recursive differential equations for a whole bunch of bypass capacitors across an ASIC's rail voltage to ground.



$$i_x = i - i_1 - i_2 - \dots - i_n$$

$$e = i_x * R_x$$

For $j = 1$ to n :

$$e_{Lj} = e - e_{Cj} - i_j * R_j$$

$$d i_j = e_{Lj} * dt / L_j$$

$$d e_{Cj} = i_j * dt / C_j$$

$$i_j = i_j + d i_j$$

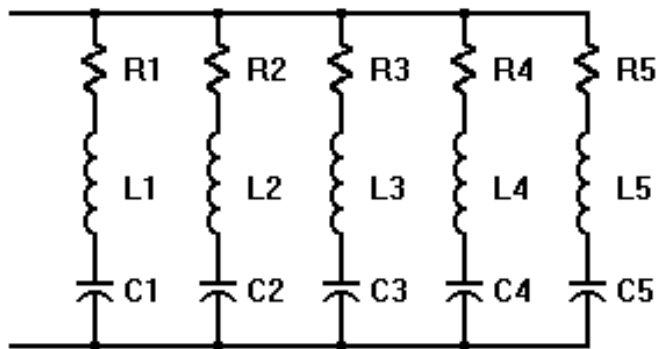
$$e_{Cj} = e_{Cj} + d e_{Cj} : \text{Next } j$$

$$t = t + dt$$

Recursive
Differential
Equations

Ambertec, P.E., P.C.

Here is the complement of bypass capacitors on the ASIC's rail voltage with respect to ground.



1 = 100 μ F capacitor of flyback output.

2 = 10 μ F tantalum capacitor.

3 = Ten paralleled 0.1 μ F ceramic capacitors.

4 = Twelve paralleled 0.01 μ F ceramic capacitors.

5 = Circuit board capacitance of 3000 pF.

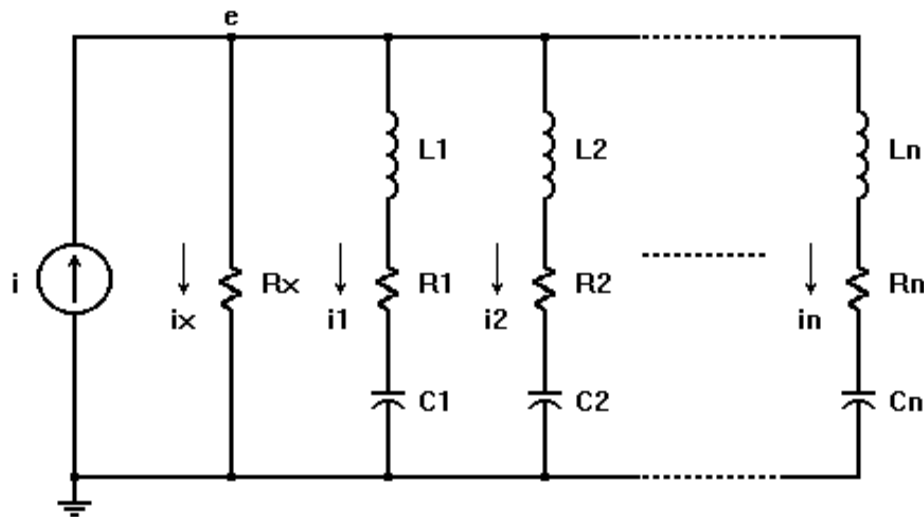
We let SRF_j be the self resonant frequency of capacitor C_j.

We find each $L_j = 1 / 4 / \pi^2 / \text{SRF}_j^2 / C_j$.

Each R_j is the equivalent series resistance (ESR) of capacitor C_j.

Ambertec, P.E., P.C.

Resistor Rx was added to the paralleled decoupling capacitors to simplify the differential equation derivation. Rx is chosen as 1000 Ohms in these calculations, a value much larger than the capacitor impedances over the frequencies of interest.

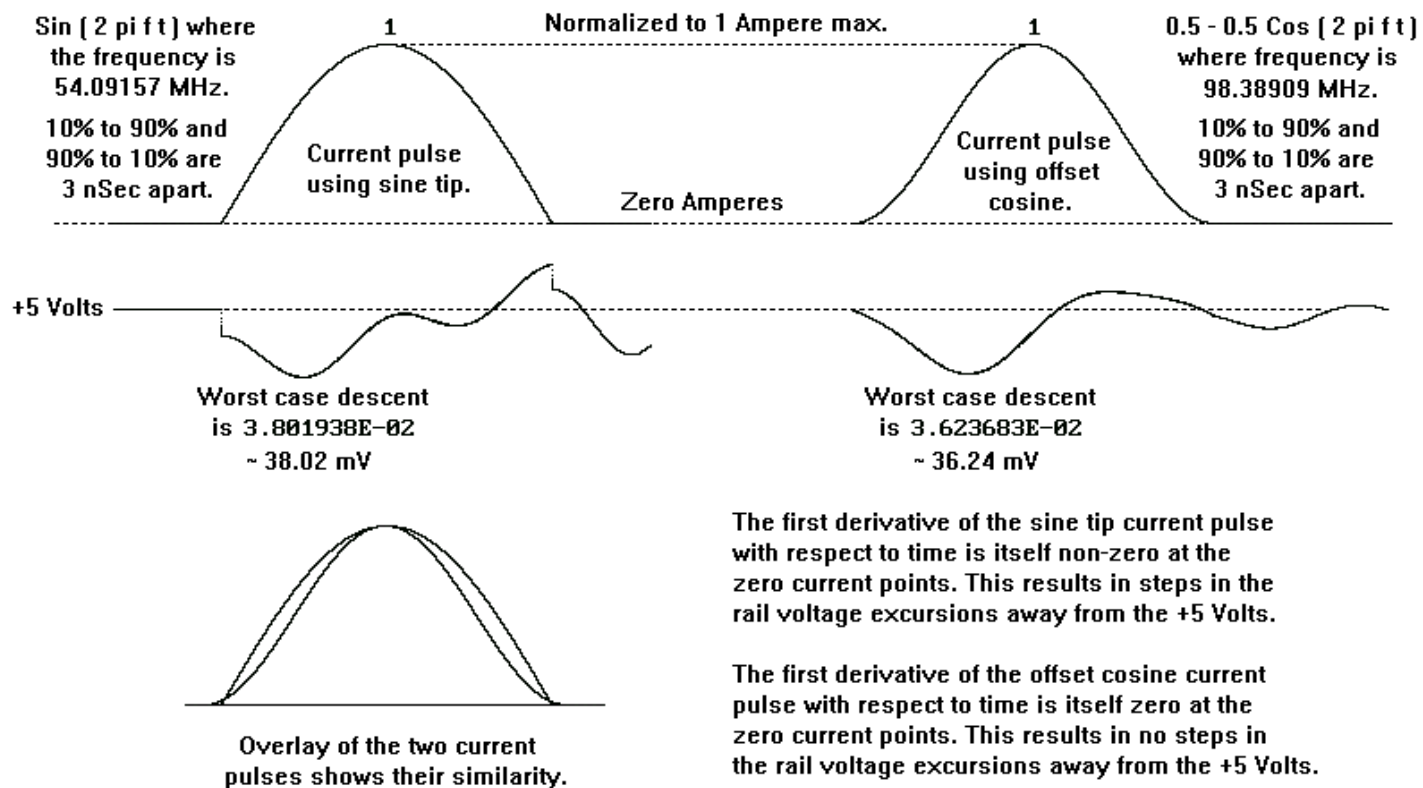


```
ix = i - i1 - i2 - ... - in
e = ix * Rx
For j = 1 to n:
  eLj = e - eCj - ij * Rj
  dij = eLj * dt / Lj
  deCj = ij * dt / Cj
  ij = ij + dij
  eCj = eCj + deCj : Next j
t = t + dt
```

Recursive
Differential
Equations

Ambertec, P.E., P.C.

We examine two versions of the ASIC's current pulse(s).



railpuls.bas

Ambertec, P.E., P.C.

The rail voltage departures from +5V are as many mV as shown per ampere of pulse current.

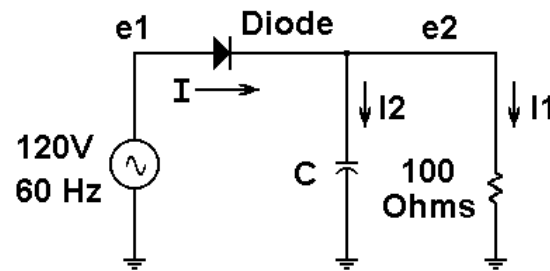
Rounded upward, the rail voltage departures may be taken as 40 mV per Ampere of pulse current.

The actual current pulses turned out to be 150 mA for which the rail voltage ripple came to 6 mV.

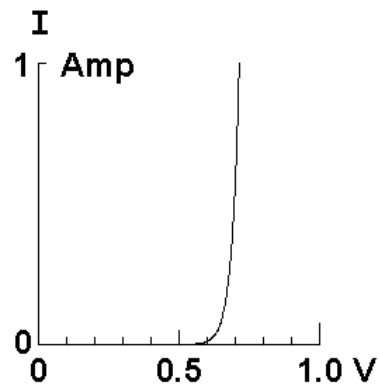
There were smiles.

Ambertec, P.E., P.C.

Half-wave rectifier including a non-linear device, the diode.



$q = 1.602 \text{ E-19 coul}$
 $k = 1.38 \text{ E-23 joules per deg K}$
 $T = 300 \text{ K}$
 $I_s = 1\text{E-12}$



$$I = I_s (\exp (qV / kT) - 1)$$

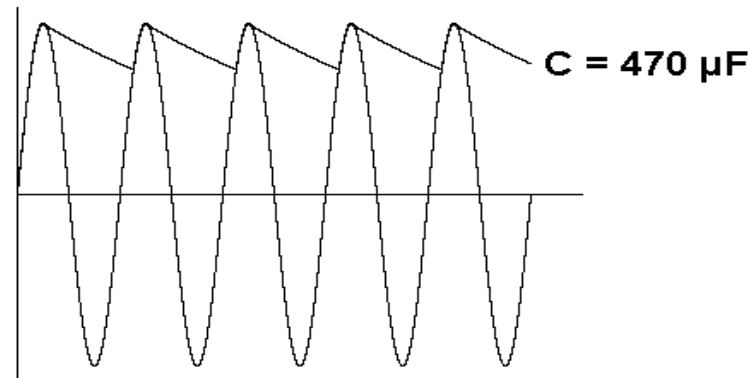
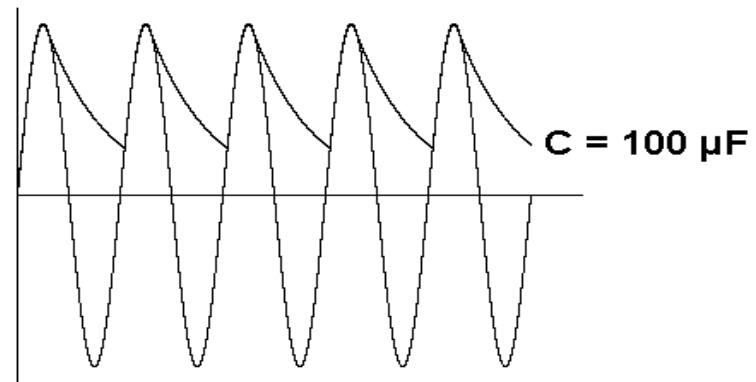
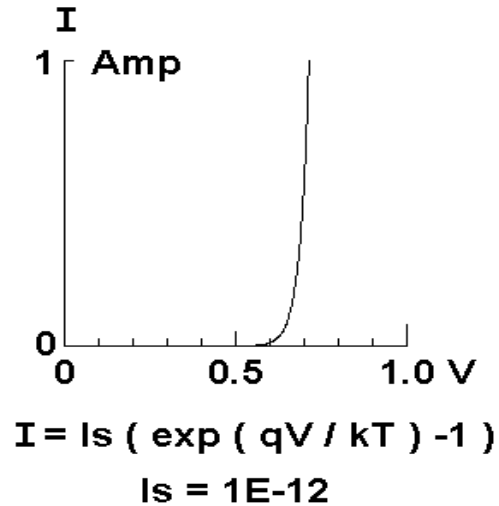
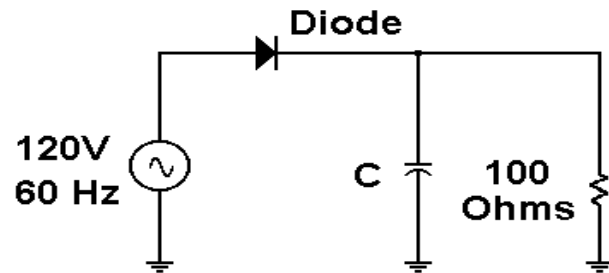
$$I_s = 1\text{E-12}$$

$I = I_s (\exp (q (e1 - e2) / kT) - 1)$
 $I1 = e2 / R$
 $I2 = I - I1$
 $de2 = I2 \, dt / C$
 $e2 = e2 + de2$
 $t = t + dt$

diodevi.bas

Ambertec, P.E., P.C.

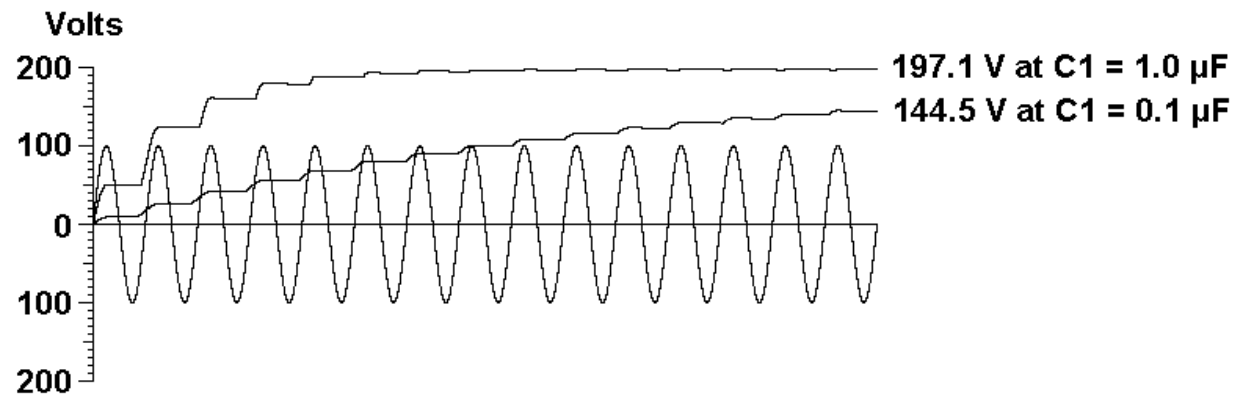
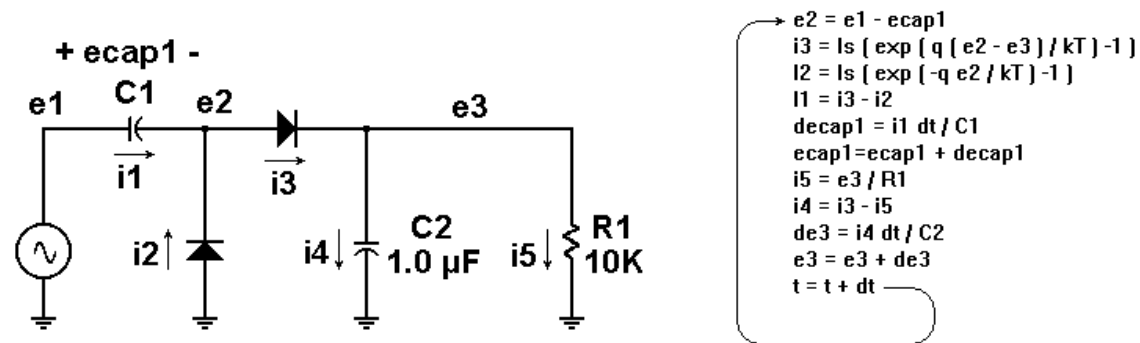
Half-wave rectifier including a non-linear device, the diode.



halfwave.bas

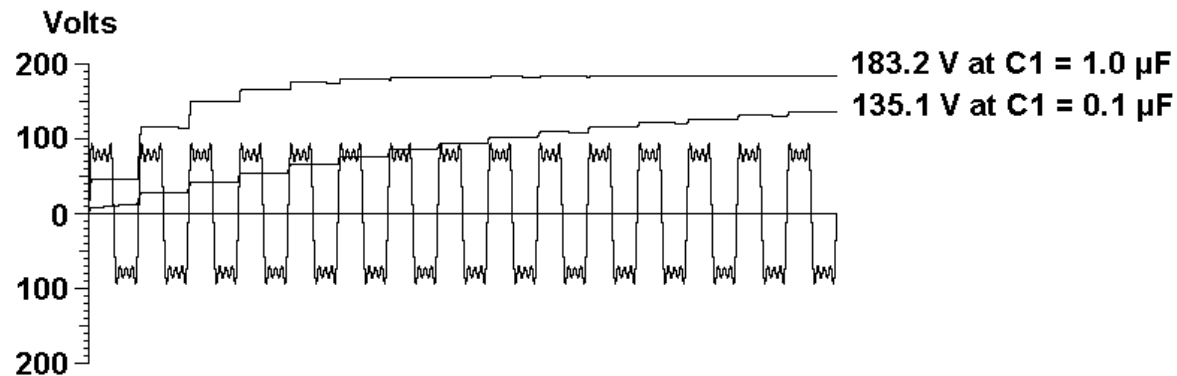
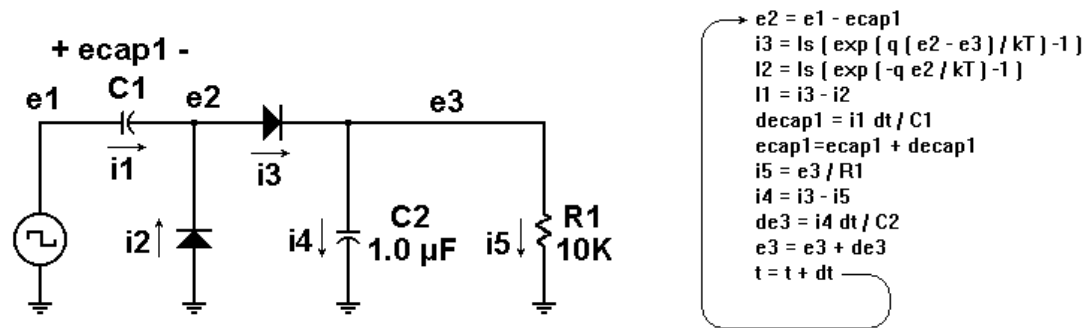
Ambertec, P.E., P.C.

Half-wave, Cockcroft-Walton voltage doubling rectifier



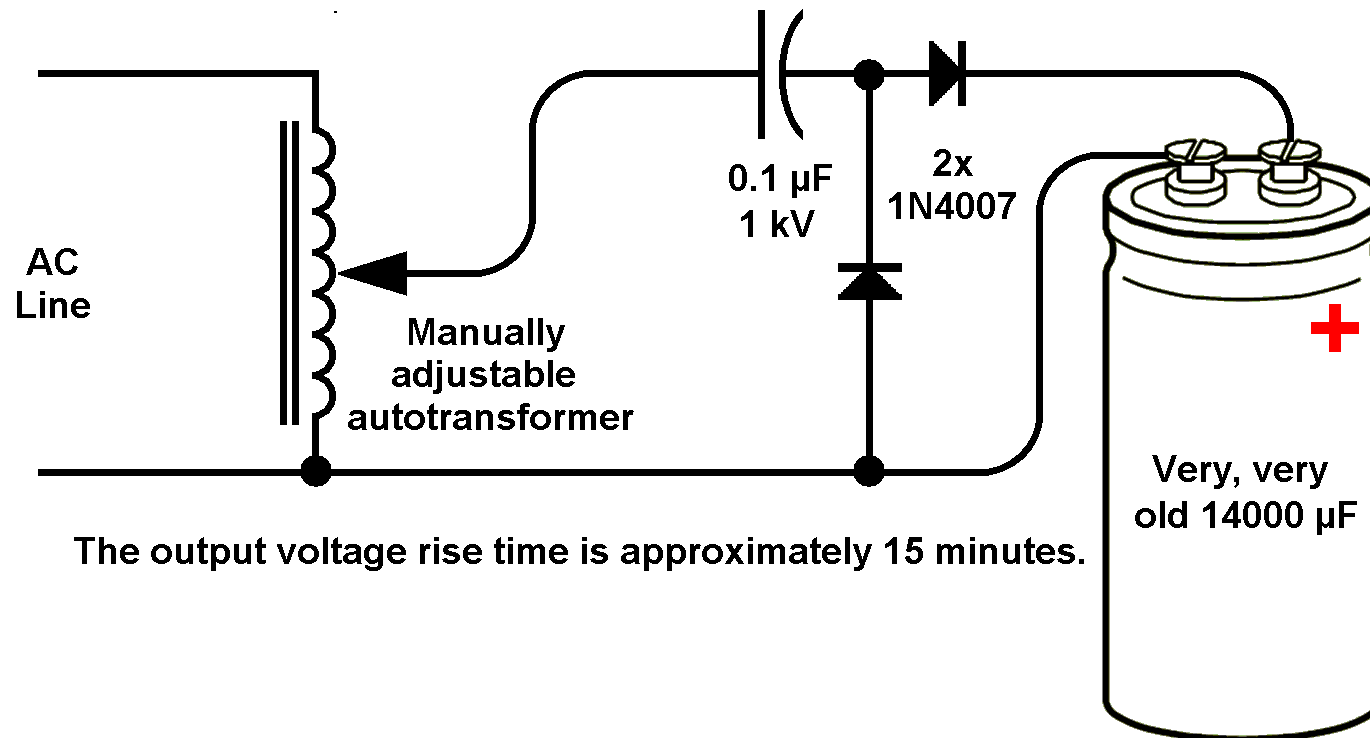
Ambertec, P.E., P.C.

Half-wave, Cockcroft-Walton voltage doubling rectifier



Ambertec, P.E., P.C.

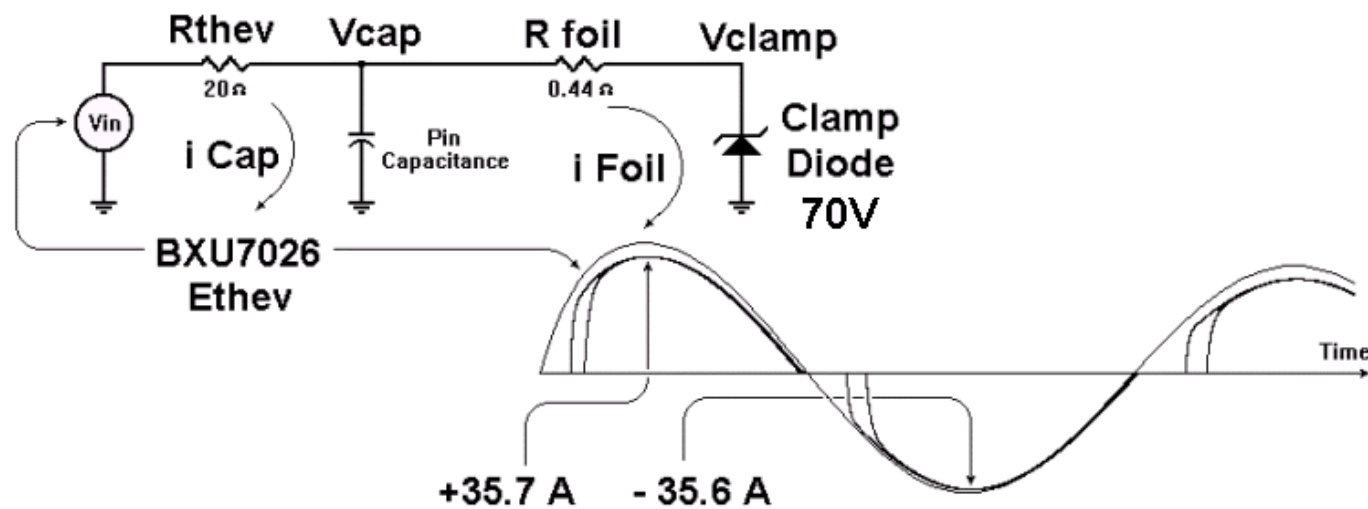
A safe method to rejuvenate an old electrolytic capacitor.



The output voltage rise time is approximately 15 minutes.

Ambertec, P.E., P.C.

Lightning strike protection using a clamp diode.

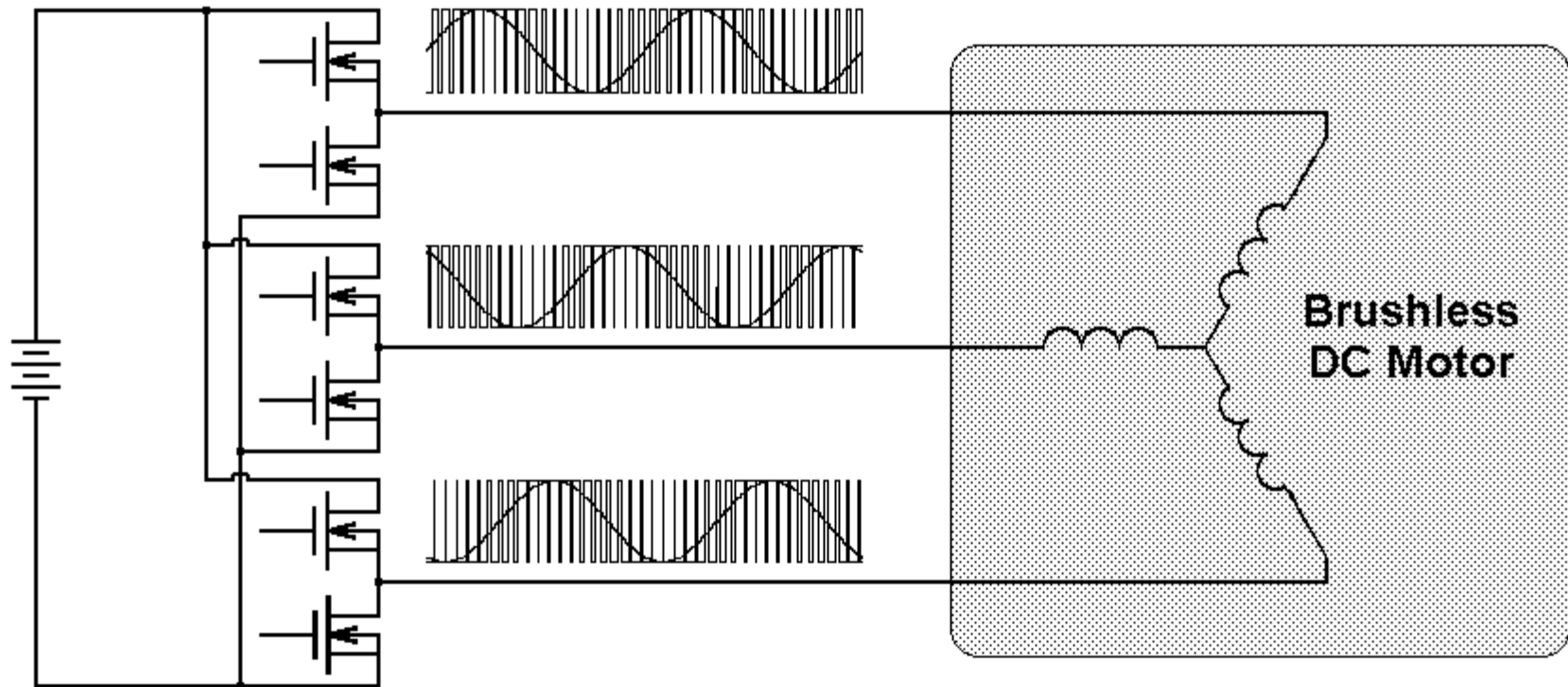


For $C_{pin} = 8000\text{ pF}$, maximum foil current = 35.7 A and -35.6 A .
If $C_{pin} = 16000\text{ pF}$, the maximum foil currents are the same, but their onsets are slightly delayed.

zlight.bas

Ambertec, P.E., P.C.

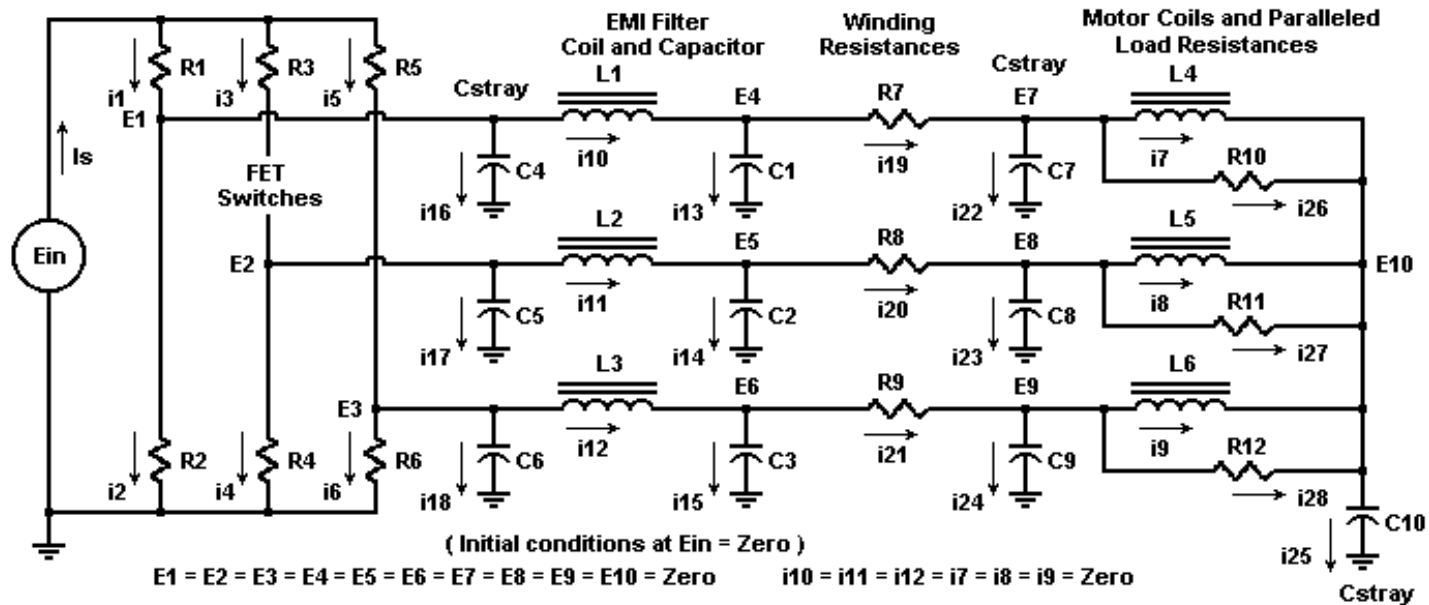
Three-phase PWM excitation of a brushless DC motor.



pwm_sine.bas

Ambertec, P.E., P.C.

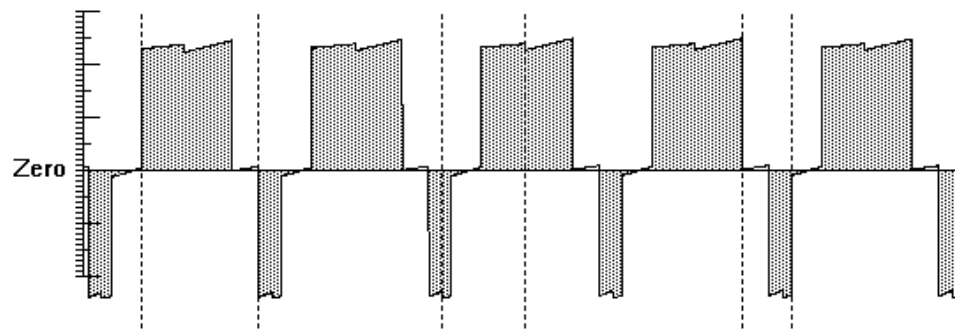
Three-phase PWM excitation of a brushless DC motor.



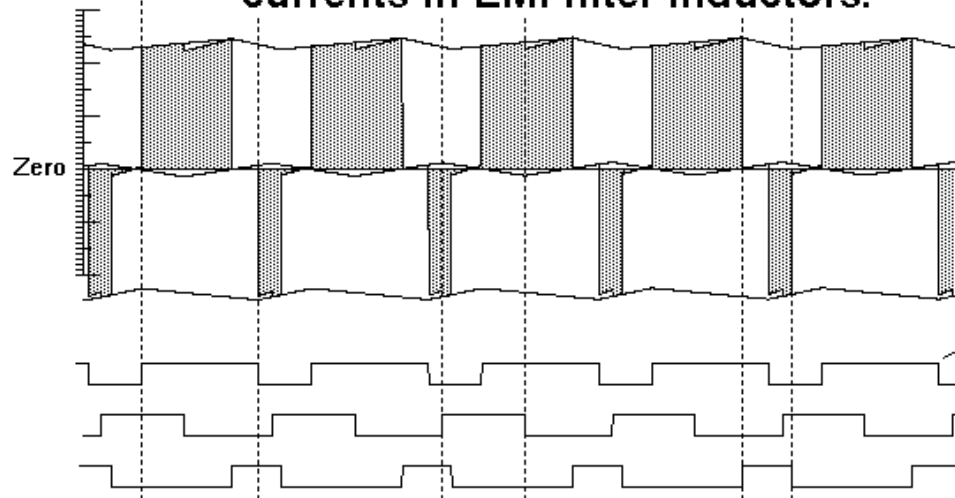
$$\begin{aligned}
 i1 &= (E_{in} - E1) * G1 ; i2 = E1 * G2 ; i16 = i1 - i2 - i10 ; dE1 = i16 * dT / C4 ; E1 = E1 + dE1 ; di10 = (E1 - E4) * dT / L1 \\
 i10 &= i10 + di10 ; i13 = i10 - i19 ; dE4 = i13 * dT / C1 ; E4 = E4 + dE4 ; i19 = (E4 - E7) * G7 ; i22 = i19 - i7 - i26 \\
 dE7 &= i22 * dT / C7 ; E7 = E7 + dE7 ; di7 = (E7 - E10) * dT / L4 ; i7 = i7 + di7 ; i26 = (E7 - E10) * G10 \\
 i3 &= (E_{in} - E2) * G3 ; i4 = E2 * G4 ; i17 = i3 - i4 - i11 ; dE2 = i17 * dT / C5 ; E2 = E2 + dE2 ; di11 = (E2 - E5) * dT / L2 \\
 i11 &= i11 + di11 ; i14 = i11 - i20 ; dE5 = i14 * dT / C2 ; E5 = E5 + dE5 ; i20 = (E5 - E8) * G8 ; i23 = i20 - i8 - i27 \\
 dE8 &= i23 * dT / C8 ; E8 = E8 + dE8 ; di8 = (E8 - E10) * dT / L5 ; i8 = i8 + di8 ; i27 = (E8 - E10) * G11 \\
 i5 &= (E_{in} - E3) * G5 ; i6 = E3 * G6 ; i18 = i5 - i6 - i12 ; dE3 = i18 * dT / C6 ; E3 = E3 + dE3 ; di12 = (E3 - E6) * dT / L3 \\
 i12 &= i12 + di12 ; i15 = i12 - i21 ; dE6 = i15 * dT / C3 ; E6 = E6 + dE6 ; i21 = (E6 - E9) * G9 ; i24 = i21 - i9 - i28 \\
 dE9 &= i24 * dT / C9 ; E9 = E9 + dE9 ; di9 = (E9 - E10) * dT / L6 ; i9 = i9 + di9 ; i28 = (E9 - E10) * G12 \\
 i25 &= i7 + i8 + i9 + i26 + i27 + i28 ; dE10 = i25 * dT / C10 ; E10 = E10 + dE10 ; Is = i1 + i3 + i5
 \end{aligned}$$

Three-phase PWM excitation of a brushless DC motor.

Input line current all by itself



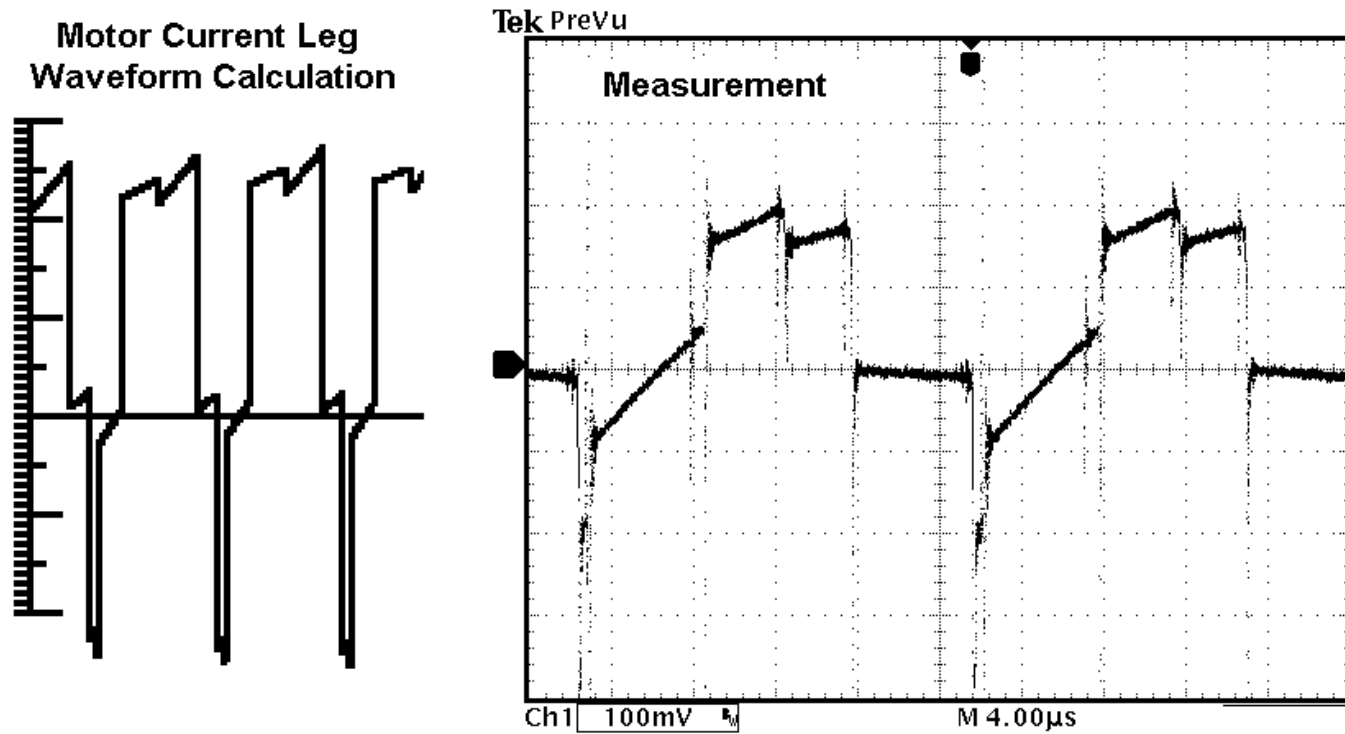
Input line current seen within boundaries of currents in EMI filter inductors.



On
Off
FET Switching

Ambertec, P.E., P.C.

Comparison of Simulation and Measurement



motstall.bas

Ambertec, P.E., P.C.



ambertec@ieee.org